Finding Max and Min – Naïve Approach

Example Array : A = [12, 7, 3, 15, 9, 20, −2, 8]   N = 8

Required 2 scans to find min and max element in the array.

Pass 1 : Find max (7 comparisons)

Start Max = 12

1. 12 vs 7 → keep 12
2. 12 vs 3 → keep 12
3. 12 vs 15 → update 15
4. 15 vs 9 → keep 15
5. 15 vs 20 → update 20
6. 20 vs -2 → keep 20
7. 20 vs 8 → keep 20

→ Max = 20

Pass 1 : Find min (7 comparisons)

Start Min = 12

1. 12 vs 7 → update 7
2. 7 vs 3 → update 3
3. 3 vs 15 → keep 3
4. 3 vs 9 → keep 3
5. 3 vs 20 → keep 3
6. 3 vs -2 → update -2
7. -2 vs 8 → keep -2

→ Min = -2

Total comparisons (Naïve) = 2 x N – 2 = 2 x 8 – 2 = 16 – 2 = 14

Finding Max and Min – Divide & Conquer

Example Array : A = [12, 7, 3, 15, 9, 20, −2, 8] N = 8

Level 0. Split into two halves

Left : [12, 7, 3, 15]

Pair [12,7]

#1) 12 vs 7 → min=7, max=12

Pair [3, 15]

#2) 3 vs 15 → min = 3, max = 15

- Combine these two pairs:

#3) max(12, 15) → 15

#4) min (7, 3) → 3

→Left result : (min=3, max=15)

Right : [9, 20, -2, 8]

Pair [9, 20]

#5) 9 vs 20 → min=9, max=20

Pair [-2, 8]

#6) -2 vs 8 → min=-2, max=8

- Combine these two pairs:

#7) max(20, 8) → 20

#8) min (9, -2) → -2

→Right result : (min=-2, max=20)

Final Combine (Left vs Right)

#9) max(15, 20) → 20 #10) min(3, -2) → -2

Total Comparisons = 10

For N = 8 , formula : 1.5 x N – 2 = 1.5 x 8 - 2 = 12 – 2 = 10

Strassen's Algorithm

Example Matrices :

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix} \quad B = \begin{bmatrix} 16 & 15 & 14 & 13 \\ 12 & 11 & 10 & 9 \\ 8 & 7 & 6 & 5 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

Step 1 : Partition into 2x2 blocks

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

Where :

$$A_{11} = \begin{bmatrix} 1 & 2 \\ 5 & 6 \end{bmatrix}, \; A_{12} = \begin{bmatrix} 3 & 4 \\ 7 & 8 \end{bmatrix}, \; A_{21} = \begin{bmatrix} 9 & 10 \\ 13 & 14 \end{bmatrix}, \; A_{22} = \begin{bmatrix} 11 & 12 \\ 15 & 16 \end{bmatrix}$$

$$B_{11} = \begin{bmatrix} 16 & 15 \\ 12 & 11 \end{bmatrix}, \; B_{12} = \begin{bmatrix} 14 & 13 \\ 10 & 9 \end{bmatrix}, \; B_{21} = \begin{bmatrix} 8 & 7 \\ 4 & 3 \end{bmatrix}, \; B_{22} = \begin{bmatrix} 6 & 5 \\ 2 & 1 \end{bmatrix}$$

Step 2 : The seven Strassen products M_1 … M_7

$$M_1 = (A_{11} + A_{22})(B_{11} + B_{22}) = \begin{bmatrix} 460 & 408 \\ 748 & 664 \end{bmatrix}$$

$$M_2 = (A_{21} + A_{22})B_{11} = \begin{bmatrix} 584 & 542 \\ 808 & 750 \end{bmatrix}$$

$$M_3 = A_{11}(B_{12} - B_{22}) = \begin{bmatrix} 24 & 24 \\ 88 & 88 \end{bmatrix}$$

$$M_4 = A_{22}(B_{21} - B_{11}) = \begin{bmatrix} -184 & -184 \\ -248 & -248 \end{bmatrix}$$

$$M_5 = (A_{11} + A_{12})B_{22} = \begin{bmatrix} 36 & 26 \\ 100 & 74 \end{bmatrix}$$

$$M_6 = (A_{21} - A_{11})(B_{11} + B_{12}) = \begin{bmatrix} 416 & 384 \\ 416 & 384 \end{bmatrix}$$

$$M_7 = (A_{12} - A_{22})(B_{21} + B_{22}) = \begin{bmatrix} -160 & -128 \\ -160 & -128 \end{bmatrix}$$

Step 3 : Combine results

$$C_{11} = M_1 + M_4 - M_5 + M_7 = \begin{bmatrix} 80 & 70 \\ 240 & 214 \end{bmatrix}$$

$$C_{12} = M_3 + M_5 = \begin{bmatrix} 60 & 50 \\ 188 & 162 \end{bmatrix}$$

$$C_{21} = M_2 + M_4 = \begin{bmatrix} 400 & 358 \\ 560 & 502 \end{bmatrix}$$

$$C_{22} = M_1 - M_2 + M_3 + M_6 = \begin{bmatrix} 316 & 274 \\ 444 & 386 \end{bmatrix}$$

Step 4 : The final 4 x 4 product C

$$C = \begin{bmatrix} 80 & 70 & 60 & 50 \\ 240 & 214 & 188 & 162 \\ 400 & 358 & 316 & 274 \\ 560 & 502 & 444 & 386 \end{bmatrix}$$

Time complexity : ≈ $O(n^{2.8})$