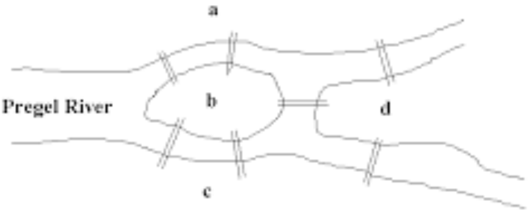
Eulerian and Hamiltonian graphs

Sylwia Cichacz

Akademia Górniczo-Hutnicza w Krakowie

December 19, 2021, Kraków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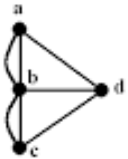
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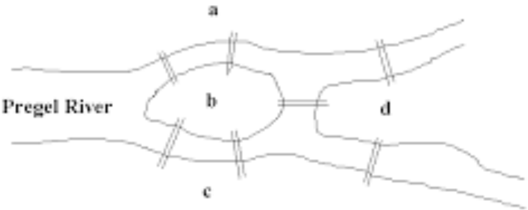

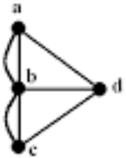

Problem: Does there exist a closed trail in the graph G that passes through all edges?

Eulerian graph

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An Eulerian cycle in a graph G is a closed walk that contains each edge exactly once (a closed trail).

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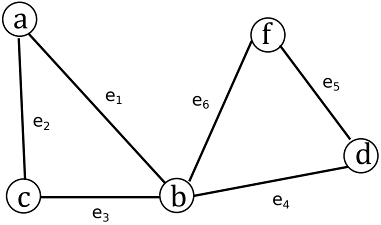

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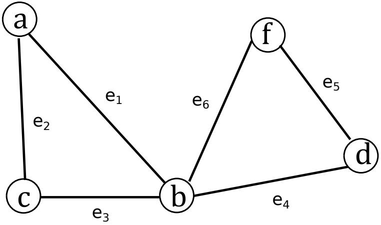

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Is G jest Eulerian?

Eulerian graph

Theorem: L. Euler; 1736

A connected graph G possesses an Eulerian cycle iff the degree of every vertex is even.

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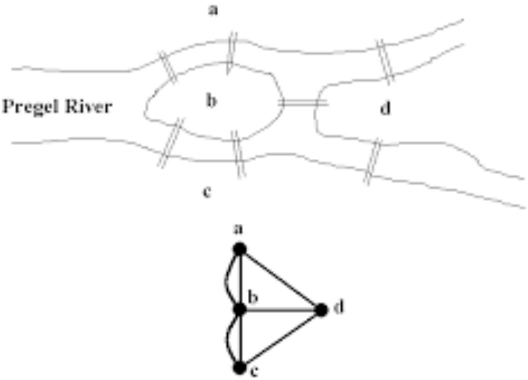

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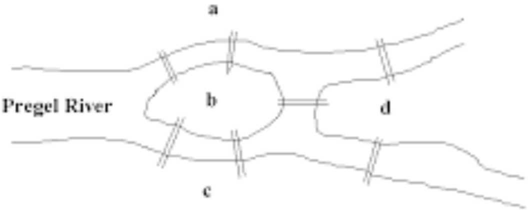

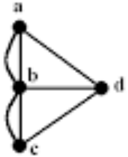

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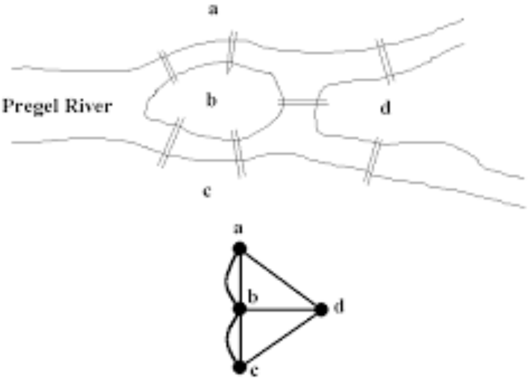

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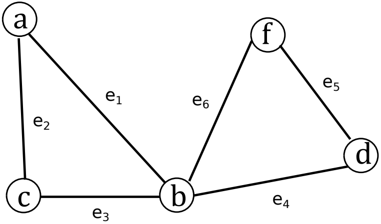

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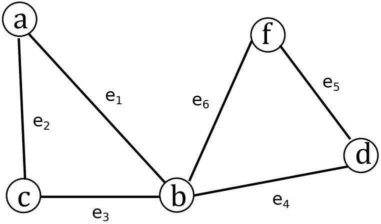

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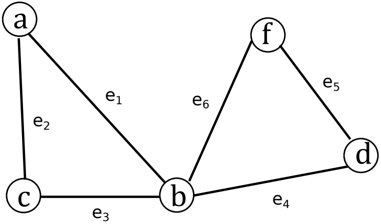

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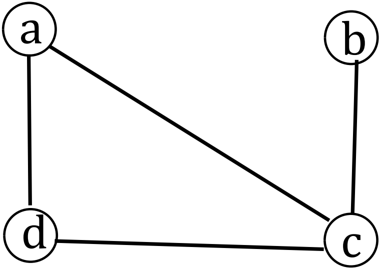

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Step 4: When you cannot travel any more, **STOP**. (You are done!)

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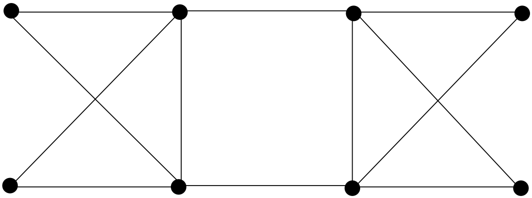

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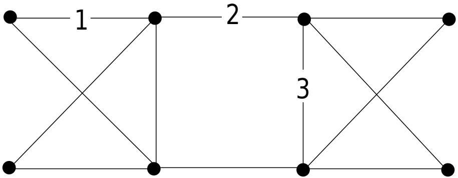

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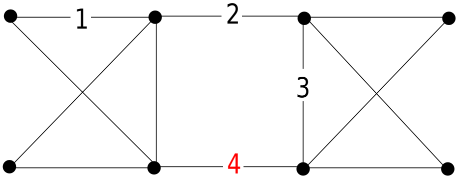

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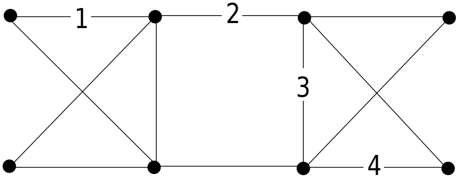

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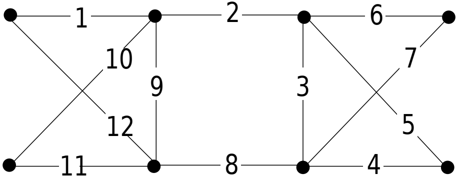

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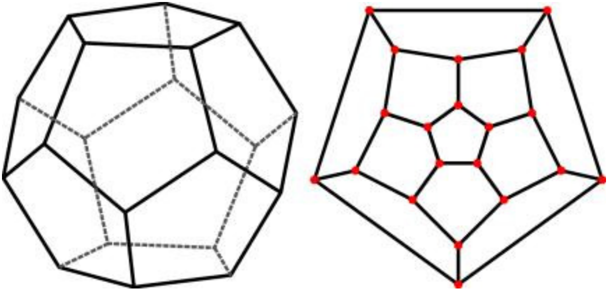

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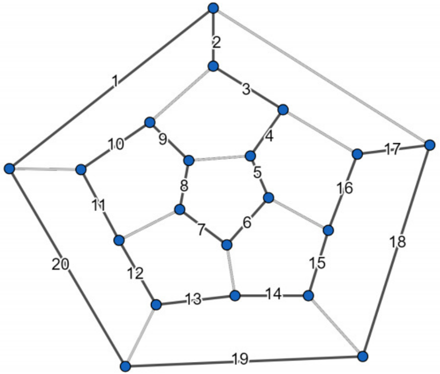

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A graph G *which possesses a hamiltonian cycle is called the* *Hamiltonian graph.*

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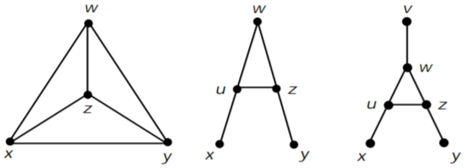

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- No such condition is known for Hamiltonicity.
- The question, whether a given graph G is Hamiltonian or not, is NP-complete.
- Hence the existence of a simple algorithm for checking it is unlikely.
- There exist easily checkable, sufficient, but not necessary conditions for Hamiltonicity.
  Many of them are are variations of "if a graph has many edges then it is Hamiltonian".

Hamiltonian graph

Theorem: Dirac, 1952

If a simple graph $G = (V, E)$ with $|V| = n > 3$ satisfies

$$\forall v \in V: \quad \deg(v) \geq \frac{n}{2}$$

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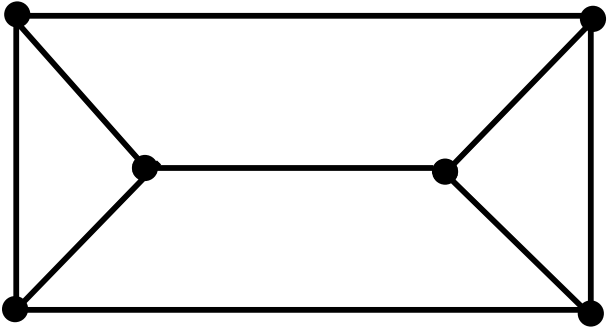

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Theorem: Ore, 1960

If a simple graph $G = (V, E)$ with $|V| = n > 3$ satisfies

$$\forall v, w \in V : \quad (\text{if } \ uv \notin E \quad \Rightarrow \quad \deg(v) + \deg(w) \geq n)$$

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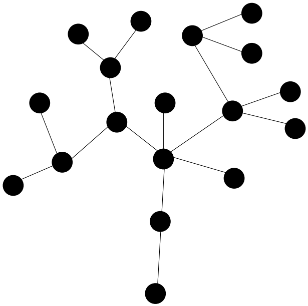

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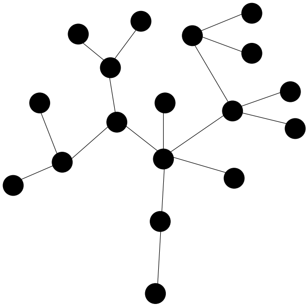

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Every tree is a bipartite graph.

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5. $|E| = |V| - 1$.

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Suppose that every tree with k vertices has precisely $k - 1$ edges.

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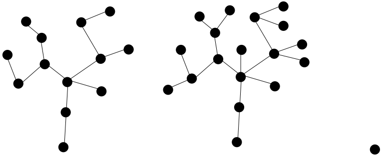

Forest

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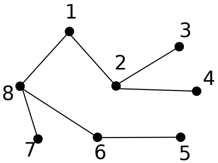

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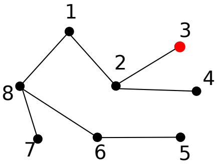

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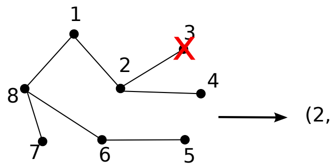

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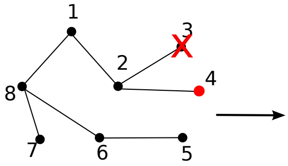

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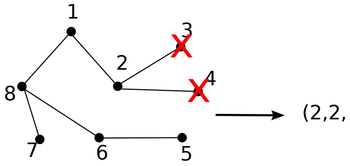

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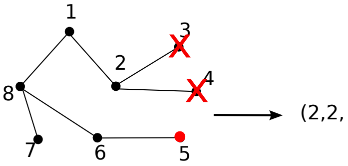

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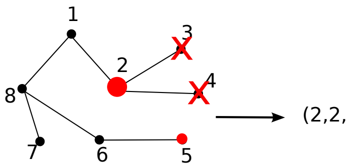

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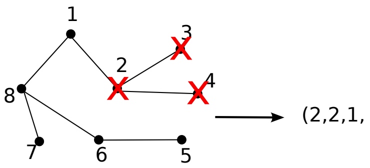

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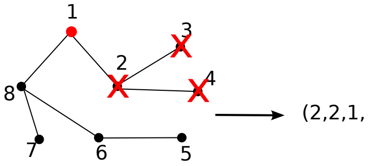

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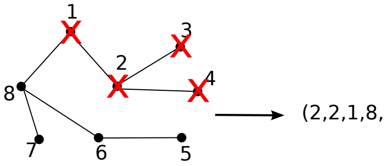

Definition:

A Prüfer sequence of length $n-2$, for $n \geq 2$, is any sequence of integers between 1 and n, with repetitions allowed.

A labeled tree gives a Prüfer sequence by:
Repeat $n-2$ times:

- Pick the leaf with the smallest label, call it v.
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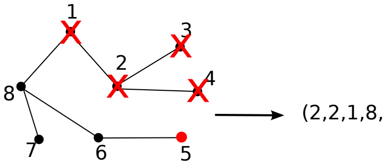

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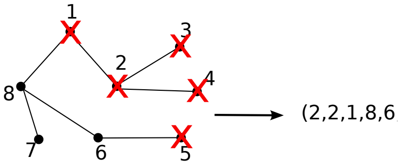

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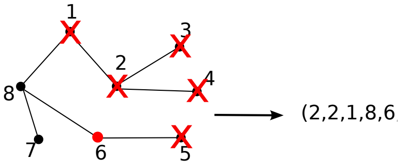

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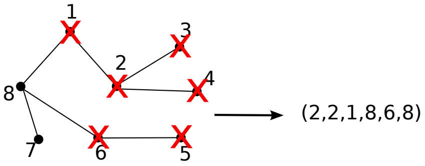

Prüfer Decoding

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A Prüfer sequence determines a labeled tree by:

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L=(1,2,3,4,5,6,7,8)

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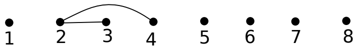
- Remove k from L and the first number in P.



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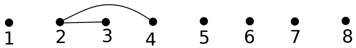

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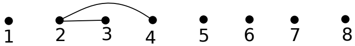

L=$(1,\boxed{2},\cancel{3},\cancel{4},5,6,7,8)$

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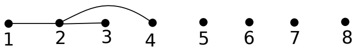

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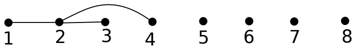

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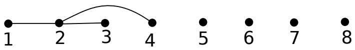

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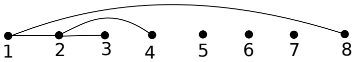

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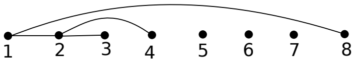

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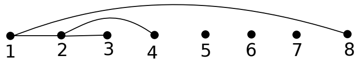

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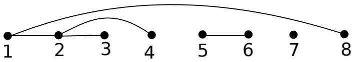

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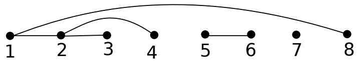

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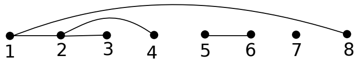

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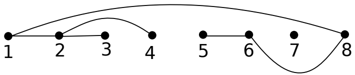

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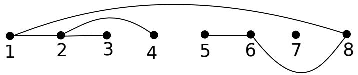
When this is completed there will be two numbers left in L, add the edge corresponding to these two numbers.



$L=(\cancel{1},\cancel{2},\cancel{3},\cancel{4},\cancel{5},\cancel{6},7,8)$

$P=(\cancel{2},\cancel{2},\cancel{1},\cancel{8},\cancel{6},\cancel{8})$

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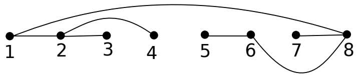
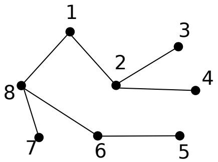
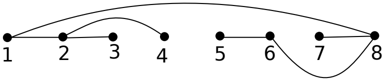


L=(~~1~~,~~2~~,~~3~~,~~4~~,~~5~~,~~6~~,7,8)

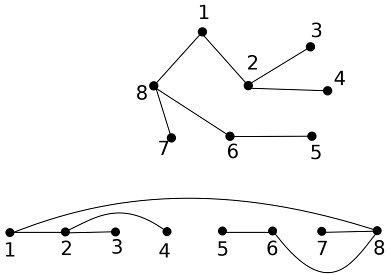
P=(~~2~~,~~2~~,~~1~~,~~8~~,~~6~~,~~8~~)

Prüfer coding and decoding





P=(2,2,1,8,6,8)



P=(2,2,1,8,6,8)

Prüfer coding and decoding are inverse operations, that means that there is a one-to-one correspondence between labeled trees with n vertices and Prüfer sequences of length $n-2$.

Definition:

A spanning tree T of a connected, undirected graph G is a tree composed of all the vertices and some (or perhaps all) of the edges of G.

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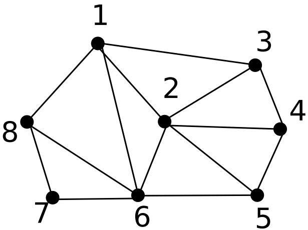

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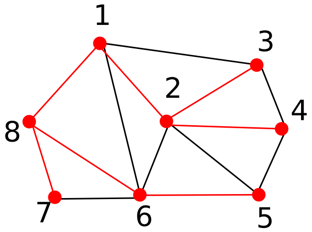
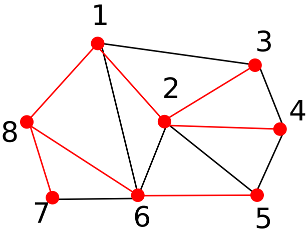


Definition:

A spanning tree T of a connected, undirected graph G is a tree composed of all the vertices and some (or perhaps all) of the edges of G.



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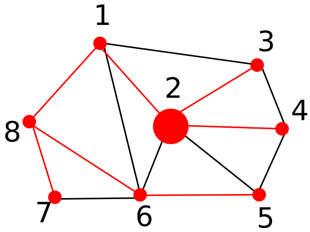

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A rooted tree is a tree with a vertex designated as root (it can be any vertex). A rooted tree introduces a parent - child relationship between the vertices and the notion of depth in the tree.

Theorem: Cayley's Formula

The number of spanning labeled trees of K_n is n^{n-2}, for $n \geq 2$.

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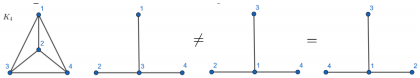

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The number of spanning trees of K_n is the same as the number of sequences with $(n-2)$ elements with repetitions from the set $\{1, 2, \ldots, n\}$. Thus each Prüfer sequence corresponds to exactly one spanning tree of K_n.

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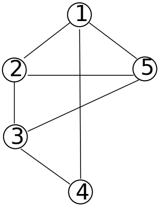

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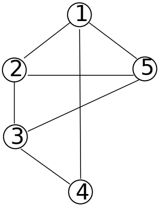

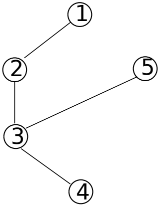

DFS

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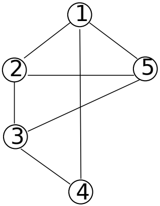

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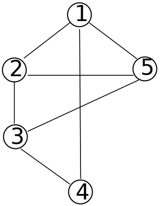

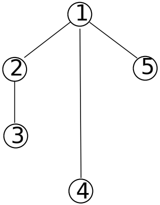

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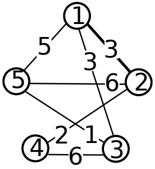

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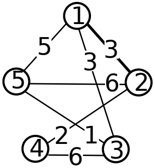

Graph G

$$w(G) = \sum_{e \in E} w(e) = 26.$$

Dijkstra's algorithm

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The shortest path from s to v is $v, p(v), p(p(v)), \dots, s$.

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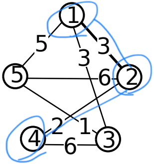
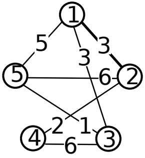


	1	2	3	4	5	S
1	0	∞	∞	∞	∞	1
2	x	3	3	∞	5	2
3	x	x	3	5	5	3
4	x	x	x	5	4	5
5	x	x	x	5	x	4

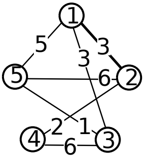
	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1
3	x	x	1	2	1
4	x	x	x	2	3
5	✓	✓	X	2	X

Dijkstra's algorithm



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	

	1	2	3	4	5
1	0	0	0	0	0



	1	2	3	4	5	S
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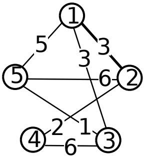
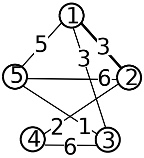


	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	

	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1



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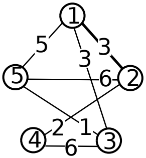

	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	2
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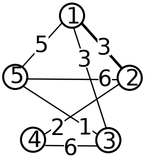

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1	0	oo	oo	oo	oo	1
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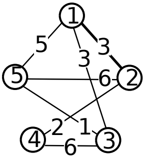

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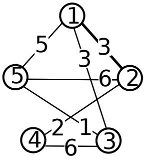

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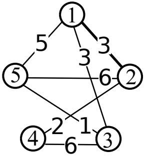

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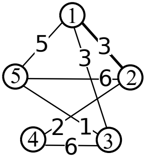

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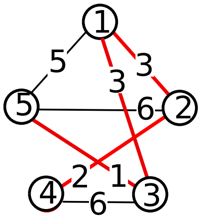

$v_3v_5, v_2v_4, v_1v_3, v_1v_2, v_1v_5, v_3v_4, v_3v_6$

$E_1 = (v_3v_5, v_2v_4, v_1v_3, v_1v_2)$

Prim algorithm

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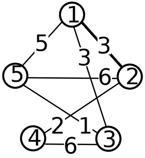

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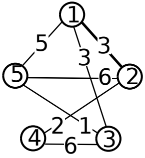
	1	2	3	4	5

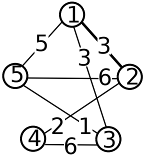
Prim algorithm



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	

	1	2	3	4	5
1	0	0	0	0	0

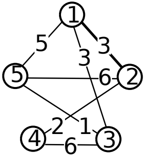
Prim algorithm



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1

	1	2	3	4	5
1	0	0	0	0	0

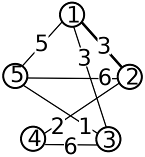
Prim algorithm



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	

	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1

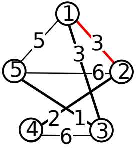
Prim algorithm



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	2

	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1

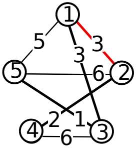
Prim algorithm



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	2
3	x	x	3	2	5	

	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1
3	x	x	1	2	1

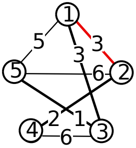
Prim algorithm



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	2
3	x	x	3	2	5	4

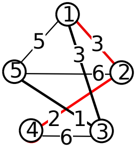
	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1
3	x	x	1	2	1

Prim algorithm



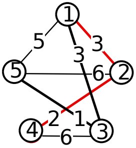
	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	2
3	x	x	3	2	5	4

	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1
3	x	x	1	2	1



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	2
3	x	x	3	2	5	4

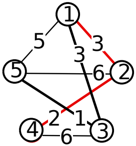
	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1
3	x	x	1	2	1



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	2
3	x	x	3	2	5	4
4	x	x	3	x	5	

	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1
3	x	x	1	2	1
4	x	x	1	x	1

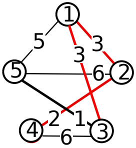
Prim algorithm



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	2
3	x	x	3	2	5	4
4	x	x	3	x	5	3

	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1
3	x	x	1	2	1
4	x	x	1	x	1

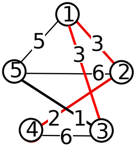
Prim algorithm



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	2
3	x	x	3	2	5	4
4	x	x	3	x	5	3

	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1
3	x	x	1	2	1
4	x	x	1	x	1

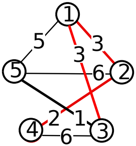
Prim algorithm



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	2
3	x	x	3	2	5	4
4	x	x	3	x	5	3
5	x	x	x	x	1	

	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1
3	x	x	1	2	1
4	x	x	1	x	1
5	x	x	x	x	3

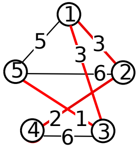
Prim algorithm



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	2
3	x	x	3	2	5	4
4	x	x	3	x	5	3
5	x	x	x	x	1	5

	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1
3	x	x	1	2	1
4	x	x	1	x	1
5	x	x	x	x	3

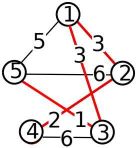
Prim algorithm



	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	2
3	x	x	3	2	5	4
4	x	x	3	x	5	3
5	x	x	x	x	1	5

	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1
3	x	x	1	2	1
4	x	x	1	x	1
5	x	x	x	x	3

Prim algorithm



$E_1=(v_3v_5,v_2v_4,v_1v_3,v_1v_2)$

	1	2	3	4	5	S
1	0	oo	oo	oo	oo	1
2	x	3	3	oo	5	2
3	x	x	3	2	5	4
4	x	x	3	x	5	3
5	x	x	x	x	1	5

	1	2	3	4	5
1	0	0	0	0	0
2	x	1	1	0	1
3	x	x	1	2	1
4	x	x	1	x	1
5	x	x	x	x	3