ESERCIZIO

Sia X una v.a. con densità $f_X(x) = c\, x\, \mathbb{1}_{(4,9)}(x)$.

$$\begin{array}{r} 81 \\ 16 \\ \hline 65 \end{array}$$

1) Trovare il valore di c

2) Trovare la funzione di distribuzione di $Y = \sqrt{X}$

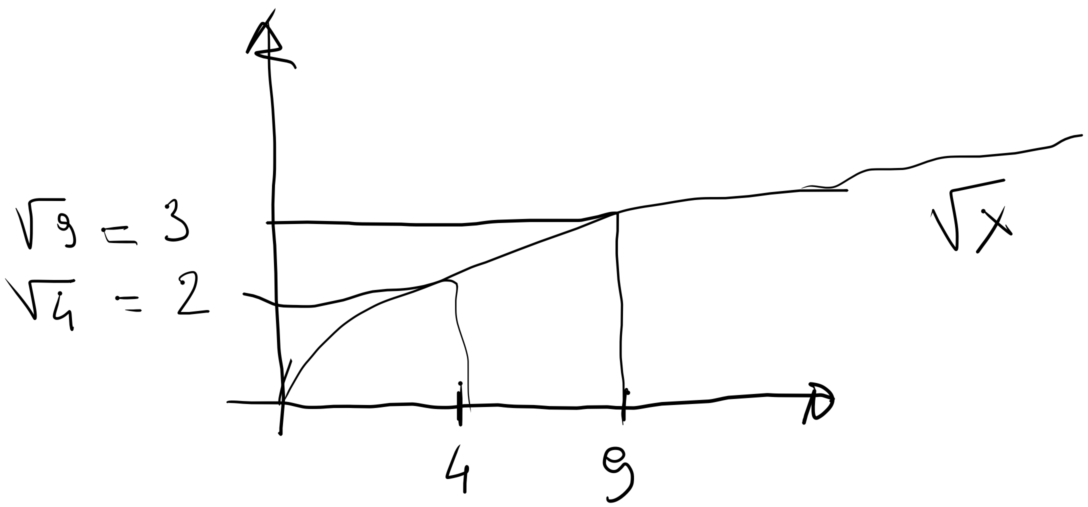
3) Calcolare $\mathbb{E}[Y]$.

SVOLGIMENTO

1) $$1 = \int_{-\infty}^{\infty} f_X(x)\,dx = c \int_4^9 x\,dx = c \left[\frac{x^2}{2}\right]_{x=4}^{x=9} = \frac{c}{2}\left(9^2 - 4^2\right) = \frac{c}{2}(81-16)$$

$$= c\,\frac{65}{2} \longrightarrow c = \frac{2}{65}$$

2)

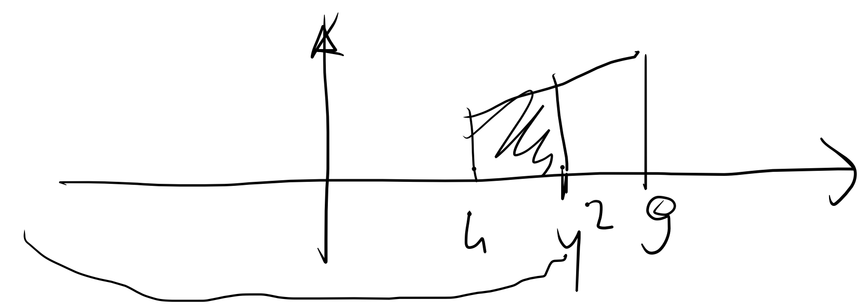


$$P(Y \in [2,3]) = 1$$

$$F_Y(y) = \begin{cases} 0 & \text{per } y \leq 2 \\ \circledast & \text{per } 2 < y < 3 \\ 1 & \text{per } y \geq 3 \end{cases}$$

$$\circledast = P(Y \leq y) = P(\sqrt{X} \leq y) = P(X \leq y^2) = \int_4^{y^2} \frac{2}{65} x \, dx = \frac{\cancel{2}}{65}\left[\frac{x^2}{\cancel{2}}\right]_{x=4}^{x=y^2} = \boxed{\frac{y^4 - 16}{65}}$$

$4 < y^2 < 9$ perché



3) 1º modo

$$\begin{array}{r} 65 \\ 5 \\ \hline 325 \end{array}$$

$$\mathbb{E}[Y] = \mathbb{E}[\sqrt{X}] = \int_4^9 \sqrt{x}\, \overset{C}{\frac{2}{65}}\, x \, dx = \frac{2}{65} \int_4^9 \underbrace{x\sqrt{x}}_{= x \cdot x^{1/2}}\, dx =$$

$$= \frac{2}{65} \int_4^9 x^{3/2}\, dx = \frac{2}{65} \left[\frac{x^{5/2}}{\frac{5}{2}} \right]_{x=4}^{x=9} = \frac{2}{65} \cdot \frac{2}{5} \left(9^{5/2} - 4^{5/2} \right) =$$

$$= \frac{4}{325} \left(3^5 - 2^5 \right) = \frac{4}{325} (243 - 32) = \frac{4}{325} \cdot 181 = \frac{724}{325}$$

$$\begin{array}{r} 181 \\ 4 \\ \hline 724 \end{array}$$

2° modo

$$\mathbb{E}[Y] = \int_2^3 y \, f_Y(y)\,dy = \int_2^3 y \underbrace{\frac{4y^3}{65}}_{=f_Y(y)} dy =$$

$= f_Y(y) \rightsquigarrow$ derivata di $\dfrac{y^4-16}{65}$

$$= \int_2^3 \frac{4}{65} y^4 \,dy = \frac{4}{65}\int_2^3 y^4\,dy =$$

$$= \frac{4}{65}\left[\frac{y^5}{5}\right]_{y=2}^{y=3} = \frac{4}{\underbrace{65 \cdot 5}_{=325}}\left[3^5 - 2^5\right] = \frac{4}{325}\cdot(243-32) = \frac{4}{325}\,181 = \frac{724}{325}$$