MFE409 Risk Management Homework 2

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Question 1

1.

Barclays Capital uses the historical simulation method with a two year unweighted historical period to measure the daily Value at Risk (DVaR). In 2008, the confidence level was changed to 95% from 98% as an increasing incidence of significant market movements made the existing measure more volatile and less effective for risk management purposes. Switching to 95% made DVaR more stable and consequently improved management, transparency and control of the market risk profile.
The historical simulation calculation can be split into three parts:
- Calculate hypothetical daily profit or loss for each position over the most recent two years, using observed daily market moves.
- Sum hypothetical profit or losses for day one, giving one total profit or loss. This is repeated for all other days in the two year history.
- DVaR is the 95th percentile selected from the two years of daily hypothetical total profit or loss.

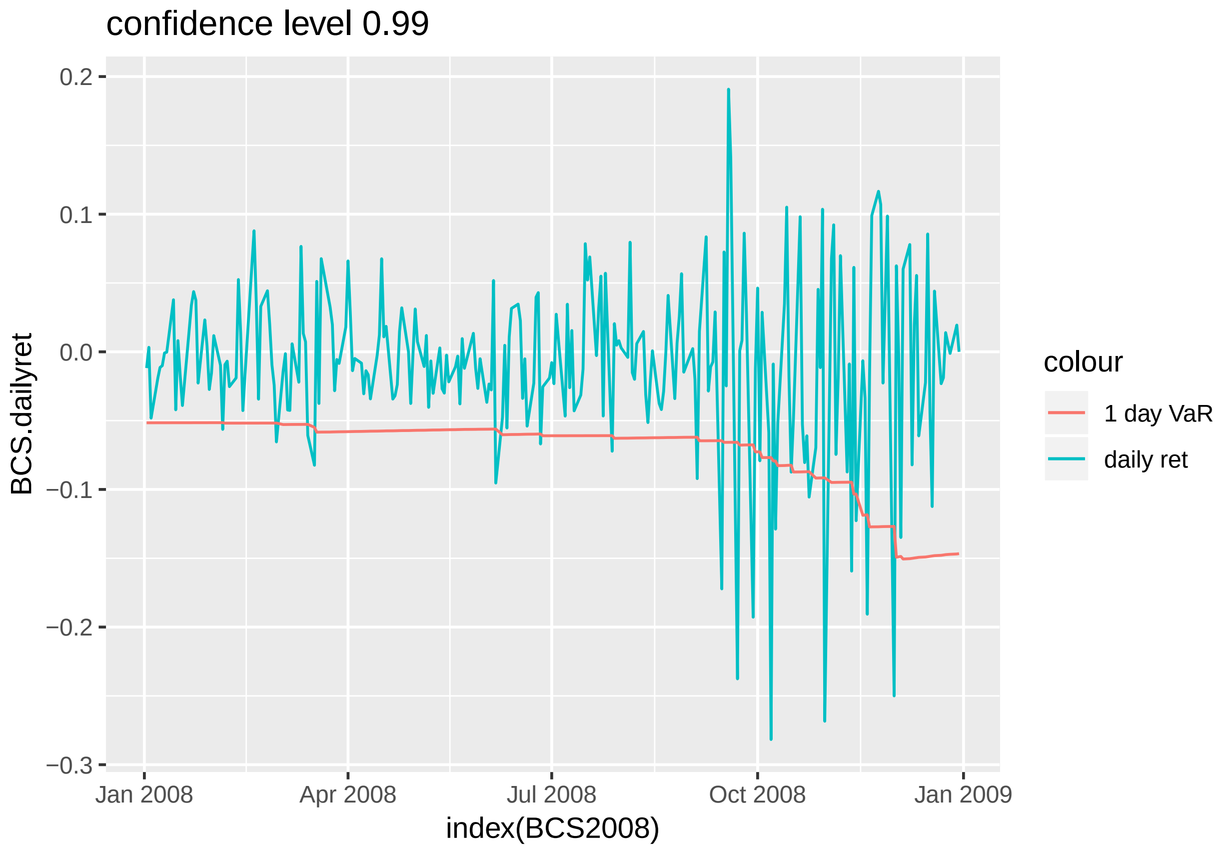
The FSA categorises a DVaR model as green, amber or red. A green model is consistent with a good working DVaR model and is achieved for models that have four or less back-testing exceptions in a 12-month period. For Barclays Capital's trading book, green model status was maintained for 2008 and 2007.

2.

```r
library(quantmod)
library(ggplot2)
symbol = c('GS', 'UBS', 'JPM', 'C', 'BCS', 'MS', 'DB', 'BAC', 'BNPQY', 'CS')
getSymbols(symbol,from = "2005-12-30", to = "2008-12-31", src="yahoo", auto.assign=T)
```

```
##  [1] "GS"    "UBS"   "JPM"   "C"     "BCS"   "MS"    "DB"    "BAC"
##  [9] "BNPQY" "CS"
```

```r
BCS$BCS.dailyret = diff(BCS$BCS.Close)/BCS$BCS.Close
BCS = na.omit(BCS)
BCS2008 = BCS[index(BCS) > '2008-01-01']
n = length(BCS2008$BCS.Close)
DVaRs = rep(0, n)
for (i in 1:n){
  temp = BCS[index(BCS) < index(BCS2008[i])]
  W0 = mean(temp$BCS.dailyret)
  VaR = quantile(temp$BCS.dailyret, 0.01)[[1]]
  DVaRs[i] = VaR
}
BCS2008$BCS.DVaR = DVaRs
ggplot(data = BCS2008, aes(x = index(BCS2008))) + geom_line(aes(y = BCS.dailyret, colour = 'daily ret'))
```



As we can see from the above figure, we can see a lot of exceptions after September 2008, where actual loss is greater than VaR. If we count them, we get

```
sum(BCS2008$BCS.dailyret < BCS2008$BCS.DVaR)
```

```
## [1] 21
```

number of exceptions. These great market movements caused DVaR to increase significantly overtime.

3.

For this portfolio, we use the return data from 2006 to 2008 to get the 1-day VaR at 2009.01.02, which is

```
VaR
```

```
##       1%
## 1.969284
```

million dollars.
DVaR and CVaR look like this:

```
library(knitr)
kable(out, caption = 'DVaR and CVaR in millions')
```

Table 1: DVaR and CVaR in millions

symbol	DVaR	CVaR
GS	0.126178830939239	0.12617895711807
UBS	0.157297251224264	0.31459465974578
JPM	0.100748339226797	0.100748439975136
C	0.134186168621042	0.268372471428253

symbol	DVaR	CVaR
BCS	-0.0600541012474309	-0.0600541613015322
MS	0.261127033551745	0.522254328230523
DB	0.147642151171468	0.147642298813619
BAC	0.0992622215534311	0.198524542369084
BNPQY	0.089570314276699	0.0895704038470133
CS	0.130726673042147	0.261453476810968

As we can see, the sum of CVaRs is

```
sum(CVaR)
```

```
## [1] 1.969285
```

which is equal to the VaR.

Question 2

1.

To derive the formula for the expected shortfall of normal distribution, we first derive the formula of conditional expectation of random variable x:

$$\frac{1}{\sqrt{2\pi\sigma^2}}\int_{-\infty}^{x_0-VaR} x\cdot\exp(-\frac{(x-\mu)^2}{2\sigma^2})dx$$

First, let

$$y=\frac{x-\mu}{\sigma}$$

Then

$$dy=\frac{dx}{\sigma}, x=\sigma y+\mu$$

Then we can transform the above equation into

$$\begin{aligned}
&\frac{1}{\sqrt{2\pi\sigma^2}}\int(\sigma y+\mu)\cdot\exp(-\frac{(\sigma y+\mu-\mu)^2}{2\sigma^2})\sigma dy\\
=&\frac{1}{\sqrt{2\pi\sigma^2}}\int(\sigma y+\mu)\exp(-\frac{y^2}{2})\sigma dy\\
=&\frac{1}{\sqrt{2\pi\sigma^2}}\int\sigma^2 y\exp(-\frac{y^2}{2})dy+\frac{1}{\sqrt{2\pi\sigma^2}}\int\sigma\mu\exp(-\frac{y^2}{2})dy\\
=&\mu\int_{-\infty}^{y_0-VaR_{(0,1)}}\frac{1}{\sqrt{2\pi}}\exp(-\frac{y^2}{2})dy+\sigma\int_{-\infty}^{y_0-VaR_{(0,1)}}\frac{1}{\sqrt{2\pi}}y\exp(-\frac{y^2}{2})dy
\end{aligned}$$

Since y is a standard normal distribution, we get $y_0=0, VaR_{(0,1)}=-z(c)$, then

$$\begin{aligned}
&\mu\int_{-\infty}^{y_0-VaR_{(0,1)}}\frac{1}{\sqrt{2\pi}}\exp(-\frac{y^2}{2})dy+\sigma\int_{-\infty}^{y_0-VaR_{(0,1)}}\frac{1}{\sqrt{2\pi}}y\exp(-\frac{y^2}{2})dy\\
=&\mu\cdot(1-c)+\sigma\cdot\frac{-1}{\sqrt{2\pi}}\cdot e^{\frac{-y^2}{2}}|_{\infty}^{z(c)}\\
=&\mu\cdot(1-c)+\sigma\cdot\frac{-1}{\sqrt{2\pi}}e^{-z(c)^2/2}
\end{aligned}$$

Then

$$ES = y_0 - \frac{\mu \cdot (1-c) + \sigma \cdot \frac{-1}{\sqrt{2\pi}} e^{-z(c)^2/2}}{1-c} = -\mu + \sigma \cdot \frac{1}{\sqrt{2\pi}(1-c)} e^{-z(c)^2/2}$$

2.

From definition, we know that $VaR_\alpha = W_0 - F^{-1}(1-\alpha)$, where F is any arbitrary cummulation distribution function. Then we can write

$$\begin{aligned} ES &= \frac{1}{1-c}\int_c^1 VaR_\alpha d\alpha \\ &= \frac{1}{1-c}\int_c^1 W_0 - F^{-1}(1-\alpha) d\alpha \\ &= W_0 - \frac{1}{1-c}\int_c^1 F^{-1}(1-\alpha) d\alpha \end{aligned}$$

Let $W = F^{-1}(1-\alpha)$, then by the theorem of derivation of an inverse function, we have

$$dW = -\frac{1}{F'(F^{-1}(1-\alpha))} d\alpha = -\frac{1}{f(F^{-1}(1-\alpha))} d\alpha$$

where f is the probability distribution function.
Then

$$d\alpha = -f(F^{-1}(1-\alpha))dW = -f(W)dW$$

Also, $F^{-1}(1-1) = -\infty$, and $F^{-1}(1-c) = W_0 - VaR$, there goes out new bounding values when we change variables. Hence, we can change our integral into:

$$\begin{aligned} &W_0 - \frac{1}{1-c}\int_c^1 F^{-1}(1-\alpha) d\alpha \\ =&W_0 - \frac{1}{1-c}\int_{W_0 - VaR}^{-\infty} -Wf(W)dW \\ =&W_0 - \frac{1}{1-c}\int_{-\infty}^{W_0 - VaR} Wf(W)dW \end{aligned}$$