170, 100, 75, 104, 97, 75, 123, 100, 75, 123, 100, 97, 120, 109

Assignment 1

Ans-1) 4 4 5 5 6 6 6 6 7 7 7 7 7 8 8 8 8 9 10

$$\text{Mean} = \frac{\Sigma x_i}{n} = \frac{128}{19} = 6.73$$

Median = 7

Mode = 7

Ans-2) 28 40 68 70 75 75 75 80 86 89 90 90 97
100 100 100 104 109 113 120 120 120 122
123 123 127 130 140 145 170 174 194

Mean = 103.81

$$\text{Median} = \frac{100+104}{2} = 102$$

Mode = 75, 100, 120 } → Multimodal distribution

Ans-3) $E(x) = \Sigma x \cdot f(x)$

$= 0\times0.09 + 1\times0.15 + 2\times0.40 + 3\times0.25 + 4\times0.10 + 5\times0.01$

$= 0 + 0.15 + 0.8 + 0.75 + 0.4 + 0.05$

$= 2.15$

$\sigma^2 = E(x^2) - [E(x)]^2$

$= 0^2\times0.09 + 1^2\times0.15 + 2^2\times0.40 + 3^2\times0.25 + 4^2\times0.10 + 5^2\times0.01$ $- (2.15)^2$

$= 0 + 0.15 + 1.6 + 2.25 + 1.6 + 0.25$

$= 5.85 - (2.15)^2$

$= 5.85 - 4.62$

$= 1.22$

Ans-4)
&
Ans-9)

4 & 9 are same qs

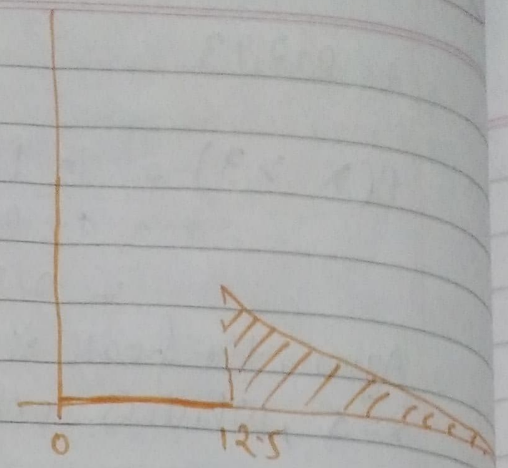
$$\int_{12.5}^{12.6} f(d)\, \partial d$$

$$= \int_{12.5}^{12.6} 20 e^{-20(d-12.5)}$$

$$= \left. \frac{20 e^{-20(d-12.5)}}{-20} \right]_{12.5}^{12.6}$$

$$= -\left[e^{-20\times 0.1} - e^{0} \right]$$

$$= 0.864$$



proportion of parts with $d > 12.6$ mm $= 1 - 0.864$
$= 0.136$

CDF $(d = 11) = 0$ } The probability of finding a hole with $d < 12.5 = 0$

The pdf is an exponential function, the graph decreases ~~slowly~~ rapidly from 12.5 to 12.6 and kind of plateaus later. So most of the holes drilled in the aluminium sheet is between 12.5 to 12.7 i.e approximately 98%. The chances of finding holes > 12.7 is very low (thin tails)

Ans.5) probability of having 2 faulty LEDs $= {}^6C_2 (0.3)^2 (0.7)^4$

$= 15 \times 0.09 \times 0.24$

$= 0.32$

Pdf $\rightarrow$ $[P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5) + P(X=6)] \times X$

} $X \rightarrow$ random variable of getting a faulty LED

$E(X) = {}^6C_0 (0.3)^0 \times (0.7)^6 \times 0 + {}^6C_1 (0.3) \times (0.7)^5 \times 1 + {}^6C_2 (0.3)^2 (0.7)^4 \times 2$
$+ {}^6C_3 (0.3)^3 \times (0.7)^3 \times 3 + {}^6C_4 (0.3)^4 \times (0.7)^2 \times 4 + {}^6C_5 (0.3)^5 (0.7) \times 5$
$+ {}^6C_6 (0.3)^6 \times (0.7)^0 \times 6$

$= 0.117 \times 0 + 0.30 \times 1 + 0.32 \times 2 + 0.185 \times 3 + 0.059 \times 4$
$+ 0.01 \times 5 + 0.000729 \times 6$

$= \cancel{0.117}\ 0 + 0.30 + 0.64 + 0.555 + 0.236 + 0.05$
0.0043

$= 1.7853$

$\sigma^2 = E(X^2) - [E(X)]^2$

$= [0 + 1^2 \times 0.3 + 2^2 \times 0.32 + 3^2 \times 0.185 + 4^2 \times 0.059$
$+ 5^2 \times 0.01 + 6^2 \times 0.000729] - (1.785)^2$

$= [0.3 + 1.28 + 1.665 + 0.944 + 0.25$
$+ 0.026] - 3.186$

$= 1.279$

$\Rightarrow \sigma = \sqrt{1.279} = 1.13$

Ans 6) $P(\text{Barkha} = 5 \ \& \ \text{Gaurav} = 5)$

$= P(B=5) \times P(G=5)$ } Independent events so joint probability = product of marginal probabilities.

$= {}^{12}C_5 (0.45)^5 (0.55)^7$

$\times \ {}^{8}C_5 (0.75)^5 (0.25)^3$

$= 0.2225 \times 0.2076$

$= 0.046$

$P(B=4 \ \& \ G=4)$

$= {}^{12}C_4 (0.45)^4 (0.55)^8 \times {}^{8}C_4 (0.75)^4 (0.25)^4$

$= 0.1699 \times 0.086$

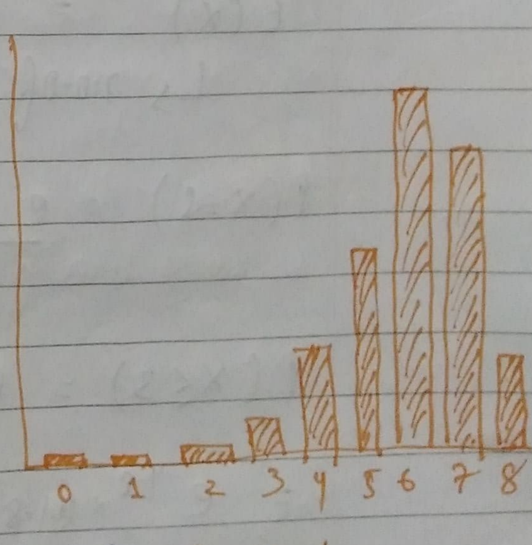
$= 0.00146$

$P(B=6 \ \& \ G=6)$

$= {}^{12}C_6 (0.45)^6 (0.55)^6 \times {}^{8}C_6 (0.75)^6 \times (0.25)^2$

$= 0.2123 \times 0.3114$

$= 0.066$



If we look at the pdf for Gaurav, the probability of getting 6 correct answers is highest. This is intuitive, as probability of getting a correct answer = 75%

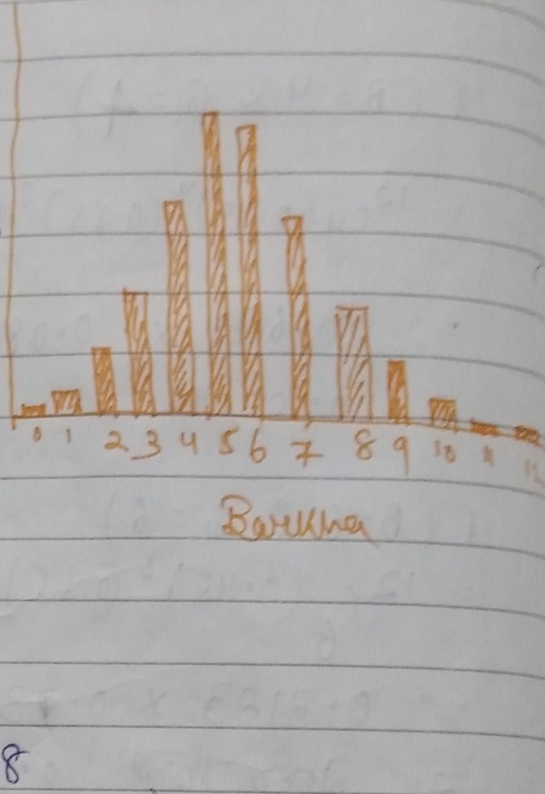
Also, $(6/8 \times 100) = 75\%$. So, it is highly likely that Gaurav will get 6/8 answers correct in majority of occassions.

Also, getting 7 answers correct is more likely to happen than getting 5 answers correct, as Gaurav is more likely to get correct answers ~~and~~ rather than wrong answers, but it can't be generalized.

What can in fact be generalized, is the peak of the distr will happen at $(x=6)$ as this is the exact prob of getting an answer correct.

The similar argument can be made for Barkha

Getting 5 or 6 correct answers is closer to the correction rate $= 45\%$ and therefore the frequency is highest for $(x=5)$ followed by $(x=6)$



-7) a) 5 customers

$$E(x) = \frac{4 \times 72}{60} = 4.8$$

↳ no. of customers arriving ~~to~~ in my shop in 4 mins

$$P(X=5) = \frac{e^{-4.8}(4.8)^5}{5!} = 0.174$$

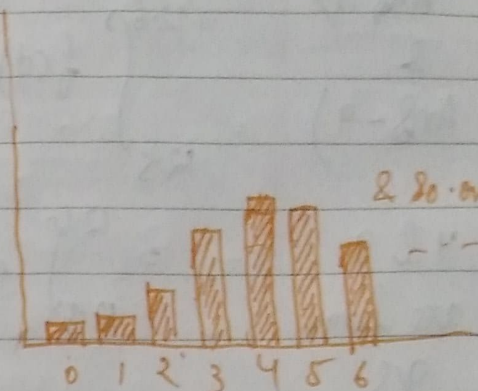
$$P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= \frac{e^{-4.8} \times (4.8)^0}{1} + \frac{e^{-4.8} \times 4.8}{1} + \frac{e^{-4.8} \times 4.8^2}{2} + \frac{e^{-4.8} \times 4.8^3}{6}$$

$$= 0.0082 + 0.039 + \frac{0.189}{2} + 0.1516$$

$= 0.293$

$P(X > 3) = 1 - P(X \leq 3)$
$= 1 - 0.293$
$= 0.707$

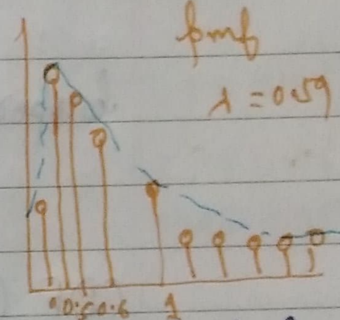


Again, the peak is at $x = 4$ followed by $x = 5$, which is consistent with $E(X)$

Ans-8)

$E(X)$
↳ no. of Errors ~~in entering a word~~ in a 455 word page

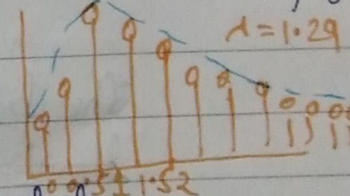
$$= \frac{6}{77 \times 60} \times 455 = 0.59$$



$$P(X = 2) = \frac{e^{-0.59} \times (0.59)^2}{2!} = 0.096$$

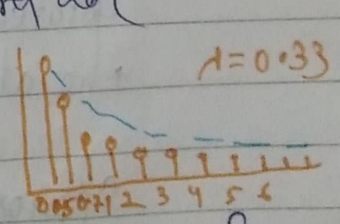
$E(X')$ $X' \rightarrow$ no. of Errors in ~~entering a~~ a 1000 words page

$$= \frac{6}{77 \times 60} \times 1000 = 1.29$$



$E(X'')$ $X'' \rightarrow$ no. of Errors in a 255 word doc

$$= \frac{6}{77 \times 60} \times 255 = 0.33$$



The λ for a poisson distribution is given by $\lambda = np$

As n increases, i.e the no. of words typed increases, the mean will shift towards right. This is because, probability of committing an Error remaining constant, the mean of Errors committed will increase.

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Ans-10) a) $P(Z > 1.26) = 1 - P(Z \leq 1.26) =$
$1 - 0.896165 = 0.10384$

$P(Z < -0.86) = 0.194894$

$P(Z > -1.37) = 1 - P(Z \leq -1.37)$
$= 1 - 0.0853 = 0.9147$

$P(-1.25 < Z < 0.37) = P(Z \leq 0.37) - P(Z \leq -1.25$
$= 0.644309 - 0.105$
$= 0.639$

$P(Z \leq -4.6) \simeq 0$

Ans-11) $P(X > 13 \text{ mA})$

$\Rightarrow P\left(\frac{X-10}{\sqrt{4}} > \frac{13-10}{\sqrt{4}}\right)$

$\Rightarrow P(Z > 1.5) = 1 - P(Z \leq 1.5)$
$= 1 - 0.933$
$= 0.067$

$P(9 < X < 11)$

$= P\left(\frac{9-10}{2} < \frac{X-10}{2} < \frac{11-10}{2}\right)$

$= P(-0.5 < Z < 0.5)$

$= P(Z \leq 0.5) - P(Z \leq -0.5)$
$= 0.69 - 0.277$
$= 0.413$

$P(Z < z) = 0.98$
$\Rightarrow z = 2.06$ → So current $= 6z + \mu = 14.12$ mA

Ans-10)
b) $P(Z > z) = 0.05$
$\Rightarrow 1 - P(Z < z) = 0.05$
$\Rightarrow P(Z < z) = 0.95$
$z = 1.65$

c) $P(-z < Z < z) = 0.99$
$\Rightarrow P(Z < z) - P(Z < -z) = 0.99$

$\Rightarrow z = 2.58$

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Ans 12) $P(0.2485 < x < 0.2515)$

$= P\left(\frac{0.2485 - 0.2508}{0.0005} < Z < \frac{0.2515 - 0.2508}{0.0005}\right)$

$= P(-4.6 < Z < 1.4)$

$= P(Z < 1.4) - P(Z < -4.6)$

$= 0.91924 - 0.000$

$= 0.91924$

$P(0.2485 < x < 0.2515)$

$= P\left(\frac{0.2485 - 0.2500}{0.0005} < Z < \frac{0.2515 - 0.2500}{0.0005}\right)$

$= P(-3 < Z\ 3)$

$= P(Z < 3) - P(Z < -3)$

$= 0.9986 - 0.0013$

$= 0.9973$

In the first case, when mean = 0.2508, the mean of shafts ~~are~~ is located closer to upper specification (0.2 and therefore which are not in sync with the specifications are unusually large. This also explains low yield

If mean is ~~increased~~ decreased to 0.2500 & the range of acceptable shaft diameter remaining same, it is possible to yield more number of shafts.