

$$\boxed{\nabla_{\phi} \mathcal{L}(\phi)} = \nabla_{\phi} \mathbb{E}_{q_{\phi}} [\log P(x, z) - \log q_{\phi}]$$

$$= \nabla_{\phi} \left[ \int q_{\phi} (\log P(x, z) - \log q_{\phi}) dz \right]$$

$$= \int \nabla_{\phi} q_{\phi} (\log P(x, z) - \log q_{\phi}) dz +$$

$$\int q_{\phi} \nabla_{\phi} [\underbrace{\log P(x, z)}_{\text{与 } \phi \text{ 无关}} - \log q_{\phi}] dz$$

$$= ① + ②$$

$$② = \int q_{\phi} \nabla_{\phi} (-\log q_{\phi}) dz$$

$$= - \int q_{\phi} \cdot \frac{1}{q_{\phi}} \cdot \nabla_{\phi} q_{\phi} dz$$

$$= - \int \nabla_{\phi} q_{\phi} dz$$

$$= - \nabla_{\phi} \underbrace{\int q_{\phi} dz}_1$$

$$= - \nabla_{\phi} 1$$

$$= 0$$

$$= ① = \int (\nabla_{\phi} q_{\phi}) (\log P(x, z) - \log q_{\phi}) dz$$

$$= \int q_{\phi} \cdot \nabla_{\phi} (\log q_{\phi}) \cdot (\log P(x, z) - \log q_{\phi}) dz$$

$$\rightarrow \mathbb{E}_{q_{\phi}} [\nabla_{\phi} \log q_{\phi} \cdot (\log P(x, z) - \log q_{\phi}) dz]$$

The gradient could be represented by an expectation,

so we could use sampling methods to estimate the gradient (Monte Carlo).