# Critical Time Analysis

In the first section of this notebook, we determine the critical time between treatments needed for periodic cycling. We do this for different species (with different parameters) of human-infecting and vet-infecting soil-transmitted helminths. In the second section “Categorizing Behaviors”, we determine the qualitative behavior of parameter sets inputted from a csv file.

Model equations

In[∘]:=
```
fW = β * Nhost * L - (μh + μa) * W;
fL = 0.5 * W * λ - (μl + β * Nhost) * L;
```

Eigenvalues, eigenvectors

In[∘]:=
```
Jacob = Simplify[D[{fW, fL}, {{W, L}}]];
```

In[∘]:=
```
eigensys = Eigensystem[Jacob]
λ1 = eigensys[[1]][[1]]
λ2 = eigensys[[1]][[2]]
```

Out[∘]=

$$\left\{\left\{0.5\left(-1.\,\text{Nhost}\,\beta-1.\,\mu a-1.\,\mu h-1.\,\mu l+1.\sqrt{\left((1.\,\text{Nhost}\,\beta+1.\,\mu a+1.\,\mu h+1.\,\mu l)^2-4.\,(-0.5\,\text{Nhost}\,\beta\,\lambda+1.\,\text{Nhost}\,\beta\,\mu a+1.\,\text{Nhost}\,\beta\,\mu h+1.\,\mu a\,\mu l+1.\,\mu h\,\mu l)\right)}\right),\right.\right.$$

$$-0.5\left(1.\,\text{Nhost}\,\beta+1.\,\mu a+1.\,\mu h+1.\,\mu l+1.\sqrt{\left((1.\,\text{Nhost}\,\beta+1.\,\mu a+1.\,\mu h+1.\,\mu l)^2-4.\,(-0.5\,\text{Nhost}\,\beta\,\lambda+1.\,\text{Nhost}\,\beta\,\mu a+1.\,\text{Nhost}\,\beta\,\mu h+1.\,\mu a\,\mu l+1.\,\mu h\,\mu l)\right)}\right)\Bigg\},$$

$$\left\{\left\{\frac{1}{\lambda}1.\left(1.\,\text{Nhost}\,\beta-1.\,\mu a-1.\,\mu h+1.\,\mu l+1.\sqrt{\left((1.\,\text{Nhost}\,\beta+1.\,\mu a+1.\,\mu h+1.\,\mu l)^2-4.\,(-0.5\,\text{Nhost}\,\beta\,\lambda+1.\,\text{Nhost}\,\beta\,\mu a+1.\,\text{Nhost}\,\beta\,\mu h+1.\,\mu a\,\mu l+1.\,\mu h\,\mu l)\right)}\right),1.\right\},\right.$$

$$\left.\left.\left\{-\frac{1}{\lambda}1.\left(-1.\,\text{Nhost}\,\beta+1.\,\mu a+1.\,\mu h-1.\,\mu l+1.\sqrt{\left((1.\,\text{Nhost}\,\beta+1.\,\mu a+1.\,\mu h+1.\,\mu l)^2-4.\,(-0.5\,\text{Nhost}\,\beta\,\lambda+1.\,\text{Nhost}\,\beta\,\mu a+1.\,\text{Nhost}\,\beta\,\mu h+1.\,\mu a\,\mu l+1.\,\mu h\,\mu l)\right)}\right),1.\right\}\right\}\right\}$$

Out[∘]=

$$0.5\left(-1.\,\text{Nhost}\,\beta-1.\,\mu a-1.\,\mu h-1.\,\mu l+1.\sqrt{\left((1.\,\text{Nhost}\,\beta+1.\,\mu a+1.\,\mu h+1.\,\mu l)^2-4.\,(-0.5\,\text{Nhost}\,\beta\,\lambda+1.\,\text{Nhost}\,\beta\,\mu a+1.\,\text{Nhost}\,\beta\,\mu h+1.\,\mu a\,\mu l+1.\,\mu h\,\mu l)\right)}\right)$$

Out[∘]=

$$-0.5\left(1.\,\text{Nhost}\,\beta+1.\,\mu a+1.\,\mu h+1.\,\mu l+1.\sqrt{\left((1.\,\text{Nhost}\,\beta+1.\,\mu a+1.\,\mu h+1.\,\mu l)^2-4.\,(-0.5\,\text{Nhost}\,\beta\,\lambda+1.\,\text{Nhost}\,\beta\,\mu a+1.\,\text{Nhost}\,\beta\,\mu h+1.\,\mu a\,\mu l+1.\,\mu h\,\mu l)\right)}\right)$$

```
In[•]:= eigenvec = Eigenvectors[Jacob];
inveigenvec = Inverse[Transpose[eigenvec]];
c1c2 = inveigenvec.{W0, L0};
c1 = c1c2[[1]];
c2 = c1c2[[2]];
```

## Solution to critical equation

A critical frequency of MDA exists only if without MDA, the worm population blows up (i.e., requires a negative determinant of the Jacobian).

Ascaris

```
In[•]:= IC = {W0 → 100, L0 → 3000};
ParmsAsc = {β → 0.04 / 365, Nhost → 0.000123,
    μh → 0.0167 / 365, μa → 1 / 365, λ → 3.65 * 10^6 / 365, μl → 6 / 365};
Tr[Jacob] /. ParmsAsc
Det[Jacob] /. ParmsAsc
λ1 /. ParmsAsc
λ2 /. ParmsAsc
eigenvec /. ParmsAsc
Out[•]= -0.0192238
Out[•]= -0.0000216085
Out[•]= 0.00106504
Out[•]= -0.0202889
Out[•]= {{3.50068 × 10^-6, 1.}, {-7.70104 × 10^-7, 1.}}
```

Hookworm

```
In[•]:= IC = {W0 → 100, L0 → 3000};
ParmsHk = {β → 0.35 / 365, Nhost → 0.000123,
    μh → 0.0167 / 365, μa → 0.5 / 365, λ → 1.095 * 10^6 / 365, μl → 30 / 365};
Tr[Jacob] /. ParmsHk
Det[Jacob] /. ParmsHk
eigenvec /. ParmsHk
Out[•]= -0.0836075
Out[•]= -0.0000605656
Out[•]= {{0.0000552734, 1.}, {-1.42257 × 10^-6, 1.}}
```

Trichuris

In[∘]:=
```
IC = {W0 → 100, L0 → 3000};
ParmsTri = {β → 0.62 / 365, Nhost → 0.000123,
   μh → 0.0167 / 365, μa → 1 / 365, λ → 7.3 * 10^5 / 365, μl → 18.25 / 365};
Tr[Jacob] /. ParmsTri
Det[Jacob] /. ParmsTri
eigenvec /. ParmsTri
```

Out[∘]= -0.0527857

Out[∘]= -0.000069657

Out[∘]= $\{\{0.0000512884, 1.\}, \{-4.07366\times 10^{-6}, 1.\}\}$

TcolubNZ

In[∘]:=
```
IC = {W0 → 100, L0 → 3000};
ParmsTc = {β → 511 / 365, Nhost → 0.0015,
   μh → 0.1 / 365, μa → 17.89 / 365, λ → 3285 / 365, μl → 8.4 / 365};
Tr[Jacob] /. ParmsTc
Det[Jacob] /. ParmsTc
λ1 /. ParmsTc
λ2 /. ParmsTc
eigenvec /. ParmsTc
```

Out[∘]= -0.0744014

Out[∘]= -0.0082122

Out[∘]= 0.060759

Out[∘]= -0.13516

Out[∘]= {{0.0190828, 1.}, {-0.0244548, 1.}}

TcolubAUS

```
In[•]:= IC = {W0 → 100, L0 → 3000};
ParmsTcA = {β → 1124.2 / 365, Nhost → 0.001,
   μh → 0.1 / 365, μa → 17.89 / 365, λ → 2463.75 / 365, μl → 28.4 / 365};
Tr[Jacob] /. ParmsTcA
Det[Jacob] /. ParmsTcA
λ1 /. ParmsTcA
λ2 /. ParmsTcA
eigenvec /. ParmsTcA

Out[•]= -0.130176

Out[•]= -0.00640821

Out[•]= 0.038085

Out[•]= -0.168261

Out[•]= {{0.0352513, 1.}, {-0.0258882, 1.}}
```

OcircNZ

```
In[•]:= IC = {W0 → 100, L0 → 3000};
ParmsOc = {β → 1080.4 / 365, Nhost → 0.0015,
   μh → 0.1 / 365, μa → 40.9 / 365, λ → 4380 / 365, μl → 8.49 / 365};
Tr[Jacob] /. ParmsOc
Det[Jacob] /. ParmsOc
eigenvec /. ParmsOc

Out[•]= -0.140029

Out[•]= -0.0235285

Out[•]= {{0.0210499, 1.}, {-0.0351546, 1.}}
```

OcircUK

```
In[•]:= IC = {W0 → 100, L0 → 3000};
ParmsOcUK = {β → 956.3 / 365, Nhost → 0.002,
   μh → 0.1 / 365, μa → 14.9 / 365, λ → 1752 / 365, μl → 11.12 / 365};
Tr[Jacob] /. ParmsOcUK
Det[Jacob] /. ParmsOcUK
eigenvec /. ParmsOcUK

Out[•]= -0.0768016

Out[•]= -0.0111086

Out[•]= {{0.0456167, 1.}, {-0.0478626, 1.}}
```

HcontNZ

In[•]:=
```
IC = {W0 → 100, L0 → 3000};
ParmsHc = {β → 584 / 365, Nhost → 0.0015,
   μh → 0.1 / 365, μa → 3.65 / 365, λ → 11 558.5 / 365, μl → 620.87 / 365};
Tr[Jacob] /. ParmsHc
Det[Jacob] /. ParmsHc
λ1 /. ParmsHc
λ2 /. ParmsHc
eigenvec /. ParmsHc
```

Out[•]= -1.71369

Out[•]= -0.0204997

Out[•]= 0.01188

Out[•]= -1.72557

Out[•]= {{0.108333, 1.}, {-0.00139918, 1.}}

HcontAUS

In[•]:=
```
IC = {W0 → 100, L0 → 3000};
ParmsHcA = {β → 890.6 / 365, Nhost → 0.001,
   μh → 0.1 / 365, μa → 3.65 / 365, λ → 5501.8 / 365, μl → 230.36 / 365};
Tr[Jacob] /. ParmsHcA
Det[Jacob] /. ParmsHcA
eigenvec /. ParmsHcA
```

Out[•]= -0.643837

Out[•]= -0.0118804

Out[•]= {{0.0864455, 1.}, {-0.00374512, 1.}}

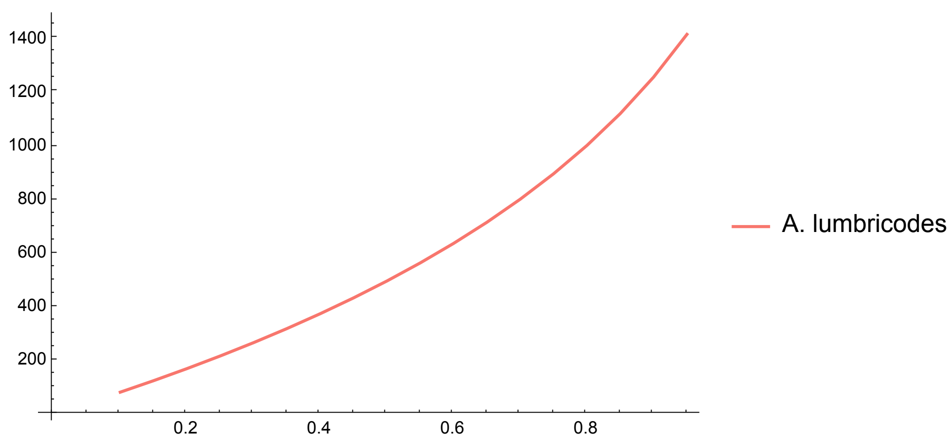
## Find Critical T as a function of mu for each species

Ascaris

In[•]:=
```
ParmsAsc = {β → 0.04 / 365, Nhost → 0.000123,
   μh → 0.0167 / 365, μa → 1 / 365, λ → 3.65 * 10^6 / 365, μl → 6 / 365};
critTvec = {};
muvec = {};
For[mu = 0.1, mu ≤ .99, mu += .05,
 AppendTo[muvec, mu];
 AppendTo[critTvec, T /. NSolve[(Exp[λ2 * T] - 1) / (1 - Exp[λ1 * T]) ==
       (eigenvec[[2]][[1]] * ((1 - μ) * Exp[λ2 * T] - 1)) / (eigenvec[[1]][[1]] *
          (1 - (1 - μ) * Exp[λ1 * T])) /. μ → mu /. ParmsAsc, T, Reals][[1]]
 ]
]
critTvec
pAsc = ListLinePlot[Transpose@{muvec, critTvec},
  PlotStyle → RGBColor["#F8766D"], PlotLegends → {"A. lumbricodes"}]
```

Out[•]=
```
{79.9298, 122.862, 167.888, 215.168, 264.923, 317.43, 373.026, 432.109, 495.154,
 562.735, 635.559, 714.509, 800.712, 895.637, 1001.25, 1120.26, 1256.58, 1416.12}
```

Out[•]=

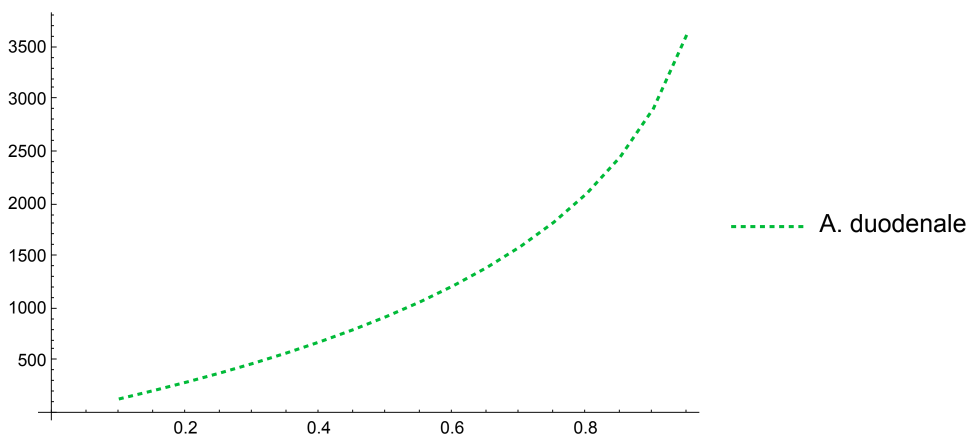



Hookworm

In[∘]:=
```
ParmsHk = {β → 0.35 / 365, Nhost → 0.000123,
   μh → 0.0167 / 365, μa → 0.5 / 365, λ → 1.095 * 10^6 / 365, μl → 30 / 365};
critTvec = {};
muvec = {};
For[mu = 0.1, mu ≤ .99, mu += .05,
 AppendTo[muvec, mu];
 AppendTo[critTvec, T /. NSolve[(Exp[λ2 * T] - 1) / (1 - Exp[λ1 * T]) ==
        (eigenvec[[2]][[1]] * ((1 - μ) * Exp[λ2 * T] - 1)) / (eigenvec[[1]][[1]] *
           (1 - (1 - μ) * Exp[λ1 * T])) /. μ → mu /. ParmsHk, T, Reals][[1]]
 ]
]
critTvec
pHk = ListLinePlot[Transpose@{muvec, critTvec},
  PlotStyle → {Dotted, RGBColor["#00BA38"]}, PlotLegends → {"A. duodenale"}]
```

Out[∘]=
```
{142.818, 220.124, 301.977, 388.945, 481.708, 581.096, 688.128, 804.078, 930.568,
 1069.71, 1224.32, 1398.26, 1597.08, 1829.1, 2107.67, 2456.27, 2922.41, 3628.21}
```

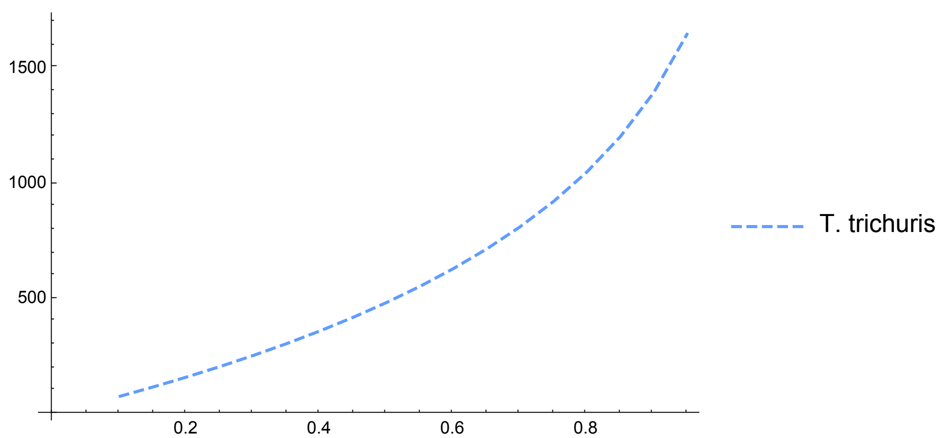
Out[∘]=




Trichuris

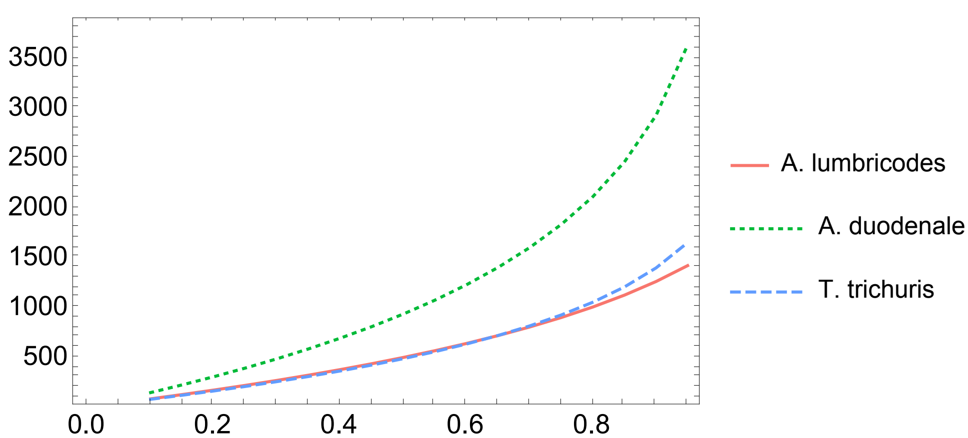
```
In[•]:= ParmsTri = {β → 0.62 / 365, Nhost → 0.000123,
          μh → 0.0167 / 365, μa → 1 / 365, λ → 7.3 * 10^5 / 365, μl → 18.25 / 365};
        critTvec = {};
        muvec = {};
        For[mu = 0.1, mu ≤ .99, mu += .05,
         AppendTo[muvec, mu];
         AppendTo[critTvec, T /. NSolve[(Exp[λ2 * T] - 1) / (1 - Exp[λ1 * T]) ==
                (eigenvec[[2]][[1]] * ((1 - μ) * Exp[λ2 * T] - 1)) / (eigenvec[[1]][[1]] *
                   (1 - (1 - μ) * Exp[λ1 * T])) /. μ → mu /. ParmsTri, T, Reals][[1]]
         ]
        ]
        critTvec
        pTri = ListLinePlot[Transpose@{muvec, critTvec},
          PlotStyle → {Dashed, RGBColor["#619CFF"]}, PlotLegends → {"T. trichuris"}]
Out[•]= {75.4591, 116.144, 159.073, 204.514, 252.78, 304.248, 359.372, 418.711, 482.965,
         553.021, 630.031, 715.532, 811.628, 921.323, 1049.11, 1202.16, 1392.99, 1646.56}
```

Out[•]=

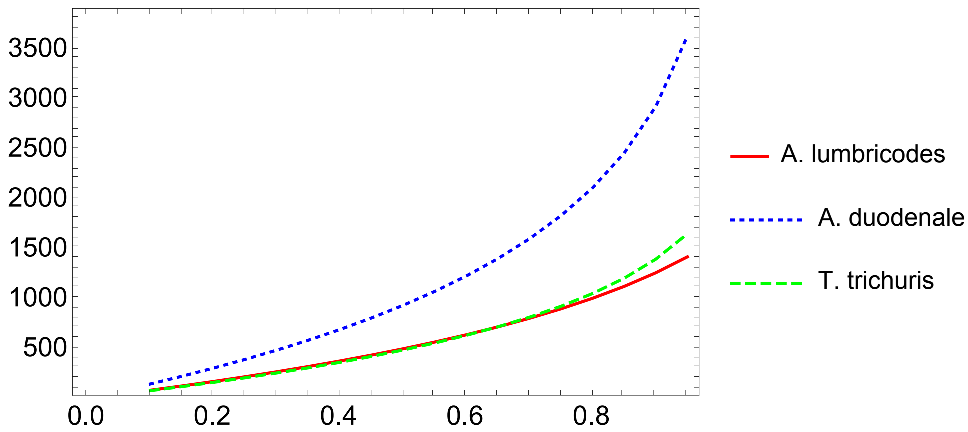



```
In[•]:= Show[pAsc, pHk, pTri, PlotRange → {100, 3700},
         Frame -> True, FrameStyle → Directive[Black, 14]]
```

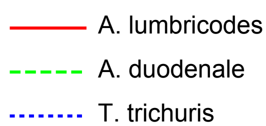
Out[•]=



In[∘]:=
```
Show[pAsc, pHk, pTri, PlotRange → {100, 3700},
 Frame -> True, FrameStyle → Directive[Black, 14],
 PlotLegends → {{Red, {Dashed, Green}, {Dotted, Blue}},
   {"A. lumbricodes", "A. duodenale", "T. trichuris"}}]
```

Out[∘]=



In[∘]:=
```
LineLegend[{Red, {Dashed, Green}, {Dotted, Blue}},
 {"A. lumbricodes", "A. duodenale", "T. trichuris"}]
```

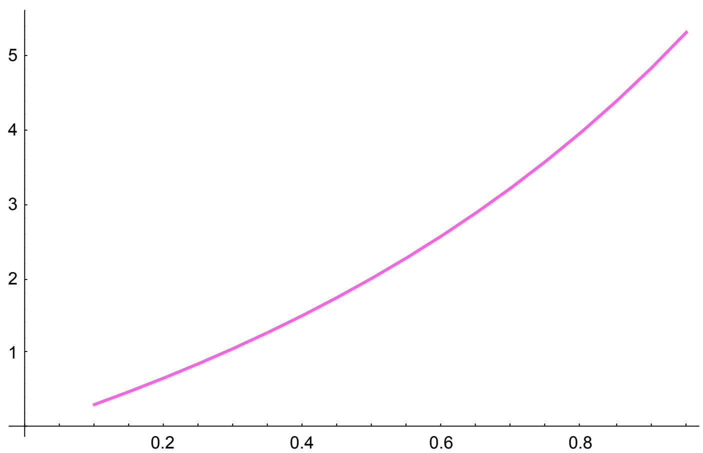
Out[∘]=



TColubNZ

In[∘]:=
```
ParmsTc = {β → 511 / 365, Nhost → 0.0015,
   μh → 0.1 / 365, μa → 17.89 / 365, λ → 3285 / 365, μl → 8.4 / 365};
critTvec = {};
muvec = {};
For[mu = 0.1, mu ≤ .99, mu += .05,
 AppendTo[muvec, mu];
 AppendTo[critTvec, T /. NSolve[(Exp[λ2 * T] - 1) / (1 - Exp[λ1 * T]) ==
        (eigenvec[[2]][[1]] * ((1 - μ) * Exp[λ2 * T] - 1)) / (eigenvec[[1]][[1]] *
           (1 - (1 - μ) * Exp[λ1 * T])) /. μ → mu /. ParmsTc, T, Reals][[1]]
 ]
]
critTvec
pTc = ListLinePlot[Transpose@{muvec, critTvec}, PlotStyle → RGBColor["#F564E3"]]
```

Out[∘]=
```
{0.32186, 0.495744, 0.679156, 0.872848, 1.07764, 1.29446, 1.52428, 1.76822, 2.02749,
 2.30344, 2.59754, 2.91143, 3.24691, 3.60599, 3.99084, 4.40389, 4.8478, 5.32548}
```

Out[∘]=

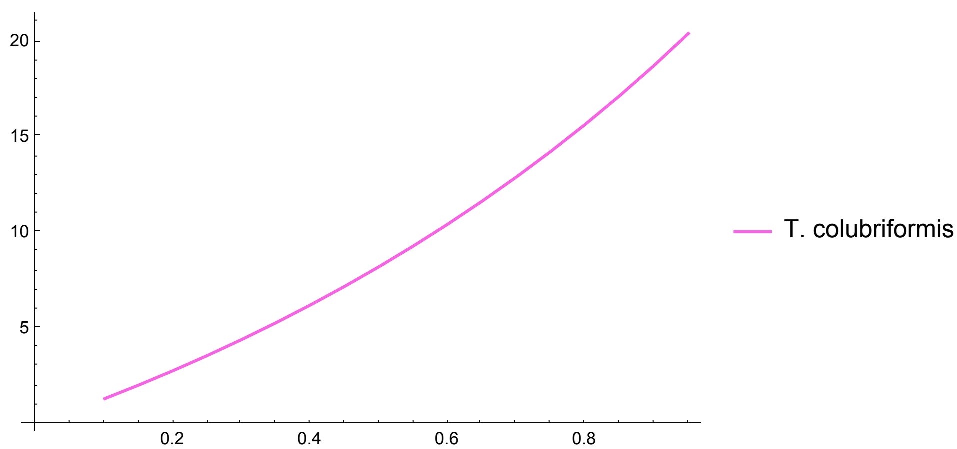



TcolubAUS

In[∘]:=
```
ParmsTcA = {β → 1124.2 / 365, Nhost → 0.001,
    μh → 0.1 / 365, μa → 17.89 / 365, λ → 2463.75 / 365, μl → 28.4 / 365};
critTvec = {};
muvec = {};
For[mu = 0.1, mu ≤ .99, mu += .05,
 AppendTo[muvec, mu];
 AppendTo[critTvec, T /. NSolve[(Exp[λ2 * T] - 1) / (1 - Exp[λ1 * T]) ==
        (eigenvec[[2]][[1]] * ((1 - μ) * Exp[λ2 * T] - 1)) / (eigenvec[[1]][[1]] *
           (1 - (1 - μ) * Exp[λ1 * T])) /. μ → mu /. ParmsTcA, T, Reals][[1]]
 ]
]
critTvec
pTcA = ListLinePlot[Transpose@{muvec, critTvec},
  PlotStyle → {RGBColor["#F564E3"]}, PlotLegends → {"T. colubriformis"}]
```

Out[∘]=
```
{1.32793, 2.04413, 2.79791, 3.59145, 4.42699, 5.30688, 6.23353, 7.20947, 8.23735,
 9.32001, 10.4605, 11.6621, 12.9287, 14.2645, 15.6747, 17.1653, 18.7435, 20.4184}
```

Out[∘]=

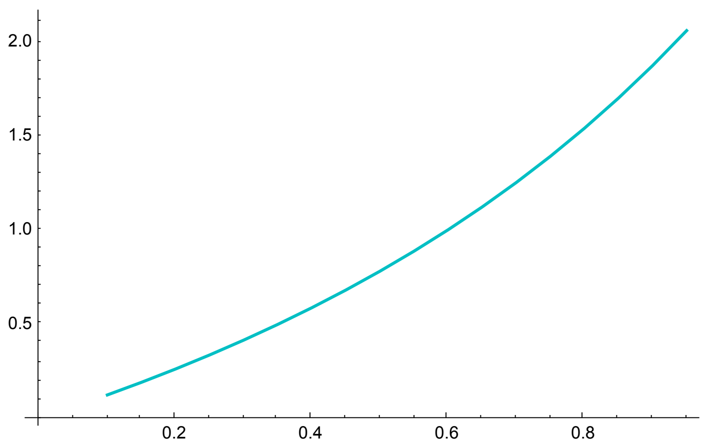



OcircNZ

In[◦]:=
```
ParmsOc = {β → 1080.4 / 365, Nhost → 0.0015,
   μh → 0.1 / 365, μa → 40.9 / 365, λ → 4380 / 365, μl → 8.49 / 365};
critTvec = {};
muvec = {};
For[mu = 0.1, mu ≤ .99, mu += .05,
 AppendTo[muvec, mu];
 AppendTo[critTvec, T /. NSolve[(Exp[λ2 * T] - 1) / (1 - Exp[λ1 * T]) ==
        (eigenvec[[2]][[1]] * ((1 - μ) * Exp[λ2 * T] - 1)) / (eigenvec[[1]][[1]] *
           (1 - (1 - μ) * Exp[λ1 * T])) /. μ → mu /. ParmsOc, T, Reals][[1]]
 ]
]
critTvec
pOc = ListLinePlot[Transpose@{muvec, critTvec}, PlotStyle → RGBColor["#00BFC4"]]
```

Out[◦]=
```
{0.123912, 0.19086, 0.261482, 0.336072, 0.414952,
 0.498478, 0.587042, 0.681079, 0.781068, 0.887543, 1.00109,
 1.12237, 1.2521, 1.39108, 1.54022, 1.7005, 1.873, 2.05893}
```

Out[◦]=

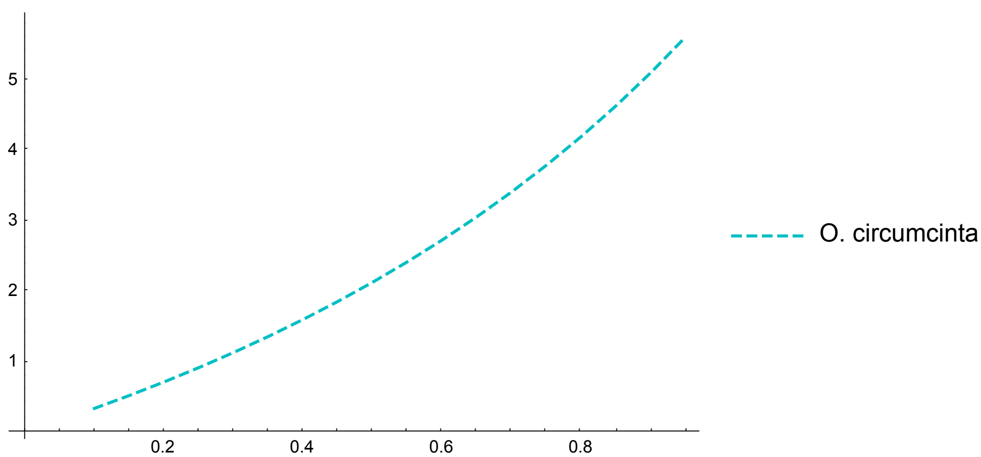



OcircUK

```
In[•]:= ParmsOcUK = {β → 956.3 / 365, Nhost → 0.002,
           μh → 0.1 / 365, μa → 14.9 / 365, λ → 1752 / 365, μl → 11.12 / 365};
        critTvec = {};
        muvec = {};
        For[mu = 0.1, mu ≤ .99, mu += .05,
         AppendTo[muvec, mu];
         AppendTo[critTvec, T /. NSolve[(Exp[λ2 * T] - 1) / (1 - Exp[λ1 * T]) ==
                (eigenvec[[2]][[1]] * ((1 - μ) * Exp[λ2 * T] - 1)) / (eigenvec[[1]][[1]] *
                   (1 - (1 - μ) * Exp[λ1 * T])) /. μ → mu /. ParmsOcUK, T, Reals][[1]]
         ]
        ]
        critTvec
        pOcUK = ListLinePlot[Transpose@{muvec, critTvec},
          PlotStyle → {RGBColor["#00BFC4"], Dashed}, PlotLegends → {"O. circumcinta"}]

Out[•]= {0.338299, 0.521076, 0.713886, 0.917529, 1.13288, 1.36093, 1.60272, 1.85947, 2.13247,
         2.42318, 2.73323, 3.0644, 3.41869, 3.79832, 4.20576, 4.64377, 5.11539, 5.62405}
```

Out[•]=

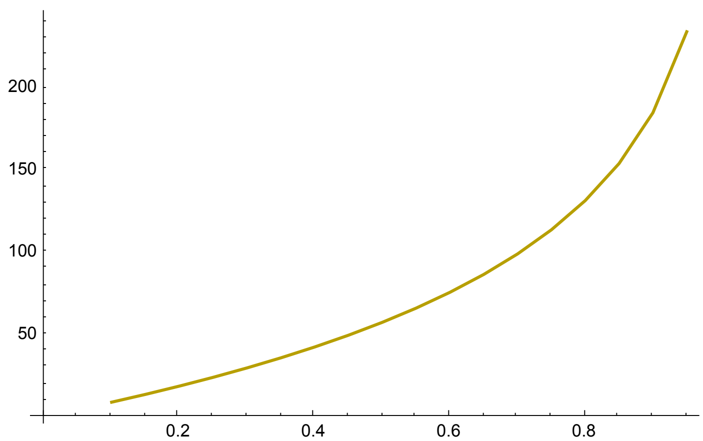



HcontNZ

In[∘]:=
```
ParmsHc = {β → 584 / 365, Nhost → 0.0015,
   μh → 0.1 / 365, μa → 3.65 / 365, λ → 11 558.5 / 365, μl → 620.87 / 365};
critTvec = {};
muvec = {};
For[mu = 0.1, mu ≤ .99, mu += .05,
 AppendTo[muvec, mu];
 AppendTo[critTvec, T /. NSolve[(Exp[λ2 * T] - 1) / (1 - Exp[λ1 * T]) ==
        (eigenvec[[2]][[1]] * ((1 - μ) * Exp[λ2 * T] - 1)) / (eigenvec[[1]][[1]] *
          (1 - (1 - μ) * Exp[λ1 * T])) /. μ → mu /. ParmsHc, T, Reals][[1]]
 ]
]
critTvec
pHc = ListLinePlot[Transpose@{muvec, critTvec}, PlotStyle → RGBColor["#B79F00"]]
```

Out[∘]=
```
{8.74957, 13.4909, 18.5152, 23.8587, 29.5644, 35.6853, 42.2863, 49.4494, 57.2793,
 65.9128, 75.5342, 86.3989, 98.8768, 113.532, 131.287, 153.818, 184.676, 233.906}
```

Out[∘]=

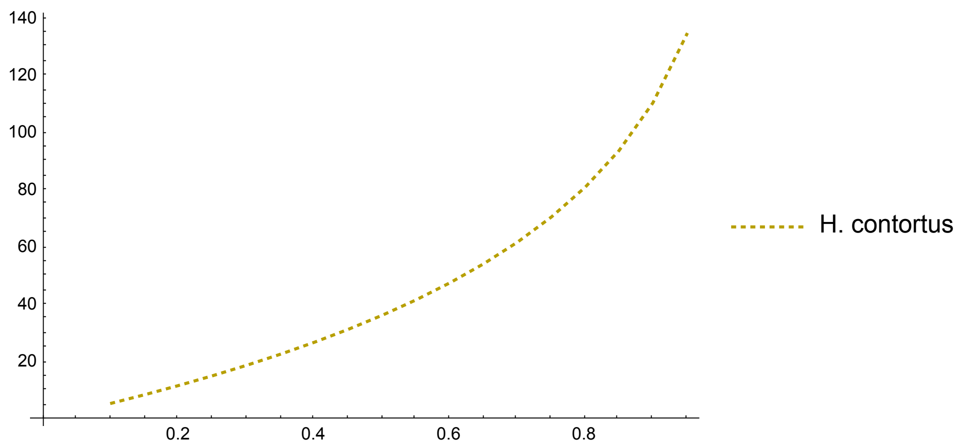



HcontAUS

In[•]:=
```
ParmsHcA = {β → 890.6 / 365, Nhost → 0.001,
    μh → 0.1 / 365, μa → 3.65 / 365, λ → 5501.8 / 365, μl → 230.36 / 365};
critTvec = {};
muvec = {};
For[mu = 0.1, mu ≤ .99, mu += .05,
 AppendTo[muvec, mu];
 AppendTo[critTvec, T /. NSolve[(Exp[λ2 * T] - 1) / (1 - Exp[λ1 * T]) ==
        (eigenvec[[2]][[1]] * ((1 - μ) * Exp[λ2 * T] - 1)) / (eigenvec[[1]][[1]] *
           (1 - (1 - μ) * Exp[λ1 * T])) /. μ → mu /. ParmsHcA, T, Reals][[1]]
 ]
]
critTvec
pHcA = ListLinePlot[Transpose@{muvec, critTvec},
  PlotStyle → {RGBColor["#B79F00"], Dotted}, PlotLegends → {"H. contortus"}]
```

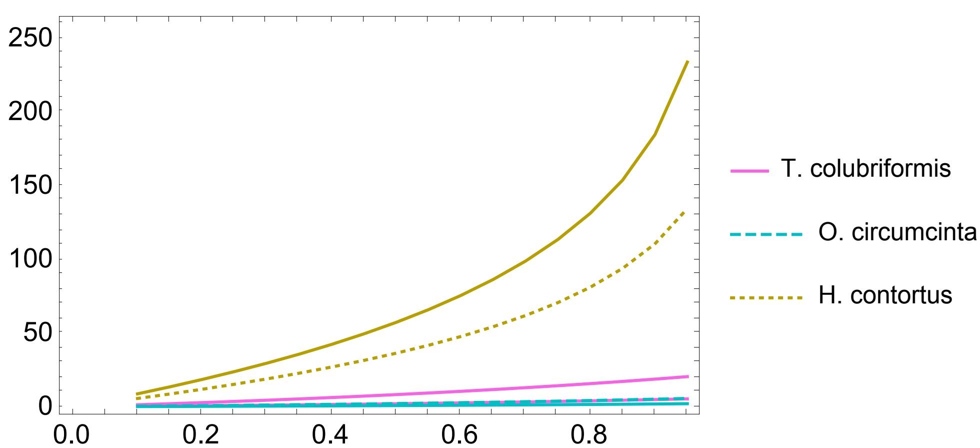
Out[•]= {5.61202, 8.64613, 11.8548, 15.2594, 18.8858, 22.7647, 26.9342, 31.4411, 36.345, 41.7227, 47.6756, 54.3416, 61.9151, 70.6828, 81.0931, 93.9067, 110.577, 134.476}

Out[•]=



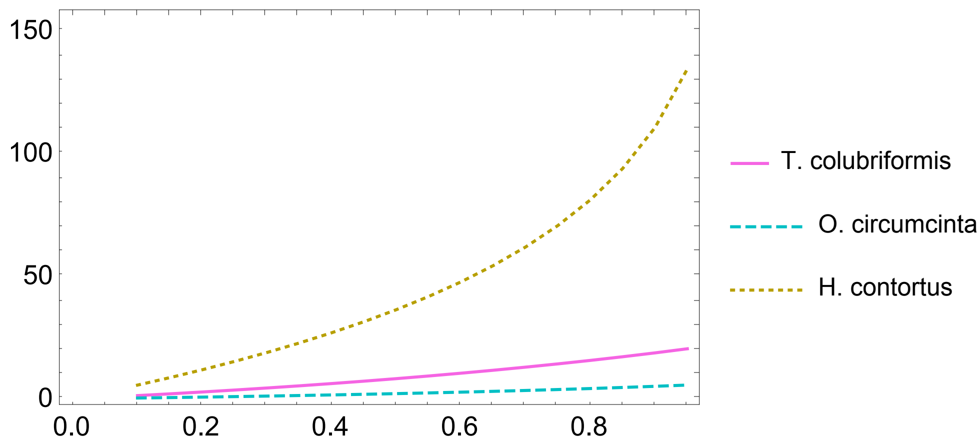

In[•]:=
```
Show[pTc, pTcA, pOc, pOcUK, pHc, pHcA, PlotRange → {0, 250},
 Frame -> True, FrameStyle → Directive[Black, 14]]
```

Out[•]=



In[•]:=
```
Show[pTcA, pOcUK, pHcA, PlotRange → {0, 150},
  Frame -> True, FrameStyle → Directive[Black, 14]]
```

Out[•]=




## Categorizing Qualitative Behaviors

Reads in a csv file in which each row is a parameter set and determines the corresponding qualitative behavior: (1) decays even after evolution of resistance, (2) evolutionary rescue, (3) growth even before evolution of resistance, and (4) decay even without treatment (see Figure 3 in main text.)

```
LHS = Import["50grid.csv"];
critTvecbb = {};
critTvecaa = {};
determ = {};
dynvec = {};
For[idx = 2, idx ≤ Dimensions[LHS][[1]], idx += 1,
 parms = LHS[[idx]];
 Parmstemp = {β → parms[[7]], Nhost → parms[[6]], μh → parms[[9]], μa → parms[[4]],
   λ → parms[[5]], μl → parms[[8]], cov → 1, bb → 0.9, ab → 0.5, aa → 0.1};
 AppendTo[critTvecbb, T /. NSolve[(Exp[λ2 * T] - 1) / (1 - Exp[λ1 * T]) ==
         (eigenvec[[2]][[1]] * ((1 - μ) * Exp[λ2 * T] - 1)) / (eigenvec[[1]][[1]] *
            (1 - (1 - μ) * Exp[λ1 * T])) /. μ → cov * bb /. Parmstemp, T, Reals][[1]]
   ] ×
   AppendTo[critTvecaa, T /. NSolve[(Exp[λ2 * T] - 1) / (1 - Exp[λ1 * T]) ==
         (eigenvec[[2]][[1]] * ((1 - μ) * Exp[λ2 * T] - 1)) / (eigenvec[[1]][[1]] *
            (1 - (1 - μ) * Exp[λ1 * T])) /. μ → cov * aa /. Parmstemp, T, Reals][[1]]
   ] ×
   freq = parms[[3]];
 categ4 = If[critTvecbb[[idx - 1]] < 0, 4, 0];
 categ3 =
  If[critTvecaa[[idx - 1]] > 365 / freq && critTvecbb[[idx - 1]] > 365 / freq, 3, 0];
 categ2 = If[critTvecaa[[idx - 1]] < 365 / freq &&
    critTvecbb[[idx - 1]] > 365 / freq, 2, 0];
 categ1 = If[critTvecaa[[idx - 1]] < 365 / freq &&
    critTvecbb[[idx - 1]] < 365 / freq, 1, 0];
 AppendTo[dynvec, categ3 + categ2 + categ1 + categ4]
]
Export["50grid_results.csv", dynvec];
```