A Simulation of Planetary Orbits

Swapnaneel Nath
swapnaneel.nath@protonmail.ch

Abstract

I present three minimalistic agent-based models built in Netlogo to replicate various phenomena we observe in planetary systems, in particular, multiplanetary systems such as the Solar System. These simulations demonstrate how simple underlying processes can generate outcomes such as the concentration of the mass of such systems near a single orbital plane, the revolution of all circumstellar objects in a single direction, and the clearing of neighborhoods around orbits. Taken together, these models show how planetary orbits can develop from very simple rules.

Keywords: *Agent-based Modeling, ABM, Solar System, Planetary System, Planetary Orbits, Multiplanetary System, Modeling, Simulation, Orbits.*

I. Introduction

Agent-based models (ABMs) have been used to simulate various systems, natural and man-made, ranging from stock markets, epidemics, warfare, predator-prey dynamics, etc [1]. At the heart of the modeling technique are agents, which are individual computational units endowed with certain properties and capable of performing certain actions [2]. ABMs enable us to run experiments with heterogeneous agents and help us observe emergent patterns that result from their behavior [3]. Agent-based modeling is particularly useful when we are unable to take meaningful averages [2]. Even in cases we are able to, ABMs help us visualize our object of study better and gain a deeper understanding of the matter we are investigating [2].

This paper attempts to simulate planetary orbits through agent-based modeling using setups that rely on minimal assumptions and very simple rules. These setups retain the most essential features of the objects under study while downplaying the less important details. The goal is to deploy a basic system out of which planetary orbits (such as those we observe in multiplanetary systems like the Solar System) emerge. Netlogo [4] is the modeling environment used to develop all the models presented below.

I focus on three phenomena associated with planetary orbits: (1) planetary orbits tend to be almost coplanar, (2) planets revolve around the central star in the same direction, and (3) the neighborhood around stable orbits are cleared of other non-stellar objects. **Circumstellar Disc** deals with the first phenomenon, showing how a process of simple exchange between objects moving in non-coplanar orbits over time can result in the formation of a single circumstellar disc in which the entire mass of the star system is concentrated. **Unity of Direction** regards the second phenomenon, demonstrating how objects in a circumstellar disc moving in opposite directions would over time develop a common direction. And finally, **Neighborhood Clearing**, using the outcomes of the previous sections as assumptions, shows how the multitude of non-stellar objects would interact with other intercepting objects, resulting in the formation of a handful of planets, each with the neighborhood of its orbit cleared, and revolving around the central star in a stable orbit.

II. Circumstellar Disc

A. Conceptual Setup

Notice that all the planets in the Solar System revolve around the Sun roughly on the same orbital plane. The orbital inclination with respect to the ecliptic (the orbital plane of Earth) ranges from 0.774 degrees (for Uranus) to 7.004 degrees (for Mercury) [5]. This pattern generalizes to other multiplanetary systems as well. The five planets orbiting Kepler-444 form inclinations between 90.62 to 92.79 degrees to the sky plane [6]. The orbital inclinations of planets in the WASP-47 system range from about 85.98 to 89.32 degrees [7]. The planets of Kepler-11 are inclined at about 88.89 to 89.87 degrees to the sky plane [8]. In order to simulate planetary orbits on a two-dimensional surface, we need to first show that it would be highly improbable for our planets to have orbits on completely distinct planes.

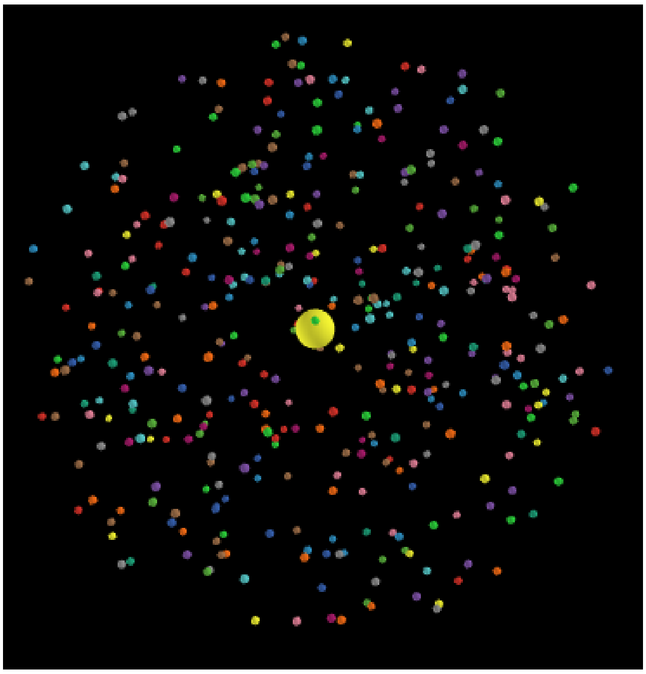

Figure 1. The initial positioning of objects around the central star.

To highlight this improbability, it would suffice to show that most of the mass of the entire star system would inevitably end up being concentrated in a single circumstellar disc. This circumstellar disc would contain the materials that would eventually form planets whose orbits we are interested in simulating.

Assume that in the beginning non-stellar objects are randomly scattered around the star as in Figure 1. At this stage, we collect every such object into circumstellar discs of various masses such that every object is a part of some disc. There would be numerous discs in various orientations. For illustrative purposes, three such discs are shown in Figure 2 so as to not clutter the view.

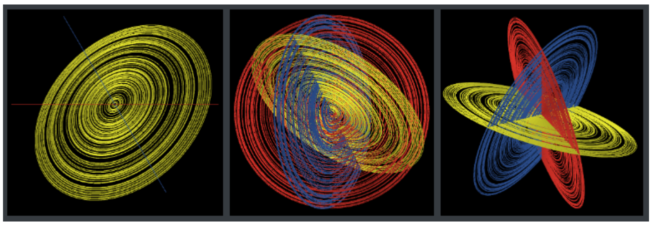

Figure 2. Left to right: front view, top view, and side view of three arbitrary circumstellar discs at various orientations. (One needs to zoom in to see the edges of the red and the blue discs in the front view.)

Now that we have these discs, we can show that through simple interactions between them, we get a singular disc.

B. The Model

This section presents the Disc Formation Model [9], the first of the models on the way to generating planetary orbits.

We begin with an arbitrary number of circumstellar discs, each with a random positive integer mass. These discs are represented by little caricatures consisting of three concentric circles as shown in Figure 3. Every disc must have the star at its center. This means that every disc intersects every other disc as we see in Figure 2. Intersections are represented by links (gray lines) between the discs. Since each disc is linked to every other in this way, we have a fully-connected graph in Figure 3.

Once the simulation begins, every disc randomly selects another disc and engages in a unit mass exchange. The outcome of this exchange stochastically depends on the relative masses of the discs. The disc with the greater mass has a greater chance of gaining a unit mass from the other disc and vice-versa. Think of this exchange as representing the pulling away of an object from a disc into another disc. Pseudocode 1 lays down this process for each individual disc with positive mass. This is central to the code presented in the Disc Formation Model.

```
Begin
        Set index to a random integer between 0
and [total number of discs] - 1
        Choose disc corresponding to this index
        If mass of chosen disc ("other's mass") is
positive
                Set p = own mass / [own mass +
other's mass]
                With probability p, add 1 unit to
own mass and subtract 1 unit from other's mass
                With probability 1-p, add 1 unit
to other's mass and subtract 1 unit from own
mass
End
```

Pseudocode 1

As the exchange continues, whenever a disc reaches zero mass, it disappears and its links to other discs are broken. The process terminates when only one disc with positive mass remains.

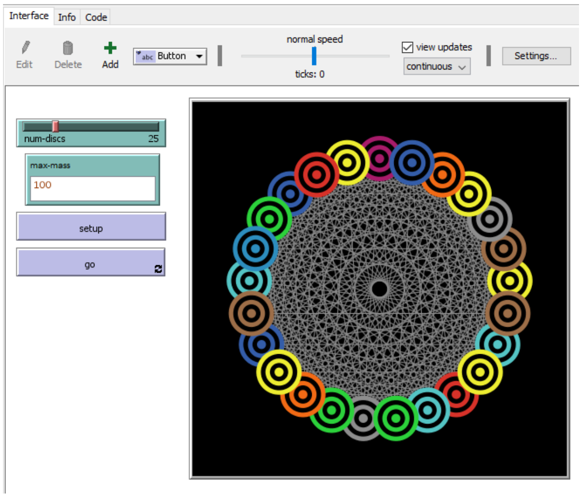

Figure 3. The Circumstellar Disc Model initialized with twenty-five discs, each with a mass between 1 to 100 units inclusive.

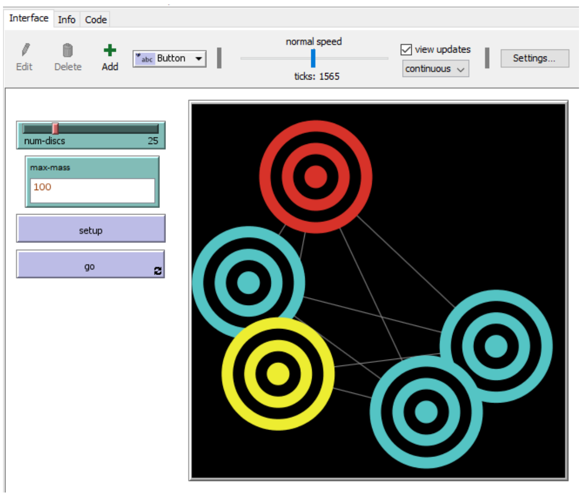

Figure 4. Output of the Circumstellar Disc Model after 1565 iterations.

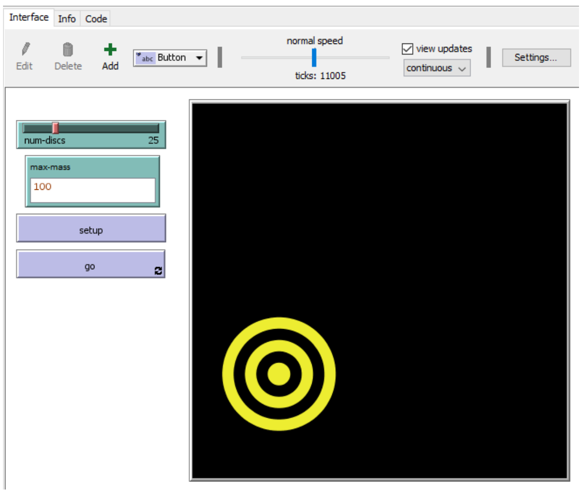

Figure 5. Final result of the Circumstellar Disc Model after 11005 iterations.

C. A Sample Run

The following are images from a sample run. We begin with twenty-five discs with integer masses between 1 and 100 inclusive. Each disc is connected to every other disc [Figure 3]. After 1565 iterations of the exchange process described above, we are left with five discs containing the entire mass [Figure 4]. The simulation continues for a total of 11005 iterations at the end of which we are left with a single disc containing the entire mass of the system [Figure 5].

D. Remarks

The sample run presented above is typical. The number of starting discs and maximum mass may be arbitrarily increased, but we always eventually get a single disc, although the computation time required to arrive at the result increases as well.

The results of this model help us justify the use of two-dimensional spatial models in the upcoming sections.

III. Unity of Direction

A. Conceptual Setup

The vast majority of non-stellar objects revolve around the Sun in the same direction. All eight planets in the Solar System revolve counterclockwise around the Sun [10], and so does the asteroid belt [11].

If we are able to demonstrate that as a result of simple interactions, all objects in the circumstellar disc end up traveling in the same direction, we can assert that the planets that would eventually form would all travel in the same direction.

For the purpose of this model, we add the assumption that objects orbit in a circular path. (While in reality, elliptical paths are the norm, using circular paths simplifies the code with little loss of generality.) Take the disc with all the objects and reshape it into a rectangular strip that wraps vertically: an object traveling up emerges at the bottom when it reaches the upper edge and vice-versa. As a result of this reshaping, objects on an annulus in the original disk would get transferred to a corresponding vertical in the rectangular strip as shown in Figure 6. Clockwise movement in the disc corresponds to downward movement in the strip, while counterclockwise movement in the disc corresponds to upward movement in the strip.

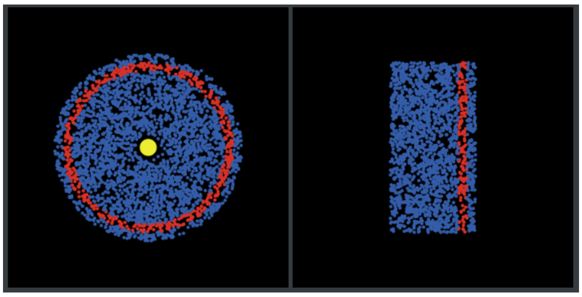

Figure 6. Objects in red in both the disc and the strip are the same.

Now, using simple repeated interactions, we can show how objects moving in opposite directions would end up moving in the same direction.

B. The Model

This section presents the Unity of Direction Model [12].

We begin with an arbitrary number of dots in a rectangular world that wraps vertically. This represents the reshaped annulus mentioned above. Each dot depicts a non-stellar object in the circumstellar disc derived from the previous model. Every one of these objects has a mass and a velocity. Objects with positive velocities move up in the rectangular world, which corresponds to moving counterclockwise in the annulus, whereas objects with negative velocities move down, or clockwise.

At the core of the model is a velocity adjustment process. Each object has a radius of influence that is proportional to the square root of its mass. When an object gets too close to another object, their velocities change in a way such that the new velocity is the average of the old velocities of the two objects weighted by mass. That is, when an object enters the radius of influence of another, the velocities get closer. This change is proportional to the ratio of the mass of the object to the total mass of both objects. The change in velocity of the object with more mass is less than the change in velocity of the object with less mass.

Pseudocode 2 describes this process from the perspective of individual objects in the simulation. The velocity adjustment process continues until either all objects have positive velocities or all objects have negative velocities, that is, all objects are moving in the same direction.

```
Begin
        If other object is inside influence radius
                Set other's new velocity =
{(other's old velocity * other's mass) + (own old
velocity * own mass)} / {other's mass + own
mass}
                Set own new velocity = other's
new velocity
End
```

Pseudocode 2

C. A Sample Run

The following are images from a sample run. We begin with 1000 randomly placed objects each with mass between 0 to 10 and velocity between 10 to 10. About half of these objects are moving in each direction: there are 506 objects going up, and 494 going down [Figure 7]. After seven iterations, we have 716 going up and 284 going down [Figure 8]. The process continues and after 48 iterations, we end up with positive velocities for all the objects; all the objects are going up [Figure 9]. Objects also tend to get closer together. This effect is particularly prominent when the radius of influence is reduced.

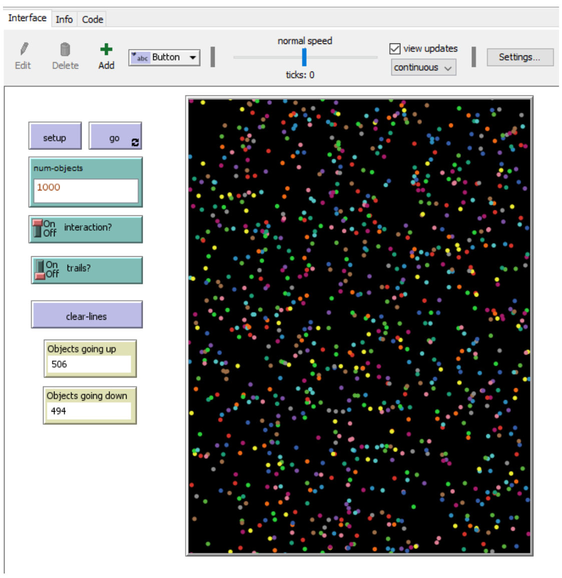

Figure 7. Unity of Direction Model initialized with 1000 objects with various masses and velocities.

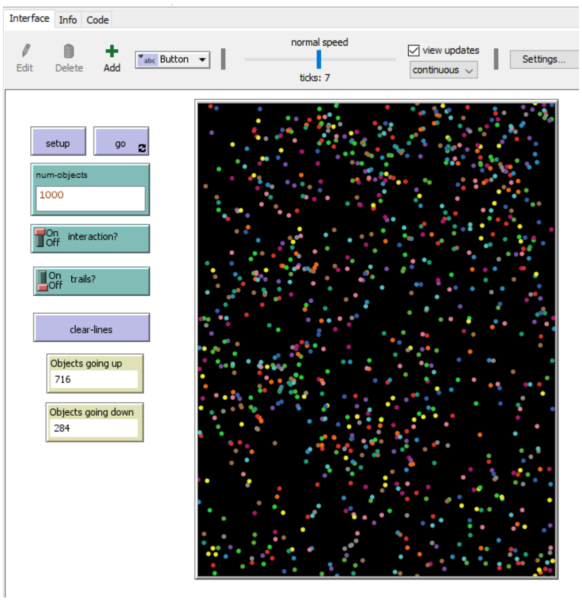

Figure 8. Output of the Unity of Direction Model after seven iterations.

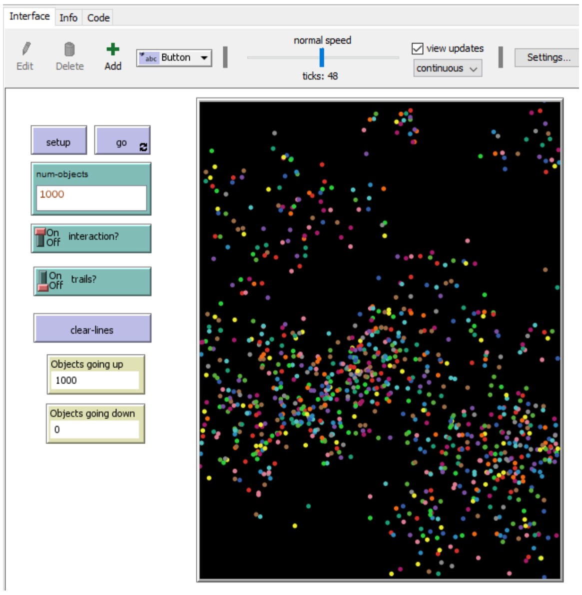

Figure 9. Final result of the Unity of Direction Model after 48 iterations.

D. Remarks

The radius of influence of an object in this model is set to the square-root of its mass. If the radius is increased, unity of direction is achieved faster. If the radius is decreased, achieving the result takes longer. Unity of direction always occurs as long as we initialize the process with enough (well-distributed) objects. If the spread is too sparse, for example, if we have only ten objects in the entire window, unity of direction does not emerge. However, this does not pose a problem with respect to validation. Recall, this rectangular region contains the multitude of objects in the entire circumstellar disc; the number of objects we begin the simulation with should be high.

The results of the Unity of Direction Model provides grounds for the use of velocities with only one sign in the next model.

IV. Neighborhood Clearing

A. Conceptual Setup

Planetary systems usually have one to eight planets revolving around a central star [13]. Planets in the Solar System revolve around the Sun in stable, roughly circular paths that have been cleared of all other big non-stellar objects. According to the International Astronomical Union's definition of a planet adopted in 2006, neighborhood clearing phenomenon is a requirement for planethood [14].

Using results from the previous two sections, we set up a plane with points traveling in one direction (clockwise or counterclockwise), and show how simple interactions lead to the formation of stable orbits with cleared neighborhoods.

B. The Model

This section presents the Neighborhood Clearing Model [15] which demonstrates how planetary orbits can form.

Generate a two-dimensional world with a fixed "sun" at the center and populate it with "planets" in random positions. These planets are represented by dots and have a velocity (in this model, we use positive velocity or counterclockwise travel only) and a mass associated with each of them. Each planet moves forward a distance proportional to its velocity, and then moves a certain distance towards the sun. This movement towards the sun simulates the effect of gravity. The net effect is that the planet spirals out, spirals in, or remains on a circular path. Unless the planet is intercepted, usually it spirals in to form an orbit closer to the sun, or spirals out to form an orbit farther away from the sun. Occasionally, it may spiral into the sun if it is too slow or spiral out of the world altogether if it is too fast; in these cases, the planet is removed from the simulation. Figure 10 depicts some of these possibilities.

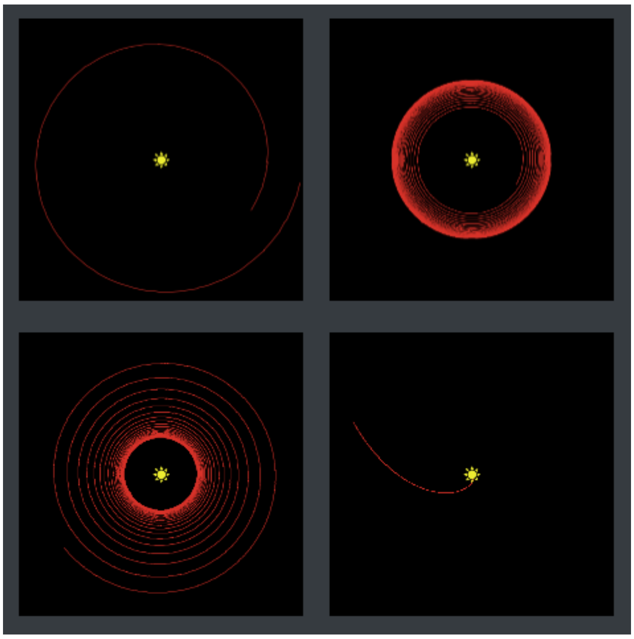

Figure 10. Various possibilities with a single planet.

When multiple planets are involved and interaction among them is allowed, the neighborhood clearing phenomenon takes place. If two planets come in contact with each other, the planet with the smaller mass gets subsumed by the planet with the larger mass. This is consistent with the clumping up of objects we observe in the Unity of Direction model, especially if we decrease the radius of influence. Pseudocode 3 provides the steps for this clearing process (from the perspective of individual planets). This forms the core of the Neighborhood Clearing Model.

```
Begin
        If other's position = own position
                If other's mass < own mass
                        Remove other
                        Set own new mass = own
mass + other's mass
End
```

Pseudocode 3

As the process continues, most planets are removed from the simulation. Eventually, we are left with a few planets in circular, non-intersecting (cleared) orbits. On a few occasions, no planet remains. But usually, we are left with a single digit number of planets, with lower numbers more frequent than the higher.

C. A Sample Run

The following are images from a sample run. We begin with 1000 planets randomly placed around the Sun [Figure 11]. These planets each have a mass between 0 and 10 units, and a velocity between 0 and 10 units. Within four iterations of the process, we see that the number of planets has decreased to 400 [Figure 12]. The simulation runs for about 16000 iterations and we are left with only four planets with stable, cleared orbits [Figure 13]. Clicking the "clear-lines" button once enables us to get a clear view of the orbits of the remaining planets.

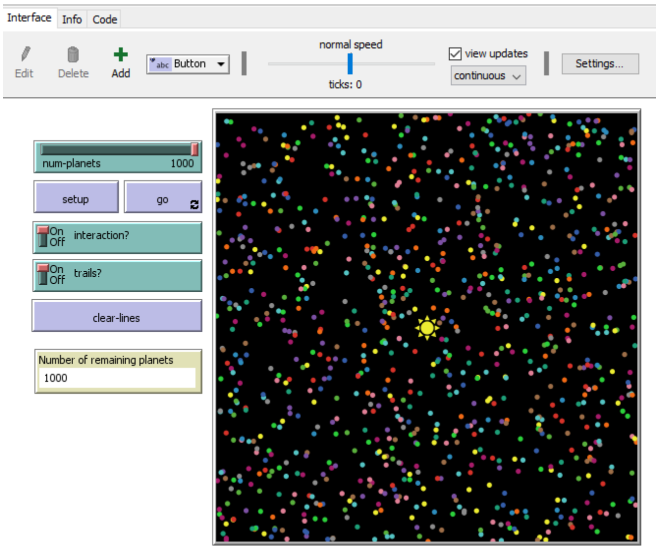

Figure 11. Initializing the Neighborhood Clearing Model with 1000 planets.

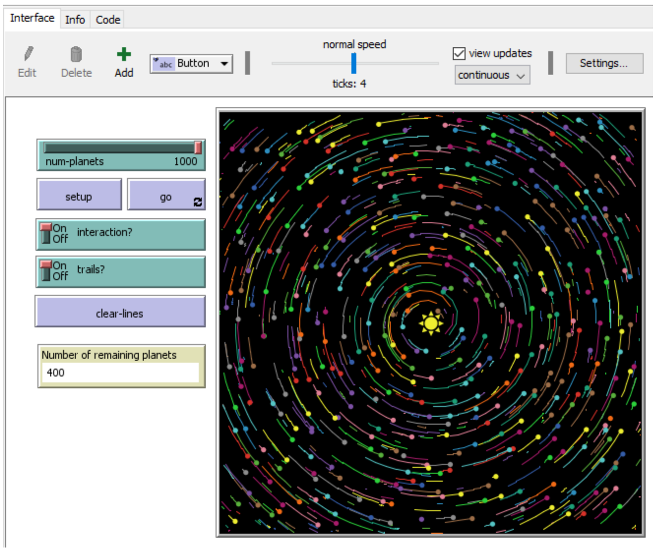

Figure 12. Neighborhood Clearing Model after four iterations.

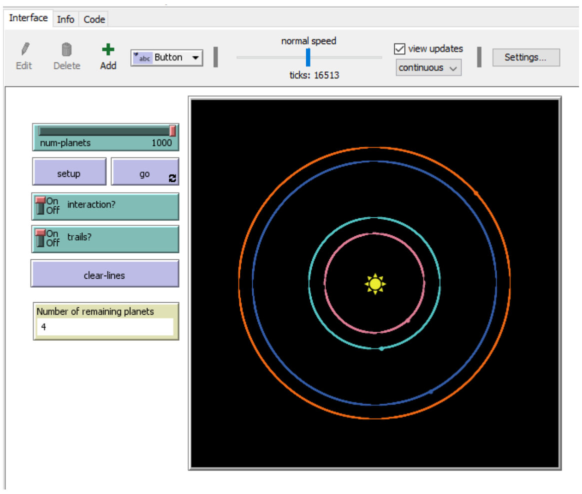

Figure 13. Final result of the Neighborhood Clearing Model after clicking Clear-Lines once.

D. Remarks

This is a common result. We are usually left with a single digit number of planets regardless of whether we start with ten planets or a hundred thousand. On a few occasions, we are left with no planets.

We now have a system that generates an approximation of planetary orbits.

V. Discussion

We begin with a state of mass scattered in three-dimensions and after a three-step process, end up with planetary orbits resembling those we observe in multiplanetary systems. For step one, Circumstellar Disc, a simple pairwise mass exchange process among the various discs is deployed. This process gives a stochastic advantage to the disc with greater mass. Regardless of the starting number of discs or the distribution of mass among these discs, the result of the exchange process iterated over time is always the same: one final disc remains. The model gives us an understanding of why in the presence of such interactions between objects in different intersecting discs, it would be unlikely for highly non-coplanar to exist in the long term.

For step two, Unity of Direction, we make the objects adjust their velocities based on the velocities of their neighbors. Here, we add the simplifying assumption that objects travel in circular paths, clockwise or counterclockwise, at different speeds. The code allows for the radius of influence of objects to be altered. In the presentation above, the radius is set proportionate to the square-root of the object mass. As long as the world is populated densely enough with the objects, the result is the same: over time, all objects move in one direction. For larger radii of influence, this result is achieved quicker. For smaller radii, it takes longer, and the gathering of objects closer together over time becomes more visible.

Step three, Neighborhood Clearing, enables us to see the formation of multiplanetary orbits. It starts with objects dispersed around the central star in a single plane, as justified by the results of the Circumstellar Disc model. It assigns mass to each of these objects, the way the previous models do. And finally, it also assigns velocity with a single sign to all these objects, as the outcome of the Unity of Direction model implies would be the case. Then we allow each object to perform three simple actions: a forward movement proportional to velocity, a fixed movement towards the sun, and removal of any intercepting object with lower mass. Whatever the starting number of objects may be, the vast majority of objects quickly get eliminated from the simulation, and we are left with stable, cleared orbits for the few objects that remain. These objects now satisfy the neighborhood clearing requirement necessary for getting designated as planets.

VI. Conclusion

This paper demonstrates how various phenomena we observe in planetary systems can be derived from simple rules. In this demonstration we apply certain simplifying assumptions, but retain the essential core characteristics of the objects under study. In none of the three models presented is the final outcome coded in; rather, outcomes are allowed to emerge. One of the strengths of agent-based modeling is that it helps us visualize and investigate emergent patterns. The collapse of materials around a star into one circumstellar disc, the movement of non-stellar objects in one direction, and the clearing of neighborhood to form non-intersecting orbits are all examples of emergent phenomena.

It is worth mentioning that this is just one way of simulating planetary orbits. One advantage of following the steps described in this paper is that it helps us perform the entire simulation while remaining in a two-dimensional simulation environment. Future setups might make use of a three-dimensional environment, or highlight other features such as the rotation of the central star or the presence of planetary moons—aspects of planetary systems the above models abstracted away.

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