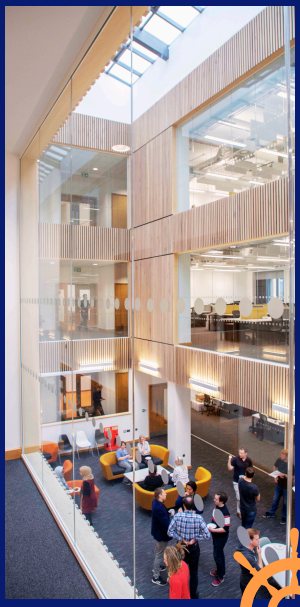
2025

Continuity Computability Constructivity

From Logic to Algorithms



Swansea

Heilbronn Institute for Mathematical Research

Swansea University
Prifysgol Abertawe

Zienkiewicz Institute for Modelling, Data and AI
Sefydliad Zienkiewicz ar gyfer Modelu, Data ac AI

Invited Speakers

Anuj Dawar
University of Cambridge

Lower Bounds for Symmetric Algebraic Circuits

Algebraic circuits (also known as arithmetic circuits) are a model for computing polynomial functions over an arbtrary field. Valiant's conjecture that VP is different from VNP is sometimes called an algebraic analogue of the conjecture that P is different from NP and has been an open question for nearly as long. It amounts to showing that algebraic circuits for computing the permanent of a matrix are necessarily of super-polynomial size. We are able to show lower bounds on circuits computing the permanent when restricted to certain symmetry conditions. With a natural notion of symmetry, this gives an exponential separation between the size of circuits computing the permanent and those computing the determinant. The result is sensitive to the choice of symmetries considered.

In this talk, I give a general introduction to algebraic circuits and Valiant's conjecture. I explain and motivate the symmetry restrictions and explore the lower bounds that can be established. I also give an overview of the proof methods, which bring together combinatorial techniques with methods from finite model theory.

Cécilia Pradic
Swansea University

How unconstructive is the Cantor-Bernstein theorem?

The Cantor-Bernstein theorem states that sizes of sets can be compared meaningfully using injections: if A injects into B and vice-versa, A and B are in bijection. This is typically proven via an explicit construction that does not involve choice, but the proof cannot be constructive. For instance, [0,1] and (0,1) can be embedded into one another but are not homeomorphic, meaning that Cantor-Bernstein is violated in a number of models of intuitionistic set theory. Faced with this state of affairs, we can still ask: how bad it is?

First, we are going to see how Cantor-Bernstein implies full excluded middle. We will then turn our attention to the Myhill isomorphism theorem, a constructive version of Cantor-Bernstein that states that, for any two subsets A, B $\subseteq \mathbb{N}$ that are inter-reducible via injections, there is a bijection $\mathbb{N} \to \mathbb{N}$ that preserves them. The theorem remains true classically if $\mathbb{N}$ is replaced by an arbitrary set X, but this is not the case constructively. Bauer asked if there is a nice class of sets X for which it does hold constructively. After checking there is no hope for this class of sets to be closed under basic operations like disjoint unions, we will see that a version of this generalized Myhill isomorphism theorem holds for the conatural numbers $\mathbb{N}^\infty$ by adapting the usual back-and-forth construction and assuming Markov's principle. However, this does not extend much: this fails for $2^\times \mathbb{N}^\infty$, $\mathbb{N} + \mathbb{N}^\infty$ as well as Cantor space. We are going to see why those failures are of different flavours, and sketch how to make this could be made more precise by using oracle modalities.

Based on joint work with Chad Brown (https://arxiv.org/abs/1904.09193 and https://arxiv.org/abs/2507.05028).

Nobuko Yoshida
University of Oxford

Multiparty Session Types: Separation and Encodability Results

The talk first introduces the history and background of types for communications and multiparty session types, relating to the history of Computer Science in Oxford.

Multiparty session types (MPST) are a type discipline for enforcing the structured, deadlock-free communication of concurrent and message-passing programs. Traditional MPST have a limited form of choice in which alternative communication possibilities are offered by a single participant and selected by another. Mixed choice multiparty session types (MCMP) extend the choice construct to include both selections and offers in the same choice. This talk presents a mixed choice synchronous multiparty session calculus and its typing system, which guarantees communication safety and deadlock-freedom. We then talk expressiveness of nine subcalcli of MCMP-calculus by examining their encodability (there exists a good encoding from one to another) and separation (there exists no good encoding from one calculus to another). The highlight is the binary (2-party) mixed sessions by Casal et al (2022) is strictly less expressive than the MCMP-calculus.

A joint work with Kirstin Peters appeared in LICS'24 (https://arxiv.org/abs/2405.08104).

Milly Maietty
University of Padova

Constructivity, computability and continuity relative to the Minimalist Foundation (Tutorial)

Issues regarding constructivity, computability, and continuity are inevitable when developing a foundation for constructive mathematics, i.e., mathematics where the underlying logical reasoning has computational contents, sets are meant as data types, and the existence of objects is shown by construction.

This tutorial aims to describe solved or still open issues of constructivity, computability, and continuity related to the Minimalist Foundation, for short MF, for constructive mathematics ideated in joint work with Giovanni Sambin in [2] and completed in [1].

[1] Maietti, M.E.: A minimalist two-level foundation for constructive mathematics. Ann. of Pure and Applied Logic, 2009
[2] Maietti, M.E., Sambin, G.: Toward a minimalist foundation for constructive mathematics. In: L. Crosilla and P. Schuster (ed.) From Sets and Types to Topology and Analysis: Practicable Foundations for Constructive Mathematics, no. 48, OUP, 2005.

Programme

All talks will take place in Robert Recorde Room (Computational Foundry, Room 102)

Monday 1st of September

09:15 - 10:15	Opening, Invited Talk: Anuj Dawar (Cambridge)
10:15 - 10:45	Coffee break
10:45 - 11:45	Contributed talks 1
11:50 - 12:30	Tutorial: Milly Maietti (Padua)
12:30 - 14:00	Lunch break
14:00 - 14:50	Invited talk: Cecilia Pradic (Swansea)
14:50 - 15:20	Coffee break
15:20 - 17:00	Contributed talks 2

Tuesday 2nd of September

09:30 - 10:15	Dieter Spreen (Siegen)
10:15 - 10:45	Coffee break
10:45 - 11:15	Pieter Collins (Maastricht)
11:15 - 11:45	Jens Blanck (Swansea)
11:50 - 12:35	Michal Konečný (Birmingham)
12:35 - 14:00	Lunch
14:00	Excursion: Caswell Bay to Bracelet Bay
18:45	Conference Dinner @ Bistrot Pierre Oyster Wharf, 3 Mumbles Rd, Mumbles, Swansea SA3 4DN

Excursion

"Bracelet Bay" by Olga Petrovska, CC BY 2.0

Wednesday 3rd of September

09:30 - 10:20	Invited talk: Nobuko Yoshida (Oxford)
10:20 - 10:50	Coffee break
10:50 - 12:30	Contributed talks 3
12:30 - 14:00	Lunch break
14:00 - 14:40	Tutorial 2: Milly Maietti (Padua)
14:45 - 15:15	Coffee break
15:15 - 16:15	Contributed talks 4
16:15	Closing

Special session

This session is dedicated to Norbert Müller on the occasion of his retirement.

Dieter Spreen *Computing with Compact Sets*

In collaboration with U. Berger, a general framework was presented for extracting algorithms that compute on elements of compact metric spaces and their compact subsets from proofs in a many-sorted intuitionistic first-order predicate logic, extended by strictly positive inductive and coinductive definitions. The approach is computationally equivalent to Weihrauch's type-two theory of effectivity. Unlike this approach, however, it is purely logical and representation-free. Representations of the computed objects are obtained via a realizability interpretation of the logic. Note that although the logic is fundamentally intuitionistic, much of classical logic is nevertheless available: any genuine disjunction-free formula can be used as an axiom.

In this talk, we discuss the mathematical framework introduced for the treatment of elements and nonempty compact subsets of compact metric spaces in formal logic. Furthermore, we present a generalization of Berger's nested coinductive inductive characterization of uniformly continuous functions of the unit interval to the general case of compact metric spaces. This characterization enables the treatment of (constructively) uniformly continuous functions in the aforementioned first-order logical calculus and the derivation of programs for computing such functions. We re-prove some well-known results in metric space theory based on the characterization. The proofs now use coinduction and induction in a nested manner and differ significantly from the usual proofs of classical topology.

Pieter Collins *Exact Real Computation and the iRRAM*

One of the main goals of research into computable analysis is to develop tools for real number computation which are both exact and efficient. Norbert Müller's iRRAM, originating from the mid 1990s, was the first system to do just this, and is still competitive today. In this talk, I'll look at the development of iRRAM and perspectives for the future of exact real computation.

Jens Blank *A Domain Theoretical Understanding of the iRRAM*

The iRRAM is a successful implementation of Exact Real Arithmetic. We will look at the implementation from a domain theoretic perspective and at some of the important choices made in its implementation.

Michal Konečný *Multi-Valued Limits in iRRAM and cAERN*

joint work with Sewon Park and Holger Thies

Some real-valued problems are only computable as non-deterministic functions. For example, the complex square root is not computable as a function but it is computable as a multi-valued function that returns one of the two roots non-deterministically. Moreover, the non-deterministic result is sometimes computed as a limit: The terms of the limit are non-deterministic and yet converge to one of the possible valid results.

I recall and contrast the multi-valued limit operators provided by iRRAM, AERN and cAERN. Each has some subtle aspects that require careful description. In summary, the iRRAM version is more efficient, the AERN version is simpler, and cAERN provides a way to formally verify uses of the AERN multivalued limit. cAERN's multivalued dependent choice axiom is similar to iRRAM's mechanism for achieving convergence of a multivalued limit.

Contributed Talks 1

Pieter Collins, Bernard Hanzon, Eike Neumann

On Positivity of Exponential-Trigonometric Polynomials and Irrationality Exponents

We study the Positivity Problem for exponential-trigonometric polynomials over ordered fields that are obtained from the rationals by adjunction of finitely many computable transcendental real constants. We show that if one of the constants is equal to π, then a solution of the Positivity Problem allows us to compute the irrationality exponents of all the constants we adjoin. In particular, a solution to the Positivity Problem over the field $\mathbb{Q}(\pi)$ would imply computability of the irrationality exponent of π - a major open problem in transcendental number theory. We further construct a field $K = \mathbb{Q}(\pi, \alpha)$ where α is a computable number, such that the Positivity problem over K is provably undecidable.

George Davie

Computing C(x) given x. Applications to left c.e. reals, relative computability, Turing-completeness and computational gaps

We show that the difficulty of computing C(x) from x plays a central role in the computability of another string y from x, and also how much information x contains about the Halting problem. We apply this to: Chaitin's theorem – that α is computable if and only if there is a $d \in \mathbb{N}$ for which $C(\alpha n) \leq C(n) + d$ for all n – and its relativisation due to Frank Stephan. Gap phenomena:We show that, when $C(C(\alpha n)|\alpha n)$ is large, there are puzzling computational gaps around initial segments αn .

Contributed Talks 2

Samuele Maschio, Davide Trotta

A topos for extended Weihrauch degrees

The main goal of this talk is to show how one can define a topos for extended Weihrauch degrees, providing a suitable universe for studying this reducibility categorically. Then, we take advantage from this categorical presentation, and we establish the precise connection between extended Weihrauch degrees and realizability.

Arno Pauly, Cécilia Pradic, Giovanni Solda, Manlio Valenti

Higman's lemma in the Weihrauch lattice

We study Weihrauch degrees associated with Higman's lemma.

Benjamin Koch, Elvira Mayordomo, Arno Pauly, Cécilia Pradic, Manlio Valenti

Computability-theoretic properties of Hausdorff oracles

We describe the computational powers of oracles that witness the Hausdorff dimension of subsets of R^n.

Contributed Talks 3

Paulo Oliva, Ulrich Berger
Uniform Realizability Interpretations

We report here on a novel framework of uniform realizability that unifies and generalises various realizability interpretations of logic, particularly focussing on the treatment of atomic formulas and quantifiers. Traditional realizability interpretations (such as Kleene's number realizability) require explicit witnesses for existential quantifiers. In contrast, newer approaches, such as in the first author's uniform Heyting arithmetic, Herbrand realizability of non-standard arithmetic, or in the "classical" realizability of arithmetic, (some) quantifiers, are treated uniformly. The proposed notion of uniform realizability abstracts these differences, parametrising the interpretation by a given treatment of atomic formulas, accounting for both classical and modern variants.

Thorsten Altenkirch
Completeness in University Algebra: A New Perspective on High School Identities

We propose a new foundational setting for reasoning about isomorphisms between types and exponential identities, called University Algebra. Building on the failure of completeness in High School Algebra (HSA) and its categorification via bicartesian closed categories, we explore how categories with families (CwFs) equipped with Π-types, Σ-types, 1, and 2 can recover the missing isomorphisms. Notably, Wilkie's counterexample---which exposes the incompleteness of exponential semirings---becomes derivable in this framework using the type-theoretic axiom of choice. We conjecture that all definable isomorphisms in University Algebra can be reduced to equational ones and outline a syntactic strategy toward proving this completeness.

Margret K Tembo, Eike Neumann
Linear and Affine Escape Problem Over the Reals

We consider a natural computational problem: Does every point of a given polyhedron escape the polyhedron under iteration of a given matrix Tiwari (2004) has shown this problem to be decidable when the matrix and polyhedron are given by rational/algebraic data. We show that the problem is maximally partially decidable over arbitrary real data.

Full abstracts for all contributed talks are available on the website: https://swansea-theory.github.io/conferences/CCC2025/programme.html

Contributed Talks 4

Fateme Ghasemi, Jeffery Zucker

WhileCC -Approximability and Acceptability of Elementary Functions

In this work, we study models of computation for partial functions on the reals. Existing work [Fu and Zucker, 2014, Tucker and Zucker, 1999, 2004] studies classes of computable partial functions on R, namely GL-computability, tracking computability, multipolynomial approximability, and WhileCC-approximability.

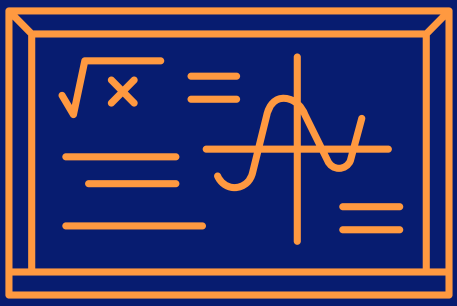
Fu and Zucker [2014] show that all these four models of computation are equivalent when we restrict our attention to a specific class of functions we call "acceptable" functions.
This means, within the realm of acceptable functions, we can work with WhileCC-approximability without giving up expressivity and transfer results amongst the models. However, it was previously unknown whether the class of acceptable functions is sufficiently large to include many common functions, such as the elementary functions.

In this work, we solve the conjecture posed by Fu and Zucker [2014] and show that all elementary functions are acceptable. We also prove that the elementary functions are WhileCC-approximable and therefore computable in all the aforementioned models of computation.

Harry Bryant, Anton Setzer, Monika Seisenberger, Andrew Lawrence

Applying Verified Z3 Proof Checking to Ladder Logic Verification of Railway Interlockings

Railway systems are safety critical and demand the highest levels of assurance for their control software. Formal verification tools, used alongside conventional testing, are essential for ensuring compliance with stringent safety and regulatory standards. In this article, we present a solution with the goal to be part of a future verification toolchain involving Z3 SAT/SMT solving and to be certified at Safety Integrity Level 4 (SIL4). We demonstrate this via a verified and formally extracted SAT proof checker in the context of Ladder Logic verification. Our approach is tailored to the needs of our industrial partner, adaptable, and also extendable to include further SMT theories. Our proof checker currently works for Z3's full propositional proof output, including Tseitin transformations. A checker for Z3 proofs of CNF formulas has been formalised and verified in Rocq, with a certified OCaml implementation extracted from the proof, whilst the full extendable framework including RUP inferences and Tseitin transformation has been fully verified in Agda. Our approach enables formal reasoning about Z3 outputs in both theorem provers. Finally, we demonstrate the entire approach with a small case study, and provide results on the scalability on an industrial level.



WiFi

Swansea University has eduroam. However, if you are not able to connect to it, you can use our visitors network instead.

Get Started – Connect to 'SwanseaUni-Visitors'

The first thing you need to do is view the list of available wireless networks on your device and then connect it to the open SwanseaUni-Visitors SSID. Once you are connected you should be prompted to sign-in. If you are not prompted, open a web browser and type in the URL of https://socialwifi.swansea.ac.uk

Log in

After you have connected, you need to log in using either Facebook credentials or your email address. Once you are logged in the window may close, or you will be redirected to the university web site.

Connection Restrictions

Once you are connected you are free to use the internet, but there are some restrictions to keep in mind. Adult and inappropriate content is blocked. Connections are limited to 4Mbs per device, and you will be disconnected after 4 hours inactivity.

Restaurants

Swansea city centre offers several restaurants and cafes. Here are some options that you might consider:

£-££ Awa Grill House (Middle Eastern):
8-10 College Street, Swansea, SA1 5AE http://awagrillhouse.restaurantwebx.com

£-££ Turkish Kitchen (Turkish):
21 High St, Swansea SA1 1LF https://www.swanseaturkishkitchen.com

£-££ Panshee (Indian):
29 Singleton St, Swansea SA1 3QN https://pansheeswansea.online

££ Ask Italian (Italian, Mediterranean):
6 Wind Street, Swansea, SA1 1DF https://www.askitalian.co.uk

££ Las Iguanas (Mexican, Latin American):
1-4 Castle Square, Swansea, SA1 1DN https://www.iguanas.co.uk

££-£££ Gallini's (Italian):
Unit 3, Fishmarket Quay, SA1 1UP http://www.gallinisrestaurant.co.uk

££-£££ Madeira (Portuguese):
46 Kingsway, Swansea, SA1 5HG https://www.madeirarestaurantswansea.co.uk

££-£££ The River House (European and British):
Ice House, Kings Rd, Swansea SA1 8AW https://www.riverhouse.co.uk/

££-£££ Lotus (Thai):
Unit 2, The Icehouse, Kings Rd, Swansea SA1 8AW
https://sites.google.com/view/lotusswansea

££-£££ The Welsh House (Welsh):
unit 5, J Shed Arcade, Swansea SA1 8PL
https://thewelsh-house.co.uk/the-welsh-house-swansea/

££-£££ Rasoi (Indian):
J Shed, Unit 3-4, Kings Rd, Swansea SA1 8PL https://www.rasoiwaterfront.co.uk/

Campws y Bae Prifysgol Abertawe

Swansea University Bay Campus

Adeiladau	/	Buildings
Sefydliad Ymchwil Diogelwch Ynni	1	Energy Safety Research Institute (ESRI)
ORACLE II	1.1	ORACLE II
Dosbarth Gwaithredol	1.2	Active Classroom
Swyddfa Weithredol	1.3	Active Office
CISM	1.4	CISM
Sefydliad Deunyddiau Strwythurol	2	Institute of Structural Materials (ISM)
Adeilad Dwyreiniol Peirianneg	3	Engineering East
Iard Nwyddau	3.1	Service Goods Yard
Adeilad Canolog Peirianneg	4	Engineering Central
Llyfrgell y Bae	5	Bay Library
Y Neuadd Fawr	6	Great Hall
Yr Ysgol Reolaeth	7	School of Management
Y Twyni	7.1	Y Twyni
Undeb Myfyrwyr	7.1	Students' Union
Y Coleg	7.2	The College
Preswylfeydd Myfyrwyr	8,9	Student Residences
Parth Gwaith Nanhyfer	8.12	Nanhyfer Work Zone
Neuadd Rod Jones	8.19	Rod Jones Hall
Canolfan Wybodaeth y Tŵr	9	Tower Information Centre (TIC)
MyUniHub	9	MyUniHub
Gwasanaethau Preswyl	9	Residential Services
Ffowndri Gyfrifiadol	11	Computational Foundry
Adeilad Gogleddol Peirianneg	12	Engineering North

Prifysgol Abertawe
Swansea University