1. Find all elements of groups $\mathbb{Z}_7$, $\mathbb{Z}_7^*$, $\mathbb{Z}_8$, $\mathbb{Z}_8^*$.
2. Using Euclidean Algorithm find the inverse of 7 in $\mathbb{Z}_{31}^*$ and $\mathbb{Z}_{137}^*$.
3. Find the smallest generator of the group $\mathbb{Z}_{31}^*$.
4. Find all subgroups of $\mathbb{Z}_6$.
5. Find all subgroups of $\mathbb{Z}_8^*$.
6. Is $\mathbb{Z}_2$ a subgroup of $\mathbb{Z}_4$?
7. Is 3 a generator of $\mathbb{Z}_{53}^*$?
8. Is the group $(\mathbb{Z}, \circ)$ cyclic, if $a \circ b = a + b - 5$. If yes, find the generators.
9. Which group is cyclic: $\mathbb{Z}_5^*$, $\mathbb{Z}_8^*$, $\mathbb{Z}_{15}^*$?
10. For all $a \in \mathbb{Z}_9^*$ find $\langle a \rangle$ and $|a|$. Is $\mathbb{Z}_9^*$ cyclic?
11. For all $a \in \mathbb{Z}_{14}^*$ find $\langle a \rangle$ and $|a|$. Is $\mathbb{Z}_{14}^*$ cyclic?
12. Prove, that $5n + 3$ and $7n + 4$ are relatively prime for any positive n.
13. Find primes p, q if $n = p \cdot q = 414847$ and $\phi(n) = 413280$.
14. Find an integer a such that $a \equiv 4 \pmod 6$ and $a \equiv 5 \pmod{35}$.
15. Find an integer a such that $a \equiv 4 \pmod 7$ and $a \equiv 1 \pmod{19}$.
16. Find an integer a such that $a \equiv 38 \pmod{103}$ and $a \equiv 81 \pmod{83}$.
17. Find an integer a such that $a \equiv 4 \pmod 6$ and $a \equiv 5 \pmod{35}$.
18. Find an integer a such that $a \equiv 4 \pmod 7$ and $a \equiv 1 \pmod{19}$.
19. Find an integer a such that $a \equiv 38 \pmod 9)1)$, $a \equiv 81 \pmod{83}$ and $a \equiv 3 \pmod{95}$.
20.
21. Knowing $n = 5133$ and

0	1	2	3	4	5	6	7	8	9
RY	SYS	TEM	O	TY	MA	GA	EK	WA	TE

encrypt the message SYSTEM, TEORY using Rabin method.

22. Using Rabin method decrypte the messege $E(M) = 17(\mathrm{mod}\, 1121)$, if you know that $1121 = 19 \cdot 59$.

0	1	2	3	4	5	6	7	8	9
A	M	L	D	F	T	Y	O	Z	K

23. Knowing that $n = 589 = 19 \cdot 31$, $e = 53$ and encrypting function for RSA cryptosystem is $E(M) = M^e(\mathrm{mod}\, n)$ find decrypting function (for RSA method).

24. Knowing that $n = 589 = 19 \cdot 31$, $d = 23$ and decrypting function for RSA cryptosystem is $D(N) = N^d(\mathrm{mod}\, n)$ find encrypting function (for RSA method).

25. Let *day-23, nice-7, good-1, have-4, luck-3, the-59, always-54, reason-47*. Using RSA method for $p = 11$, $q = 13$, $e = 11$ decrypt the message "$113, 1$".