1. Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive 0s. How many such bit strings are there of length five?

2. A vending machine dispensing books of stamps accepts only one-dollar coins, \$1 bills, and \$5 bills.
   a) Find a recurrence relation for the number of ways to deposit n dollars in the vending machine, where the order in which the coins and bills are deposited matters.
   b) What are the initial conditions?
   c) How many ways are there to deposit \$10 for a book of stamps?
   d) Find the solution of the recurrence relation.

3. a) Find a recurrence relation for the number of ways to completely cover a $2 \times n$ checkerboard with 1×2 dominoes. [Hint: Consider separately the coverings where the position in the top right corner of the checkerboard is covered by a domino positioned horizontally and where it is covered by a domino positioned vertically.]
   b) What are the initial conditions for the recurrence relation in part (a)?
   c) Find the solution of the recurrence relation.

4. a) Find a recurrence relation for the number of ways to lay out a walkway with slate tiles if the tiles are red, green, or gray, so that no two red tiles are adjacent and tiles of the same color are considered indistinguishable.
   b) What are the initial conditions for the recurrence relation in part (a)?
   c) How many ways are there to lay out a path of seven tiles as described in part (a)?
   d) Find the solution of the recurrence relation.

5. Solve these recurrence relations together with the initial conditions given.:

   (a) $a_n = a_{n-1} + 6a_{n-2}$ for $n \geq 2$, $a_0 = 3$, $a_1 = 6$
   (b) $a_n = 7a_{n-1} - 10a_{n-2}$ for $n \geq 2$, $a_0 = 2$, $a_1 = 1$
   (c) $a_n = 6a_{n-1} - 8a_{n-2}$ for $n \geq 2$, $a_0 = 4$, $a_1 = 10$
   (d) $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 2$, $a_0 = 4$, $a_1 = 1$

(e) $a_n = a_{n-2}$ for $n \geq 2$, $a_0 = 5$, $a_1 = -1$

(f) $a_n = -6a_{n-1} - 9a_{n-2}$ for $n \geq 2$, $a_0 = 3$, $a_1 = -3$

(g) $a_{n+2} = -4a_{n+1} + 5a_n$ for $n \geq 0$, $a_0 = 2$, $a_1 = 8$

(h) $an = 2a_{n-1} + a_{n-2} - 2a_{n-3}$ for $n \geq 3$, $a_0 = 3$, $a_1 = 6$, and $a_2 = 0$

(i) $a_n = 7a_{n-2} + 6a_{n-3}$ for $n \geq 3$ $a_0 = 9$, $a_1 = 10$, and $a_2 = 32$

(j) $a_n = 5a_{n-2} - 4a_{n-4}$ for $n \geq 4$, $a_0 = 3$, $a_1 = 2$, $a_2 = 6$, and $a_3 = 8$.

6. What is the general form of the particular solution ofthe linear nonhomogeneous recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3} + F(n)$ if

   (a) $F(n) = n^2$?

   (b) $F(n) = 2^n$?

   (c) $F(n) = n2^n$?

   (d) $F(n) = (-2)^n$?

   (e) $F(n) = n^2 2^n$?

   (f) $F(n) = n^3(-2)^n$?

   (g) $F(n) = 3$?

7. What is the general form of the particular solution of the recurrence relation $a_n = 8a_{n-2} - 16a_{n-4} + F(n)$ if

   (a) $F(n) = n^3$?

   (b) $F(n) = (-2)^n$?

   (c) $F(n) = n2^n$?

   (d) $F(n) = n^2 4^n$?

   (e) $F(n) = (n^2 - 2)(-2)^n$?

   (f) $F(n) = n^4 2^n$?

   (g) $F(n) = 2$?