Exercise 1:

1) To prove that $\boldsymbol{q}_1^T\boldsymbol{x}(t)$ is a constant of motion, we have to take the derivatative w.r.t time and prove it equal to zero.

$$\frac{d(\boldsymbol{q}_1^T\boldsymbol{x})}{dt} = \boldsymbol{q}_1^T\dot{\boldsymbol{x}}$$

We know that $\dot{x} = -\boldsymbol{L}x$, then

$$\frac{d(\boldsymbol{q}_1^T\boldsymbol{x})}{dt} = -\boldsymbol{q}_1^T\boldsymbol{L}\boldsymbol{x}$$

Also, $\boldsymbol{q}_1^T\boldsymbol{L} = 0$.

Since, $\boldsymbol{q}_1^T$ is left eigen vector of $\boldsymbol{L}$. Therefore,

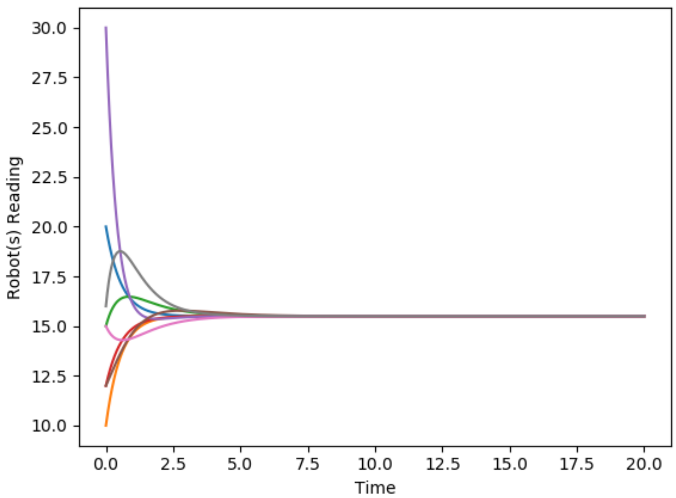
$$\frac{d(\boldsymbol{q}_1^T\boldsymbol{x})}{dt} = 0$$

This proves that $\boldsymbol{q}_1^T\boldsymbol{x}(t)$ is a constant of motion.

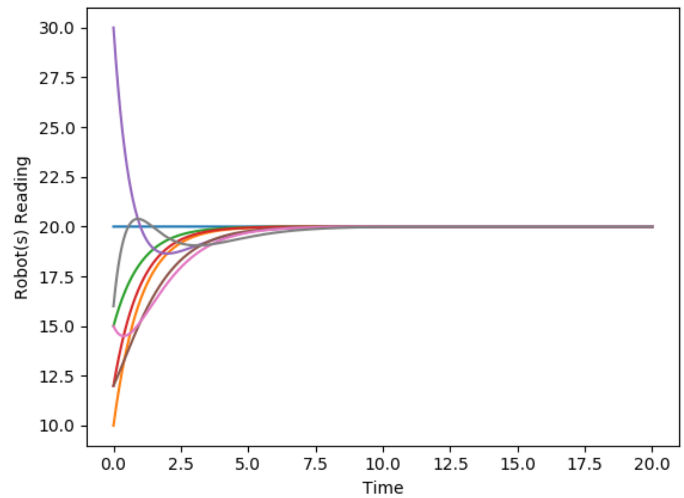
2) Graphs (a), (b) and (d) will converge because all the three graphs contain rooted out branching. Yes, the **consensus value** depends on the initial condition. **However, the convergence(whether the graph will converge or not) doesn't depend on the initial condition.**

3) For graph (a):
Initial condition = [20, 10, 15, 12, 30, 12, 15, 16]
Agreed Value = [15.5]
For graph (b):
Initial condition = [20, 10, 15, 12, 30, 12, 15, 16]
Agreed Value = [20]
For graph (d):
Initial condition = [20, 10, 15, 12, 30, 12, 15, 16]
Agreed Value = [16.25]

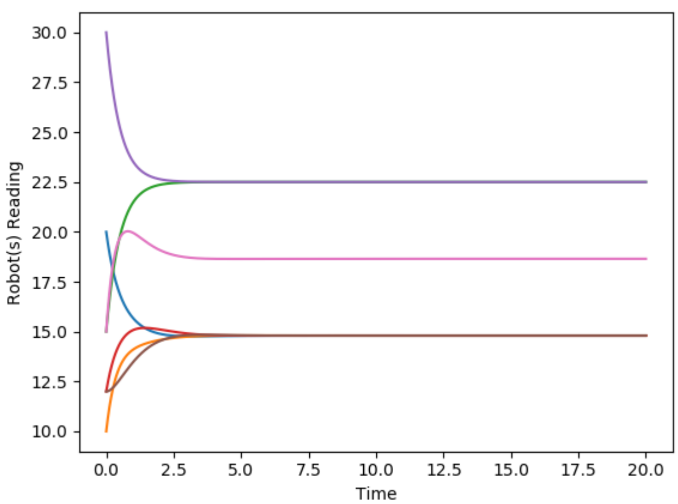
4) For the code please refer to the *“hw2_1_3.py”* file
Following the plots:



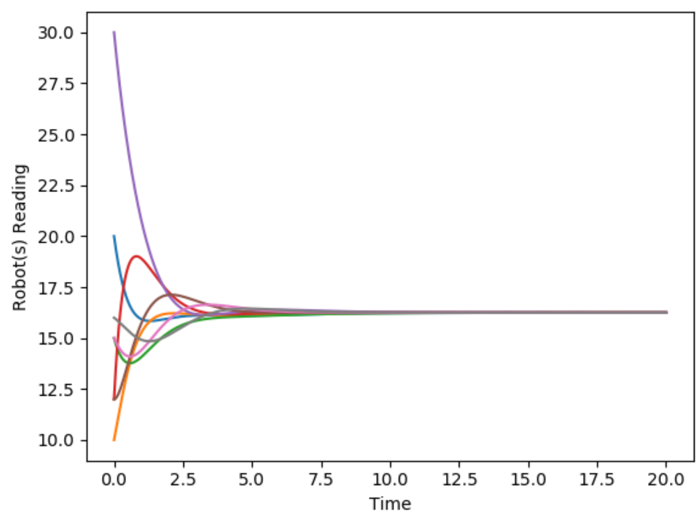
Graph (a)



Graph (b)



Graph (c)



Graph (d)

Exercise 2:

1) To calculate the fixed points of this system, we need to find the solution for $\dot{x} = 0 \; and \; \dot{y} = 0$

$$\dot{x} = 0 \;\Rightarrow\; -x + y = 0 \Rightarrow y = x$$
$$\dot{y} = 0 \;\Rightarrow\; -x - y = 0 \Rightarrow y = -x$$

The only solution for the above two equations is $x = 0 \; and \; y = 0 \; i.e. (x, y) = (0,0)$

2) Given: $\dot{x} = -x + y = 0 \; and \; \dot{y} = -x - y$

For $V_1(x, y) = \frac{1}{2}x^2 + y^2$

Let us evaluate $\dot{V}_1$

$$\dot{V}_1 = x\dot{x} + 2y\dot{y}$$
$$\Rightarrow \dot{V}_1 = x(-x + y) + 2y(-x - y)$$
$$\Rightarrow \dot{V}_1 = -x^2 - xy - 2y^2$$
$$\Rightarrow \dot{V}_1 = -\frac{x^2}{2} - \frac{3}{2}y^2 - \left(\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}}\right)^2$$

Since, $\dot{V}_1 < 0$ and , V_1 is positive definite. Therefore, it is a valid Lypunov function.

For $V_2(x, y) = -x^2 + 100y^2$

Since, $V_2 \ngtr 0$ i.e. it is not strictly positive definite . Therefore, it is not a valid Lypunov function.

For $V_3(x, y) = \left(x - \frac{1}{2}y\right)^2 + \frac{7}{4}y^2$

Let us evaluate $\dot{V}_3$

$$\dot{V}_3 = 2\left(x - \frac{1}{2}y\right)\left(\dot{x} - \frac{1}{2}\dot{y}\right) + \frac{7}{2}y\dot{y}$$
$$\Rightarrow \dot{V}_3 = 2\left(x - \frac{1}{2}y\right)\left((-x + y) - \frac{1}{2}(-x - y)\right) + \frac{7}{2}y(-x - y)$$
$$\Rightarrow \dot{V}_3 = -x^2 + \frac{7}{2}xy - 5y^2 - \frac{7}{2}xy$$
$$\Rightarrow \dot{V}_3 = -x^2 - 5y^2$$

Since, $\dot{V}_3 < 0$ and , V_3 is positive definite. Therefore, it is a valid Lypunov function.

With $\boldsymbol{V_1 \; and \; V_3}$ we can say that fixed point is **asymptotically stable.**

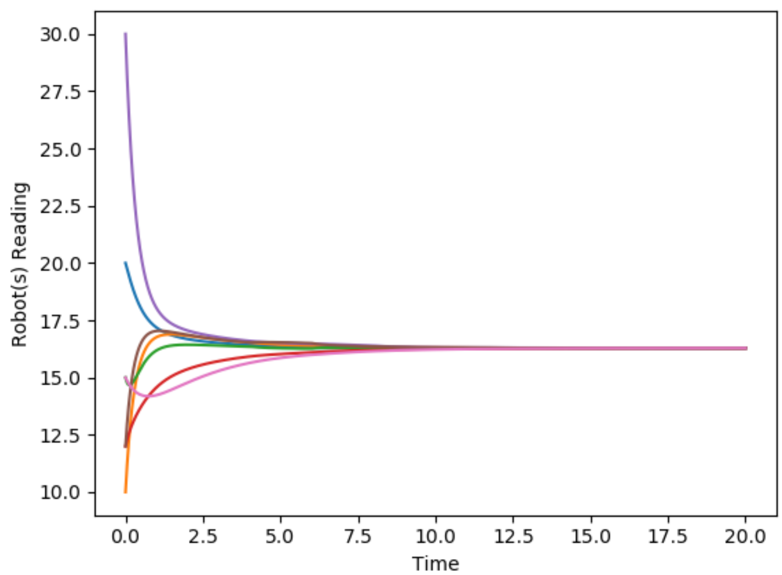
Exercise 4:

1) **Yes**, for each graph the consensus protocol will **converge** since, in all the cases the graph is **connected**. Since, it is also a **balanced digraph**, the consensus protocol will converge to **average value**.

2) **Yes**, even with switching the consensus protocol will **converge** because at all the time the graph is connected.

3) Please refer to *'simulateConsensusSwitch.py'*

4) We can see that with switch time = 2s plot is smooth in comparison to switch time = 0.1s
When switch time = 2s



When switch time = 0.1s