INVERSE OF A MATRIX

Logic → $AX = I$

X → is the Inverse matrix to be found.

and, $A = LU$

So; $LUX = I$

$X = [x_0 \; x_1 \; x_2 \; \dots \; x_{n-1}]$

x_k → k^{th} column of X matrix

Step-1 : Find L, U from A.

Step-2 : Use forward, backward sub using L & U matrices.

$L \cdot y_k = e_k$

e_k → eigen vector column

find y_k from here.

4. Forward Substitution (Solving $L\mathbf{y} = \mathbf{b}$)

Since L is lower-triangular, you can solve $\mathbf{y}$ component-by-component from **top row to bottom row**:

For a 4×4 example:

$$\begin{pmatrix} l_{00} & 0 & 0 & 0 \\ l_{10} & l_{11} & 0 & 0 \\ l_{20} & l_{21} & l_{22} & 0 \\ l_{30} & l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix} \begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \end{pmatrix}.$$

1. $l_{00}\, y_0 = b_0 \quad \rightarrow \quad y_0 = b_0 / l_{00}.$
2. $l_{10}\, y_0 + l_{11}\, y_1 = b_1 \quad \rightarrow \quad y_1 = (b_1 - l_{10}\, y_0) / l_{11}.$
3. And so on, "moving down" each row.

If $\mathbf{b} = \mathbf{e}_k$, then that sets up the **forward substitution** to get $\mathbf{y}_k$.

Using $U x_k = y_k$

$\boxed{\text{Find } \underline{x_k} \text{ to construct the inverse matrix } X.}$

Similarly, because U is upper-triangular, you solve from **bottom row up**:

$$\begin{pmatrix} u_{00} & u_{01} & u_{02} & u_{03} \\ 0 & u_{11} & u_{12} & u_{13} \\ 0 & 0 & u_{22} & u_{23} \\ 0 & 0 & 0 & u_{33} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \begin{pmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{pmatrix}.$$

1. $u_{33}\, x_3 = y_3 \quad \rightarrow \quad x_3 = y_3 / u_{33}.$
2. $u_{22}\, x_2 + u_{23}\, x_3 = y_2 \quad \rightarrow \quad x_2 = (y_2 - u_{23}\, x_3) / u_{22}.$
3. And so forth, "moving upward" each row.

If $\mathbf{y} = \mathbf{y}_k$ (the result of forward-sub), this step yields $\mathbf{x}_k$. Then $\mathbf{x}_k$ is exactly the kth column of A^{-1}.