CS 325 Project 2: Coin Change
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10/28/15

1)
Two tables are used in the dynamic programming algorithm both are a single array of length A+1 indexed from 0 to A, one for storing the minimum coins needed to make each value less than and equal to A (array T) and another for storing the last denomination of coin used to reach this value in an optimum way (array C) stored as an index into the set V.

Both tables are filled starting from 1 (with a 0 value placed in T[0] as it takes 0 coins to reach amount 0). determining the value of T[i] depends on the values of T[0..i-1], the minimum number of coins to make value i is min(T[i-V[j]]) + 1 where i-V[j] >= 0. An inductive proof of this way of determining T[i] will follow in question 3 but informally we can see that this is equivalent to choosing what the first valued coin should be used first by evaluating the relevant subproblems of the minimum amount of coins needed to generate A-V[i] for the chosen i.

This optimum choice of first coin is what is recorded in the C[1...A] array, at each optimum solution calculated for T[i] the denomination used to achieve that minimum is recorded in C[i]. Then when the minimum number of coins required a solution is back traced by starting with index A of C and then subtracting this index by the value of the coin indexed in V at C[backtrace-Index], tallying the amount used for each denomination.

2)

ChangeSlow Algorithm Pseudo Code:

```
slowHelper(Values[1...N],Amount)
     for i = 1 to N
          if Amount == Values[i]
               return new array [I]

     bestKI = INFINITY
     bestKICoins = NULL

     for i = 1 to k//2 + 1
          leftCoins = slowHelper(Values,i)
          rightCoins = slowHelper(Values,amount-i)
          if leftCoins.length + rightCoins.length < bestKI
               bestKI =  leftCoins.length + rightCoins.length
               bestKICoins = leftCoins + rightCoins

     return bestKICoins

Slow(Values[1...N],Amount)
     let Result[1..N] be a new array [0,0..,0]

     coinsUsed = slowHelper(Values,Amount)

     for i = 1 to coinsUsed.length
          result[coinsUsed[i]] += 1

     return result
```

GreedyChange Algorithm Pseudo Code:

```
Greedy(Values[1...N],Amount)
     let Result[0..N] be a new array

     for i=N down to 1
          Result[i]= Amount//Values[i]
          Amount = Amount%Values[i]

     return Result
```

ChangeDP Algorithm Pseudo Code:

```
DP(Values[1...N],Amount)

     let minCoins[0..Amount] be a new array
     minCoins[0] = 0
     minCoins[1-Amount] = INFINITY

     let trace[1..Amount] be a new array


     for i = 1 to Amount
          coinIndex = 0
          while coinIndex <= N and Values[coinIndex] <= I

               if minCoins[i-Values[coinIndex]] + 1 < minCoins[i]
                    minCoins[i] = minCoins[i-Values[coinIndex]] + 1
                    trace[i] = coinIndex

               coinIndex += 1


     let Result[1..N] be a new array [0,0..,0]
     traceIndex = amount
     while traceIndex > 0:
          coinUsed = trace[traceIndex]
          result[coinUsed] +=1
          traceIndex -= coins[coinUsed]
     return result
```

3)

Base case: T[0] = 0

Strong Inductive Hypothesis: T[0-k] = fewest number of coins needed to make change for T[i]

0 <= i <= k

fewest coins needed to make change for k+1 using any individual coin as the last coin will be the that one coin plus the fewest needed to make k+1 minus the value of that coin. Formally

T [k+1] using V[i] = 1 + fewest(k+1 - V[i])

the fewest possible using any coin will be the min of the above formula using all coins that are less than or equal to k+1, Therefore:

$T[k+1] = \min_{v[i]<=k+1} \{T[k+1 – V[i]] +1\}$

proving the inductive hypothesis.

EXPERIMENTAL ANALYSIS
please see attached recorded results and times and plot charts, all times recorded in milliseconds

It can be seen by comparing the output of the ChangeSlow algorithm to that of the ChangeDP algorithm for low values that it is providing optimum solutions. However due to its running time the number of coins as a function of A for problems 4-6 are compared on amounts less than 36. but are not plotted as they have the same relationship to ChangeGreedy as ChangeDP

4) With these denominations of coins the dynamic programming and greedy algorithm generate the same values of minimum number of coins, In other words they both find an optimal solution.

5) The greedy algorithm produces a non-optimal solution for both sets of denominations, the amount of non-optimal answers was measured experimentally by comparing the output of the dynamic programming algorithm. Both sets of values produced very similar ratios of optimal to non optimal solutions.
V_1 greedy non-optimal: 41 out of: 201 tests for an average of: 20.39% non-optimal solutions
V_2 greedy non-optimal: 37 out of: 201 tests for an average of: 18.40% non-optimal solutions

however the greedy algorithm produced non-optimal solutions repeatedly in groups of 12 for sequentially inputs of Amount on the denomination set V_1 however on values set V_2 they were more sporadicly dispersed.

6) Once again the greedy algorithm always produces optimal solutions for this set of denominations of coins.

7) see attached plots of recorded running time. ChangeSlow has such an explosive runtime that only amounts less than 36 could be recorded (12 hours of computer time were expended attempting to record the time required to calculate the minimum coins for amount=40). A separate comparison of all 3 algorithms for amounts less than 36 on all problems denomination sets is included. At these low values there are outliers for the ChangeSlow algorithm as if the amount being calculated is one of the coin denominations then the base case will return that single coin in constant time without ever entering the base case. ChangeSlow on Problem 6, with the even coin denominations up to 30 has an apparent constant running time for amount 1 through 30, but after amounts greater than the largest denomination the exponential time would begin.
Therefore the 3 algorithms are only plotted for problem V=[1,5,10,25,50] and V=[1] for amount 1 to 35.

It can be determined from the experimental analysis that the running times can be characterized by a function of A as such:

ChangeSlow: $f(A) = O(C^A)$ Exponential
ChangeDP = $f(A) = O(A)$ - Linear Time
ChangeGreedy = $f(A) = O(1)$ – Constant Time

The plot of ChangeGreedy appears to have a number of outliers but upon careful look at the scale of the plot it can be seen that the running times never vary by more than a hundredth of a millisecond. This much variance is insignificant and it can be accurately claimed that A has no effect on the running time of the ChangeGreedy algorithm.

8) The size of n has an effect on both ChangeGreedy and ChangeDP, for changeDP it is only one of the factors, but for ChangeGreedy n is the only determinant of its running time.

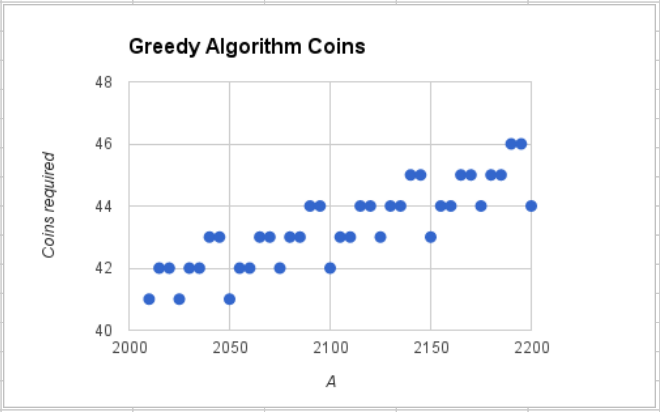
Evaluating the algorithms experimental and theoretically it can be seen that the running time of ChangeGreedy is $\Theta(n)$, and the running time of ChangeDP is $O(An)$ there are A iterations to fill the dynamic programming table and on each one at most n comparisons are made.

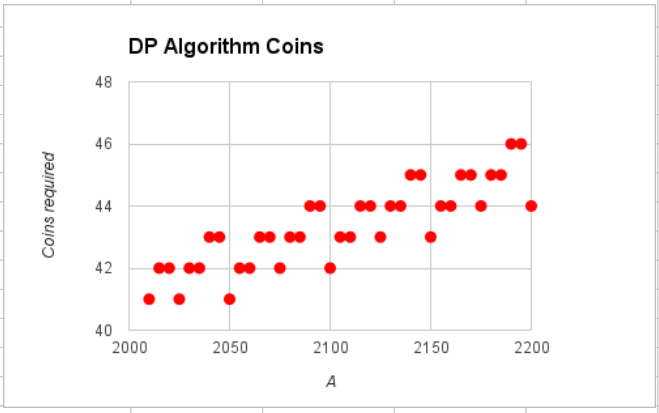
9) In such a case the greedy algorithm would always produce an optimal solution. This can be ascertained by considering that a locally optimal solution will be globally optimal.
For whatever original or remaining amount being consider, A, it will be $p^k <= A < p^{k+1}$ and for any value exponent k, to sum up to p^k with the values $p^{k-1} .. p^0$ the sum of the coefficients will always be greater than 1, at a minimum they will be p, therefore the globally optimal strategy is to repeatedly divide by the greatest possible denomination.

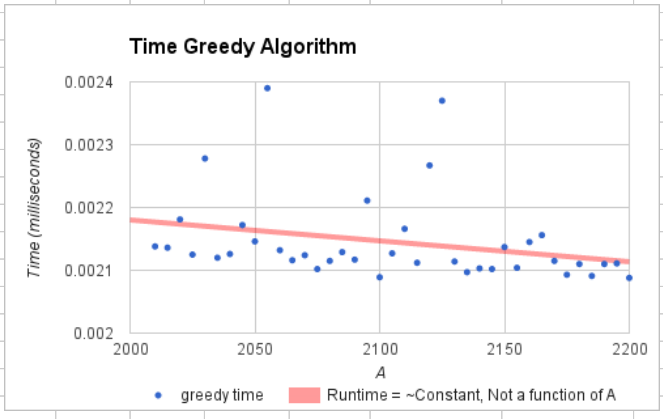
Therefore the greedy algorithm would be a preferable algorithm as its running time would be better. As the greedy algorithms runtime is dependent on the number of denominations (and therefore the number of divisions and mod operations to perform) its runtime is $\Theta(n)$.
the number of denominations in this imaginary scenario could be very large, larger than A, but if the greedy algorithm was modified to start its divisions at the largest denomination less than or equal to A then it would be upper bound by the number of denominations less than A, which is $\Theta(\log_p A)$, making the greedy algorithm $O(\log_p A)$ while the dynamic programming solution would still be $\Theta(An)$

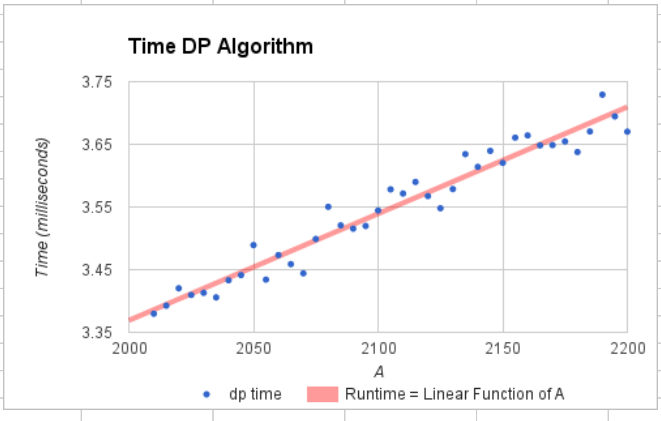
problem 4

amount	greedy coins	dp coins	greedy time	dp time
2010	41	41	0.002138	3.37967
2015	42	42	0.002136	3.39256
2020	42	42	0.002181	3.42028
2025	41	41	0.002125	3.4096
2030	42	42	0.002278	3.41309
2035	42	42	0.00212	3.40577
2040	43	43	0.002126	3.43308
2045	43	43	0.002172	3.4413
2050	41	41	0.002146	3.4892
2055	42	42	0.00239	3.43409
2060	42	42	0.002132	3.47326
2065	43	43	0.002116	3.45883
2070	43	43	0.002124	3.44417
2075	42	42	0.002102	3.49894
2080	43	43	0.002115	3.55028
2085	43	43	0.002129	3.5207
2090	44	44	0.002117	3.51547
2095	44	44	0.002211	3.5197
2100	42	42	0.002089	3.54442
2105	43	43	0.002127	3.57812
2110	43	43	0.002166	3.57149
2115	44	44	0.002112	3.59001
2120	44	44	0.002267	3.56738
2125	43	43	0.00237	3.54797
2130	44	44	0.002114	3.5786
2135	44	44	0.002097	3.63435
2140	45	45	0.002103	3.61402
2145	45	45	0.002102	3.63954
2150	43	43	0.002137	3.62034
2155	44	44	0.002104	3.66065
2160	44	44	0.002145	3.66425
2165	45	45	0.002156	3.64831
2170	45	45	0.002115	3.64878
2175	44	44	0.002093	3.6546
2180	45	45	0.00211	3.6377
2185	45	45	0.002091	3.67051
2190	46	46	0.00211	3.72922
2195	46	46	0.002111	3.6946
2200	44	44	0.002088	3.67028



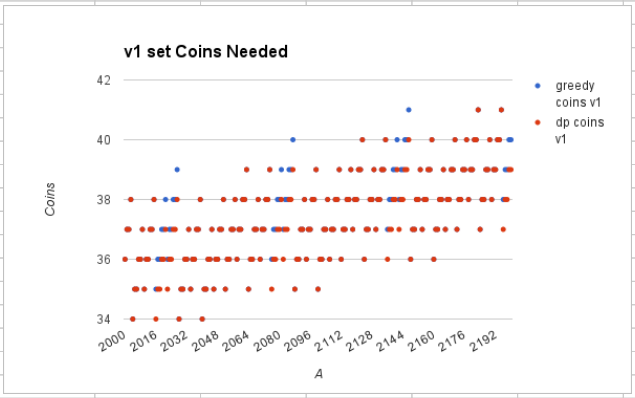


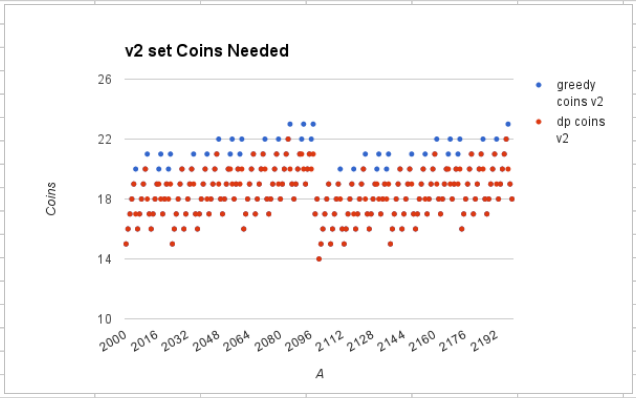




problem5

amount	greedy coins v1	dp coins v1	greedy coins v2	dp coins v2	greedy time v1	dp time v1	greedy time v2	dp time v2
2000	36	36	15	15	0.002612	4.5041	0.002134	3.33621
2001	37	37	16	16	0.002546	4.53779	0.002077	3.34719
2002	37	37	17	17	0.002542	4.58974	0.00209	3.33804
2003	38	38	18	18	0.002515	4.52578	0.002098	3.34602
2004	34	34	19	19	0.002531	4.52378	0.002065	3.42077
2005	35	35	20	17	0.002505	4.52924	0.002083	3.35307
2006	35	35	16	16	0.002531	4.54393	0.002155	3.36062
2007	36	36	17	17	0.002515	4.59742	0.002086	3.35109
2008	36	36	18	18	0.002562	4.53805	0.002119	3.37131
2009	37	37	19	19	0.002519	4.53429	0.002167	3.39628
2010	35	35	20	20	0.00262	4.54345	0.002175	3.35019
2011	36	36	21	18	0.002529	4.53946	0.002081	3.37578
2012	36	36	17	17	0.002558	4.58039	0.002105	3.36554
2013	37	37	16	16	0.002528	4.54679	0.002116	3.38168
2014	37	37	17	17	0.00256	4.55601	0.002086	3.41001
2015	38	38	18	18	0.002561	4.56733	0.002095	3.36592
2016	35	34	19	19	0.002562	4.55655	0.002069	3.37806
2017	36	35	20	19	0.002557	4.59917	0.002133	3.35195
2018	36	35	21	18	0.002857	4.5528	0.002145	3.38384
2019	37	36	17	17	0.00258	4.5664	0.002108	3.40682
2020	37	36	18	18	0.002563	4.58181	0.002088	3.35147
2021	38	37	19	19	0.002556	4.56365	0.002155	3.37303
2022	36	35	20	18	0.002565	4.63397	0.002112	3.38221
2023	37	36	21	19	0.002521	4.5609	0.002361	3.4769
2024	37	36	15	15	0.002597	4.61627	0.002212	3.40244
2025	38	37	16	16	0.002525	4.57187	0.002098	3.36811
2026	38	37	17	17	0.002556	4.57731	0.002081	3.40174
2027	39	38	18	18	0.002852	4.63461	0.002094	3.40157
2028	34	34	19	19	0.002578	4.58557	0.002088	3.37895
2029	35	35	20	20	0.002546	4.62954	0.002071	3.41891
2030	35	35	16	16	0.002561	4.59335	0.002109	3.39321
2031	36	36	17	17	0.002542	4.59465	0.002064	3.42299
2032	36	36	18	18	0.002581	4.59573	0.002178	3.41925
2033	37	37	19	19	0.002578	4.59779	0.002114	3.3814
2034	35	35	20	20	0.002555	4.61773	0.002112	3.39676
2035	36	36	21	19	0.002583	4.60599	0.002106	3.41067
2036	36	36	17	17	0.002591	4.59812	0.002146	3.4272
2037	37	37	16	16	0.002614	4.61561	0.002144	3.41287
2038	37	37	17	17	0.002571	4.61743	0.002127	3.42957
2039	38	38	18	18	0.002533	4.64917	0.002078	3.39273
2040	34	34	19	19	0.00251	4.62383	0.002132	3.40762
2041	35	35	20	20	0.002512	4.62823	0.002087	3.45159
2042	35	35	21	18	0.002539	4.5988	0.002107	3.42339
2043	36	36	17	17	0.002566	4.62957	0.002121	3.41854
2044	36	36	18	18	0.002535	4.67738	0.002128	3.39847
2045	37	37	19	19	0.002547	4.61033	0.002174	3.42951
2046	35	35	20	20	0.002629	4.62725	0.002315	3.45374
2047	36	36	21	21	0.002803	4.62132	0.002117	3.41125
2048	36	36	22	19	0.002545	4.62097	0.002143	3.43473
2049	37	37	18	18	0.002574	4.66469	0.002211	3.43088
2050	37	37	17	17	0.002556	4.62793	0.002084	3.4222
2051	38	38	18	18	0.002538	4.66544	0.002207	3.43582
2052	35	35	19	19	0.002593	4.63771	0.002093	3.42998
2053	36	36	20	20	0.002536	4.61741	0.002245	3.48655
2054	36	36	21	20	0.002547	4.64218	0.002083	3.43323
2055	37	37	22	19	0.00253	4.65938	0.002089	3.41302
2056	37	37	18	18	0.00261	4.6656	0.002097	3.44581
2057	38	38	19	19	0.002641	4.63485	0.002112	3.46618
2058	36	36	20	20	0.002575	4.65057	0.002131	3.45916
2059	37	37	21	19	0.002528	4.65201	0.00213	3.45397
2060	37	37	22	20	0.002585	4.66782	0.002348	3.45367
2061	38	38	16	16	0.002589	4.70884	0.002104	3.43842
2062	38	38	17	17	0.002602	4.65386	0.00209	3.46123
2063	39	39	18	18	0.002537	4.66852	0.002201	3.47443
2064	35	35	19	19	0.0026	4.68123	0.002096	3.43882
2065	36	36	20	20	0.002546	4.67021	0.002123	3.48161
2066	36	36	21	21	0.002757	4.68669	0.002083	3.45249
2067	37	37	17	17	0.002525	4.67245	0.002139	3.45116
2068	37	37	18	18	0.002589	4.73114	0.002087	3.46136
2069	38	38	19	19	0.002525	4.66983	0.002134	3.4644
2070	36	36	20	20	0.002542	4.68232	0.0021	3.4769
2071	37	37	21	21	0.002529	4.69328	0.002122	3.46135
2072	37	37	22	20	0.002544	4.68235	0.00208	3.47545
2073	38	38	18	18	0.00259	4.72279	0.002097	3.54734
2074	38	38	17	17	0.002561	4.70824	0.002109	3.49037



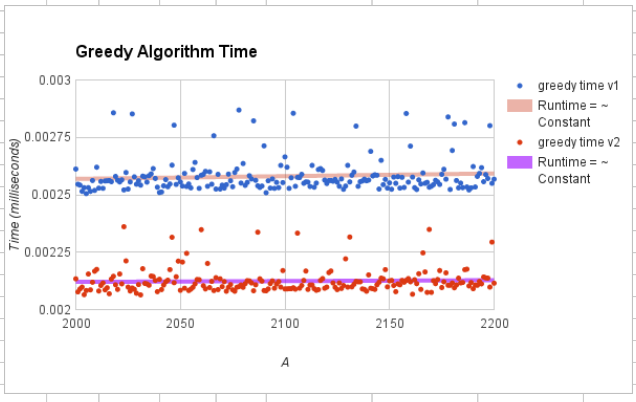


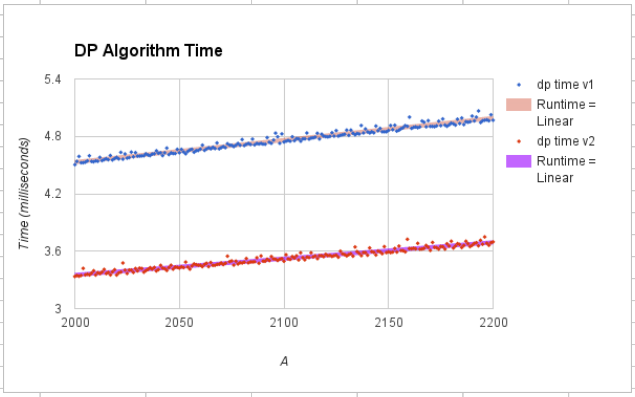
problem5

amount	greedy coins v1	dp coins v1	greedy coins v2	dp coins v2	greedy time v1	dp time v1	greedy time v2	dp time v2
2075	39	39	18	18	0.002535	4.70784	0.002086	3.49976
2076	36	35	19	19	0.002597	4.69409	0.00208	3.45523
2077	37	36	20	20	0.002612	4.70705	0.002097	3.49473
2078	37	36	21	21	0.002869	4.72525	0.0021	3.46581
2079	38	37	22	19	0.002551	4.70731	0.002107	3.47994
2080	38	37	18	18	0.002636	4.7669	0.002097	3.4768
2081	39	38	19	19	0.002541	4.70445	0.002113	3.48472
2082	37	36	20	20	0.002554	4.7164	0.002126	3.52339
2083	38	37	21	21	0.002583	4.72583	0.002125	3.47799
2084	38	37	22	22	0.002569	4.71551	0.002116	3.48849
2085	39	38	23	20	0.002822	4.77136	0.002082	3.48815
2086	39	38	19	19	0.002563	4.71105	0.002108	3.48892
2087	40	39	18	18	0.002539	4.72644	0.002337	3.53081
2088	35	35	19	19	0.002532	4.72044	0.002134	3.4733
2089	36	36	20	20	0.002538	4.72629	0.002124	3.55101
2090	36	36	21	21	0.002713	4.71892	0.002083	3.49592
2091	37	37	22	21	0.002509	4.71741	0.002082	3.49439
2092	37	37	23	20	0.002561	4.78795	0.00209	3.51711
2093	38	38	19	19	0.002529	4.73864	0.002113	3.50127
2094	36	36	20	20	0.002569	4.72838	0.002125	3.54471
2095	37	37	21	21	0.002528	4.7618	0.002093	3.51867
2096	37	37	22	20	0.002554	4.83387	0.002111	3.51535
2097	38	38	23	21	0.002531	4.79621	0.002122	3.5033
2098	38	38	17	17	0.002628	4.72647	0.002098	3.51575
2099	39	39	18	18	0.002553	4.82635	0.00217	3.52242
2100	35	35	14	14	0.002665	4.74157	0.00209	3.49477
2101	36	36	15	15	0.00262	4.74939	0.00209	3.56221
2102	36	36	16	16	0.002528	4.75702	0.002091	3.51967
2103	37	37	17	17	0.002574	4.75026	0.002091	3.51254
2104	37	37	18	18	0.002855	4.79635	0.002095	3.54462
2105	38	38	19	19	0.002537	4.75578	0.002089	3.52692
2106	36	36	15	15	0.002576	4.77613	0.002333	3.55155
2107	37	37	16	16	0.002575	4.77325	0.002093	3.50755
2108	37	37	17	17	0.002589	4.74936	0.002105	3.5834
2109	38	38	18	18	0.00255	4.77426	0.002127	3.53345
2110	38	38	19	19	0.002556	4.7742	0.002168	3.50947
2111	39	39	20	18	0.002555	4.83524	0.002088	3.53878
2112	36	36	16	16	0.002536	4.77109	0.002107	3.53355
2113	37	37	15	15	0.002628	4.78157	0.002107	3.58334
2114	37	37	16	16	0.002533	4.78446	0.002091	3.53939
2115	38	38	17	17	0.002551	4.79105	0.002084	3.55088
2116	38	38	18	18	0.00261	4.83284	0.002087	3.52141
2117	39	39	19	19	0.002614	4.77274	0.002127	3.55323
2118	37	37	20	17	0.002606	4.82349	0.002135	3.55369
2119	38	38	16	16	0.002564	4.802	0.002093	3.54262
2120	38	38	17	17	0.002549	4.79645	0.002098	3.56665
2121	39	39	18	18	0.002562	4.79545	0.002161	3.55235
2122	39	39	19	19	0.002562	4.80067	0.002168	3.55649
2123	40	40	20	20	0.002559	4.83809	0.002116	3.54398
2124	36	36	21	18	0.002567	4.80339	0.002097	3.55401
2125	37	37	17	17	0.002555	4.86062	0.002087	3.55955
2126	37	37	16	16	0.002537	4.8019	0.002116	3.53592
2127	38	38	17	17	0.002569	4.81001	0.002092	3.59929
2128	38	38	18	18	0.002524	4.81572	0.002105	3.57233
2129	39	39	19	19	0.002538	4.82169	0.002221	3.56527
2130	37	37	20	19	0.002628	4.86633	0.002098	3.5516
2131	38	38	21	18	0.002556	4.81173	0.002316	3.563
2132	38	38	17	17	0.002585	4.86552	0.002099	3.57368
2133	39	39	18	18	0.002599	4.83364	0.00211	3.54851
2134	39	39	19	19	0.002799	4.81678	0.002123	3.64396
2135	40	40	20	18	0.002537	4.83095	0.002121	3.59592
2136	37	36	21	19	0.002557	4.82481	0.002121	3.57669
2137	38	37	15	15	0.00256	4.91306	0.00209	3.55381
2138	38	37	16	16	0.002568	4.83508	0.002088	3.58633
2139	39	38	17	17	0.002561	4.87766	0.00208	3.58528
2140	39	38	18	18	0.002564	4.84615	0.002151	3.55833
2141	40	39	19	19	0.002689	4.8357	0.002093	3.63364
2142	38	37	20	20	0.002582	4.84738	0.002091	3.59513
2143	39	38	16	16	0.002587	4.83779	0.002088	3.5871
2144	39	38	17	17	0.002585	4.90767	0.002108	3.56085
2145	40	39	18	18	0.002525	4.85964	0.002119	3.60076
2146	40	39	19	19	0.00265	4.88308	0.002137	3.59351
2147	41	40	20	20	0.002544	4.84902	0.002134	3.57413
2148	36	36	21	19	0.002567	4.84913	0.002174	3.64874
2149	37	37	17	17	0.002529	4.8479	0.002099	3.58399

problem5

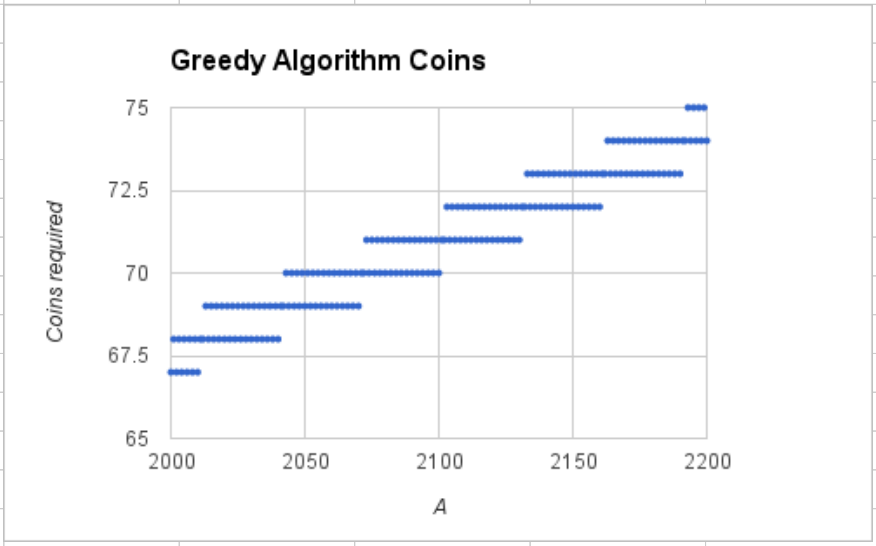
amount	greedy coins v1	dp coins v1	greedy coins v2	dp coins v2	greedy time v1	dp time v1	greedy time v2	dp time v2
2150	37	37	16	16	0.002552	4.86186	0.002125	3.59447
2151	38	38	17	17	0.002524	4.91263	0.002127	3.58541
2152	38	38	18	18	0.002577	4.86896	0.002105	3.59116
2153	39	39	19	19	0.00255	4.91762	0.002078	3.62992
2154	37	37	20	20	0.002579	4.85493	0.002112	3.5908
2155	38	38	21	18	0.002529	4.86414	0.002108	3.65317
2156	38	38	17	17	0.002576	4.8795	0.002127	3.60604
2157	39	39	18	18	0.002532	4.89012	0.002119	3.6054
2158	39	39	19	19	0.002854	4.91268	0.002127	3.59254
2159	40	40	20	20	0.002555	4.90522	0.002135	3.72378
2160	36	36	21	21	0.002712	5.00189	0.002121	3.62728
2161	37	37	22	19	0.002577	4.88349	0.002067	3.62603
2162	37	37	18	18	0.002531	4.89382	0.00213	3.63153
2163	38	38	17	17	0.002523	4.89925	0.002113	3.62482
2164	38	38	18	18	0.002565	4.88991	0.002156	3.68245
2165	39	39	19	19	0.002547	4.89926	0.002088	3.62015
2166	37	37	20	20	0.002595	4.96024	0.002247	3.62371
2167	38	38	21	20	0.00255	4.94747	0.002164	3.63005
2168	38	38	22	19	0.002554	4.89977	0.002075	3.66086
2169	39	39	18	18	0.002533	4.9647	0.002349	3.64062
2170	39	39	19	19	0.002537	4.90342	0.002075	3.6084
2171	40	40	20	20	0.002544	4.9044	0.002126	3.69187
2172	37	37	21	19	0.002563	4.90246	0.002113	3.63994
2173	38	38	22	20	0.002556	4.91048	0.002131	3.63675
2174	38	38	16	16	0.002545	4.95906	0.00217	3.62124
2175	39	39	17	17	0.002589	4.91153	0.002096	3.65004
2176	39	39	18	18	0.002611	4.98222	0.002121	3.64554
2177	40	40	19	19	0.002568	4.91872	0.00216	3.62267
2178	38	38	20	20	0.002839	4.90543	0.002106	3.67795
2179	39	39	21	21	0.002523	4.92037	0.002106	3.65344
2180	39	39	17	17	0.002693	4.93809	0.002116	3.70333
2181	40	40	18	18	0.002808	4.93194	0.002095	3.63318
2182	40	40	19	19	0.002562	4.9173	0.002107	3.65414
2183	41	41	20	20	0.00253	4.97942	0.002137	3.67897
2184	37	37	21	21	0.002551	4.93482	0.002116	3.66983
2185	38	38	22	20	0.002531	4.99242	0.002118	3.63789
2186	38	38	18	18	0.002814	4.92257	0.002174	3.66002
2187	39	39	17	17	0.002534	4.95327	0.002176	3.70431
2188	39	39	18	18	0.002521	4.94779	0.002103	3.67814
2189	40	40	19	19	0.00253	4.95383	0.002118	3.6478
2190	38	38	20	20	0.002623	5.01309	0.002091	3.6617
2191	39	39	21	21	0.002534	4.96183	0.002107	3.68117
2192	39	39	22	19	0.00258	5.00182	0.002106	3.68837
2193	40	40	18	18	0.002593	5.06586	0.002123	3.65662
2194	40	40	19	19	0.002618	4.94497	0.002112	3.71317
2195	41	41	20	20	0.002557	4.96347	0.002142	3.67278
2196	38	37	21	21	0.002588	4.96738	0.002138	3.74838
2197	39	38	22	22	0.002571	4.97251	0.002137	3.68343
2198	39	38	23	20	0.002801	4.96317	0.002098	3.66129
2199	40	39	19	19	0.00255	5.02663	0.002294	3.68828
2200	40	39	18	18	0.002568	4.96749	0.002115	3.696

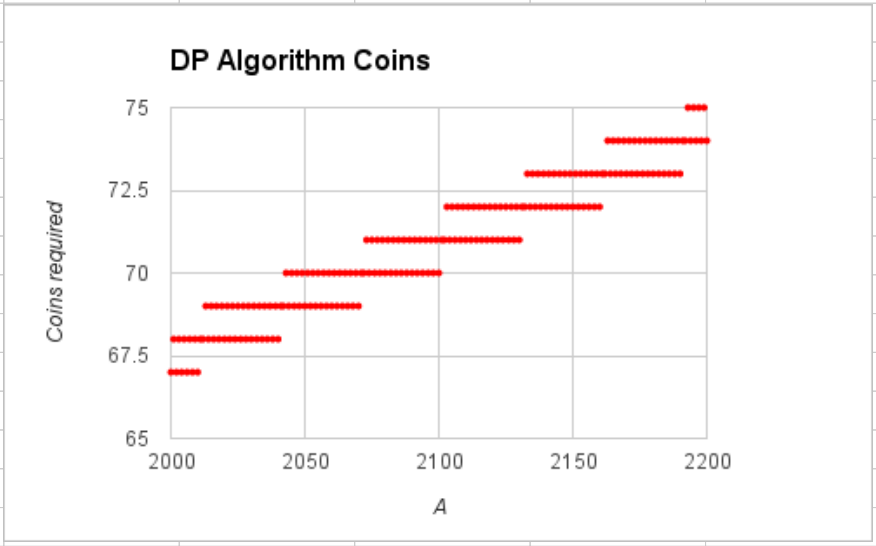


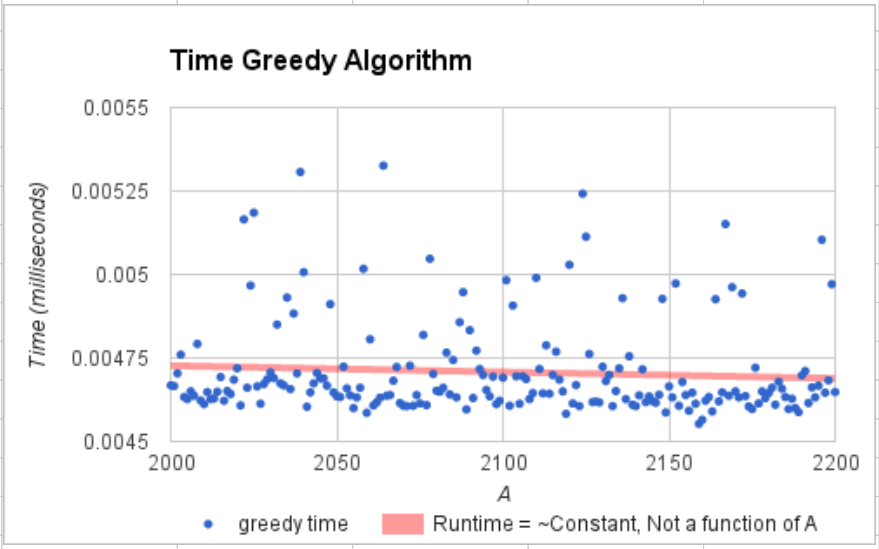


problem6

amount	greedy coins	dp coins	greedy time	dp time
2000	67	67	0.004668	9.14199
2001	68	68	0.004666	9.07287
2002	67	67	0.004704	9.13838
2003	68	68	0.00476	9.09008
2004	67	67	0.004633	9.1965
2005	68	68	0.004627	9.12943
2006	67	67	0.00465	9.25391
2007	68	68	0.004637	9.14449
2008	67	67	0.004792	9.10587
2009	68	68	0.004623	9.1516
2010	67	67	0.004612	9.10162
2011	68	68	0.004648	9.1659
2012	68	68	0.004627	9.12151
2013	69	69	0.004629	9.18581
2014	68	68	0.004649	9.11992
2015	69	69	0.004693	9.19712
2016	68	68	0.004622	9.14917
2017	69	69	0.004651	9.19147
2018	68	68	0.004642	9.14143
2019	69	69	0.004685	9.21675
2020	68	68	0.004719	9.15636
2021	69	69	0.004608	9.20107
2022	68	68	0.005165	9.17452
2023	69	69	0.004661	9.18765
2024	68	68	0.004967	9.20762
2025	69	69	0.005185	9.15405
2026	68	68	0.004665	9.23819
2027	69	69	0.004613	9.18618
2028	68	68	0.004672	9.23532
2029	69	69	0.004686	9.22774
2030	68	68	0.004707	9.29129
2031	69	69	0.00469	9.34514
2032	68	68	0.00485	9.33325
2033	69	69	0.004674	9.23718
2034	68	68	0.004668	9.3331
2035	69	69	0.004931	9.28317
2036	68	68	0.004657	9.36545
2037	69	69	0.004883	9.34838
2038	68	68	0.004704	9.27761
2039	69	69	0.005307	9.34114
2040	68	68	0.005007	9.2732
2041	69	69	0.004604	9.35948
2042	69	69	0.004647	9.34525
2043	70	70	0.004674	9.36761
2044	69	69	0.004705	9.36521

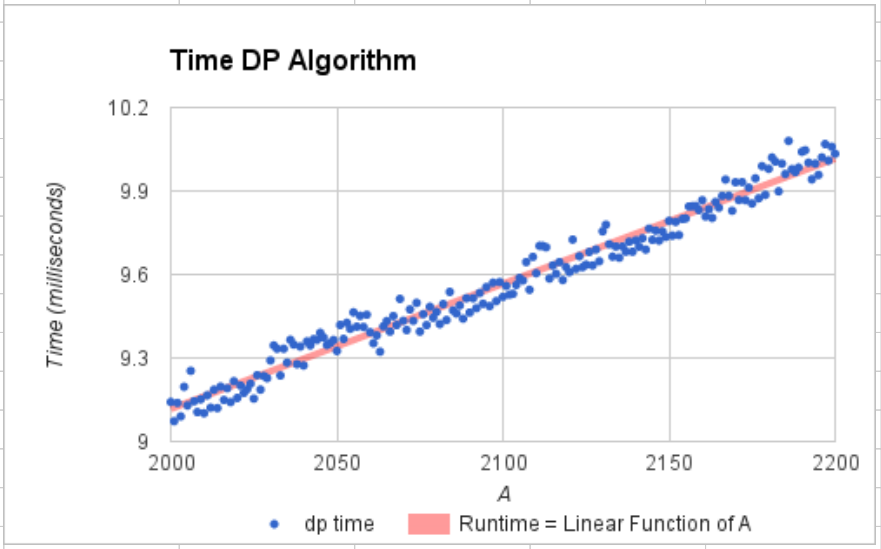






problem6

amount	greedy coins	dp coins	greedy time	dp time
2045	70	70	0.004689	9.39055
2046	69	69	0.004689	9.37382
2047	70	70	0.004667	9.34666
2048	69	69	0.004911	9.35353
2049	70	70	0.004647	9.36467
2050	69	69	0.004635	9.32516
2051	70	70	0.004633	9.41865
2052	69	69	0.004724	9.36851
2053	70	70	0.004659	9.42641
2054	69	69	0.004638	9.40486
2055	70	70	0.0046	9.46423
2056	69	69	0.004632	9.41257
2057	70	70	0.004661	9.45141
2058	69	69	0.005017	9.41142
2059	70	70	0.004586	9.45545
2060	69	69	0.004806	9.39254
2061	70	70	0.004608	9.35301
2062	69	69	0.004617	9.38122
2063	70	70	0.004631	9.3226
2064	69	69	0.005326	9.41352
2065	70	70	0.004637	9.43236
2066	69	69	0.004639	9.39467
2067	70	70	0.004682	9.45049
2068	69	69	0.004723	9.41853
2069	70	70	0.004615	9.51238
2070	69	69	0.004607	9.4337
2071	70	70	0.004606	9.40039
2072	70	70	0.004727	9.47459
2073	71	71	0.004607	9.43358
2074	70	70	0.004639	9.49813
2075	71	71	0.004614	9.3948
2076	70	70	0.004819	9.45741
2077	71	71	0.004609	9.41814
2078	70	70	0.005047	9.48418
2079	71	71	0.004702	9.44443
2080	70	70	0.004652	9.46565
2081	71	71	0.004649	9.42179
2082	70	70	0.004661	9.49367
2083	71	71	0.004766	9.43641
2084	70	70	0.004641	9.53738
2085	71	71	0.004744	9.47083
2086	70	70	0.004632	9.45993
2087	71	71	0.004857	9.48902
2088	70	70	0.004947	9.44133
2089	71	71	0.004596	9.51466



problem6

amount	greedy coins	dp coins	greedy time	dp time					
2090	70	70	0.004833	9.46325					
2091	71	71	0.00463	9.5145					
2092	70	70	0.004772	9.47885					
2093	71	71	0.004717	9.53379					
2094	70	70	0.004699	9.49491					
2095	71	71	0.004654	9.5548					
2096	70	70	0.004635	9.48636					
2097	71	71	0.004694	9.57038					
2098	70	70	0.004612	9.50449					
2099	71	71	0.004622	9.57274					
2100	70	70	0.004691	9.51959					
2101	71	71	0.004983	9.55905					
2102	71	71	0.004607	9.52749					
2103	72	72	0.004907	9.53038					
2104	71	71	0.004695	9.56253					
2105	72	72	0.004613	9.58728					
2106	71	71	0.004695	9.57842					
2107	72	72	0.004687	9.64434					
2108	71	71	0.004627	9.54503					
2109	72	72	0.004645	9.66319					
2110	71	71	0.00499	9.6046					
2111	72	72	0.004717	9.70295					
2112	71	71	0.004644	9.70272					
2113	72	72	0.004788	9.69787					
2114	71	71	0.004643	9.58558					
2115	72	72	0.004699	9.6326					
2116	71	71	0.004769	9.60247					
2117	72	72	0.004685	9.64459					
2118	71	71	0.00465	9.58014					
2119	72	72	0.004583	9.62628					
2120	71	71	0.005029	9.60836					
2121	72	72	0.004614	9.7254					
2122	71	71	0.004669	9.62013					
2123	72	72	0.004606	9.66556					
2124	71	71	0.005242	9.62647					
2125	72	72	0.005113	9.63535					
2126	71	71	0.004762	9.681					
2127	72	72	0.004618	9.63262					
2128	71	71	0.004619	9.68954					
2129	72	72	0.004617	9.64742					
2130	71	71	0.004724	9.75501					
2131	72	72	0.004681	9.77895					
2132	72	72	0.004699	9.70856					
2133	73	73	0.004606	9.66388					
2134	72	72	0.004651	9.70047					

problem6

amount	greedy coins	dp coins	greedy time	dp time					
2135	73	73	0.004719	9.66018					
2136	72	72	0.004929	9.70051					
2137	73	73	0.004627	9.68195					
2138	72	72	0.004755	9.71761					
2139	73	73	0.00461	9.68161					
2140	72	72	0.004606	9.72251					
2141	73	73	0.004638	9.69982					
2142	72	72	0.004716	9.73006					
2143	73	73	0.004618	9.68934					
2144	72	72	0.004635	9.76477					
2145	73	73	0.004621	9.72464					
2146	72	72	0.004616	9.75743					
2147	73	73	0.00464	9.72187					
2148	72	72	0.004927	9.75469					
2149	73	73	0.004587	9.73494					
2150	72	72	0.004665	9.79246					
2151	73	73	0.004632	9.73945					
2152	72	72	0.004973	9.78896					
2153	73	73	0.004607	9.74168					
2154	72	72	0.004678	9.79955					
2155	73	73	0.004639	9.8014					
2156	72	72	0.004592	9.84439					
2157	73	73	0.004646	9.84563					
2158	72	72	0.004613	9.84622					
2159	73	73	0.004553	9.83154					
2160	72	72	0.004565	9.86692					
2161	73	73	0.004622	9.80824					
2162	73	73	0.004633	9.83452					
2163	74	74	0.00459	9.80326					
2164	73	73	0.004926	9.86044					
2165	74	74	0.00462	9.84001					
2166	73	73	0.004647	9.88212					
2167	74	74	0.005151	9.94104					
2168	73	73	0.004637	9.88266					
2169	74	74	0.004962	9.82895					
2170	73	73	0.00465	9.93141					
2171	74	74	0.004632	9.86744					
2172	73	73	0.004943	9.93177					
2173	74	74	0.004636	9.86703					
2174	73	73	0.004605	9.91215					
2175	74	74	0.004597	9.85472					
2176	73	73	0.004721	9.94546					
2177	74	74	0.004614	9.87384					
2178	73	73	0.00465	9.9893					
2179	74	74	0.00463	9.88603					

problem6

amount	greedy coins	dp coins	greedy time	dp time					
2180	73	73	0.004645	9.97963					
2181	74	74	0.004661	10.02038					
2182	73	73	0.00461	10.00576					
2183	74	74	0.004679	9.89876					
2184	73	73	0.004658	9.99825					
2185	74	74	0.004633	9.96107					
2186	73	73	0.004597	10.07991					
2187	74	74	0.004628	9.97831					
2188	73	73	0.0046	9.96791					
2189	74	74	0.004588	9.98425					
2190	73	73	0.004698	10.04097					
2191	74	74	0.004711	10.04625					
2192	74	74	0.004615	10.00162					
2193	75	75	0.004662	9.94201					
2194	74	74	0.004632	9.99835					
2195	75	75	0.004667	9.95757					
2196	74	74	0.005104	10.02062					
2197	75	75	0.004646	10.06864					
2198	74	74	0.004683	10.00938					
2199	75	75	0.004971	10.05886					
2200	74	74	0.004648	10.03387					

LowValues

Slow Algorithm

amount	[1, 5, 10, 25, 50]:Coins	[1, 5, 10, 25, 50]:Time	[1, 2, 6, 12, 24, 48, 60]:Coins	[1, 2, 6, 12, 24, 48, 60]:Time	[1, 6, 13, 37, 150]:Coins	[1, 6, 13, 37, 150]:Time	[1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30]:Coins	[1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30]:Time
10	1	0.008	3	0.294	5	0.518	1	0.003
11	2	0.63	4	0.549	6	1.006	2	0.075
12	3	1.133	1	0.002	2	1.842	1	0.003
13	4	2.083	2	1.015	1	0.002	2	0.146
14	5	4.491	2	2.237	2	3.759	1	0.003
15	2	8.619	3	4.437	3	7.883	2	0.293
16	3	16.725	3	8.446	4	14.802	1	0.002
17	4	33.34	4	16.084	5	29.587	2	0.579
18	5	65.313	2	32.174	3	59.23	1	0.003
19	6	128.785	3	63.587	2	118.774	2	1.08
20	2	258.035	3	122.771	3	229.5	1	0.002
21	3	502.905	4	251.62	4	458.397	2	2.128
22	4	1008.897	4	499.097	5	915.975	1	0.002
23	5	2002.284	5	987.249	6	1818.773	2	4.098
24	6	4021.655	1	0.002	4	3654.59	1	0.003
25	1	0.002	2	1974.664	3	7298.311	2	8.223
26	2	8031.583	2	3965.648	2	14554.762	1	0.003
27	3	16503.043	3	8529.867	3	30064.505	2	18.04
28	4	33163.29	3	16257.053	4	58806.331	1	0.006
29	5	65255.188	4	32021.954	5	116771.299	2	35.518
30	2	128.766689	2	62.3953	5	231.93289	1	0.000004
35	2	4072.610423	5	1990.510881	6	7411.213062	3	1.04513

DP Algorithm

amount	[1, 5, 10, 25, 50]:Coins	[1, 5, 10, 25, 50]:Time	[1, 2, 6, 12, 24, 48, 60]:Coins	[1, 2, 6, 12, 24, 48, 60]:Time	[1, 6, 13, 37, 150]:Coins	[1, 6, 13, 37, 150]:Time	[1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30]:Coins	[1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30]:Time
10	1	0.01099	3	0.014	5	0.012	1	0.01873
11	2	0.01376	4	0.01596	6	0.01287	2	0.0202
12	3	0.01455	1	0.01752	2	0.01348	1	0.02139
13	4	0.0161	2	0.01845	1	0.01421	2	0.02458
14	5	0.01859	2	0.02111	2	0.01598	1	0.02605
15	2	0.01843	3	0.0215	3	0.01635	2	0.02863
16	3	0.02022	3	0.02543	4	0.01941	1	0.03285
17	4	0.02096	4	0.02503	5	0.01894	2	0.03487
18	5	0.02132	2	0.02733	3	0.02053	1	0.0378
19	6	0.02354	3	0.03055	2	0.02184	2	0.04351
20	2	0.02371	3	0.0289	3	0.02302	1	0.04429
21	3	0.02502	4	0.03071	4	0.02522	2	0.05124
22	4	0.02803	4	0.0327	5	0.02568	1	0.05064
23	5	0.02831	5	0.03393	6	0.02622	2	0.05642
24	6	0.02862	1	0.03757	4	0.02895	1	0.05852
25	1	0.0295	2	0.03697	3	0.02892	2	0.06388
26	2	0.03197	2	0.04065	2	0.02943	1	0.06531
27	3	0.03315	3	0.04313	3	0.03267	2	0.07273
28	4	0.03466	3	0.0427	4	0.03246	1	0.07688
29	5	0.04022	4	0.04646	5	0.03384	2	0.07857
30	2	0.03878	2	0.04668	5	0.03874	1	0.08696
35	2	0.04848	5	0.06122	6	0.04483	3	0.11115

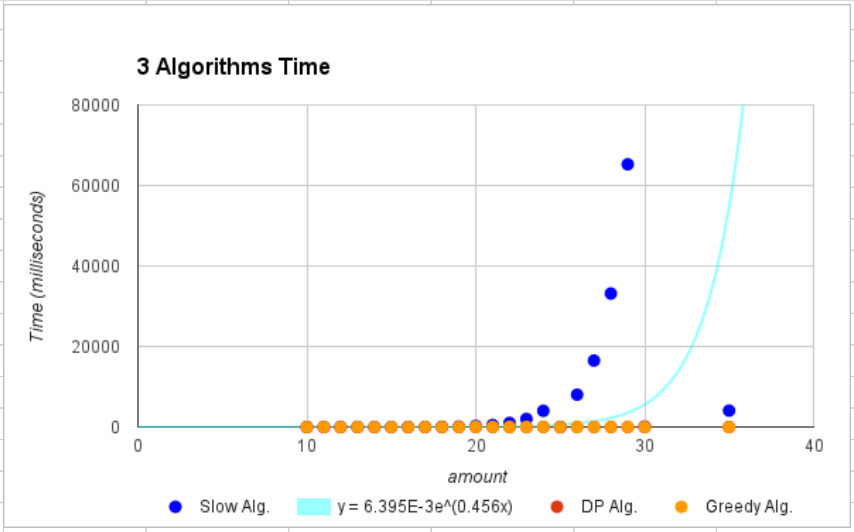
LowValues

Greedy Algorithm

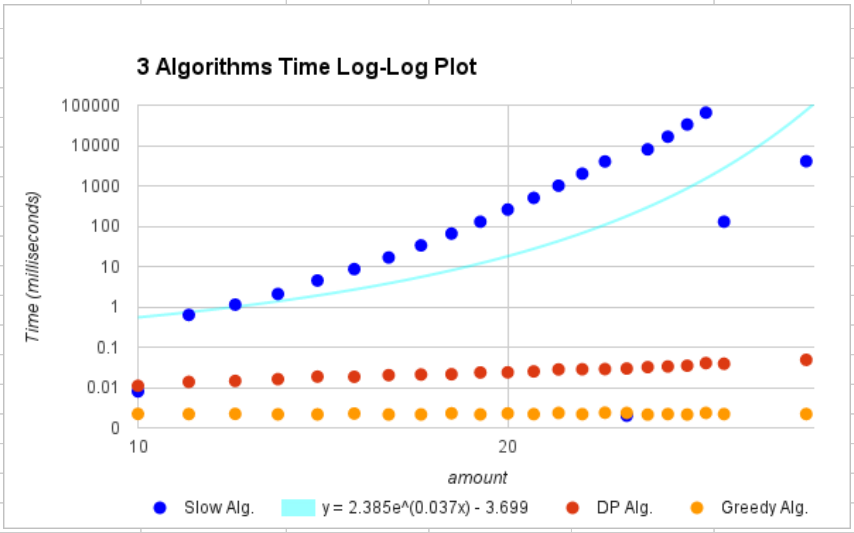
amount	[1, 5, 10, 25, 50]:Coins	[1, 5, 10, 25, 50]:Time	[1, 2, 6, 12, 24, 48, 60]:Coins	[1, 2, 6, 12, 24, 48, 60]:Time	[1, 6, 13, 37, 150]:Coins	[1, 6, 13, 37, 150]:Time	[1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30]:Coins	[1, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30]:Time
10	1	0.002211	3	0.002641	5	0.002167	1	0.005297
11	2	0.002194	4	0.002679	6	0.002238	2	0.005189
12	3	0.002216	1	0.00283	2	0.00226	1	0.00483
13	4	0.002161	2	0.002624	1	0.002162	2	0.004793
14	5	0.002153	2	0.002656	2	0.002125	1	0.005272
15	2	0.002258	3	0.002632	3	0.002163	2	0.005031
16	3	0.002141	3	0.002704	4	0.002116	1	0.004928
17	4	0.002144	4	0.00292	5	0.002324	2	0.00488
18	5	0.002291	2	0.002804	3	0.002258	1	0.004814
19	6	0.002149	3	0.002685	2	0.002283	2	0.005025
20	2	0.002282	3	0.002787	3	0.002086	1	0.004758
21	3	0.002182	4	0.002596	4	0.002121	2	0.004896
22	4	0.002341	4	0.002943	5	0.002111	1	0.004979
23	5	0.00218	5	0.002921	6	0.002363	2	0.005284
24	6	0.002371	1	0.002926	4	0.002366	1	0.005191
25	1	0.002338	2	0.002825	3	0.00235	2	0.004895
26	2	0.002132	2	0.002647	2	0.00218	1	0.005055
27	3	0.002192	3	0.002759	3	0.002225	2	0.004858
28	4	0.002138	3	0.00262	4	0.002165	1	0.004757
29	5	0.002346	4	0.002733	5	0.002224	2	0.004945
30	2	0.002187	2	0.002612	5	0.002183	1	0.004741
35	2	0.002196	5	0.002606	6	0.00217	3	0.005238

Problem 1 time comparisons

Amount	Slow Alg.	DP Alg.	Greedy Alg.
10	0.008	0.01099	0.002211
11	0.63	0.01376	0.002194
12	1.133	0.01455	0.002216
13	2.083	0.0161	0.002161
14	4.491	0.01859	0.002153
15	8.619	0.01843	0.002258
16	16.725	0.02022	0.002141
17	33.34	0.02096	0.002144
18	65.313	0.02132	0.002291
19	128.785	0.02354	0.002149
20	258.035	0.02371	0.002282
21	502.905	0.02502	0.002182
22	1008.897	0.02803	0.002341
23	2002.284	0.02831	0.00218
24	4021.655	0.02862	0.002371
25	0.002	0.0295	0.002338
26	8031.583	0.03197	0.002132
27	16503.043	0.03315	0.002192
28	33163.29	0.03466	0.002138
29	65255.188	0.04022	0.002346
30	128.766689	0.03878	0.002187
35	4072.610423	0.04848	0.002196

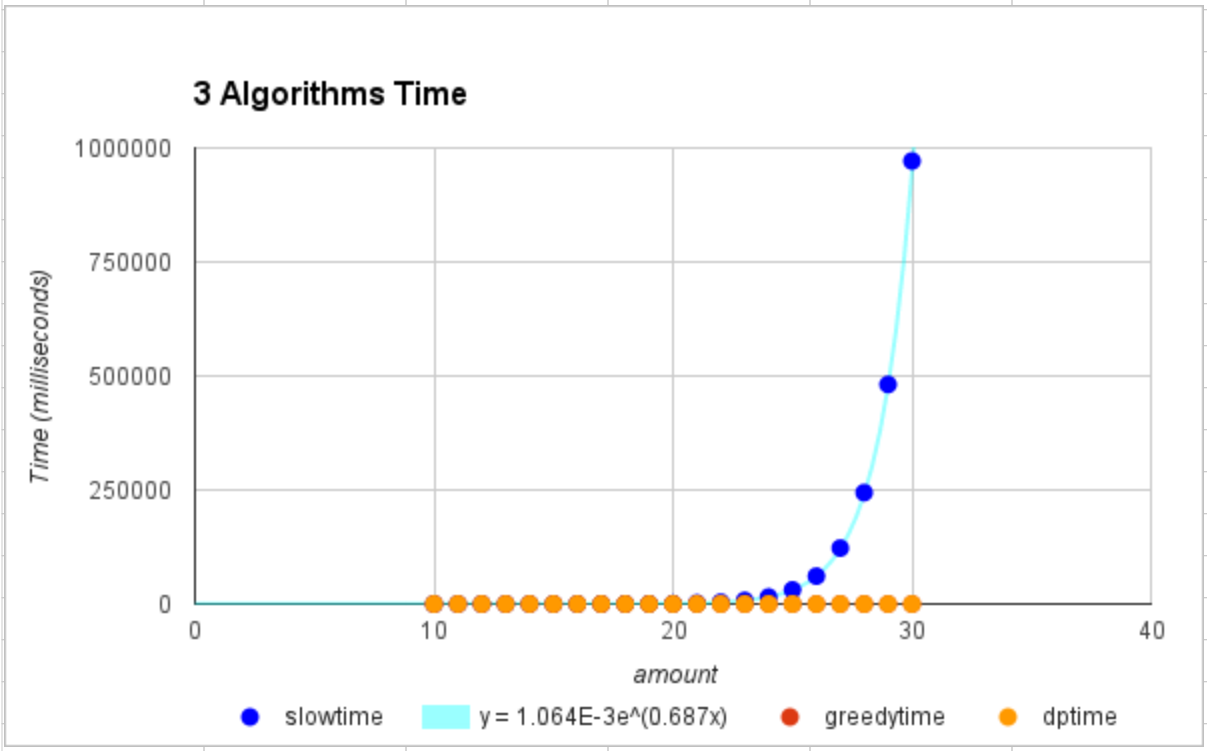


LowValues


3 Algorithms Time Log-Log Plot
100000
10000
1000
100
10
1
0.1
0.01
0
Time (milliseconds)
10
20
amount
Slow Alg.
y = 2.385e^(0.037x) - 3.699
DP Alg.
Greedy Alg.

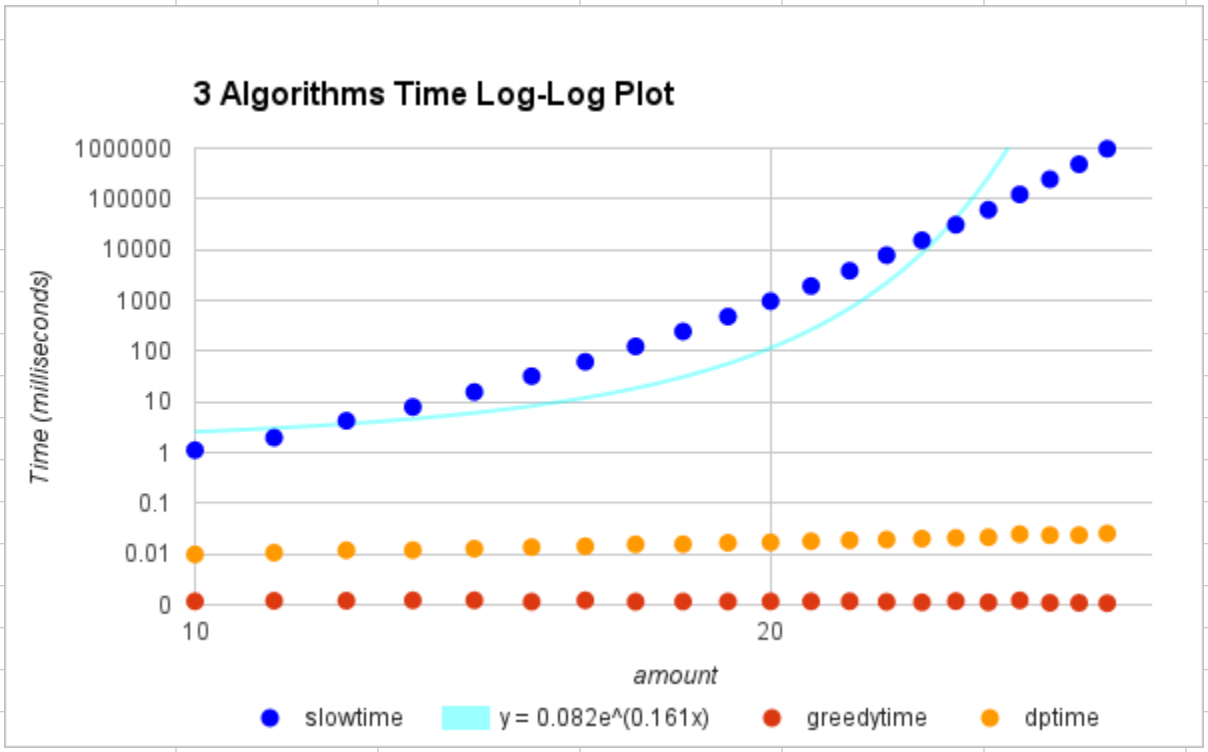
Penny Only

amount	slowtime	greedytime	dptime
10	1.117	0.00117	0.00984
11	1.967	0.001199	0.01065
12	4.241	0.001202	0.01189
13	7.931	0.001225	0.01196
14	15.672	0.001229	0.01278
15	32.054	0.001151	0.01366
16	61.709	0.001228	0.01426
17	124.06	0.001149	0.01555
18	243.982	0.00116	0.01566
19	480.214	0.001158	0.01667
20	963.19	0.001165	0.01704
21	1916.949	0.001166	0.01793
22	3838.9	0.001171	0.01869
23	7768.46	0.001144	0.01929
24	15313.587	0.001117	0.02015
25	30947.424	0.001178	0.02086
26	61082.748	0.001116	0.02142
27	122581.667	0.001222	0.02469
28	244188.997	0.001093	0.02356
29	481349.882	0.001092	0.02366
30	971419.625	0.001075	0.02547



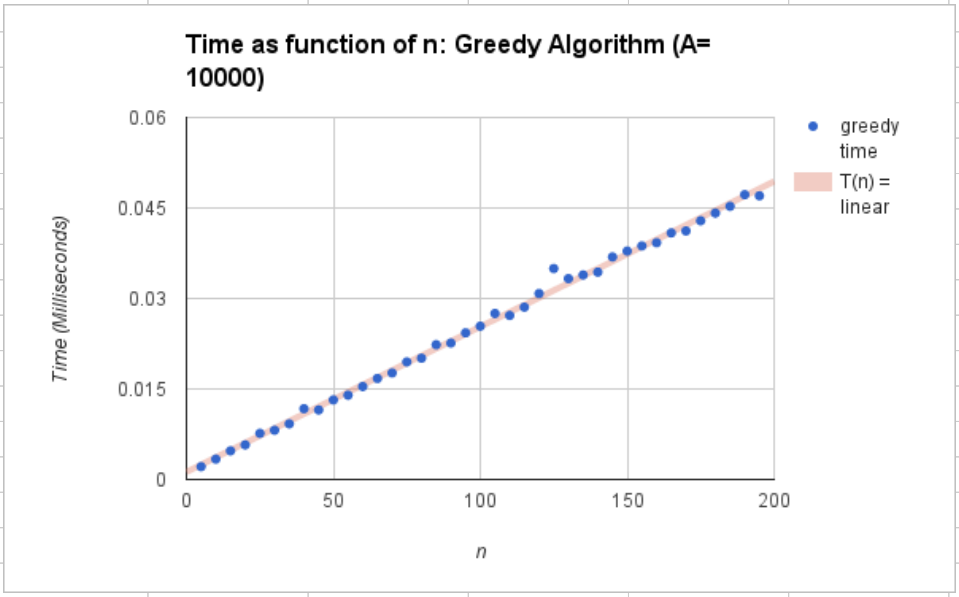
Penny Only

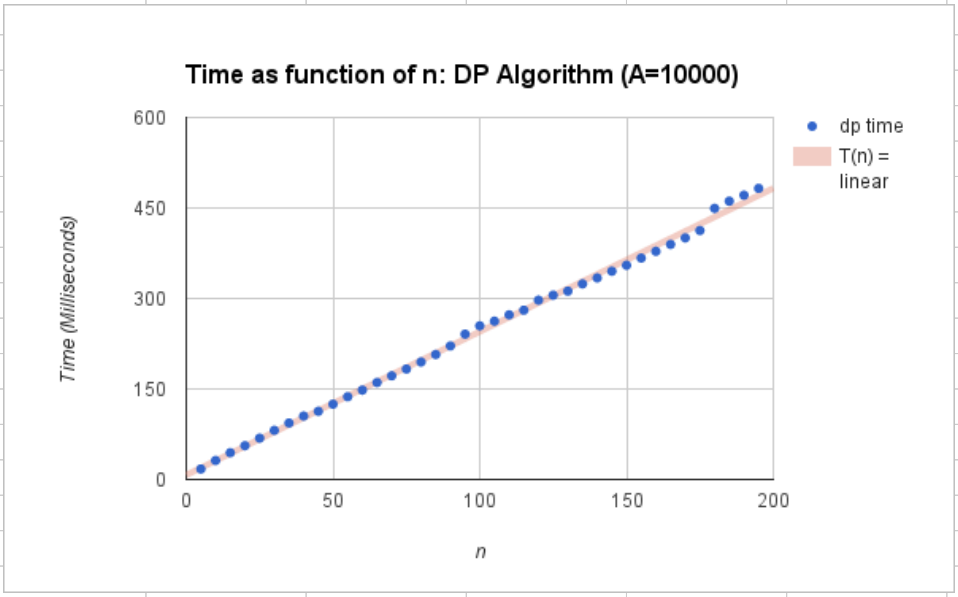
amount	slowtime	greedytime	dptime			



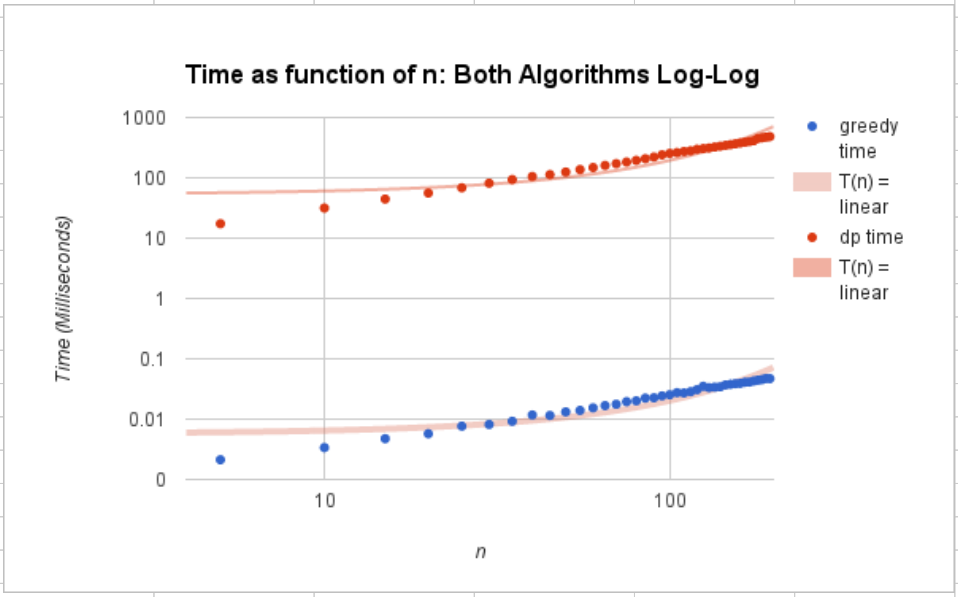
Problem 8

n	greedy time	dp time
5	0.002125	17.3555
10	0.003371	31.5552
15	0.004746	44.3823
20	0.00573	56.0113
25	0.007645	68.4563
30	0.008162	81.4617
35	0.009213	93.587
40	0.011733	105.1997
45	0.011521	113.0595
50	0.013189	124.8159
55	0.013972	137.1366
60	0.015398	148.1501
65	0.016747	160.6969
70	0.017662	171.8906
75	0.019478	183.0565
80	0.020109	194.7379
85	0.022343	207.1717
90	0.022623	221.4406
95	0.024308	240.8097
100	0.025404	254.6596
105	0.027513	262.4102
110	0.027187	272.9284
115	0.028556	280.623
120	0.030817	297.3106
125	0.034971	305.5011
130	0.033297	312.2042
135	0.033903	324.2806
140	0.034365	334.1839
145	0.036891	345.2021
150	0.037856	355.1331
155	0.038717	367.0361
160	0.039239	378.2038
165	0.040875	389.6809
170	0.041195	400.499
175	0.042885	412.6342
180	0.044148	449.4021
185	0.045284	461.3423
190	0.047214	471.194
195	0.047025	482.5027





Problem 8


Time as function of n: Both Algorithms Log-Log
Time (Milliseconds)
1000
100
10
1
0.1
0.01
0
10
100
n
greedy time
T(n) = linear
dp time
T(n) = linear