ESERCIZIO

Consideriamo la seguente densità congiunta

$\frac{1}{10}$ per ogni punto.

$\frac{1}{2}$
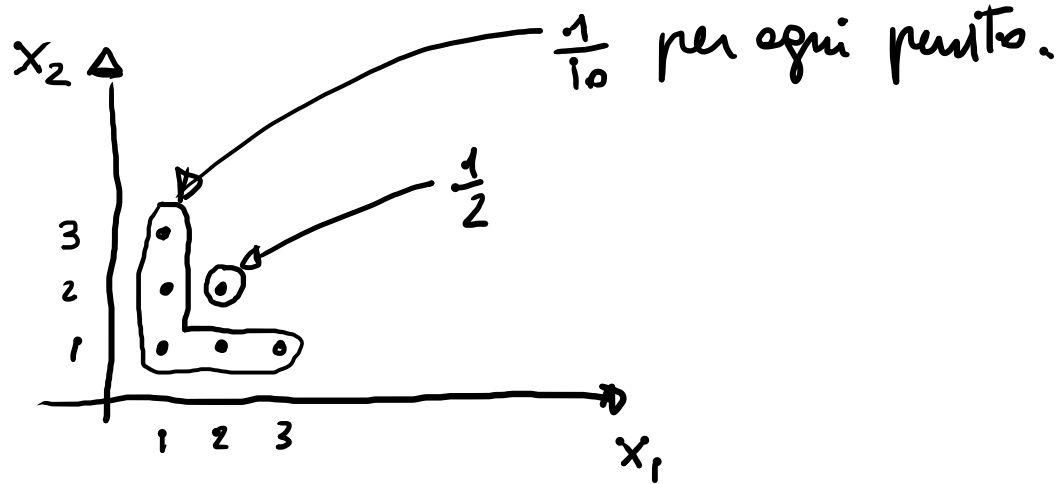
Calcolare $\mathbb{E}[X_1 + X_2]$

SVOLGIMENTO

Y assume valori in $\{2, 3, 4\}$

$$P_Y(2) = P_{\underline{x}}(1,1) = \boxed{\frac{1}{10}}$$

$$P_Y(3) = P_{\underline{x}}(1,2) + P_{\underline{x}}(2,1) = \frac{1+1}{10} = \boxed{\frac{2}{10}}$$

$$P_Y(4) = P_{\underline{x}}(1,3) + P_{\underline{x}}(2,2) + P_{\underline{x}}(3,1) = \frac{1}{10} + \frac{1}{2} + \frac{1}{10} = \cdots = \boxed{\frac{7}{10}}$$

SOMMA $= 1$ ok

$$\mathbb{E}[Y] = 2 \cdot P_Y(2) + 3 \cdot P_Y(3) + 4 \cdot P_Y(4) = 2 \cdot \frac{1}{10} + 3 \cdot \frac{2}{10} + 4 \cdot \frac{7}{10} =$$

$$= \frac{2 + 6 + 28}{10} = \frac{36}{10} = \frac{18}{5}$$

METODO ALTERNATIVO

$$\mathbb{E}[Y] = \mathbb{E}[X_1 + X_2] = (1+1)\, p_{\underline{x}}(1,1) + (1+2)\, p_{\underline{x}}(1,2) + (2+1)\, p_{\underline{x}}(2,1) +$$

$$+ (3+1)\, p_{\underline{x}}(3,1) + (2+2)\, p_{\underline{x}}(2,2) + (1+3)\, p_{\underline{x}}(1,3) =$$

$$= 2 \cdot \frac{1}{10} + 3 \cdot \frac{1}{10} + 3 \cdot \frac{1}{10} +$$

$$+ 4 \cdot \frac{1}{10} + 4 \cdot \frac{1}{2} + 4 \cdot \frac{1}{10} = \frac{2+3+3+4+20+4}{10} =$$

$$= \frac{36}{10} = \frac{18}{5}$$

METODO ALTERNATIVO ULTERIORE

$$\mathbb{E}[Y] = \mathbb{E}[X_1] + \mathbb{E}[X_2].$$

$\uparrow$ linearità

Bisogna trovare le marginali di X_1 e X_2

Si dovrebbe avere

$$P_{X_1}(1) = \frac{3}{10} \quad , \quad P_{X_1}(2) = \frac{6}{10} \quad , \quad P_{X_1}(3) = \frac{1}{10}$$

P_{X_2} stessa cosa

$$\mathbb{E}[X_1] = \mathbb{E}[X_2] = 1 \cdot \frac{3}{10} + 2 \cdot \frac{6}{10} + 3 \cdot \frac{1}{10} = \frac{3+12+3}{10} = \frac{18}{10}$$

$$\mathbb{E}[Y] = \frac{18}{10} + \frac{18}{10} = \frac{36}{10} = \frac{18}{5}.$$