IMPORTANT FORMULAE FOR COMPETITIVE EXAMS

1. Important formulae used in simplification:
(1) $(a + b)^2 = a^2 + b^2 + 2ab$
(2) $(a - b)^2 = a^2 + b^2 - 2ab$
(3) $(a + b)^2 = (a - b)^2 + 4ab$
(4) $a^2 - b^2 = (a - b)(a + b)$
(5) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
(6) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
(7) $a^2 + b^2 = \frac{1}{2}[(a+b)^2 + (a-b)^2]$

2. Rules of counting numbers
1. Sum of first n natural numbers $= \frac{n(n+1)}{2}$
2. Sum of first n odd natural numbers $= n^2$
3. Sum of first n even natural numbers $= n(n + 1)$
4. Sum of the squares of first n natural numbers $= \frac{n(n+1)(2n+1)}{6}$
5. Sum of the cubes of first n natural numbers $= \left[\frac{n(n+1)}{2}\right]^2$

PERCENTAGES

1. Two successive percentage changes of a% and b% is an effective change of $\left(a+b+\frac{ab}{100}\right)\%$.

2. If A is r% more/less than B, B is $\frac{100\,r}{100 \pm r}\%$ less/more than A.

INTEREST

1. P = Principal, A = Amount, I = Interest, n = no. of years, r% = rate of interest

The Simple Interest (S.I.) $= \frac{P \times r \times n}{100}$

2. If P is the principal kept at Compound Interest (C.I.) @ r% p.a., amount after n years

$$= P\left(1+\frac{r}{100}\right)^n$$

3. Amount = Principal + Interest

4. Let P = Original Population, P′ = Population after n years, r% = rate of anual growth

$$P' = P\left(1+\frac{r}{100}\right)^n$$

5. Difference between CI and SI for 2 and 3 years respectively:
$(CI)_2 - (SI)_2 = Pa^2$ for two years
$(CI)_3 - (SI)_3 = Pa^2(a + 3)$ for three years

where, $a = \frac{r}{100}$

6. A principal amounts to X times in T years at S.I. It will become Y times in:

$$\text{Years} = \left(\frac{Y-1}{X-1}\right) \times T$$

7. A principal amounts to X times in T years at C.I. It will become Y times in:
Years = T × n
where n is given by $X^n = Y$

PROFIT AND LOSS

1. Profit % $= \frac{\text{Profit}}{\text{CP}} \times 100$

2. SP = CP + P% of CP $= CP\left(1+\frac{P}{100}\right)$

3. Discount = Marked Price – Selling Price

4. Discount % $= \frac{\text{Discount}}{\text{Marked Price}} \times 100$

5. The selling price of two articles is same. If one is sold at X% profit and the other at loss of X%, then there is always a loss of $\frac{X^2}{100}\%$

RATIO & PROPORTION

1. It a : b : : c : d, then ad = bc

2. If a < b and x is a positive quantity, then

$$\frac{a}{b} < \frac{a+x}{b+x} \text{ and } \frac{a}{b} > \frac{a-x}{b-x}$$

3. If a > b and x is a positive quantity, then

$$\frac{a}{b} > \frac{a+x}{b+x} \text{ and } \frac{a}{b} < \frac{a-x}{b-x}$$

4. If $\frac{a}{b} = \frac{c}{d}$ then:

(a) $\frac{a+b}{b} = \frac{c+d}{d}$ – Componendo Law

(b) $\frac{a-b}{b} = \frac{c-d}{d}$ – Dividendo Law

(c) $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ – Componendo & Dividendo Law

(d) $\frac{a \pm c}{b \pm d} = \frac{a}{b}$

5. If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$ = K, then:

(a) $\frac{a+c+e}{b+d+f} = K$

(b) $\frac{pa+qc+re}{pb+qd+rf} = K$

(p, q and r are not all zero)

ALLIGATION, MIXTURES AND MEAN

1. Alligation is a method of calculating weighted averages. The ratio of the weights of the two items mixed will be inversely proportional to the difference of each of these two items from the average attribute of the resultant mixture.

$$\begin{array}{ccccc} x_1 & & & & x_2 \\ & \searrow & & \swarrow & \\ & & x & & \\ & \swarrow & & \searrow & \\ x_2 - x & & : & & x - x_1 \\ w_1 & & : & & w_2 \end{array}$$

$$\frac{w_1}{w_2} = \frac{(x_2 - x)}{(x - x_1)}$$

2. Arithmetic Mean = $\frac{x_1 + x_2 + x_3 + ... + x_n}{n}$

3. Geometric Mean = $\sqrt[n]{x_1 \times x_2 \times x_3 \times ...... \times x_n}$

4. Harmonic Mean =

$$\frac{n}{\left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + ... + \frac{1}{x_n}\right)}$$

5. Let K_o be the initial concentration of a solution and K is the final concentration after n dilutions. V is the original volume and x is the volume of the solution replaced each time, then

$$K = K_o\left(\frac{V-x}{V}\right)^n$$

TIME, SPEED AND DISTANCE

1. 1 km/hr = $\frac{5}{18}$ m/s and 1m/s = $\frac{18}{5}$ km/hr

2. Average Speed = $\frac{\text{Total Distance Travelled}}{\text{Total Time Taken}}$

3. When the distance is constant, the average speed is the harmonic mean of the two speeds

$$S_{avg} = \frac{2S_1S_2}{S_1 + S_2}$$

4. When the time is constant, the average speed is the arithmetic mean of the two speeds.

$$S_{avg} = \frac{S_1 + S_2}{2}$$

5. D – Speed of the boat downstream
U – Speed of the boat upstream
B – Speed of the boat in still water
R – Speed of the stream
D = B + R and
U = B – R.
Further, by adding and subtracting these equations we get,

B = $\frac{D+U}{2}$ and R = $\frac{D-U}{2}$

6. When the distance covered by a boat in downstream is same as the distance covered by the boat upstream then

$$\frac{\text{Time taken downstream}}{\text{Time taken upstream}} = \frac{\text{Upstream speed}}{\text{Downstream speed}}$$

7. If 'H' is the hours and 'M' is the minutes then the angle between the hour hand and minute hand is

$$\theta = \left|30H - \frac{11}{2}M\right|$$

NUMBER SYSTEM

1. 1 is not a prime number

2. If two numbers a and b are given, and their LCM and HCF are L and H respectively, then L × H = a × b.

3. (a) $\text{LCM of fractions} = \dfrac{\text{LCM of numerators}}{\text{HCF of denominators}}$

   (b) $\text{HCF of fractions} = \dfrac{\text{HCF of numerators}}{\text{LCM of denominators}}$

   Note: Fractions should be in the lowest form.

4. The least number leaving remainder 'r' in each case when divided by 'x', 'y' and 'z' = (LCM of x, y, z) + r

   The series of such numbers will be
   (LCM of x, y, z) × n + r

5. In general, for any composite number C, which can be expressed as $C = a^m \times b^n \times c^p \times \ldots$, where a, b, c, … are all prime factors and m, n, p are positive integers, then:

   Number of factors is equal to
   (m + 1) (n + 1)(p + 1) …

6. If N is not a perfect square,
   No. of ways of writing N as a product of two factors

   $= \frac{1}{2}\{(p + 1)(q + 1)(r + 1) \ldots\}$

7. If N is a perfect square,
   No. of ways in which N can be expressed as a prodcut of two different factors

   $= \frac{1}{2}\{(p + 1)(q + 1)(r + 1) \ldots - 1\}$ ways

   and as a product of two factors

   $= \frac{1}{2}\{(p + 1)(q + 1)(r + 1) \ldots + 1\}$ ways

8. Sum of the factors of

   $$N = \left[\frac{a^{p+1}-1}{a-1}\right]\left[\frac{b^{q+1}-1}{b-1}\right]\left[\frac{c^{r+1}-1}{c-1}\right]\ldots$$

9. Totient function is given by

   $$\phi(N) = N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)\ldots$$

   Here $\phi(N)$ is the number of numbers less than and prime to N. If P is some other natural number which is prime to N, then the remainder when $P^{\phi(N)}$ is divided by N is 1.

10. Sum of numbers less than and co-prime to a number $N = \phi(N) \times \frac{N}{2}$

11. Number of ways of writing a number N as a product of two co-prime numbers = 2^{n-1}

    where, n is the number of prime factors of a number

12. Product of all the factors of

    $$N = N^{\left(\frac{\text{Number of factors}}{2}\right)}$$

    $$= N^{\left(\frac{(p+1).(q+1).(r+1)\ldots}{2}\right)}$$

LINEAR EQUATION IN TWO VARIABLE

For the two simultaneous equations,

$ax + by = c$

$px + qy = r$

where a, b, c, p, q and r are constants

- $\frac{a}{p} = \frac{b}{q} = \frac{c}{r}$: The same equation/ Just one line/ Infinite Solutions
- $\frac{a}{p} = \frac{b}{q} \neq \frac{c}{r}$: Inconsistent Equations/ Two parallel lines/ No Solutions
- $\frac{a}{p} \neq \frac{b}{q}$: Two intersecting lines/ Unique Solution

BINOMIAL THEOREM

1. $(x+y)^n = K_0x^n + K_1x^{n-1}.y^1 + K_2x^{n-2}.y^2 + ... + K_nx^0.y^n$
where K_0, K_1, K_2, ... K_n are constants (called coefficients of binomial expansion)

2. Sum of exponents of x and y in any term = n

3. Any term is given by

$$T_{r+1} = K_r x^{n-r}.y^r = (r+1)^{th} \text{ Term}$$

4. K_r = binomial coefficient of $(r + 1)^{th}$ term

$$= {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

QUADRATIC EQUATIONS

1. **General Form:**
$ax^2 + bx + c = 0$, where $a \neq 0$
Such an equation has two roots, usually denoted by α and β.

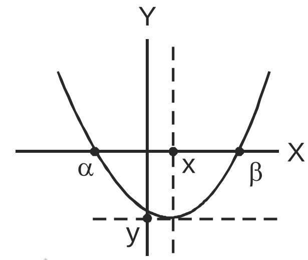
$$\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

2. Sum of roots: $\alpha + \beta = -\frac{b}{a}$

3. Product of roots: $\alpha \times \beta = \frac{c}{a}$

4. In $ax^2 + bx + c$, if **a > 0**



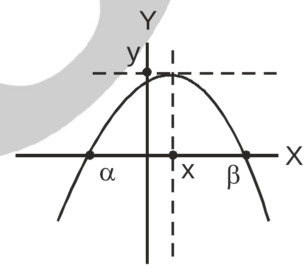
The minimum value of $ax^2 + bx + c$ will be

$$y = \frac{4ac - b^2}{4a}$$

at, $x = \frac{-b}{2a} = \frac{\alpha + \beta}{2}$

where, α, β are the roots of the equation

5. In $ax^2 + bx + c$, if **a < 0**



The maximum value of $ax^2 + bx + c$ will be

$$y = \frac{4ac - b^2}{4a}$$

at, $x = \frac{-b}{2a} = \frac{\alpha + \beta}{2}$

where, α, β are the roots of the equation

6. If the roots of a quadratic equation are α and β, the equation can be re-constructed as
x^2 – (sum of roots) × x + (product of roots) = 0

CUBIC & HIGHER DEGREE EQUATIONS

Consider the cubic equation $ax^3 + bx^2 + cx + d = 0$.

The equation would have 3 roots (equal to the degree of the equation). Some of them can be imaginary. If the roots are denoted as α, β and γ, we have

1. $\alpha + \beta + \gamma = -\frac{b}{a}$

2. $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$

3. $\alpha\beta\gamma = -\frac{d}{a}$

4. The above can be extended for higher degree equations as well. For an 'n' degree equation, Sum of roots $= -\frac{\text{co-efficient of } x^{n-1}}{\text{co-efficient of } x^n}$

5. Sum of roots taken two at a time

$= \frac{\text{co-efficient of } x^{n-2}}{\text{co-efficient of } x^n}$

6. Sum of roots taken three at a time

$= -\frac{\text{co-efficient of } x^{n-3}}{\text{co-efficient of } x^n}$

7. And, sum of roots taken 'r' at a time

$=(-1)^r \frac{\text{coefficient of } x^{n-r}}{\text{coefficient of } x^n}$

8. Product of roots

$= (-1)^n \frac{\text{constant term}}{\text{co-efficient of } x^n}$.

9. Remainder Theorem:
To identify whether a given expression is a factor of another expression, we can take help of Remainder Theorem.

According to the remainder theorem, when any expression f(x) is divided by (x – a), the remainder is f(a). (a is any constant in this example).

10. Factor Theorem:
An expression is said to be a factor of another expression only when the remainder is 0 when the latter is divided by the former.
(x – a) is a factor of f(x) if and only if f(a) = 0.

ARITHMETIC PROGRESSION (AP)

Let,
a = The first term,
d = Common difference,
T_n = The n^{th} term
ℓ = The last term,
S_n = Sum of n terms,

1. The n^{th} term is given by,
$T_n = a + (n - 1)d$

2. The sum of n terms is given by,

$S_n = \frac{n}{2}[2a + (n-1)d]$

or,

$S_n = \left(\frac{a+\ell}{2}\right) \times n$

3. $T_n = S_n - S_{n-1}$

GEOMETRIC PROGRESSION (GP)

Let,
a = The first term,
r = The common ratio
T_n = The n^{th} term and
S_n = The sum of n terms we have the following

1. $T_n = ar^{n-1}$

2. $S_n = a\frac{(1-r^n)}{(1-r)}$, where r < 1

3. $S_n = \frac{a(r^n - 1)}{(r-1)}$, where r > 1

4. Sum of infinite number of terms $= \frac{a}{1-r}$

HARMONIC PROGRESSION (HP)

1. n^{th} term of a HP is given by

$$T_n = \frac{1}{a+(n-1)d}$$

2. Harmonic Mean (HM) of two numbers a and b

$$= \frac{2ab}{a+b}$$

3. For any set of n positive numbers, the following relationship always holds true.
(AM, GM and HM have been defined earlier)

AM $\geq$ GM $\geq$ HM
$(GM)^2 = (AM)\cdot(HM)$

GEOMETRY

Triangle

1. The area of a triangle can be determined in the following ways:

(a) Area of a triangle = $\frac{1}{2} \times b \times h$, where b is base and h is height

(b) Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b and c are the sides of the triangle and s is the semi-perimeter i.e. $s = \frac{a+b+c}{2}$

This formula of area is known as Heron's formula

(c) Area of triangle = $\frac{1}{2} ab \sin\theta$, where a and b are the sides of the triangle and θ is the included angle i.e. angle between sides of length a and b.

(d) Area of a triangle = r × s, where r is the in-radius and s is the semi-perimeter

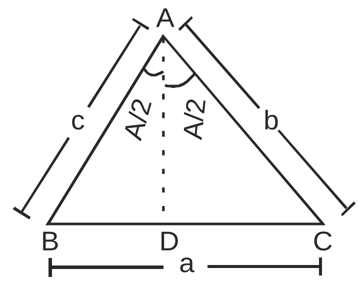
(e) **Cosine rule:** If a, b and c are the three sides of a triangle and if θ is the included angle between the sides of length a and b, then

$$\cos\theta = \frac{a^2+b^2-c^2}{2ab}$$

or $c^2 = a^2 + b^2 - 2\,ab\cos\theta$

(f) $A = \frac{abc}{4R}$ where R is circum-radius and A is area of the triangle

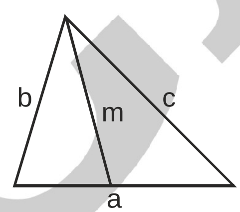
2. **Angle – Bisector Theorem:**



Angle bisector divides the opposite side in the ratio of sides containing the angle. So $\frac{BD}{DC} = \frac{AB}{AC}$

3. **Apollonius Theorem:**
Let a, b, c be the sides of a triangle and m is the length of the median to the side with length a. Then

$$b^2 + c^2 = 2m^2 + \frac{1}{2}a^2$$

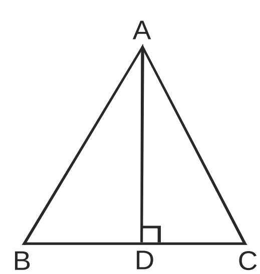


Special case:
If b = c (the triangle is isosceles), then we have

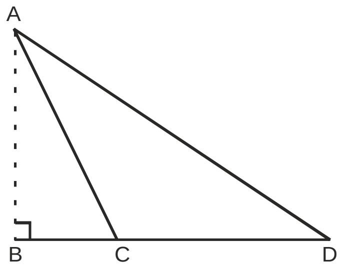
$$2b^2 = 2m^2 + \frac{1}{2}a^2$$

$$m^2 = \left(b^2 - \frac{a^2}{4}\right)$$

4. For acute triangle ABC
$AC^2 = AB^2 + BC^2 - 2 \times BC \times BD$

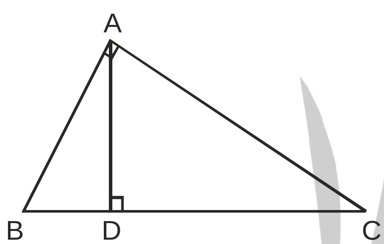


5. For obtuse triangle ABC
$AC^2 = AB^2 + BC^2 + 2 \times BC \times BD$



6. The following are some properties of a triangle right angled at A, where $AD \perp BC$:

(i) $AD^2 = BD \times DC$
(ii) $AB^2 = BD \times BC$
(iii) $AC^2 = CD \times BC$



Polygon
In a polygon of 'n' no. of sides,

1. Total number of diagonals = $\frac{n(n-3)}{2}$

2. Exterior angle of a regular polygon $= \frac{360°}{n}$

3. Interior angle of a convex polygon

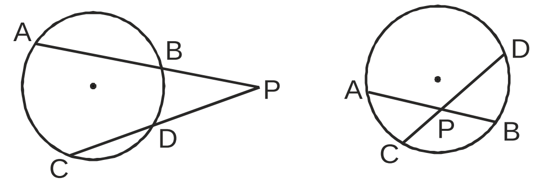
$= 180° - \frac{360°}{n}$

4. Sum of all the exterior angles of a convex polygon = 360°

5. Sum of interior angles of a n sided polygon = (n – 2) × 180°

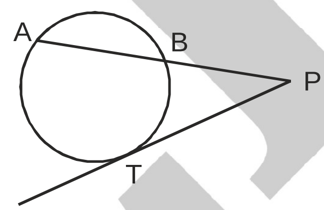
Circles

1. If two chords, AB and CD intersect inside or outside the circle at a point P,

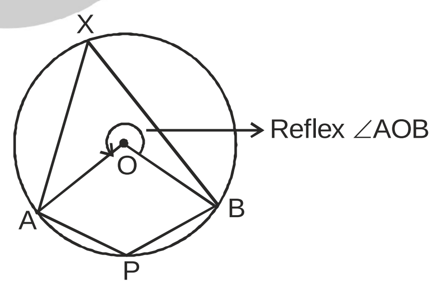


Then, PA × PB = PC × PD

2. If AB is any chord of a circle which is extended to P, and PT is a tangent drawn from P on to the circle, then
$PA \times PB = PT^2$

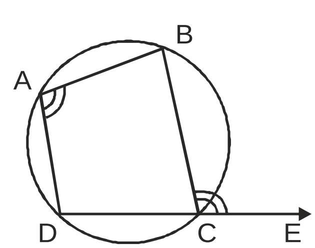


3. Angle subtended by the chord at the center of a circle is twice of that subtended at the circumference.



Thus ∠AOB = 2 × ∠AXB

4. An exterior angle of a cyclic quadrilateral is equal to the angle opposite to its adjacent interior angle.

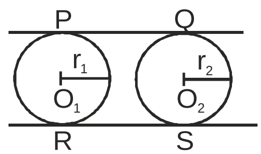


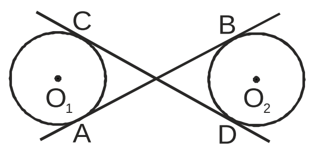
i.e. ∠BCE = ∠DAB

5. This means that a parallelogram inscribed in a circle is always a rectangle/square.

6. Also, when a square or rectangle is inscribed in a circle, the diagonal of the square / rectangle is equal to the diameter of the circle.

7. Common Tangents for a pair of circles:
For the two circles with centres O_1 and O_2 and radius r_1 & r_2





PQ, RS are Direct common tangents & AB, CD are Transverse common tangents.

Length of PQ or RS

$$= \sqrt{(\text{distance between centres})^2 - (r_2 - r_1)^2}$$

Length of AB or CD

$$= \sqrt{(\text{distance between centres})^2 - (r_2 + r_1)^2}$$

(a) When two circles touch externally
Distance between centres $C_1\ C_2 = r_1 + r_2$ and 2 direct common tangents and one transverse common tangents are possible.

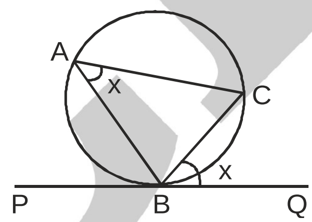
(b) When two circles touch internally
Only one common tangent is possible

(c) When two circles intersect.
Two direct common tangents are possible.

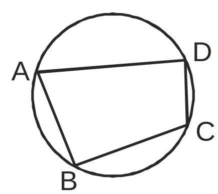
(d) When one circle is completely inside the other without touching each other.
No common tangent is possible

(e) When two circles are apart i.e. not touching each other
Two direct and two transverse tangents are possible.

8. Alternate segment theorem:
Angle between any chord passing through the tangent point and tangent is equal to the angle subtended by the chord to any point on the other side of circumference (alternate segment)



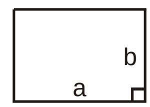
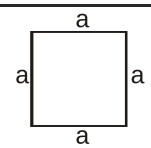
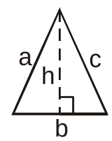
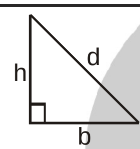
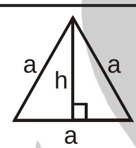
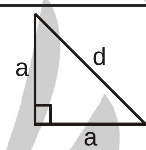
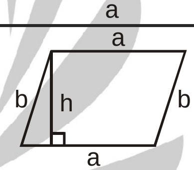
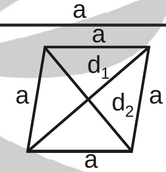
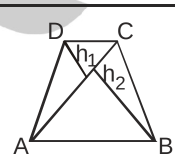
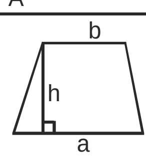
9. Ptolmey's theorem:
For a cyclic quadrilateral, the sum of products of two pairs of opposite sides equals the product of the diagonals

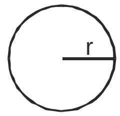
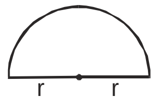
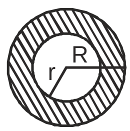
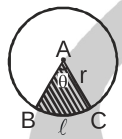
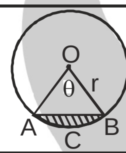
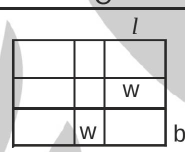
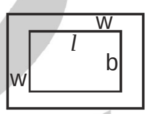
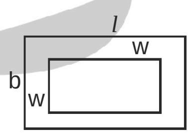


AB × CD + BC × DA = AC × BD

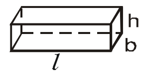
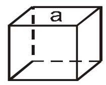
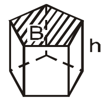
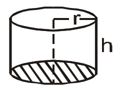
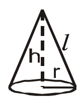
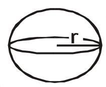
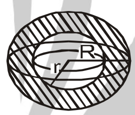
Mensuration

Two dimensional Figures

S.No.	Name	Figure	Perimeter	Area	Nomenclature
1.	Rectangle		$2(a + b)$	ab	a = Length b = Breadth
2.	Square		$4a$	a^2	a = Side
3.	Triangle		$a + b + c = 2s$	1. $\frac{1}{2} b \times h$ 2. $\sqrt{s(s-a)(s-b)(s-c)}$	b is the base and h is the altitude. a, b, c are three sides of Δ's is the semiperimeter
4.	Right angled triangle		$b + h + d$	$\frac{1}{2} bh$	d (hypotenuse) $= \sqrt{b^2 + h^2}$
5.	Equilateral triangle		$3a$	1. $\frac{1}{2} ah$ 2. $\frac{\sqrt{3}}{4} a^2$	a = side h = Altitude $= \frac{\sqrt{3}}{2} a.$
6.	Isosceles right angled triangle		$2a + d$	$\frac{1}{2} a^2$	d (hypotenuse) = $a\sqrt{2}$ a = Each of equal sides.
7.	Parallelogram		$2(a + b)$	ah	a = Side b = Side adjacent to a h = Distance between the parallel sides
8.	Rhombus		$4a$	$\frac{1}{2} d_1 \times d_2$	a = Side of rhombus, d_1, d_2 are the two diagonals.
9.	Quadrilateral		Sum of its four sides	$\frac{1}{2}(AC)(h_1 + h_2)$	AC is one of its diagonals and h_1, h_2 are the altitudes on AC from D, B respectively.
10.	Trapezium		Sum of its four sides	$\frac{1}{2} h(a + b)$	a, b are parallel sides and h is the perpendicular distance between parallel sides.

S.No.	Name	Figure	Perimeter	Area	Nomenclature
11.	Circle		Circumference $= 2\pi r$	πr^2	r = Radius of the circle $\pi = \frac{22}{7}$ or 3.416 (approx.)
12.	Semicircle		$\pi r + 2r$	$\frac{1}{2}\pi r^2$	r = Radius of the circle
13.	Ring (shaded region)		$2\pi(R + r)$	$\pi(R^2 - r^2)$	R = Outer radius r = Inner radius
14.	Sector of a circle		$l + 2r$ where $l = \left(\frac{\theta}{360°}\right) \times 2\pi r$	$\left(\frac{\theta}{360°}\right) \times \pi r^2$	θ = Central angle of the sector r = Radius of the sector l = Length of the arc
15.	Segment of a circle		$\left(\frac{\theta}{360°}\right) \times 2\pi r + 2r\sin\frac{\theta}{2}$	Area of segment ACB (Minor segment) $= r^2\left[\frac{\pi\theta}{360°} - \frac{\sin\theta}{2}\right]$	r = Radius θ = Angle subtended at the center by the arc ACB
16.	Pathways running across the middle of a rectangle		– – – –	$A = w(l + b - w)$	l = Length b = Breadth w = Width of the path
17.	Pathways outside		$2[l + b + 4w]$	$A = 2w(l + b + 2w)$	
18.	Pathways inside		$2[l + b - 4w]$	$A = 2w(l + b - 2w)$	

Solids

S.No.	Name	Figure	Lateral/curved surface area	Total surface area	Volume	Nomenclature
1.	Cuboid		_ _ _ _	$2(lb+bh+lh)$	lbh	l = Length b = Breadth h = Height
2.	Cube		_ _ _ _	$6a^2$	a^3	a = Edge
3.	Right prism		(Perimeter of base) × Height	2(Area of base) + Lateral surface area	(Area of base) × (Height)	–
4.	Right circular cylinder		$2\pi rh$	$2\pi r(r + h)$	$\pi r^2 h$	r = Radius of base h = Height of the cylinder
5.	Right pyramid		$\frac{1}{2}$(Perimeter of the base) × (Slant height)	Area of the base + Lateral surface area	$\frac{1}{3}$(Area of the base) × Height	–
6.	Right circular cone		πrl	$\pi r(l + r)$	$\frac{1}{3}\pi r^2 h$	h = Height r = Radius l = Slant height $=\sqrt{r^2 + h^2}$
7.	Sphere		–	$4\pi r^2$	$\left(\frac{4}{3}\right)\pi r^3$	r = Radius
8.	Hemisphere		$2\pi r^2$	$3\pi r^2$	$\left(\frac{2}{3}\right)\pi r^3$	r = Radius
9.	Spherical shell		–	$4\pi(R^2 + r^2)$	$\left(\frac{4}{3}\right)\pi(R^3 - r^3)$	R = Outer radius r = Inner radius

Trigonometry

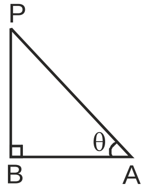
1. Angle Measures:

Angle are measured in many units viz. degree, minute, seconds, radians. We have

1 degree = 60 minutes, 1 minute = 60 seconds, π radians = 180°

Trigonometrical Ratios:

In a right angled triangle ABP, if θ be the angle between AP and AB we define



(i) $\sin\theta = \dfrac{\text{Height}}{\text{Hypotenuse}} = \dfrac{PB}{AP}$

(ii) $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{AB}{AP}$

(iii) $\tan\theta = \dfrac{\text{Height}}{\text{Base}} = \dfrac{PB}{AB}$

(iv) $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\text{Base}}{\text{Height}} = \dfrac{AB}{PB}$

(v) $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{AP}{AB}$

(vi) $\text{cosec}\,\theta = \dfrac{1}{\sin\theta} = \dfrac{\text{Hypotenuse}}{\text{Height}} = \dfrac{AP}{PB}$

2. Important Formulae:

(i) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

(ii) $\sin^2\theta + \cos^2\theta = 1$

(iii) $1 + \tan^2\theta = \sec^2\theta$

(iv) $1 + \cot^2\theta = \text{cosec}^2\theta$

3. Trigonometric measures of certain angles:

Angle	$\sin\theta$	$\cos\theta$	$\tan\theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	∞

Signs of trigonometric ratios

II^nd quadrant Here, only sin and cosec are positive.	**I^st quadrant** Here all ratios (sin, cos, tan, sec, cosec, cot) are positive.
III^rd quadrant Here, only tan and cot are positive.	**IV^th quadrant** Here, only cos and sec are positive.

You can remember above table as

School	After
To	College

LOGARITHM

1. $\log_a (XY) = \log_a X + \log_a Y$

2. $\log_a \left(\dfrac{X}{Y}\right) = \log_a X - \log_a Y$

3. $\log_a (X^k) = k \log_a X$

4. $\log_{a^k} X = \dfrac{1}{k}\log_a X$

5. $\log_a \sqrt[k]{X} = \dfrac{1}{k}\log_a X$

6. $\log_{a^{1/k}} X = k\log_a X$

7. $\log_a 1 = 0$ [As $a^0 = 1$]

8. $\log_x X = 1$

9. $\log_a X = \dfrac{1}{\log_x a}$

10. $\log_a X = \dfrac{\log_b X}{\log_b a}$

11. $a^{(\log_a X)} = X$

12. When base is not mentioned, it will be taken as 10.

MODERN MATHS

Permutations & Combinations

1. ${}^nP_r = \dfrac{n!}{(n-r)!}$

2. ${}^nC_r = \dfrac{n!}{(n-r)!\,r!}$

3. ${}^nC_r = \dfrac{{}^nP_r}{r!}$

4. ${}^nC_r = {}^nC_{n-r}$

5. ${}^nC_0 + {}^nC_1 + {}^nC_2 + {}^nC_3 + ... + {}^nC_n = 2^n$

6. Number of ways of distributing ‘n’ identical things among ‘r’ persons such that each person may get any no. of things = ${}^{n+r-1}C_{r-1}$

7. If out of n things, p are exactly alike of one kind, q exactly alike of second kind and r exactly alike of third kind and the rest are different, then the number of permutations of n things taken all at a time = $\frac{n!}{p!q!r!}$

8. Total number of ways in which a selection can be made by taking some or all out of (p + q + r + ...) items where p are of one type, q are of second type and r are of another type and so on

= {(p + 1) (q +1) (r + 1) ...} – 1

9. The number of different relative arrangement for n different things arranged on a circle = (n – 1)!

10. The number of ways in which (m + n) things can be divided into two groups containing m and n things respectively = $\frac{(m+n)!}{m!\,n!}$

11. If the numbers of things are equal, say m = n, total ways of grouping = $\frac{(2m!)}{2!(m!)^2}$

Probability

1. Probability of an event

$= \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$

2. The probability of E not occuring, denoted by P (not E), is given by P (not E) or P ($\bar{E}$) = 1 – P (E)

3. Odds in favour

$= \frac{\text{Number of favourable cases}}{\text{Number of unfavourable cases}}$

4. Odds against

$= \frac{\text{Number of unfavourable cases}}{\text{Number of favourable cases}}$

5. If two events are said to be mutually exclusive then if one happens, the other cannot happen and vice versa. In other words, the events have no simultaneous occurence.

In general P(A or B) = P(A) + P(B) – P ($A \cap B$)

If A, B are mutually exclusive then

P ($A \cap B$) = 0

If A, B are independent then

P($A \cap B$) = P(A)$\cdot$P(B)

6. Additional law of probability:
If E and F are two mutually exclusive events, then the probability that either event E or event F will occur in a single trial is given by:
P(E or F) = P(E) + P(F)

If the events are not mutually exclusive, then
P(E or F) = P(E) + P(F) – P(E and F together).

7. Multiplication law of probability:
If the events E and F are independent, then P(E and F) = P (E) × P (F)

Coordinate Geometry

Some fundamental formulae:

1. Distance between the points (x_1, y_1) and (x_2, y_2) is

$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

2. The area of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3)

$= \frac{1}{2}\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

3. The point that divides the line joining two given points (x_1, y_1) and (x_2, y_2) in the ratio m : n internally and externally are

$\left(\frac{mx_2 \pm nx_1}{m \pm n}, \frac{my_2 \pm ny_1}{m \pm n}\right)$

Note: It would be '+' in the case of internal division and '–' in the case of external division.

4. The coordinate of the mid-point of the line joining the points (x_1, y_1) and (x_2, y_2)

$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

5. The centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3)

$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

6. Slope of the line joining the points (x_1, y_1) and (x_2, y_2) is $\frac{y_2 - y_1}{x_2 - x_1}$

The slope is also indicated by m.

7. If the slopes of two lines be m_1 and m_2, then the lines will be

(i) parallel if $m_1 = m_2$

(ii) perpendicular if $m_1 m_2 = -1$

Standard forms:

1. All straight lines can be written as

$y = mx + c$,

where m is the slope of the straight line, c is the Y intercept or the Y coordinate of the point at which the straight line cuts the Y-axis.

2. The equation of a straight line passing through (x_1, y_1) and having a slope m is

$y - y_1 = m(x - x_1)$.

3. The equation of a straight line passing through two points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

4. The point of intersection of any two lines of the form $y = ax + b$ and

$y = cx + d$ is same as the solution arrived at when these two equations are solved.

5. The lenght of perpendicular from a given, point (x_1, y_1) to a given line $ax + by + c = 0$ is

$$\left|\frac{ax_1 + by_1 + c}{\sqrt{(a^2 + b^2)}}\right| = p,$$

where p is the length of perpendicular. In particular, the length of perpendicular from origin (0,0) to the line $ax + by + c = 0$ is

$$\frac{c}{\sqrt{a^2 + b^2}}$$

6. Distance between two parallel lines

$ax + by + c_1 = 0$

$ax + by + c_2 = 0$

is $\left|\frac{c_2 - c_1}{\sqrt{a^2 + b^2}}\right|$