Jensen's Inequality

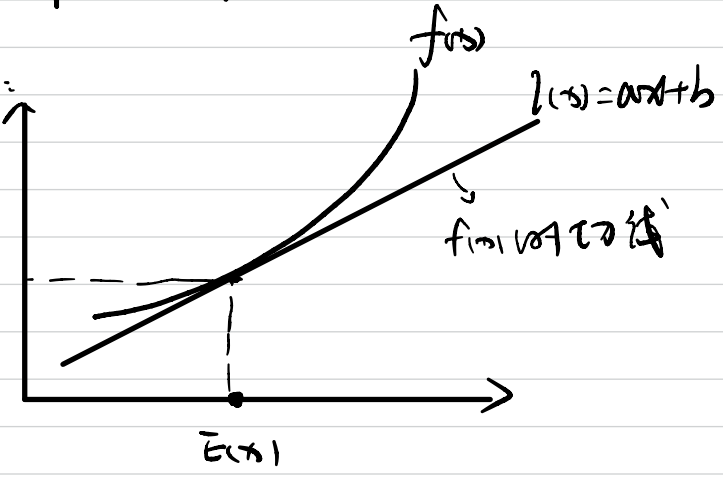
詹森不等式.

Jensen's inequality

if $f(x)$ is a convex function (凸函数)

则 $E[f(x)] \geq f[E(x)]$

prove:



Then: $f[E(x)] = l(E(x)) = aE(x)+b$

Because. $f(x)$ is convex

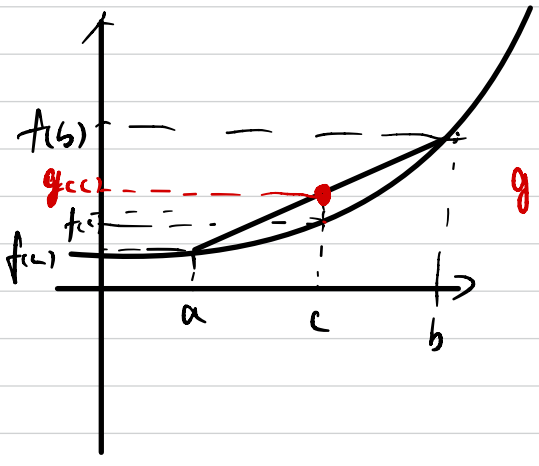
So $\forall x . f(x) \geq l(x)$

Then $E[\cdot]$ then:

$$E[f(x)] \geq E[l(x)]$$
$$= E[ax+b]$$
$$= a[E[x]+b$$
$$= f[E[x]]$$

So $E[f(x)] \geq f(E[x])$

Jensen's inequality in ML:



$g(c) \geq f(c)$

$c = ta + (1-t)b, \quad t \in (0,1)$

$g(c) = t f(a) + (1-t) f(b)$

$$t f(a) + (1-t) f(b) \geq f(ta + (1-t)b)$$