Homework 7

Due: December 16 (Thur), 2021

Name:

Please explain and show your work!

1. Let $T = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} : a, b, c \in \mathbb{R} \text{ and } ac \neq 0 \right\}$ and $U = \left\{ \begin{bmatrix} 1 & x \\ 0 & 1 \end{bmatrix} : x \in \mathbb{R} \right\}$. The sets are subgroups of $GL_2(\mathbb{R})$.

   (a) Prove that U is normal in T.

   (b) Show that T/U is abelian.

   Ans:

2. Let $A = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$ and $\phi : \mathbb{R}^2 \to \mathbb{R}^2$ be defined as $\phi(v) = Av$.

   (a) Prove that ϕ is a homomorphism.

   (b) What is the kernel of ϕ?

   (c) What is the image of ϕ?

   Ans:

$\phi(v) = Av$

let $x, y \in \mathbb{R}$.

$\phi(x+y) = A(x+y)$

$= Ax + Ay$

$= \phi(x) + \phi(y)$

$\therefore \phi$ is a homomorphism.