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# Homework 3

1) Mass - $m$ sampling rate: $\Delta t$

Treating $m$ as point mass, upwards as +ve $z$

$$\ddot{z}(t) = \frac{1}{m}\left(T(t) + \tilde{\varepsilon}_T(t)\right) - g$$

where $T(t)$ is thrust force
$\tilde{\varepsilon}_T(t)$ is thrust input noise

State vector $x(t) = \begin{bmatrix} z(t) \\ v(t) \end{bmatrix}$ → height → velocity

$$\dot{x}(t) = \begin{bmatrix} \dot{z}(t) \\ \dot{v}(t) \end{bmatrix} = \begin{bmatrix} v_k \\ \frac{T_k}{m} - g \end{bmatrix}$$

$$z_{k+1} = z_k + v_k \Delta t + \frac{1}{2}\left(\frac{T_k}{m} - g\right) \cdot \Delta t^2$$

( using equation of motion $s = ut + \frac{1}{2} at^2$ )

$$v_{k+1} = v_k + \left( \frac{T_k}{m} - g \right) \Delta t$$

( using equation of motion $v = u + at$ )

State Space Model:

$$\boxed{\begin{aligned} x_k &= A x_{k-1} + B u_k + G w_k \\ h_k &= C x_k + \tilde{\eta}_h(k) \end{aligned}}$$

$h_k \rightarrow$ sensor measurement (height)  $u_k \rightarrow$ input

$\tilde{\eta}_h \rightarrow$ sensor noise $\rightarrow \mathcal{N}(0, \sigma_h^2)$  $G w_k \rightarrow$ input noise

Where,

$$A = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{2} \Delta t^2 \\ \Delta t \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Input $u_k = \frac{T_k}{m} - g$

Thrust noise $\tilde{\eta}_T = \mathcal{N}(0, \sigma_T^2)$

$$\boxed{\text{input noise } \omega_k \sim \mathcal{N}(0, \Sigma_u) \quad \text{where } \Sigma_u = \left(\frac{\sigma_T}{m}\right)^2}$$

The input covariance matrix can be computed as

$$Q = B \Sigma_u B^T$$

$$= \begin{bmatrix} 0.5t^2 \\ \Delta t \end{bmatrix} \left(\frac{\sigma_T}{m}\right)^2 \begin{bmatrix} 0.5t^2 & \Delta t \end{bmatrix}$$

$$\boxed{Q = \left(\frac{\sigma_T}{m}\right)^2 \begin{bmatrix} \frac{\Delta t^2}{4} & \frac{\Delta t^3}{2} \\ \frac{\Delta t^3}{2} & \Delta t^2 \end{bmatrix}}$$

The sensor directly measures altitude, so sensor noise $\boxed{\Sigma_z = \sigma_n^2}$

as the state vector is $2 \times 1$, the state covariance $cov(x_k) = \mathbb{E}((x_k - \mu_k)(x_k - \mu_k)^T)$ should be $\mathbb{R}^{2 \times L}$

hence, dimension of state covariance is $2 \times 2$