Swapneel Bhatt
sdb2183

Homework 3

1) mass - m sampling rate: Δt

Treating m as point mass, upwards as +ve z

$$\ddot{z}(t) = \frac{1}{m}\left(T(t) + \tilde{\tau}_T(t)\right) - g$$

where $T(t)$ is thrust force

$\tilde{\tau}_T(t)$ is thrust input noise

State vector $x(t) = \begin{bmatrix} z(t) \\ v(t) \end{bmatrix}$ → height, → velocity

$$\dot{x}(t) = \begin{bmatrix} \dot{z}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} v_k \\ \frac{T_k}{m} - g \end{bmatrix}$$

$$z_{k+1} = z_k + v_k \Delta t + \frac{1}{2}\left(\frac{T_k}{m} - g\right) \cdot \Delta t^2$$

(using equation of motion $s = ut + \frac{1}{2}at^2$)

$$v_{k+1} = v_k + \left(\frac{T_k}{m} - g\right)\Delta t$$

(using equation of motion $v = u + at$)

State Space Model:

$$x_k = Ax_{k-1} + Bu_k + G_1 w_k$$

$$h_k = Cx_k + z_h(k)$$

$h_k \rightarrow$ sensor measurement (height) $\quad u_k \rightarrow$ input

$z_h \rightarrow$ sensor noise $\sim N(0, \sigma_h^2)$ $\quad w_k \rightarrow$ input noise

Where,

$$A = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{2}\Delta t^2 \\ \Delta t \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Input $u_k = \frac{T_k}{m} - g$

Thrust noise $z_T = N(0, \sigma_T^2)$

input noise $w_k \sim N(0, \Sigma_u)$

where $\Sigma_u = \left(\frac{\sigma_T}{m}\right)^2$

The input covariance matrix can be computed as

$$Q = B \Sigma_u B^T$$

$$= \begin{bmatrix} 0.5\Delta t^2 \\ \Delta t \end{bmatrix} \left(\frac{\sigma_T}{m}\right)^2 \begin{bmatrix} 0.5\Delta t^2 & \Delta t \end{bmatrix}$$

$$Q = \left(\frac{\sigma_T}{m}\right)^2 \begin{bmatrix} \frac{\Delta t^4}{4} & \frac{\Delta t^3}{2} \\ \frac{\Delta t^3}{2} & \Delta t^2 \end{bmatrix}$$

The sensor directly measures altitude, so sensor noise

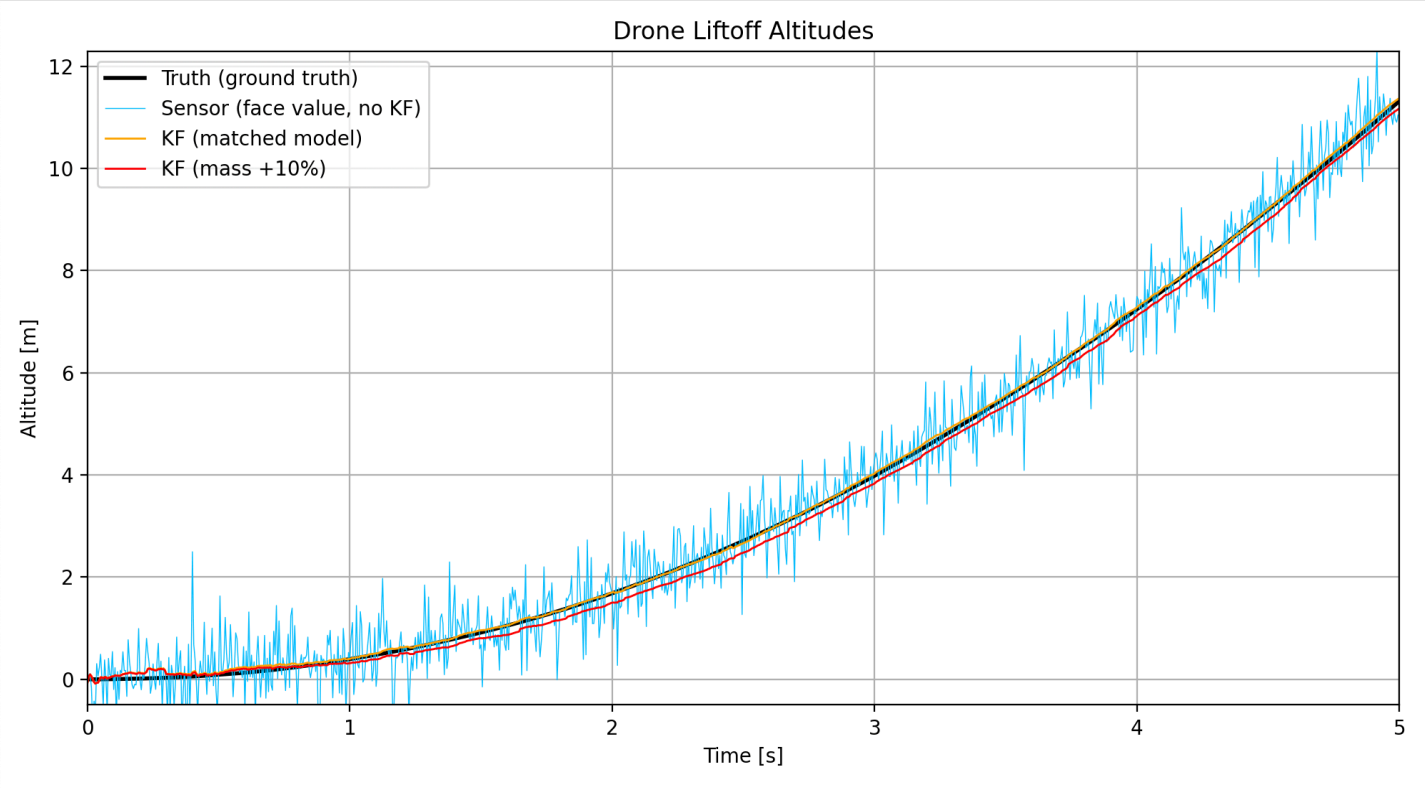
$$\Sigma_z = \sigma_n^2$$

As the state vector is 2×1, the state covariance $\text{cov}(x_k) = \mathbb{E}\left((x_k - \mu_k)(x_k - \mu_k)^T\right)$ should

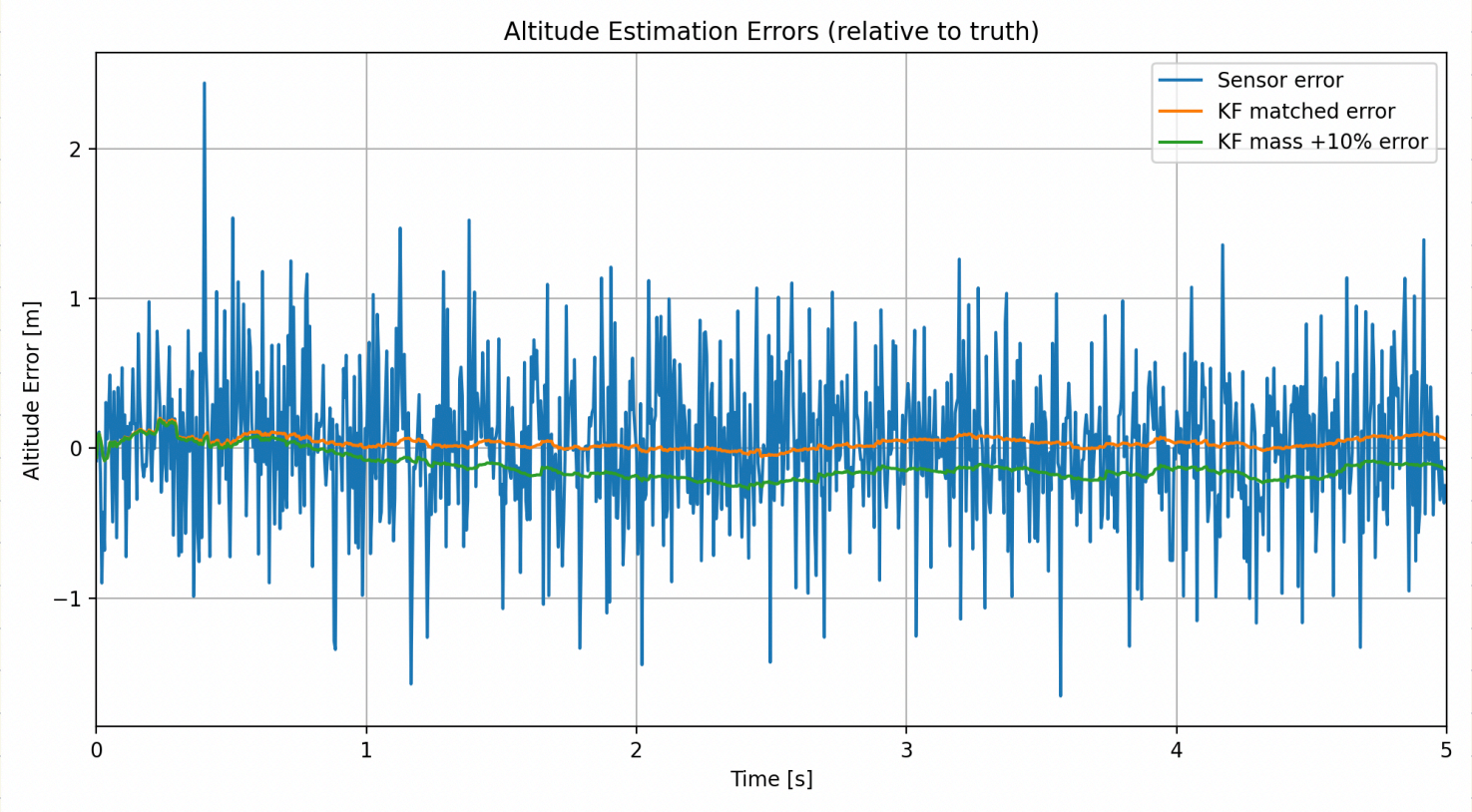
be $\mathbb{R}^{2\times 2}$

hence, dimension of the covariance is 2×2

2) Drone liftoff altitudes:



Estimation error plot:



Effect of model mismatch:

The effect of the model mismatch is that it adds a bias to the estimation. The prediction step underestimate upward acceleration, hence the filter thinks that the drone should rise slower than it actually does.