Astronomy and Astrophysics Assignment - 1

Swanith Upadhye

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1 Question 1 - Gaia Telescope Range

Calculation of Distance Limit

Gaia can measure parallax with an accuracy of 0.5 milliarcseconds (mas). Converting milliarcseconds to arcseconds:

$$0.5\,\text{mas} = 0.0005\,\text{arcseconds}$$

The distance d in parsecs is given by:

$$d = \frac{1}{\theta}$$

where θ is the parallax in arcseconds. For a parallax of 0.0005 arcseconds:

$$d = \frac{1}{0.0005} = 2000\,\text{parsecs}$$

Volume Calculation

The volume V of a sphere with radius d (in parsecs) is:

$$V = \frac{4}{3}\pi d^3$$

Substituting $d = 2000\,\text{pc}$:

$$V = \frac{4}{3}\pi(2000)^3 \approx 33.51 \times 10^9\,\text{pc}^3$$

Number of Stars Gaia Can Measure

Given a mean stellar number density of 0.1 stars per cubic parsec, the number of stars N is:

$$N = \text{density} \times \text{volume}$$

$$N = 0.1 \times 33.51 \times 10^9 \approx 3.351 \times 10^9$$

Thus, approximately 3.35 billion stars can have their distances measured by Gaia within this volume.

Comparison with Gaia DR3

Gaia DR3 reports data for 1.46 billion stars. This is consistent with our estimate of around 3.35 billion stars, modulo the practical limits of data collection, observational constraints, and most importantly the actual distribution of stars.

2 Question 2 - HCT and VBO

Best time to observe object of interest at HCT

The RA of the object of interest is $\alpha_{OI} = 17^h 45^{min} 40^s$. We want sun to be directly opposite or 12^h apart. Thus RA of sun is required to be $\alpha_{sun} = 5^h 45^m 40^s$.
The Sun is at $\gamma-$vernal equinox on 21st March. And heach month adds 2^h to its RA. So sun will have RA $= 4^h$ on 21st May, and the rest:-

$$1^h 45^m 40^s = 1 + 3/4 + 40/3600h = 1.761^h -> 1.761x30/2 \approxeq 26.415 days$$

Thus the observation time is $\approxeq 16^{th}$ June.This will be around midnight local time at HCT.

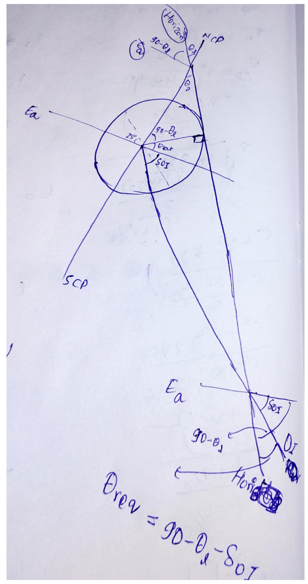

Figure 1: Angles at HCT

Altitude above the horizon

The figure shows that the angle between object of interest(OI) with declination $\delta_{OI} = -29°00'28''$ and the southern horizon is:-

$$altitude_{OI} = 90° - \delta_{oi} - \theta_{latitude} \approxeq 90 - (29 + 40/3600) - (32 + 46/60 + 46/3600) \approxeq 28.21°$$

Hence at midnight object of interest is going to be$28°$ **above the sothern horizon at HCT**.

Acrux from HCT

From the above formula, the altitude at horizon for Acrux at HCT at its suitable time of year, will be:

$$altitude_{Acrux} = 90° - \delta_{Acrux} - \theta_{latitude} = 90 - (63 + 5/60 + 56.7/3600) - (32 + 46/60 + 46/3600)$$

Thus at HCT - Acrux lies $5.88°$ **below its horizon, and hence is invisible.**

Acrux from VBO

The latitude at VBO 12.58° North. From the above formula, the altitude at horizon for Acrux at VBO at its suitable time of year, will be:

$$altitude_{Acrux} = 90° - \delta_{Acrux} - \theta_{latitude} = 90 - (63 + 5/60 + 56.7/3600) - (12.58) \approxeq = 14.32°$$

Thus at BO - Acrux lies $14.32°$ **above its horizon, and hence is visible.**

3 Question 3 - Jupiter Temperature

(a) Reflected light

Consider the case of Jupiter, with radius $R_J = 7.1 \times 10^9$ cm and mean orbital radius $a_J = 7.8 \times 10^{13}$ cm. Given that Jupiter reflects 10% of the light coming from the Sun.

The incident solar flux on Jupiter:

$$F_{\text{incident}} = \frac{L_\odot}{4\pi a_J^2}$$

Where: - $L_\odot = 3.828 \times 10^{33}$ erg/s is the Sun's luminosity, - $a_J = 7.8 \times 10^{13}$ cm is the distance between Jupiter and the Sun.

Substituting values:

$$F_{\text{incident}} = \frac{3.828 \times 10^{33}}{4\pi(7.8 \times 10^{13})^2}$$

$$F_{\text{incident}} \approx \frac{3.828 \times 10^{33}}{7.63 \times 10^{28}} = 4.3 \times 10^4 \text{ erg/cm}^2\text{/s}$$

Next, the total power received by Jupiter is the solar flux times the cross-sectional area of Jupiter:

$$L_{\text{received}} = F_{\text{incident}} \times \pi R_J^2$$

$$L_{\text{received}} = 4.3 \times 10^4 \times \pi \times (7.1 \times 10^9)^2 \approxeq 6.8 \times 10^{24} \text{erg/s}$$

Since Jupiter reflects 10% of the received energy:

$$L_{\text{reflected}} = 0.1 \times L_{\text{received}} = 0.1 \times 6.8 \times 10^{24} = 6.8 \times 10^{23}\,\text{erg/s}$$

Thus, the reflected luminosity of Jupiter is approximately $6.8{\times}10^{23}$ erg/s.

Now, we calculate the peak wavelength of the reflected light using Wien's Law. Since the Sun's temperature is approximately 5800 K(same wavelength upon reflection from jupiter):

$$\lambda_{\text{reflected}} = \frac{b}{T_\odot}$$

Where: - $b = 2.897 \times 10^{-1}\,\text{cm K}$ is Wien's constant, - $T_\odot = 5800\,\text{K}$.

Substitute the values:

$$\lambda_{\max} = \frac{2.897 \times 10^{-1}}{5800} \approx 4.99 \times 10^{-5}\,\text{cm} = 4990\,\text{Å}$$

The reflected light peaks at $4990\,\text{Å}$, which is in the visible spectrum (blue-green).

(b) Re-emitted Light

For the re-emitted radiation, we calculate the temperature of Jupiter assuming it absorbs and re-emits the remaining 90% of the incident solar flux.

The Re-emitted luminosity is:

$$L_{\text{reemitted}} = 0.90 \times L_{\text{recieved}} = 0.90 \times 6.8 \times 10^{24} \approxeq 6.1 \times 10^{24} erg/s$$

From Stefan-Boltzmann Law:

$$L_{\text{emitted}} = \sigma A T_J^4$$

where, $\sigma = 5.77 \times 10^{-5} ergcm^{-2}s^{-1}K^{-4}$, $A = 4\pi R_J^2 = 6.3 \times 10^{20}\text{cm}^2$

The effective temperature of Jupiter is:-

$$T_J = [\frac{6.1 \times 10^{24}}{5.8 \times 10^{-5} \times 6.3 \times 10^{20}}]^{1/4} \approxeq 113.7K$$

Using Wien's Law again to find the peak wavelength of the re-emitted radiation:

$$\lambda_{\max} = \frac{b}{T_J} = \frac{2.897 \times 10^{-1}}{113.7}$$
$$\lambda_{\max} \approx 0.00254\,\text{cm} = 25.4\,\mu\text{m}$$

Thus, the re-emitted radiation peaks at $25.4\,\mu$m, which lies in the far-infrared region.

4 Question -4 Solar Radiation

Solar Luminosity and Flux

The Bolometric Luminosity assuming Black body behavior is given by Stefan Boltzmann Law:

$$L_\odot = \sigma A T_\odot^4 = 5.77 \times 10^{-5} \times 4\pi \times (R_\odot = 6.96 \times 10^{10})^2 \times 5772^4 \approxeq 3.9 \times 10^{33}\text{erg/s}$$

The flux is $L_\odot$ per unit (as asked -solar) surface area:

$$F_\odot = \frac{L_\odot}{4\pi \times R_\odot^2} = \frac{3.9 \times 10^{33}}{4\pi(6.96 \times 10^{10})^2} \approxeq 6.41 \times 10^{10}\text{erg}/cm^2/s$$

The specific Intensity on solar surface is given by;

$$I_{\nu\odot} = F_\odot/4\pi \approxeq 5.1 \times 10^9\, erg/(cm^2\, s\, Sr)$$

The specific intensity in forward direction:

$$I_{\nu\odot}\big|_{forward} = F_\odot/\pi \approxeq 2.04 \times 10^10\, erg/(cm^2\, s\, Sr)$$

Solar Constants on Earth, Mars and Venus

On earth, Flux recieved:

$$F_{earth} = \frac{L_\odot}{4\pi(AU)^2} = \frac{3.9 \times 10^{33}}{4\pi.(1.5 \times 10^{13})^2} \approxeq 1.38 \times 10^6\, erg/(cm^2\, s)$$

(Solar Constant) On Earth intensity:

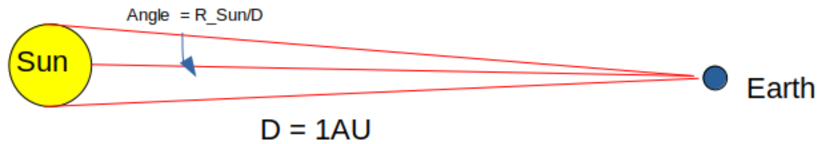

Figure 2: Solar intensity

$$I_{Earth} = \frac{F_E}{\int_0^{R_S/D} cos(\theta) d\Omega} \approx \frac{F_E}{\pi (R_S/D)^2}$$

(for $R_S << D$)

$$\Rightarrow I_E = \frac{1.38 \times 10^6}{\pi \times (\frac{6.96\times10^{10}}{1.5\times10^{13}})^2} \approxeq 2.04 \times 10^{10}\, erg/(\,cm^2\, s\, Sr)$$

(which is same as$I_{\nu\odot}|_{forward}$)

Flux at Venus:

$$F_V = \frac{L_\odot}{4\pi(0.72 \times AU)^2} = \frac{3.9 \times 10^{33}}{4\pi.(0.72 \times 1.5 \times 10^{13})^2} \approxeq 2.7\times10^6\, erg/(cm^2\, s) \approxeq 1.93\times F_E$$

Flux at Mars:

$$F_{Mars} = \frac{L_\odot}{4\pi(1.52 \times AU)^2} = \frac{3.9 \times 10^{33}}{4\pi.(1.52 \times 1.5 \times 10^{13})^2} \approxeq 6.0\times10^5\, erg/(cm^2\, s) \approxeq 0.43\times F_E$$

Intensity at Venus (Diameter of Venus(R$_V$ $\approxeq 1.2 \times 10^9 cm$):

$$I_{Venus} = \frac{F_V}{\int_0^{R_S/D_V} cos(\theta) d\Omega} \approx \frac{F_V}{\pi (R_S/D_V)^2}$$

(for $R_S << D$)

$$\Rightarrow I_V = \frac{2.7 \times 10^6}{\pi \times (\frac{6.96\times10^{10}}{0.72\times1.5\times10^{13}})^2} \approxeq 2.07 \times 10^{10}\, erg/(\,cm^2\, Sr)$$

(same as $I_E = I_{\nu\odot}$) Intensity at Mars (Diameter of Mars(R_D) $\approxeq 6.8 \times 10^8 cm$)

$$I_{Mars} = \frac{F_M}{\int_0^{R_S/D} cos(\theta) d\Omega} \approx \frac{F_M}{\pi (R_S/D)^2}$$

(for $R_S << D$)

$$\Rightarrow I_M = \frac{6.0 \times 10^5}{\pi \times (\frac{6.96\times10^{10}}{1.52\times1.5\times10^{13}})^2} \approx 2.05 \times 10^{10}\, erg/(\,cm^2\, Sr) \approx I_E\, or I_{\nu\odot}$$

Effect of Sun's Radius

The Flux and Specific intensity on Solar surface will not change. The Bolometric Luminosity on sun's surface increases by a factor of 100.
The Flux on the respective planets increases by the factor of 100 assuming the change in radius doesn't change planet-Sun distances.
The intensities on the planets remain identical to the unchanged radius cases.

5 Question 5 - Photon Mean free path

Derivation of Mean free path

Known Beer-Lamberts Law

$$\frac{dI_\nu}{ds} = -\alpha_\nu I_\nu$$

which is the rate of decrease of intensity of radiation with distance 's' traveled inside an absorbing medium of absorption coefficient($Length^{-1}$).

$$\alpha_n u := \kappa_\nu \rho$$

α_ν can be written as a product of an area and density to give the correct dimensions. Area is κ_ν and a number density is ρ. Since, the only characteristic number density can be of that of the absorbent particles, implies that the associated characteristic area becomes the absorption cross section. The area within which, if photon sees an absorbent, then it is absorbed.

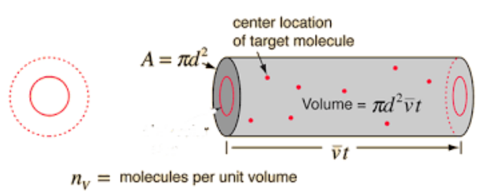

Figure 3: Mean free path (molecules | absorbent)

If photon travels a distance ct in a cylinder of cross section area $\kappa_n u$ then average collisons suffered $= vt \times A \times \kappa_n u\rho$. Thus, mean free path l =

$$l = \frac{vt}{(avg\, number\, of\, collisions)} = \frac{vt}{A \times \times k_\nu \rho vt} = 1/\kappa_\nu \rho$$

Mean free path calculation

Given $\kappa_\nu = 5.1 \times 10^{-27} cm^2$, and $\rho = 2.5 \times 10^{19} cm^{-3}$

$$l = \frac{1}{\kappa_\nu \rho} = \frac{1}{5.1 \times 2.5 \times 10^{-8}} \approxeq 7.8 \times 10^6 cm$$

Thomas scattering mean free path

Given, $\kappa_\nu = 6.65 \times 10^{-25} cm^2$ and ρ twice as above upon dissociation into ions (for O_2 and N_2)

$$l\Big|_{Thomas} = \frac{1}{6.65 \times 2.5 \times 10^{-6}} \approxeq 6.0 \times 10^4 cm$$

Mean free path in Sun

Assuming Thomas Scattering from H^+ ions, $\kappa_\nu = 6.65 \times 10^{-25} cm^2$ and
$\rho = \frac{mass\, density}{mass\, of\, single\, H^+\, ion} = \frac{1400}{1.6 \times 10^{-27}} \approxeq 8.75 \times 10^{29} m^{-3} = 8.75 \times 10^{23} cm^{-3}$

$$l\Big|_S = \frac{1}{8.75 \times 6.65 \times 10^{-2}} \approxeq 1.7 cm$$

6 Acknowledgements

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