Regret-equality in Stable Marriage

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Outline

- Matching problems
- Fairness
- Finding fair stable matchings
- Experiments
- Future work

Matching Problems

- Assign one set of entities to another set of entities
- Based on preferences and capacities

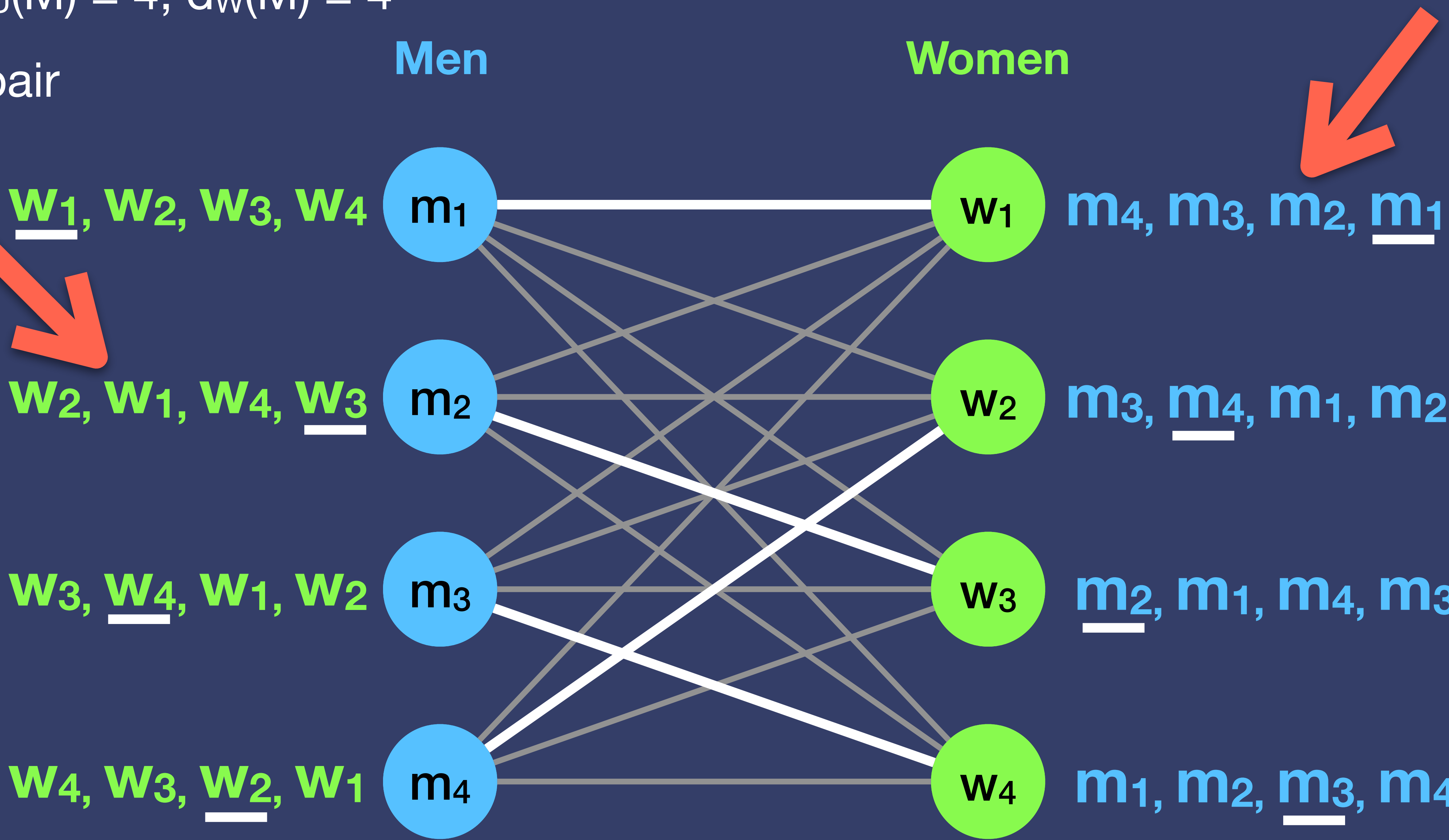
Stable Marriage

Rank

Cost: $c_U(M) = 10$, $c_W(M) = 10$

Degree: $d_U(M) = 4$, $d_W(M) = 4$

Blocking pair



A **stable matching** is a matching with no blocking pairs

Stable Marriage

- A **stable matching** is a matching with no blocking pairs
- Many stable matchings per instance
- We can find a stable matching in linear time using the man-oriented or woman-oriented Gale-Shapley Algorithm. O(m) time where m is total length of preference lists
- Man-oriented Gale-Shapley Algorithm: finds a man-optimal (woman-pessimal) stable matching (and vice versa)

Fairness

- Want to find a stable matching that provides some kind of equality between men and women

- Several different fairness measures

Fairness measures

Among all stable matchings, find the stable matching that…

	Cost: $c_U(M)$, $c_W(M)$	Degree: $d_U(M)$, $d_W(M)$
Minimises the maximum	**balanced score** Balanced stable matching NP-hard	**degree** Minimum-regret stable matching Poly
Minimises the difference	**sex-equal score** Sex-equal stable matching NP-hard	**regret-equal score** * Regret-equal stable matching ?
Minimises the sum	**egalitarian cost** Egalitarian stable matching Poly	**regret sum score** * Min-regret sum stable matching ?

Fairness measures (degree based)

10 stable matchings for this instance

m_1: $w_1, w_2, \underline{w_3}, w_4$ w_1: $m_4, m_3, \underline{m_2}, m_1$
m_2: $w_2, \underline{w_1}, w_4, w_3$ w_2: $m_3, \underline{m_4}, m_2, m_1$
m_3: $w_3, \underline{w_4}, w_1, w_2$ w_3: $m_2, \underline{m_1}, m_4, m_3$
m_4: $w_4, w_3, \underline{w_2}, w_1$ w_4: $m_1, m_2, \underline{m_3}, m_4$

Min-regret & Regret-equal

Degree: 3

Regret-equality score: 0

Min-regret sum score: 6

m_1: $w_1, w_2, \underline{w_3}, w_4$ w_1: $m_4, \underline{m_3}, m_2, m_1$
m_2: $w_2, w_1, \underline{w_4}, w_3$ w_2: $m_3, \underline{m_4}, m_2, m_1$
m_3: $w_3, w_4, \underline{w_1}, w_2$ w_3: $m_2, \underline{m_1}, m_4, m_3$
m_4: $w_4, w_3, \underline{w_2}, w_1$ w_4: $m_1, \underline{m_2}, m_3, m_4$

Min-regret & Min-regret sum

Degree: 3

Regret-equality score: 1

Min-regret sum score: 5

m_1: $\underline{w_1}, w_2, w_3, w_4$ w_1: $m_4, m_3, m_2, \underline{m_1}$
m_2: $\underline{w_2}, w_1, w_4, w_3$ w_2: $m_3, m_4, \underline{m_2}, m_1$
m_3: $\underline{w_3}, w_4, w_1, w_2$ w_3: $m_2, m_1, m_4, \underline{m_3}$
m_4: $\underline{w_4}, w_3, w_2, w_1$ w_4: $m_1, m_2, m_3, \underline{m_4}$

Min-regret sum

Degree: 4

Regret-equality score: 3

Min-regret sum score: 5

Over all stable matchings:
Minimum degree = 3
Minimum regret-equality score = 0
Minimum regret sum score = 5

Finding a Regret-Equal Stable Matching

Rotations

- Rotation - series of man-woman pairs that take us from one stable matching to another when permuted

R_1 $\begin{array}{cc} m_1 & m_4 \\ w_2 & w_3 \end{array}$

M_1 $\begin{array}{cccc} m_1 & m_2 & m_3 & m_4 \\ w_2 & w_1 & w_4 & w_3 \end{array}$

- Can only eliminate *exposed* rotations

$\downarrow R_1$

R_2 $\begin{array}{cc} m_1 & m_2 \\ w_1 & w_2 \end{array}$

M_2 $\begin{array}{cccc} m_1 & m_2 & m_3 & m_4 \\ w_3 & w_1 & w_4 & w_2 \end{array}$

- $O(n^2)$ algorithm to find all rotations

- Rotations form a structure to allow enumeration of all stable matchings. All rotation makes some men worse off and some women better off

Algorithm

1. Find the man-optimal stable matching M_0

- Each man has their best partner in any stable matching. Say **$d_U(M_0) = 2$** and **$d_W(M_0) = 5$** $d(M_0) = (2, 5)$

- Then, a regret equal stable matching must exist within the following degrees pairs:

r-e score: 3	(2, 5)
r-e score: 2	(2, 4) (3, 5)
r-e score: 1	(2, 3) (3, 4) (4, 5)
r-e score: 0	(2, 2) (3, 3) (4, 4) (5, 5)
r-e score: 1	(2, 1) (3, 2) (4, 3) (5, 4) (6, 5)
r-e score: 2	(3, 1) (4, 2) (5, 3) (6, 4) (7, 5)

why are these the only possible degrees?

- M_0 has a r-e score of 3
- men can only get worse
- women can only get better

Algorithm

2. If $d_U(M_0)$ >= $d_W(M_0)$ then exit with M_0
3. For each man m and for each column c:
   1. rotate m down to c (if possible)
   2. rotate women down column c who have worst rank

r-e score: 3 (2, 5)
r-e score: 2 (2, 4) (3, 5)
r-e score: 1 (2, 3) (3, 4) (4, 5)
r-e score: 0 (2, 2) (3, 3) (4, 4) (5, 5)
r-e score: 1 (2, 1) (3, 2) (4, 3) (5, 4) (6, 5)
r-e score: 2 (3, 1) (4, 2) (5, 3) (6, 4) (7, 5)

- Stop iterating women up the column when $d_U(M)$ >= $d_W(M)$
- Save the best matching as you go

Time complexity

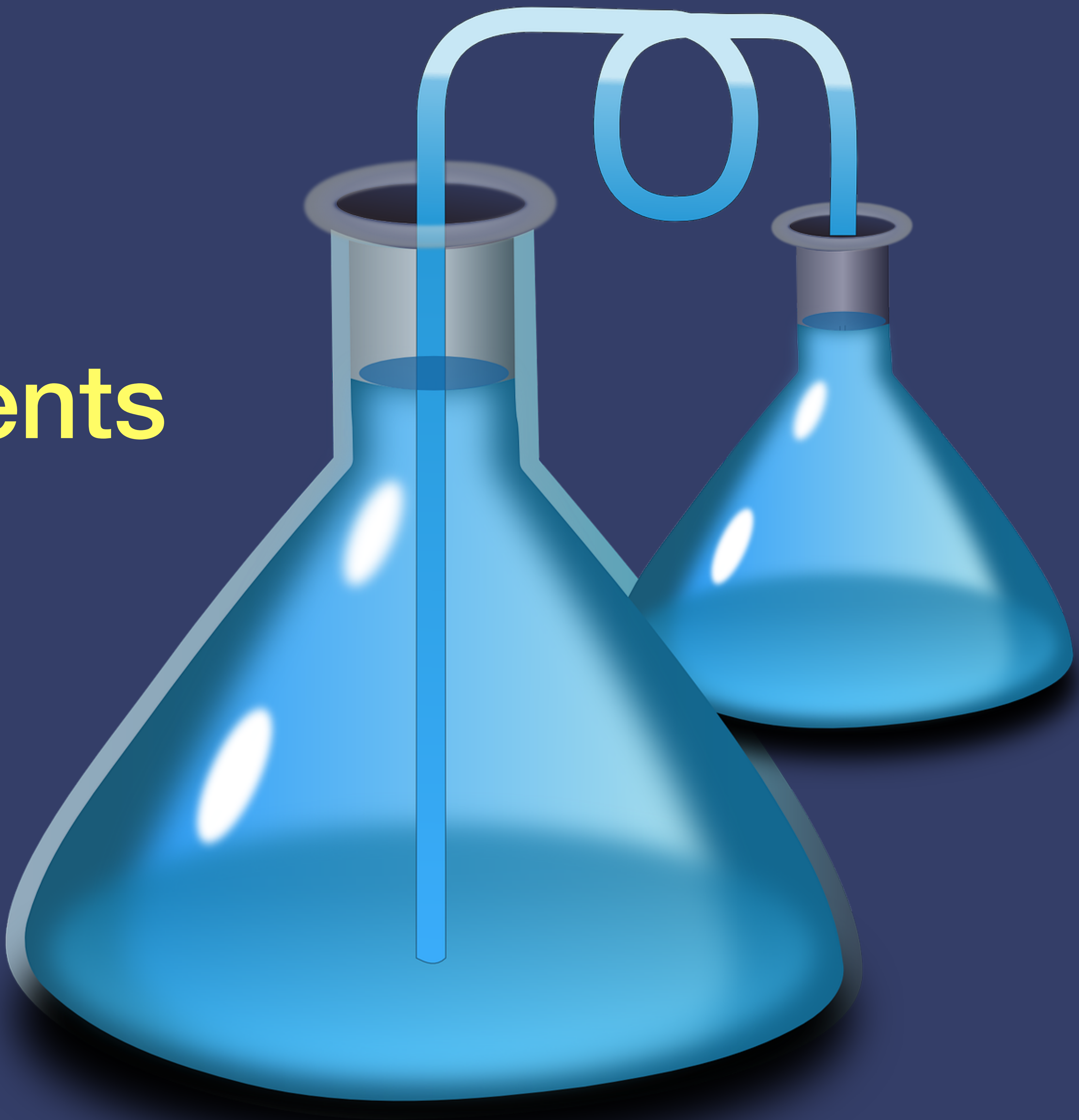
- Find man-optimal stable matching & all rotations **$O(n^2)$**
- For each man **$O(n)$**

  2 * man-optimal difference

  - For each column **$O(2 * |d_U(M_0) - d_W(M_0)|) = O(c)$**
    - Rotate man down and women down **$O(n^2)$**

★ Total $O(n^3c)$ ★

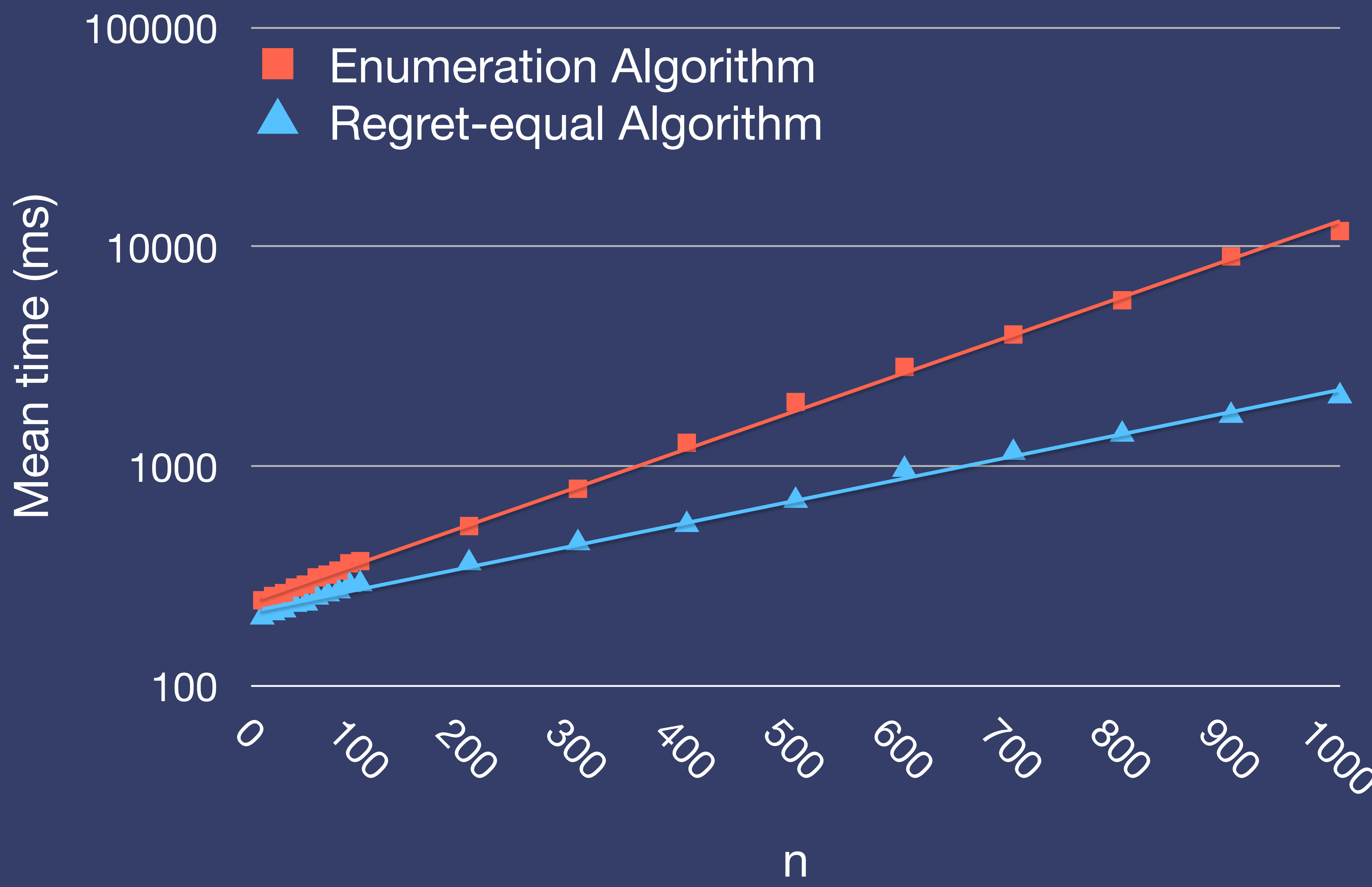
Experiments



Methodology

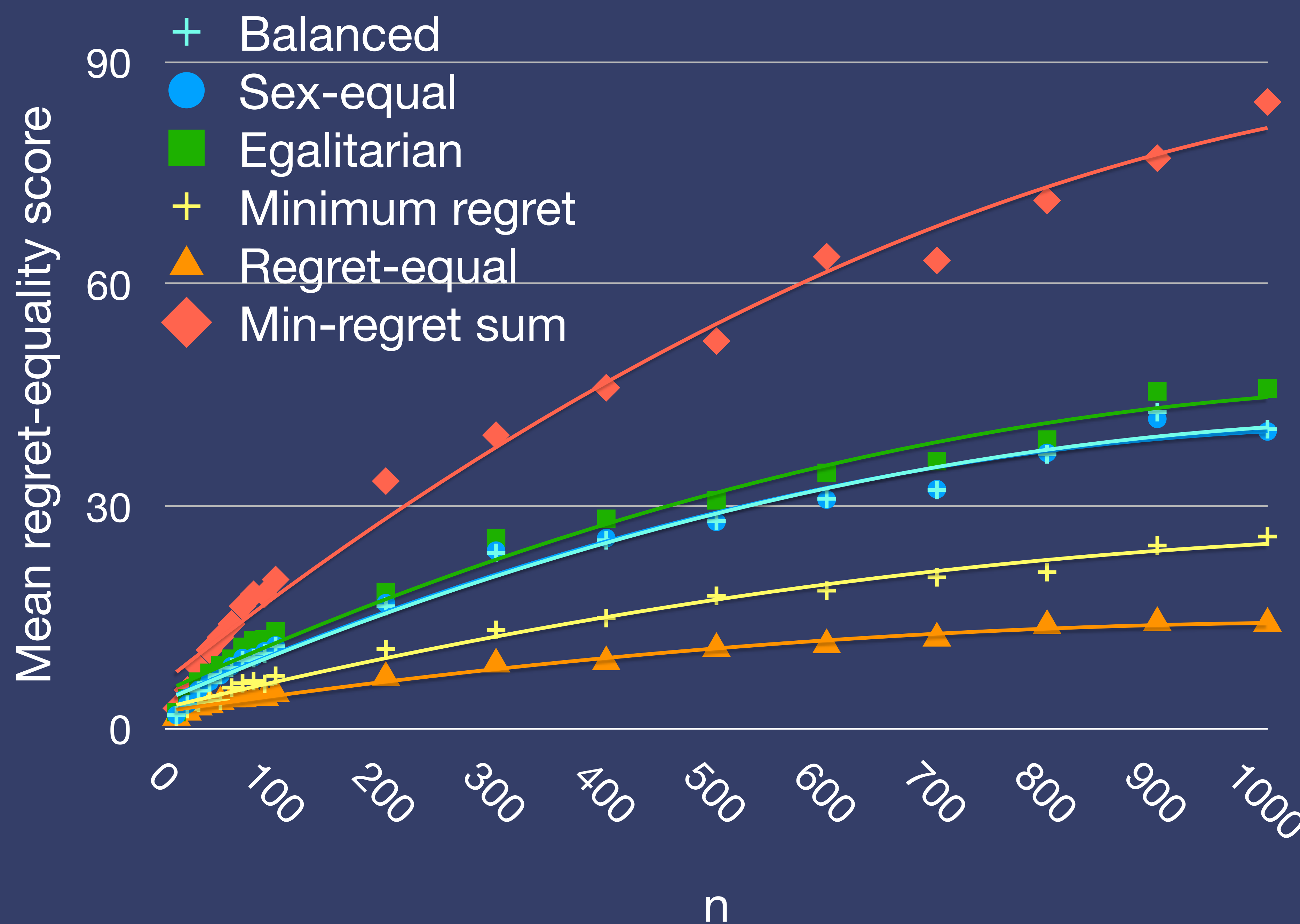
- Performance of the Regret-equal Algorithm compared to an Enumeration algorithm (exponential in worst case)
- Instances size {10, 20, …, 100, 200, …, 1000}, complete preference lists, 500 instance per size.
- looked at properties over several types of optimal stable matching (balanced, sex-equal, egalitarian, minimum regret, regret-equal, min-regret sum)
- Java, Python, Bash, GNU parallel
- Correctness
    - all matchings found were stable
    - Regret-equality scores matched
    - CPLEX up to size n = 50 for the enumeration algorithm

Time taken

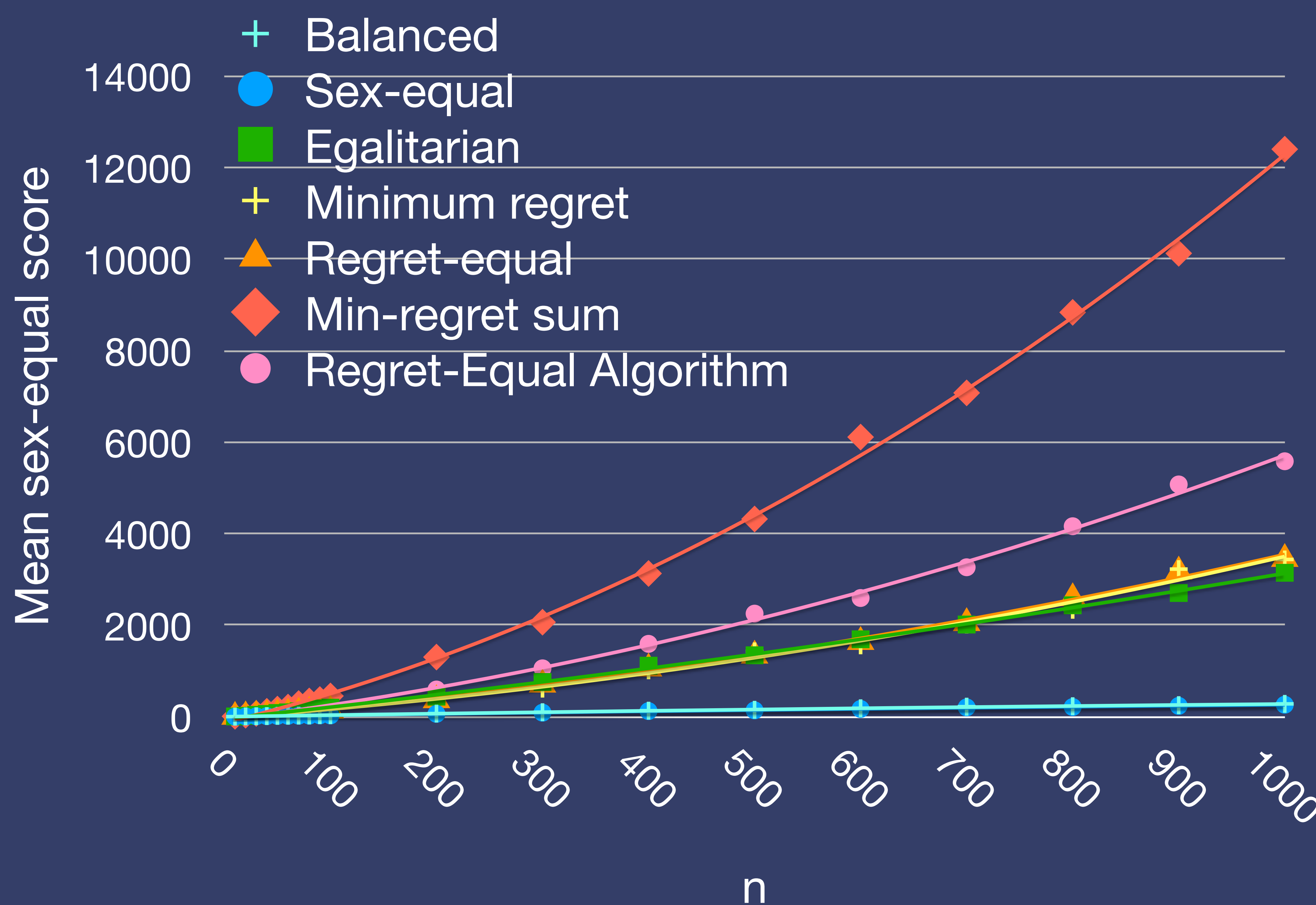


100000
10000
1000
100
Mean time (ms)
Enumeration Algorithm
Regret-equal Algorithm
0
100
200
300
400
500
600
700
800
900
1000
n

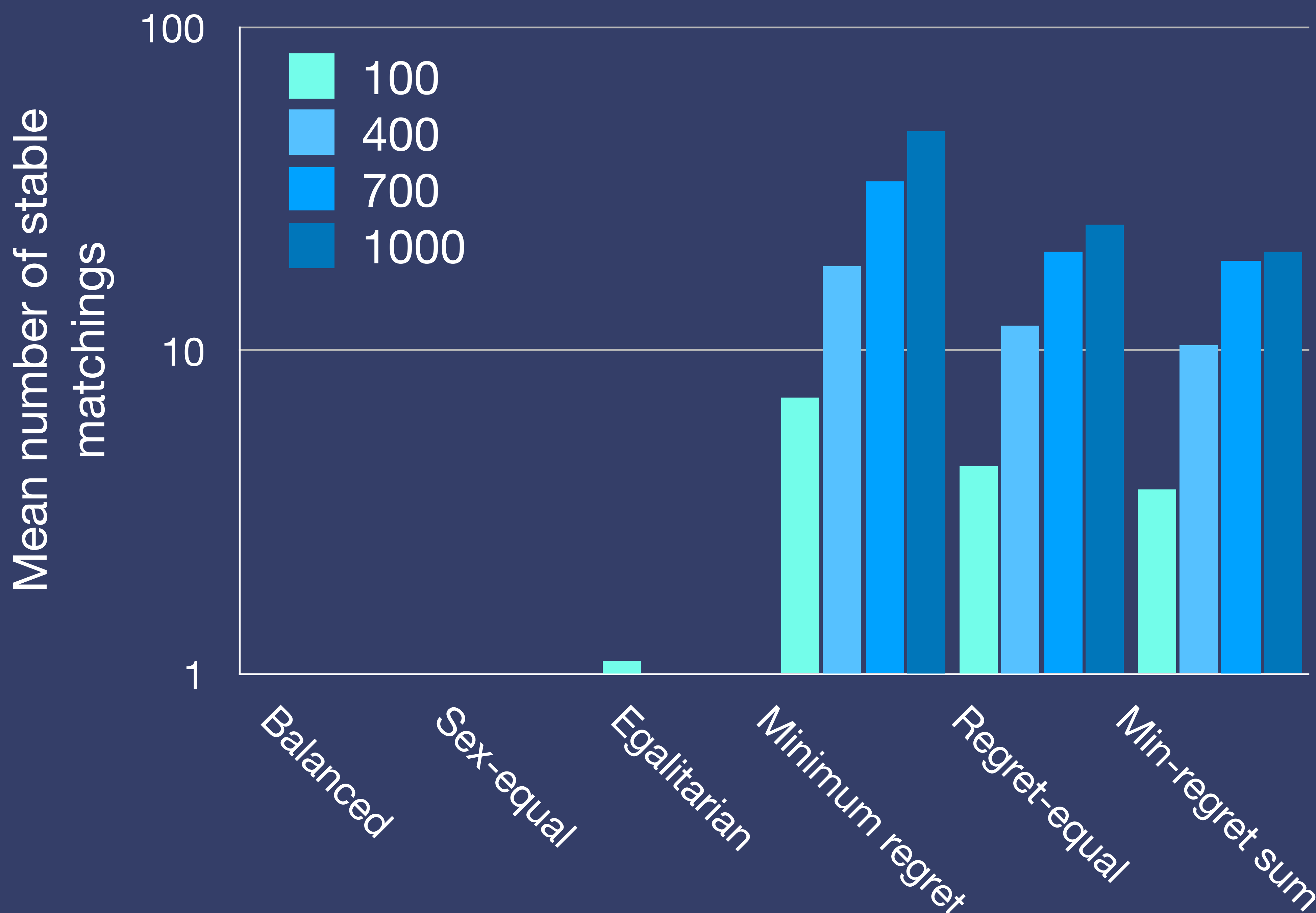
Regret-equality score for different optimal matchings



Sex-equal score for different optimal matchings



Frequency of different optimal stable matchings



Future Work

- Improving the $O(n^3c)$ Regret-equal Algorithm, where $c = |d_U(M_0) - d_W(M_0)|$
- Grouping women - e.g. women are workers and men are jobs to assign to workers.
    - Woman optimal stable matching would naturally satisfy ‘balanced’, ‘min-regret', ‘egalitarian’ and ‘min-regret sum’ criteria
    - Can find a ‘regret-equal’ stable matching in $O(n^4)$ time
    - Open problem for ‘sex-equality’ -> grouped-women-equality

Thank you

Summary

- Matching problems
- Fairness
- Finding fair stable matchings
- Experiments
- Future work: finding improved algorithms

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