# Practice Midterm 2

1. True or False.
   (a) $S_3$ is cyclic.
   (b) $(\mathbb{Z}_2 \times \mathbb{Z}_3, +)$ is a cyclic group.
   (c) If $\sigma$ is a cycle, then $\sigma^3$ can also be expressed as a cycle.
   (d) If $\sigma$ and $\mu$ are permutations, then $\sigma\mu = \mu\sigma$.
   (e) The order of $S_5$ is 100.

2. Let $H$ be a subgroup of $G$ and
$$C(H) = \{g \in G; gh = hg \text{ for all } h \in H\}.$$
Prove that $C(H)$ is a subgroup of $G$.

3. Consider the group $GL_2(\mathbb{R})$. Let $A = \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 \\ -1 & 0 \end{bmatrix}$, and $C = AB$.
   (a) Find $|A|$, $|B|$, and $|C|$.
   (b) If the order is finite, then please list all the elements of the cyclic group generated by the element.

4. Consider the group $(\mathbb{Z}_{18}, +_{18})$ under modular addition.
   (a) Find all generators of $\mathbb{Z}_{18}$.
   (b) Find all distinct subgroups of $\mathbb{Z}_{18}$.

5. Let $\sigma = \begin{pmatrix} 2 & 1 & 5 \end{pmatrix}\begin{pmatrix} 3 & 4 \end{pmatrix}$ and $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 4 & 2 & 1 \end{pmatrix}$.
   (a) Evaluate $\sigma\mu$.
   (b) Find $\sigma^{-1}$.
   (c) What is the order of $\mu$?

6. Let $\sigma = \begin{pmatrix} 2 & 1 & 5 & 3 \end{pmatrix}\begin{pmatrix} 3 & 2 & 6 \end{pmatrix}$.
   (a) Write $\sigma$ as a multiplication of disjoint cycles.
   (b) Write $\sigma$ as a multiplication of transpositions.
   (c) Is $\sigma$ an element in $A_6$?