ABtree: An Algorithm For Subgroup-Based Treatment Assignment

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Joint work with Derek Feng.

Obama Campaign 2008

https://blog.optimizely.com/2010/11/29/how-obama-raised-60-million-by-running-a-simple-experiment/

Obama Campaign 2008

Existing button:

https://blog.optimizely.com/2010/11/29/how-obama-raised-60-million-by-running-a-simple-experiment/

Obama Campaign 2008

Existing button:

Other buttons to consider:

https://blog.optimizely.com/2010/11/29/how-obama-raised-60-million-by-running-a-simple-experiment/

Obama Campaign 2008

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Traditional A/B Testing

Consider two options A and B (e.g. buttons, lines of text, pictures)

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- 2-sample t-test? 2-sample z-test?

Drawbacks of A/B Testing

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VisitId	OS	Date/Time	Referrer	Checkout
340	Windows XP	9/18 5:03pm	Google	1
341	Windows 8.1	9/18 5:04pm	Google	0
342	Windows XP	9/18 5:04pm	(Direct)	0
343	OS X Yosemite	9/18 5:06pm	Google	1
344	OS X Yosemite	9/18 5:06pm	Yahoo	0
...				

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Checkout
1
0
0
1
0
...

Drawbacks of A/B Testing

1. Ignores other information that we have on individual users.

Checkout
1
0
0
1
0
...

2. Misses potential *subgroup effects*, in which subgroups of the population benefit from one version whereas others benefit from another.

Solution?

A (poor) solution: Divide up the population into different *market segments*.

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Windows Segment:

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...		

Mac Segment:

VisitId	OS	Checkout
343	OS X Yosemite	1
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...		

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Mac Segment:

Checkout
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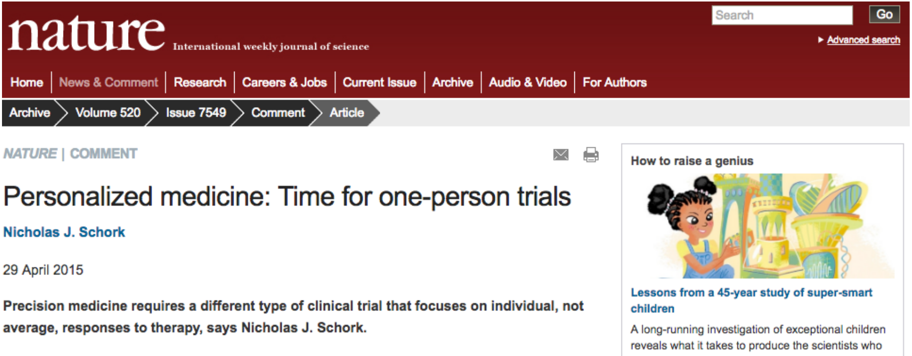
Better: **Automatically** detect *market segments* exhibiting differences in response to treatment.

Formal Question

- n individuals $i = 1, \ldots, n$
- k **treatments**
- individual i with covariates X_i gets randomly assigned to treatment T_i
- observe $Y_i | T_i$ (quantitative or binary)

Goal: For $i = 1, \ldots, n$, determine τ_i corresponding to the treatment which maximizes $E(Y_i | T_i = \tau_i)$.

Relationship to Precision Medicine



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NATURE | COMMENT

Personalized medicine: Time for one-person trials

Nicholas J. Schork

29 April 2015

Precision medicine requires a different type of clinical trial that focuses on individual, not average, responses to therapy, says Nicholas J. Schork.

How to raise a genius

Lessons from a 45-year study of super-smart children

A long-running investigation of exceptional children reveals what it takes to produce the scientists who

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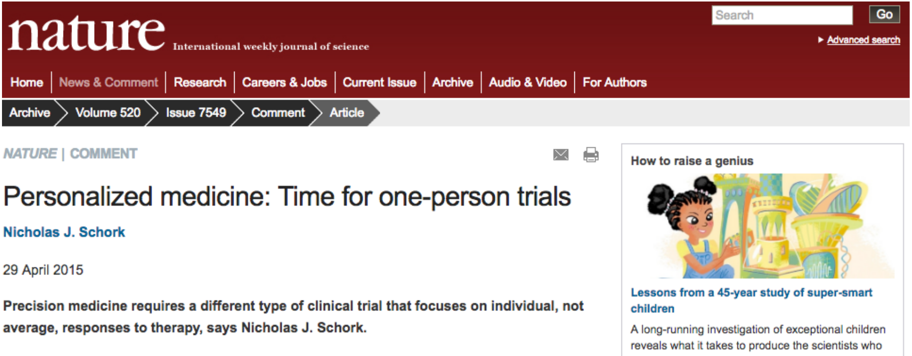
http://www.nature.com/news/personalized-medicine-time-for-one-person-trials-1.17411

Relationship to Precision Medicine



"For instance, the drug Gleevec (imatinib) was found to double survival rates of leukaemia patients with a chromosomal abnormality in their tumours called the Philadelphia translocation."

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Trees

Classification and Regression Trees (CART)

[1] `ptitanic` dataset in `library(rpart.plot)`

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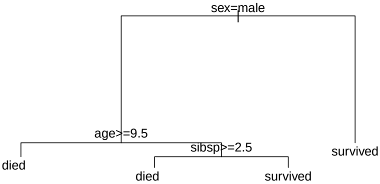
Titanic dataset[1]:



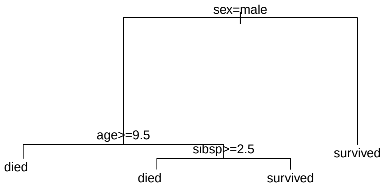
[1] ptitanic dataset in `library(rpart.plot)`

Classification and Regression Trees (CART)

Breiman et al. 1984:

- Used to model a categorical or quantitative Y using predictors X where the relationships are possibly *non-linear*
- Easy to interpret
- Fast computation
- Extends to random forests

Titanic dataset:



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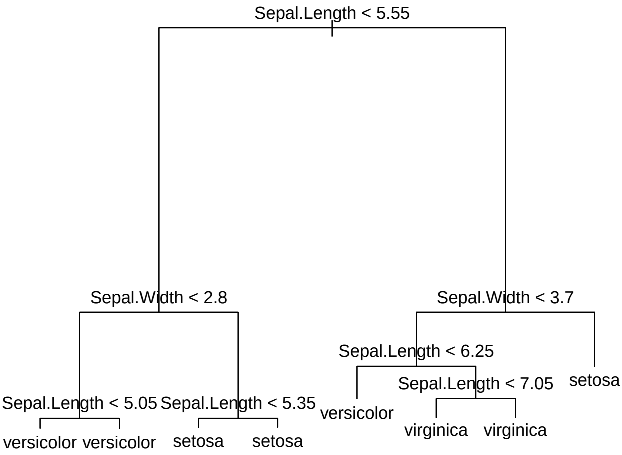

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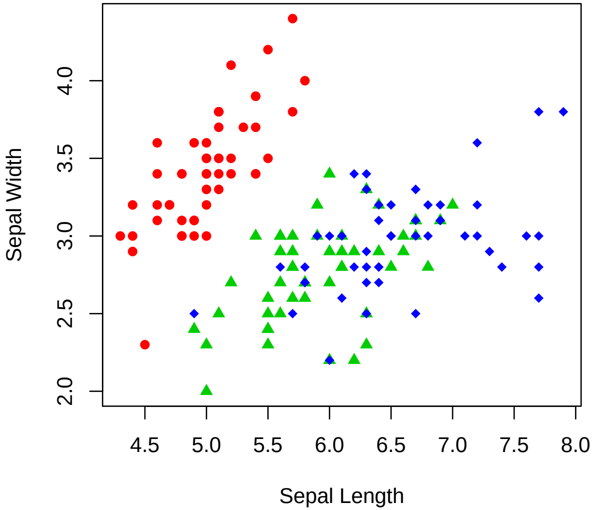

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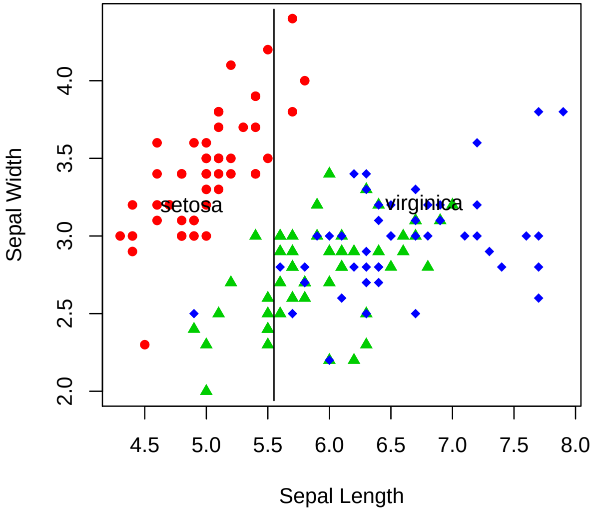

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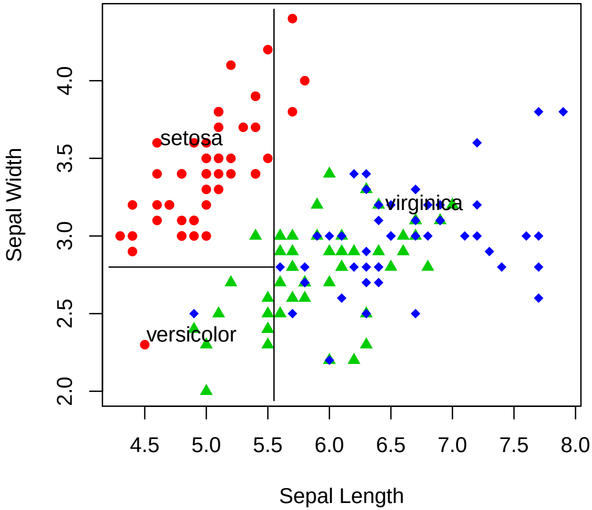

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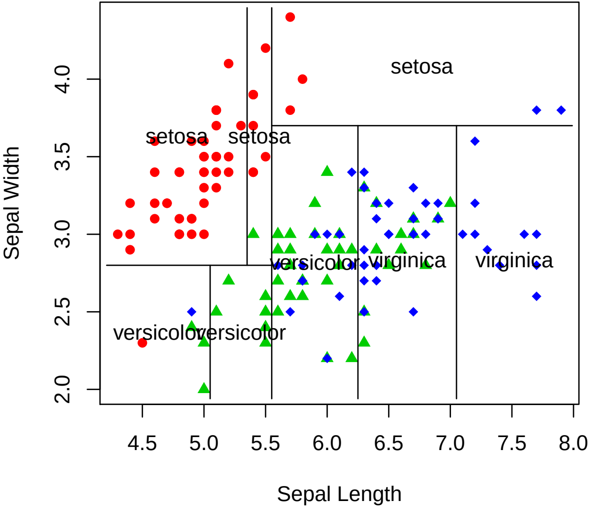

Related Work (Using Trees)

- Interaction Trees (Su et al. 2009) - continuous Y
  - Maximize significance of interaction between **treatment** and **splits**.

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  - Maximize significance of interaction between **treatment** and **splits**.
- Virtual Twins (Foster et al. 2011) - binary Y
  - Use a random forest to estimate treatment effect Z:

$$Z = P(Y = 1 | T = 1) - P(Y = 1 | T = 0)$$

  - Use CART to estimate Z for all individuals.
  - Use arbitrary cutoff c such that leafs with $Z > c$ are flagged as subgroups.

Related Work (ct'd)

- SIDES (Lipkovich et al. 2011)
  - Multiple trees
  - Splitting criterion is a function of Z, test statistic for H_0 : treatment effect $= 0$ in subgroup, e.g.

$$|Z_L - Z_R|$$

  - Each split results in **good** and **bad** subgroup.
  - Only continue splitting **good** nodes.
  - Controls for overall Type I error rate.

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  - Each split results in **good** and **bad** subgroup.
  - Only continue splitting **good** nodes.
  - Controls for overall Type I error rate.
  - Implemented in `SIDES` R package.
  - Numerous tuning parameters; slow.

Related Work (ct'd)

- QUINT (Dusseldorp & Van Mechelen 2013)
  - Splitting criterion is a weighted function of:
    - effect size
    - subgroup size
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Related Work (ct'd)

- QUINT (Dusseldorp & Van Mechelen 2013)
  - Splitting criterion is a weighted function of:
    - effect size
    - subgroup size
  - Requires the presence of subgroups where A beats B and subgroups where B beats A.
  - Implemented in `quint` R package. Buggy.

ABtree: Splitting

Tree growth procedure uses binary recursive partitioning into subgroups $S_1, S_2, \ldots$ to **maximize**:

$$\sum_{S_j} Q(S_j) \tag{1}$$

for some measure Q.

What should we use for Q?

ABtree: Idea #1

- Goal is **profit maximization**

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- Q should be a function of the *expected profit derived from the i-th individual conditional on receiving treatment t*

ABtree: Idea #1

An example:

$\bar{Y}_A$: 68
(n_A : 41)
$\bar{Y}_B$: 45
(n_B : 20)

- Total profit $= 68(41) + 45(20) = 3,688$.
- A is more profitable in this subgroup.
- Expect to gain $(68 - 45) \times 20 = 460$ in profit if everyone in this subgroup was assigned to A.

ABtree: Idea #1

An example:

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$\bar{Y}_A$: 68
(n_A : 41)
$\bar{Y}_B$: 45
(n_B : 20)

- Total profit $= 68(41) + 45(20) = 3,688$.
- A is more profitable in this subgroup. Assign treatment A.
- Expect to gain $(68 - 45) \times 20 = 460$ in profit if everyone in this subgroup was assigned to A.

Implied splitting criterion:

$$Q_{max}(S_j) := |S_j| \max_t \bar{y}_{j|t}$$

where S_j is the j-th subgroup and $\bar{y}_{j|t}$ is the average profit of individuals in this subgroup receiving treatment t.

ABtree: Idea #2

L_2 Maximization:

- Choose splits that maximize total squared distance between best average profit $\max_t \bar{y}_{j|t}$ and average profit of other treatments $\bar{y}_{j|t}$:

$$Q_{L_2} := |S_j| \sum_{t'} (\max_t \bar{y}_{j|t} - \bar{y}_{j|t'})^2.$$

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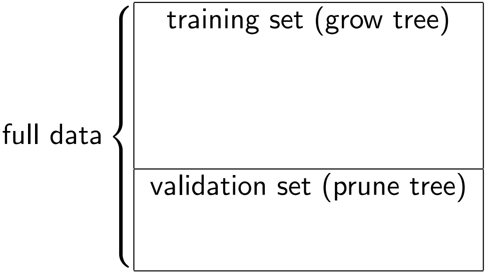
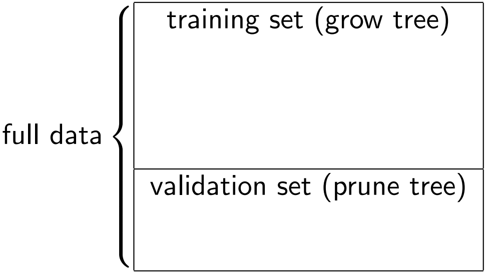
This measure outperforms Q_{max} in simulation settings.

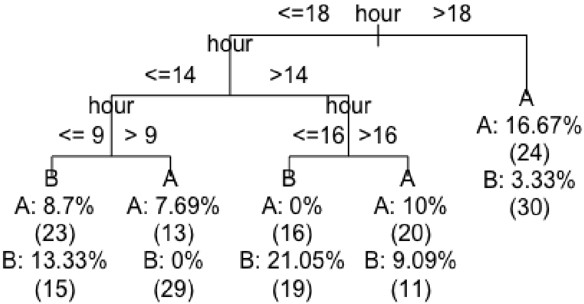
ABtree Pruning



- Pick subtree that does well



- Pick subtree that does well
- e.g. maximizes $\sum_{S_j} Q(S_j)$ in *validation set*



<=18 hour >18
hour
<=14
>14
hour
<= 9
> 9
hour
<=16
>16
B
A: 8.7%
(23)
B: 13.33%
(15)
A
A: 7.69%
(13)
B: 0%
(29)
B
A: 0%
(16)
B: 21.05%
(19)
A
A: 10%
(20)
B: 9.09%
(11)
A
A: 16.67%
(24)
B: 3.33%
(30)

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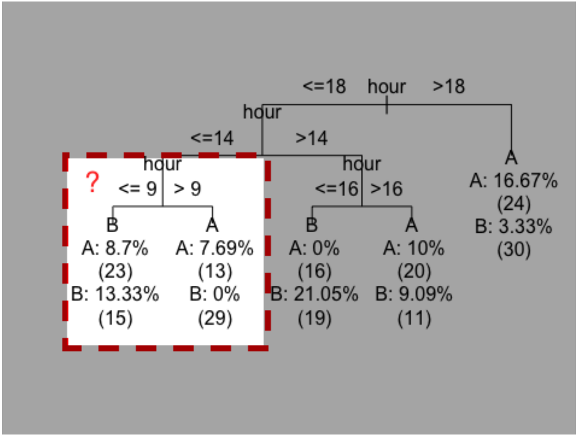

$$\sum Q_{L_2}(S_j) = (0.13 - 0.08)^2(23 + 15) + \cdots + (0.17 - 0.03)^2(24 + 30)$$
$$= 2.84$$

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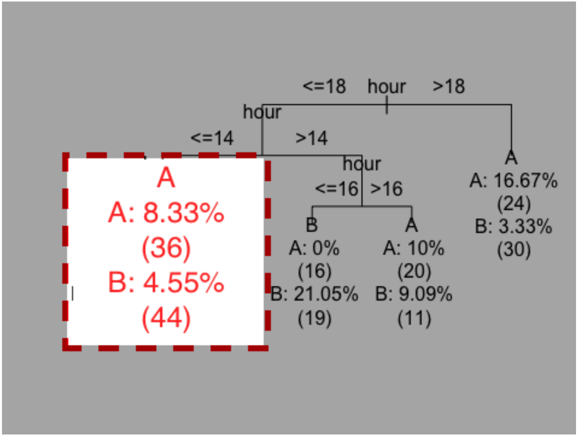
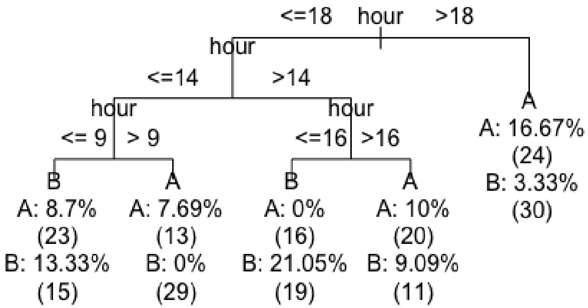


$$\sum Q_{L_2}(S_j) = (0.08 - 0.05)^2(36 + 44) + \cdots + (0.17 - 0.03)^2(24 + 30)$$
$$= 2.63$$

ABtree Pruning



Since $2.84 > 2.63$, we do not prune this branch!

Example: National Supported Work Study (LaLonde, 1986)

- 1970's large-scale national and private program designed to provide work experience for $n = 722$ disadvantaged workers
- Treatments: A (control) and B (receiving benefits to improve employability)
- Sample sizes: $n_A = 425$ and $n_B = 297$
- Y (binary): whether individual earnings increased after the completion of experiment

Example: National Supported Work Study (LaLonde, 1986)

Covariates X:

Name	Description	Type
age	age (yrs)	quantitative
educ	education (yrs)	quantitative
race	(black, hispanic, white)	categorical
marr	married flag	categorical
nodegr	no degree flag	categorical
log.income75	log income in 1975	quantitative
u75	unemployment flag in 1975	categorical

Example: National Supported Work Study (LaLonde, 1986)

Assessment:

training set $n = 500$ (grow tree)
validation set $n = 150$ (prune tree)
test set $n = 72$ (assessment)

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$y = 1$	d	e
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Tree works well if:

$$\frac{d}{d+f} > \frac{e}{e+g}$$

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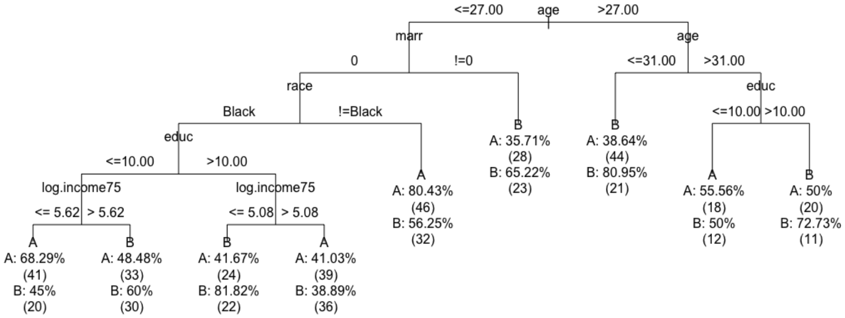

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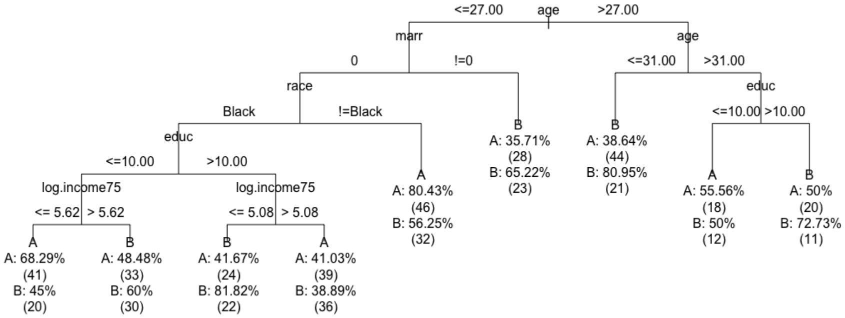

	match	no match
$y = 1$	28	21
$y = 0$	10	13
$\%success$	73.7%	61.8%

Summary

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Thank You!

References

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