

# ASSIGNMENT 3

## INTRO TO COSMOLOGY

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Deadline: → 14th April 25 -

### Ques 1 Appendix

$$\therefore T_{\mu\nu}^{\alpha} = \frac{\partial x^{\alpha}}{\partial x'^{\lambda}} \frac{\partial^2 x'^{\lambda}}{\partial x^{\mu} \partial x^{\nu}}$$

Coordinate transformation

$$T_{\tilde{m}\tilde{n}}^{\tilde{\alpha}} = \frac{\partial \tilde{x}^{\tilde{\alpha}}}{\partial x'^{\lambda}} \frac{\partial^2 x'^{\lambda}}{\partial \tilde{x}^{\tilde{m}} \partial \tilde{x}^{\tilde{n}}}$$

$$= \frac{\partial x^{\alpha}}{\partial x'^{\lambda}} \frac{\partial \tilde{x}^{\tilde{\alpha}}}{\partial x^{\alpha}} \frac{\partial x^{\nu}}{\partial \tilde{x}^{\tilde{n}}} \frac{\partial}{\partial x^{\nu}} \left( \frac{\partial x'^{\lambda}}{\partial x^{\mu}} \frac{\partial x^{\mu}}{\partial \tilde{x}^{\tilde{m}}} \right)$$

$$= \frac{\partial x^{\alpha}}{\partial x'^{\lambda}} \frac{\partial \tilde{x}^{\tilde{\alpha}}}{\partial x^{\alpha}} \frac{\partial x^{\nu}}{\partial \tilde{x}^{\tilde{n}}} \left( \frac{\partial^2 x'^{\lambda}}{\partial x^{\mu} \partial x^{\nu}} \frac{\partial x^{\mu}}{\partial \tilde{x}^{\tilde{m}}} + \frac{\partial x'^{\lambda}}{\partial x^{\mu}} \frac{\partial^2 x^{\mu}}{\partial x^{\nu} \partial \tilde{x}^{\tilde{m}}} \right)$$

$$= \frac{\partial \tilde{x}^\alpha}{\partial x^\alpha} \frac{\partial x^\nu}{\partial \tilde{x}^\nu} \frac{\partial x^\mu}{\partial \tilde{x}^\mu} \Gamma_{\mu\nu}^\alpha$$

$$+ \delta_{\mu}^\alpha \frac{\partial \tilde{x}^\alpha}{\partial x^\alpha} \frac{\partial x^\nu}{\partial \tilde{x}^\nu} \frac{\partial^2 x^\mu}{\partial \tilde{x}^\nu \partial \tilde{x}^\mu}$$

Second term

$$\frac{\partial \tilde{x}^\alpha}{\partial x^\mu} \frac{\partial^2 x^\mu}{\partial \tilde{x}^\nu \partial \tilde{x}^\mu}$$

$$\therefore \Gamma_{\tilde{\mu}\tilde{\nu}}^{\tilde{\alpha}} \Rightarrow \frac{\partial \tilde{x}^\alpha}{\partial x^\alpha} \frac{\partial x^\nu}{\partial \tilde{x}^\nu} \frac{\partial x^\mu}{\partial \tilde{x}^\mu} \Gamma_{\mu\nu}^\alpha$$

$$+ \frac{\partial \tilde{x}^\alpha}{\partial x^\mu} \frac{\partial^2 x^\mu}{\partial \tilde{x}^\nu \partial \tilde{x}^\mu}$$

→ Not a tensor transformation.