

1. Show that a finite simple graph with more than one vertex has at least two vertices with the same degree.
2. A graph G has 13 edges and 3 vertices of degree 1, 2 and 3. All remaining vertices have degree 4. How many vertices does the graph G have?
3. Check which sequences are graphic (if yes then draw a graph):

$$(4, 4, 4, 3, 2, 1, 1), (4, 4, 3, 2, 2, 1), (4, 4, 3, 2, 1)$$

4. The incidence matrix is given for a graph G :

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}.$$

Find the degree sequence, order and size of G . Draw the graph G .

5. Find the incidence matrix and the adjacency for the graph $K_{2,3}$.
6. Give two examples of graphs that are not isomorphic but they have the same degree sequence.
7. Find a graph of order 4 that is isomorphic with its complement.
8. Find a graph of order 5 that is isomorphic with its complement. Is it possible for graphs of order 2 or 3?
9. Find a complement for the complete bipartite graph $K_{m,n}$.
10. Prove that a graph is bipartite if and only if it contains no odd cycles.