ASTRO 410 — Extra-credit Assignment

DUE: 04/09/2021

IMPORTANT: This extra-credit assignment may reward you up to 15 extra points toward your final grade. Solutions to this assignment should include a write up in pdf format (using Latex or Word) detailing your answers to the questions and results (e.g., plots or tables), all your programs and all output files generated by your programs, as well as a README file that lists all the files included, and instructions to compile and run your programs. All the files should be submitted to CANVAS as a single tarred, gzipped (tgz) file named hwextra-lastname.tgz. The file when expanded should yield a directory named hwextra-lastname.

Grading Instructions:
The assignment report should include the following sections:
1. Describe the equations and algorithms to solve the assignment;
2. Answer the questions in the assignment;
3. Present the results with figures and tables (if applicable).
4. Points: (1) The methods: the equations of motion and the Leap-frog equations (3 points). (2) The code: the students can write their own Leap-frog code (5 points), or uses third-party or built-in Leap-frog function (0 point). (3) The results: a plot of the Sun-Earth orbit over a long period of time of at least 1 million years (4 points), and a plot of the total energy vs time over a long period of time of at least 1 million years (3 points).

Write a N-body code by implementing the leap-frog algorithm to solve the equation of motion, and apply it to the Sun-Earth system and solve the Kepler orbit of the Earth. Make sure to record also the total energy (sum of gravitational and kinetic energy) of the system at each time step, and plot total energy vs time over a long period of time (e.g., 1 million years) to check the energy conservation. Leapfrog should have very good accuracy and stability over a long time duration.

Tips: Start with the fundamental equations of motion for two bodies; the force of one body (say, the Sun) on the other (say, the Earth) is simply given by:

$$F = ma = \frac{GMm}{r^2} \tag{1}$$

where M and m are the masses of the two bodies (the latter for the object whose motion we are interested in), a is the acceleration of the object with mass m, G is the gravitational constant, and r is their mutual separation. This equation turns into two first order differential equations which can be solved with the Leap-frog algorithm:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{GM}{r^2} \tag{2}$$

$$\frac{\mathrm{d}r}{\mathrm{d}t} = v \tag{3}$$