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## Homework 2

1) The predicted belief will be the same as initial conditions. The prediction model will change nothing.

Probability when we do nothing:

$$P(x_{t+1} = \text{open} \mid x_t = \text{open}, u = \text{none}) = 1$$

$$P(x_{t+1} = \text{closed} \mid x_t = \text{open}, u = \text{none}) = 0$$

$$P(x_{t+1} = \text{closed} \mid x_t = \text{closed}, u = \text{none}) = 0$$

$$P(x_{t+1} = \text{open} \mid x_t = \text{closed}, u = \text{none}) = 0$$

Our initial belief (initial cond, no prior knowledge)

$$\text{bel}(\text{open}) = 0.5$$

$$\text{bel}(\text{closed}) = 0.5$$

Prediction if no action is taken:

$$\begin{aligned} \text{bel}(\text{open}) = & P(\text{open} \mid \text{open}, u = \text{none}) \text{bel}(\text{open}) + \\ & P(\text{open} \mid \text{closed}, u = \text{none}) \text{bel}(\text{closed}) \end{aligned}$$

$$= 1 \cdot 0.5 + 0 \cdot 0.5$$

$$\Rightarrow \bar{\text{bel}}(\text{open}) = 0.5$$

$$\text{Similarly, } \bar{\text{bel}}(\text{closed}) = 0.5$$

hence, prediction is same as initial condition.

2) Sensor model:

$$P(z = \text{open} | x = \text{open}) = 0.9$$

$$P(z = \text{closed} | x = \text{open}) = 0.1$$

$$P(z = \text{open} | x = \text{closed}) = 0.5$$

$$P(z = \text{closed} | x = \text{closed}) = 0.5$$

Action model:

$$P(x_{t+1} = \text{open} | x_t = \text{closed}, a = \text{push}) = 0.6$$

$$P(x_{t+1} = \text{open} | x_t = \text{open}, a = \text{push}) = 1$$

Push policy:

$$\text{If } \text{bel}(\text{open}) < 0.9$$

$a_t = \text{push}$ , then measure

$$\text{If } \text{bel}(\text{open}) > 0.9$$

$a_t = \text{none}$ , measure

$$\text{If } \text{bel}(\text{open}) \geq 0.99$$

$\mu_c = \text{none}$ ,  $z = \text{none}$ ,  
robot proceed through.

from class, at step 0

$$\text{bel}(\text{open}) = 0.5$$

action: push

$$\text{bel}^f(\text{open}) = 0.878$$

$$\text{bel}(\text{closed}) = 0.5$$

$$\overline{\text{bel}}(\text{open}) = 0.8$$

$$\overline{\text{bel}}(\text{closed}) = 0.2$$

$$\text{bel}^f(\text{closed}) = 0.122$$

at step 1

$$\text{bel}(\text{open}) = 0.878$$

$$< 0.99$$

action: push

$$\text{bel}(\text{closed}) = 0.122$$

$$\overline{\text{bel}}(\text{open}) = P(\text{open}|\text{open}, \text{push}) \text{bel}(\text{open})$$

$$+ P(\text{open}|\text{closed}, \text{push}) \text{bel}(\text{closed})$$

$$= 1 \cdot 0.878 + 0.6 \cdot 0.122$$

$$= 0.951$$

$$\overline{\text{bel}}(\text{closed}) = 0.049$$

$$P(x=\text{open} | z=\text{open}) = \eta P(z=\text{open} | x=\text{open}) \bar{be} | (\text{open})$$

$$= \eta 0.9 \cdot 0.951$$

$$P(x=\text{open} | z=\text{open}) = \eta 0.8559 \quad - (1)$$

$$P(x=\text{closed} | z=\text{open}) = \eta P(z=\text{open} | x=\text{closed}) \bar{be} | (\text{closed})$$

$$= \eta (0.5) (0.049)$$

$$= \eta 0.0245 \quad - (2)$$

$$(1) + (2) = 1 \Rightarrow \eta = 1.13584$$

$\Downarrow$

$$bel^+(\text{open}) = 0.9717 \quad \text{+ continue meaning}$$

$$bel^+(\text{closed}) = 0.0278$$

now, at step 2

$$bel(\text{open}) = 0.9717 \quad \rightarrow \text{no push, only measure}$$

$$bel(\text{closed}) = 0.0278 \quad bel = \bar{be}$$

measurement step

$$P(x=\text{open} | z=\text{open}) = h \cdot P(z=\text{open} | x=\text{open}) \cdot \bar{bel}(\text{open})$$
$$= h \cdot 0.9 \cdot 0.9717$$

$$P(x=\text{open} | z=\text{open}) = 0.87453 \cdot h$$

$$P(x=\text{closed} | z=\text{open}) = h \cdot P(z=\text{open} | x=\text{closed}) \cdot \bar{bel}(\text{closed})$$
$$= h \cdot 0.5 \cdot 0.0278$$
$$= 0.0139 \cdot h$$

$$\Rightarrow h = 1.1255$$

$$bel^+(open) = 0.9842 \rightarrow \text{continue measuring}$$

$$bel^+(closed) = 0.0156$$

at Step 3

$$bel(open) = 0.9842 \rightarrow \text{just measure}$$

$$bel(closed) = 0.0156$$

$$\bar{bel} = bel$$

$$P(x = \text{open} | z = \text{open}) = h \cdot P(z = \text{open} | x = \text{open}) \cdot \bar{bel}(\text{open})$$

$$= h \cdot 0.9 \cdot 0.9842$$

$$P(x = \text{open} | z = \text{open}) = 0.88578 h$$

$$P(x = \text{closed} | z = \text{open}) = h \cdot P(z = \text{closed} | x = \text{open}) \cdot \bar{bel}(\text{closed})$$

$$= h \cdot 0.5 \cdot 0.0156$$

$$= 0.0078 h$$

$$\Rightarrow h = 1.1909$$

$$bel^f(\text{open}) = 1.1909 \times 0.88578$$

$$= 0.99126$$

Go on at the next  
step!!

Table on next page

Step	bel (open)	bel (closed)	action	bel (open)	bel (closed)	bel <sup>T</sup> (open)	bel <sup>T</sup> (closed)
0	0.5	0.5	push	0.8	0.2	0.878	0.122
1	0.878	0.122	push	0.951	0.049	0.977	0.0228
2	0.977	0.0228	measure	0.977	0.0228	0.9842	0.0156
3	0.9842	0.0156	measure	0.9842	0.0156	0.99126	0.00874
4	0.99126	0.00874	go!!	?	?	?	?

The robot needs 2 actions & 4 measurements before it can proceed through the door.

3) Prediction model in matrix form

$$M = \begin{bmatrix} P(\text{open}|\text{open}) & P(\text{open}|\text{closed}) \\ P(\text{closed}|\text{open}) & P(\text{closed}|\text{closed}) \end{bmatrix}$$

$$\begin{bmatrix} \bar{b}_1(\text{open}) \\ \bar{b}_1(\text{closed}) \end{bmatrix} = M \begin{bmatrix} b_1(\text{open}) \\ b_1(\text{closed}) \end{bmatrix}$$

at action: push

at no push just measure

$$M: \begin{bmatrix} 1 & 0.6 \\ 0 & 0.4 \end{bmatrix}$$

$$M: \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Code submitted as hw2q3.py in Github  
HW2 directory

