Homework 6

Due: December 8 (Wed), 2021

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Please explain and show your work!

1. Consider S_4 and the dihedral group D_4.
   (a) Find $[S_4 : D_4]$.
   (b) List all distinct left cosets of D_4.
   (c) List all distinct right cosets of D_4.

   Ans:

a. There are three left cosets of D_4 in S_4.

$D_4 = \{ i, (1234), (13)(24), (1432), (12)(34), (14)(23), (13), (24) \}$

$(12)D_4 = \{ (12), (234), (2413), (143), (34), (1423), (132), (124) \}$

$(14)D_4 = \{ (14), (123), (1342), (243), (1243), (23), (134), (142) \}$

(b) The left cosets of D_4 are

$eH = \{e, b\} = bH = H$

$aH = \{a, ba^3\} = ba^3H$

$a^2H = \{a^2, ba^2\} = ba^2H$

$a^3H = \{a^3, ba\} = baH$

(c) The right cosets of D_4 are

$He = \{e, b\} = Hb = H$

$Ha = \{a, ba\} = Hba$

$Ha^2 = \{a^2, ba^2\} = Hba^2$

$Ha^3 = \{a^3, ba^3\} = Hba^3$

2. Let G be a group and H be a subgroup of G. Define a relation $\sim$ on G by $a \sim b$ if there exists $h \in H$ such that $a = hb$.
   (a) Prove that $\sim$ is an equivalence relation.
   (b) What are the equivalence classes?

   Ans:

(a) Reflexive. For any $a \in G$, $a^{-1}a = 1 \in H$ because it is a subgroup of G, where $a \sim a$.

· Symmetric: If $a \sim b$, then $b^{-1}a \in H$.
Since H is closed under inverse,

$a^{-1}b = (b^{-1}a)^{-1} \in H$, where $b \sim a$

Transitivity: If $a \sim b$ and $b \sim c$, then $b^{-1}a \in H$ and
Since H is closed under multiplication,

$c^{-1}a = (c^{-1}b)(b^{-1}a) \in H$

(b). The equivalence class $[a]$ of any $a \in G$ is the left coset of aH.

$b \in [a] \Leftrightarrow b \sim a$.

$b \in [a] \Leftrightarrow b^{-1}a \in H$

$b \in [a] \Leftrightarrow b^{-1}a = h$, for some $h \in H$.

$b \in [a] \Leftrightarrow b = ah^{-1}$, for some $h \in H$.

$b \in [a] \Leftrightarrow b = ah$, for some $h \in H$

$b \in [a] \Leftrightarrow b \in aH$.