Practice Midterm 4

1. Let $z_1 = 1+2i$ and $z_2 = 4-3i$. Please evaluate the following expressions and express your solution as a sum of its real and imaginary parts.
   (a) $z_1 + z_2$
   (b) $\overline{z_1}$
   (c) $z_1 \times z_2$
   (d) $\frac{z_1}{z_2}$
2. Let $z = -4-4i$.
   (a) Please find $|z|$.
   (b) Please find $\arg z$.
   (c) Please express z in polar form.
   (d) Please express z in exponential form.
3. Please express your solutions as a sum of its real and imaginary parts.
   (a) Let $z = 1-\sqrt{3}i$. Please find z^{11}.
   (b) Please find all solutions of $z^6 = -i$.
4. Let $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, and $F = \{aI + bJ; a, b \in \mathbb{R}\}$.
   (a) Prove that F is a ring.
   (b) Prove that F is an integral domain.
5. The set $\mathbb{Z}_7$ is a commutative ring with an identity.
   (a) Does $\mathbb{Z}_7$ has a zero divisor?
   (b) If yes, find all zero divisors. If no, show that $\mathbb{Z}_7$ is a field.