Practice Midterm 1

1. Let $A = \{a,b,c\}$, $B = \{b,c,d\}$, $C = \{c,d,e\}$.
   (a) $A \cap B =$?
   (b) $B \cup C =$?
   (c) $C \backslash A =$?

2. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = (x-1)(x-2)(x-3)$.
   (a) Is f an one to one function? Prove or disprove your answer.
   (b) What is the range of f?
   (c) Is f an onto function?

3. (a) Let R be a relation defined on $\{a,b,c,d,e\}$. The truth table of the relation xRy is given by the following. Is R an equivalence relation?

$x \backslash y$	a	b	c	d	e
a	T	F	F	T	T
b	F	T	F	F	T
c	F	F	T	F	F
d	T	F	F	T	F
e	T	T	F	F	T

   Ans: (Yes or No)
   (b) If R is an equivalence relation, list all the distinct equivalence classes given by it. If R is not an equivalence relation, state why it fails to be one.

4. Evaluate the following.
   (a) Find an integer x such that $2x \equiv 1 (\mod 5)$ and $0 \leq x < 5$.
   (b) $\gcd(4208, 288) =$?

5. Prove that
$$\sum_{i=1}^{n} (2i-1) = n^2$$
   for $n = 1, 2, \cdots$ by induction.

6. Please complete the multiplication (Cayley) table of the group $(\mathbb{Z}_5, +)$.

$+$	0	1	2	3	4
0					
1					
2					
3					
4					

7. Let $G = \left\{ \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} ; a \in \mathbb{R} \right\}$.
   (a) Prove that G is a group under the matrix multiplication.
   (b) Is G abelian?