Hedonic Diversity Games

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joint work with

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At a university far, far away…

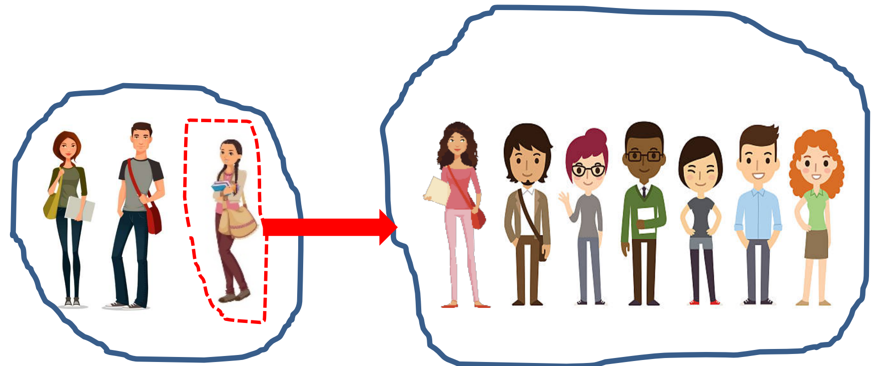
- 20 visiting students (), 80 local students ()
- Students need to split into study groups of size between 1 and 6
  - Claire (Vis): I want to practice my English, so I want to be in a group with no French students
  - Nicolas (Vis): my English is not great, I want to be in an evenly mixed group
  - Andrew (Loc): I will not learn anything in a mixed group…
  - Jen (Loc): I want to meet new people
- Can we split students into groups so that this partition is stable?

Formal model

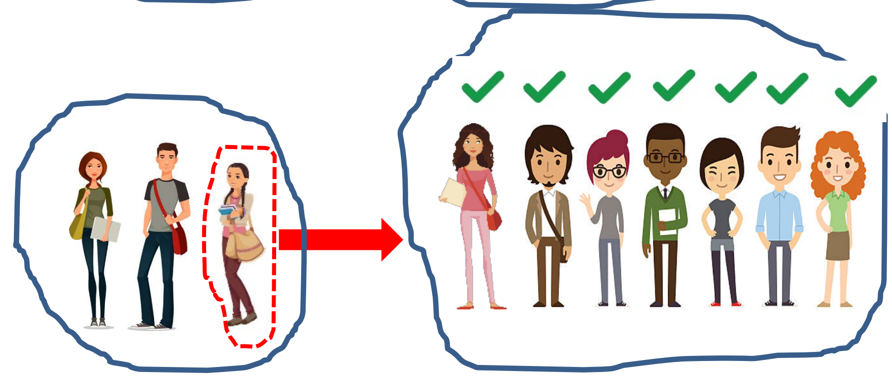
- A set of players N, $|N| = n$
- Each player is either red or blue ($N = R \cup B$)
- Outcome: partition of N into coalitions
- Preferences: each player has a preference over the fraction of red players in her group
  - (1R, 2B) ~ (2R, 4B) ~ (5R, 10B)
  - succinct: preferences are defined on $\Theta = \{i/j : i, j \leq n\}$
- Special case: single-peaked preferences
  - each player i has a preferred ratio θ_i
  - if $\theta < \theta' \leq \theta_i$ or $\theta_i < \theta' \leq \theta$, player i prefers θ' to θ

Notions of stability

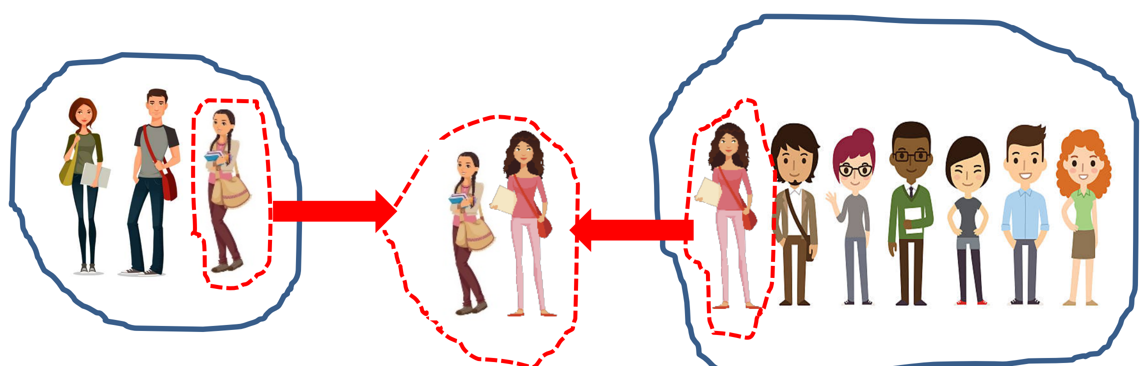
- Nash stability:
  no agent wants to move to another group



- Individual stability:
  no agent wants to move to another group that accepts her



- Core stability:
  no group wants to move



Complexity

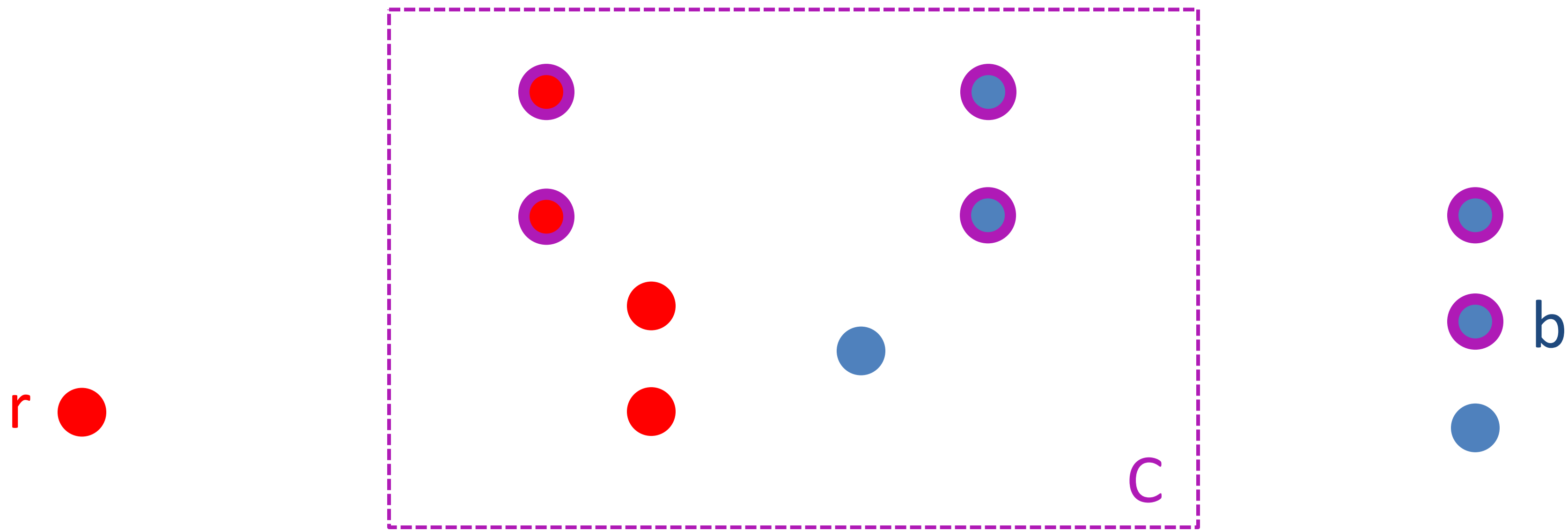
- Nash stable outcomes:
  - may fail to exist [BrEl'19]
  - can be NP-hard to find [BoE'20]
- Individually stable outcomes:
  - always exist, can be found in polynomial time
    - [BrEl'19] for single-peaked preferences
    - [BoE'20] for general preferences
- Core stable outcomes:
  - may fail to exist
  - can be NP-hard to find [BrEl'19]

Individual stability: an algorithm

<u>Definition</u>: i is *nice* if mixed pair $\geq_i$ {i}

1. Form a max balanced coalition of nice players: C
2. Allow IS deviations to C
3. Output C + remaining singletons

Individual stability: proof (1/3)

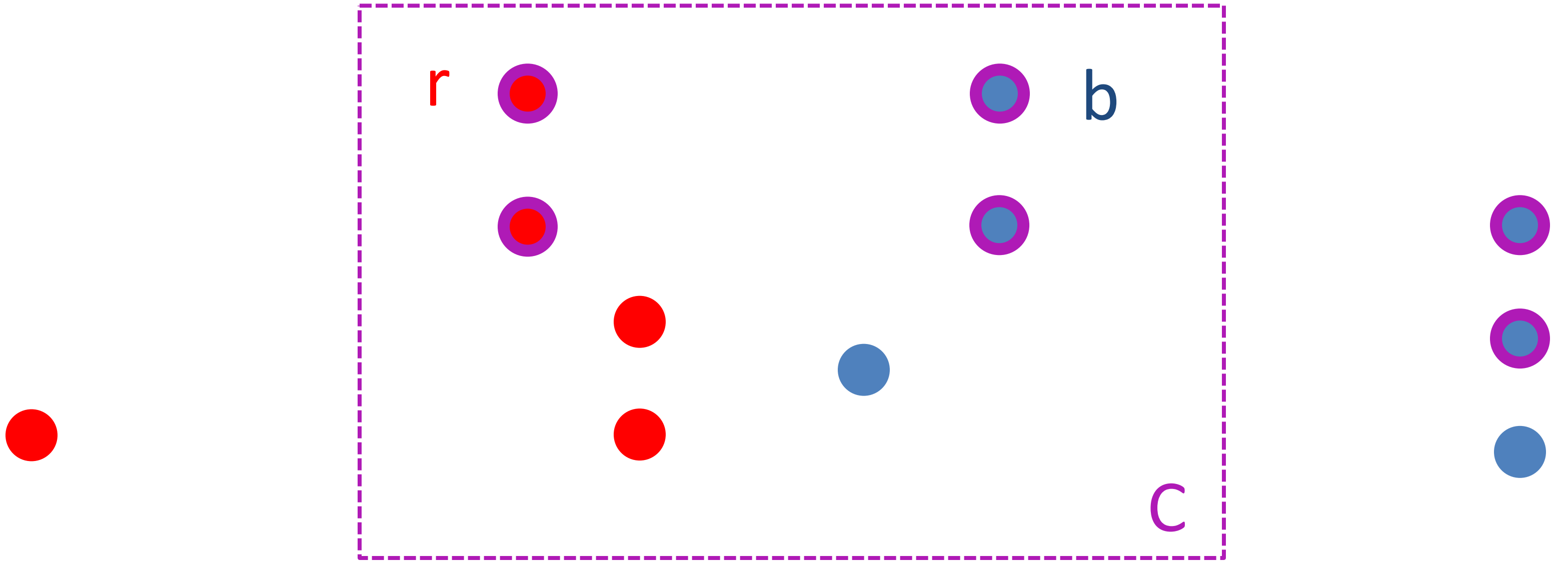


Proof of stability: singletons

- r and b have no IS deviations to C
- r does not want to join b
- b is not allowed to join r

because r is not nice

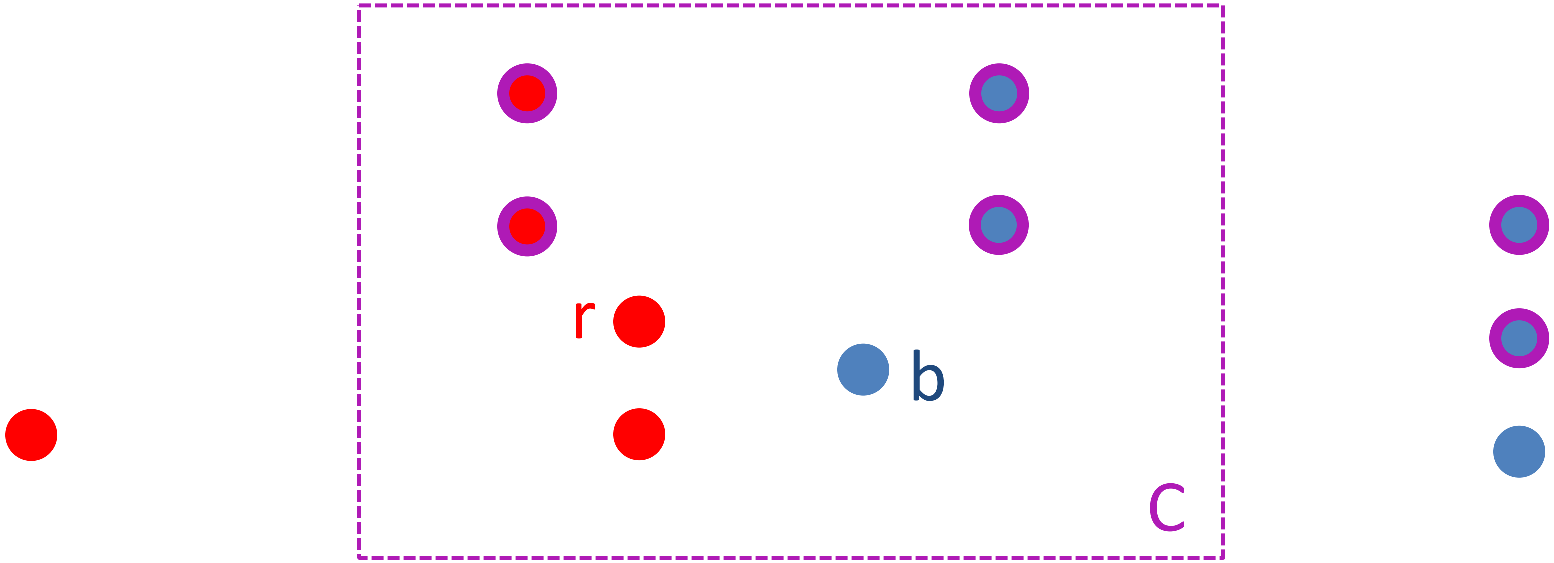
Individual stability: proof (2/3)



Proof of stability: nice players in C

- r can deviate to {r} or to a mixed pair
- r weakly prefers a mixed pair to {r}
- r approved changes to C,
  so weakly prefers C to a mixed pair

Individual stability: proof (3/3)

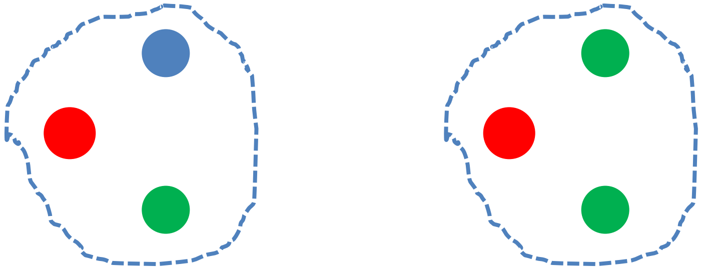


Proof of stability: non-nice players in C

– when r joined, she preferred C to {r}
– r approved all changes to C, so weakly prefers it to {r}
– r prefers {r} to a mixed pair

Beyond two types

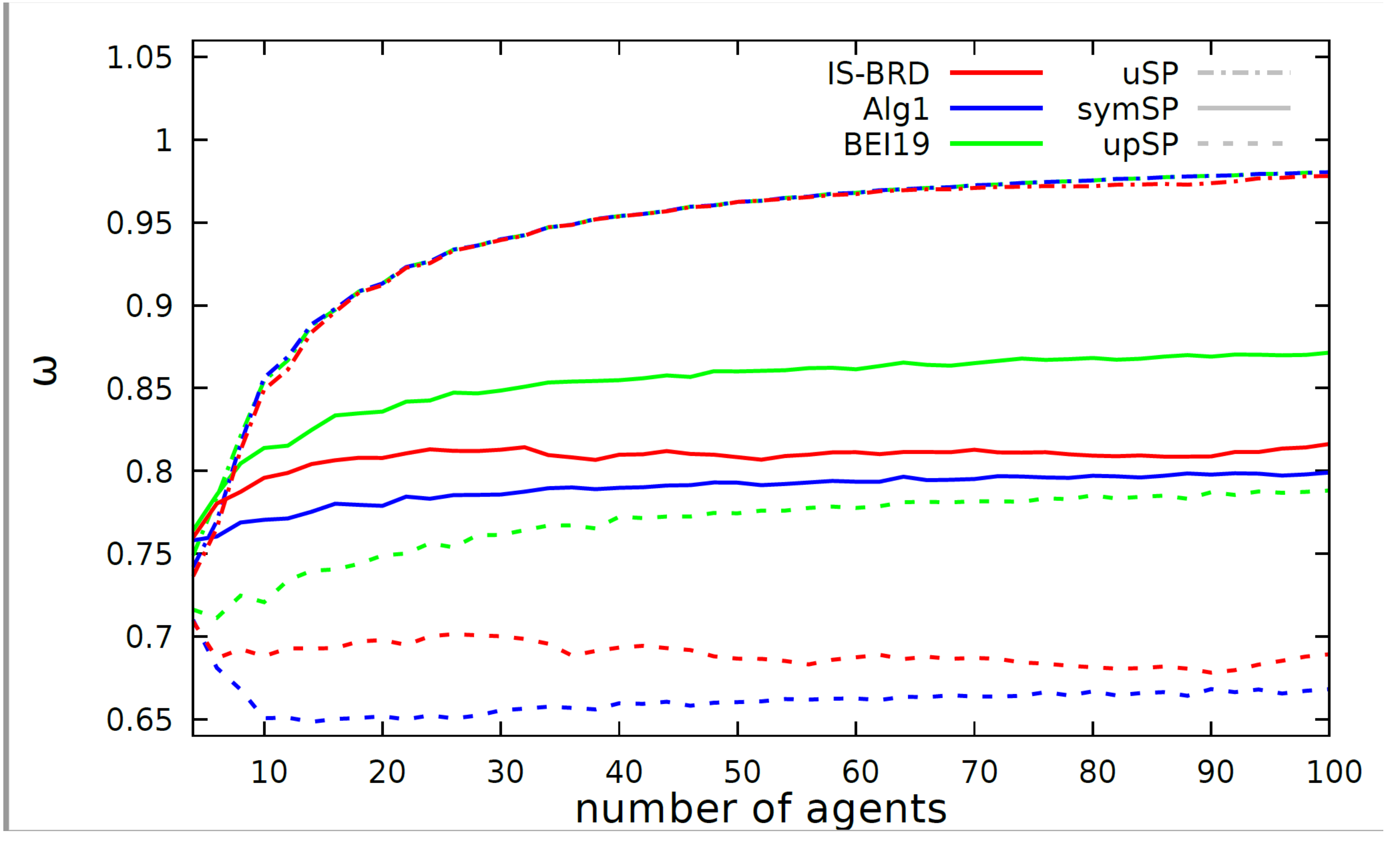
- What if we have red, green and blue players?
- Model 1: r cares about R:B, R:G, and G:B
- Model 2: r only cares about R:(R+G+B)
- Individual stability:
  - Model 1:
    - non-existence for 3 types
    - hardness for ≥ 5 types
  - Model 2:
    - subsumes anonymous games ⇨ non-existence, hardness



Better response dynamics

- Natural better response dynamics for IS:
  - if some player has an IS-deviation, let her perform it
- Does this always converge to an IS outcome?
  - empirically, yes
  - theoretically?
  - at least for some initial partition?
- Same question for (single-peaked) anonymous games

Experiments



Conference dinner problem

- A (): I do not want any alcohol at my table
- B (): I do not drink, but drinkers are amusing
- C (): I feel weird around non-drinkers
- D (): the fewer people drink, the more is left for me (but I do not want to drink alone)

Roommate problem with diversity preferences

- (Multidimensional) roommate problem:
  - k rooms of size s each
  - ks agents who need to be assigned to rooms
- Can we find an outcome that is
  - core stable?
  - swap stable?
  - Pareto optimal?
- Our work [BoE'20]:
  - existence of good outcomes for $s=2$
  - algorithmic results (FPT wrt s)
    - ILP with poly(s) variables