Homework 8

$-\sqrt{3} - i$

Due: January 19, 2021

Name:

Please explain and show your work!

1. Let $z = -\sqrt{3} - i$.

   (a) Please find $|z|$.

   $|z| = \sqrt{(-\sqrt{3})^2 + (-1)^2}$

   $= \sqrt{3+1} = \sqrt{4} = 2 = r$

   $\cos\theta = 30'$

   $x = r\cos\theta$

   (b) Please find the argument of z.

   arg of $z = \frac{11\pi}{6} + \frac{\pi}{2}$

   $180'$

   $-\sqrt{3} = r\cos\theta$

   $-\sqrt{3} = 2\cos\theta$

   $\cos\theta = -\frac{\sqrt{3}}{2}$

   $i = r\sin\theta$

   $r = 2$

   $i = 2\sin\theta$

   $\sin\theta = \frac{i}{2}$

   $30'$

   150

   $66'$

   $\frac{6\pi + 3\pi + 2\pi}{6} = \frac{11\pi}{6}$

   (c) Please express z in polar form.

   $z = r(\cos\theta + i\sin\theta)$

   $r = |z| = 2$

   $\theta = \tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

   $\tan\theta = -\frac{1}{\sqrt{3}}$

   $\theta = \frac{5\pi}{6}$ (or) $\frac{11\pi}{6}$

   complex number is in 2 quadrant, $\theta = \frac{5\pi}{6}$

   $z = 2\left[\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right]$

   (d) Please express z in exponential form.

   $z = 2e^{i\left(\frac{5\pi}{6}\right)}$

   (e) Use de Moivre's formula to find z^{-10}. Please express your answer in terms of the sum of the real and imaginary parts.

   $z^{-10} = r^{-10}(\cos(-10\theta) + i\sin(-10\theta))$

   $r = 2, \theta = \frac{5\pi}{6}$

   $z^{-10} = 2^{-10}\cos\left(-10\cdot\frac{5\pi}{6}\right) + i\sin\left(-10\cdot\frac{5\pi}{6}\right))$

$= 2^{-10}\left(\cos\left(-\frac{25\pi}{3}\right) + i\sin\left(-\frac{25\pi}{3}\right)\right)$

$= 2^{-10}\left(\cos\frac{25\pi}{3} - i\sin\frac{25\pi}{3}\right)$

$\cos\frac{25\pi}{3} = \cos\left(8\pi + \frac{\pi}{3}\right) = \cos\frac{\pi}{3} = \frac{1}{2}$

$\sin\frac{25\pi}{3} = \sin\left(8\pi + \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$\therefore\ z^{-10} = 2^{-10}\left(\frac{1}{2} - i\left(\frac{\sqrt{3}}{2}\right)\right)$

$z^{-10} = \frac{1}{2^{11}} - \frac{\sqrt{3}}{2^{11}}i$