Week 2

1) If A and B are sets and $A \cup B = A \cap B$, then which of the following is(are) true?

    A. Either one of A or B is an empty set.
    B. B is a proper subset of A.
    C. $A = B$
    D. A is a proper subset of B.

Correct Answer : C

Solution: For each element $x \in A$, we have $x \in (A \cup B)$. Since $(A \cup B) = (A \cap B)$, it follows that $x \in (A \cap B)$. Now, $(A \cap B)$ denotes the elements that belong to both A and B. Hence $x \in B$. Therefore every element of A must be in B, which implies that $A \subseteq B$. Similarly for each element $y \in B$, we have $y \in (A \cup B)$. Since $(A \cup B) = (A \cap B)$, it follows that $y \in (A \cap B)$. Hence $y \in A$. This implies that every element of B must be in A. Hence $B \subseteq A$. $A \subseteq B$ and $B \subseteq A$, together implies $A = B$.

Lecture 61:- A proof technique

2) If $A = \{3,3,4,5,6,6,9\}$, $B = \{\{5,9\},12,\{1,\{x,y\}\},\{a,b,c\}\}$, $C = \{\phi\}$, $X = \{x,y,z\}$ and $Y = \{A,B\}$. Which statement is true?

    A. Cardinality of A is 7
    B. Cardinality of B is 4
    C. Cardinality of C is 0
    D. Cardinality of X is 2
    E. Cardinality of Y is 3

Correct Answer: B

Solution: Elements of set B are $\{5,9\}$, 12, $\{1,\{x,y\}\}$, $\{a,b,c\}$. Hence the cardinality of set B is 4
Lecture 45:- Subset part 3, time: 1:00

3) Let $A = \{3,5,6,7,8,9\}$, $B=\{2,4,5,11,14\}$ and $C = \{-1,0,1\}$. Which of the following is not a disjoint set?

    A. A and C
    B. B and C
    C. C and $A \cup B$
    D. A and B

Correct Answer: D

Solution: Given sets are A = {3,5,6,7,8,9}, B={2,4,5,11,14} and C = {-1,0,1}, here we can see that all elements in set A are distinct from elements in set C, hence option A is incorrect, similarly, all elements in set B are distinct from elements in set C, hence option B is incorrect. Elements in A ∪ B are {2,3,4,5,6,7,8,9,11,14} and Set C = {-1,0,1}, therefore clearly these sets are disjoint hence option C is incorrect. Set A and B have number 5 common in them, hence they are not disjoint.

Lecture 49:- Union and intersection part, time: 2:20

4) Let set A = {4,5,6,7,9} and P(A) denote the power set of A. What is the cardinality of P(P(A))?

    A. 2^{16}
    B. 2^{32}
    C. 2^{10}
    D. 2^5

Correct Answer: B

Solution: Cardinality of P(P(A)) is $2^{|P(A)|} = 2^{2^A} = 2^{2^5} = 2^{32}$
Lecture 57:- Power set problem, time: 1:50

5) Let A ={x | x ∈ Z; -2 ≤ x ≤ 5} and B ={ x | x ∈ N; x is a prime number}, what is A − B?

    A. {-2,-1,0,1}
    B. {1,4}
    C. {-2,-1,0,1,4}
    D. {-2,-1,0,1,2,3,4,5}

Correct Answer: C

Solution:A={-2,-1,0,1,2,3,4,5} and B={2,3,5,7,11,......} thus A - B = {-2,-1,0,1,4}

Lecture 61:- A Proof technique , time: 1:00

6) In a group of 120 people, 50 people love to eat dairy milk chocolate, 70 people love to eat Kitkat, and 20 people do not like any type of chocolate. How many people love to eat both types of chocolate?

    A. 20
    B. 30
    C. 10
    D. 40

Correct Answer: A

Solution: $n(A \cup B) = 100$, $n(A) = 50$, $n(B) = 70$ as $n(A \cup B) = n(B) + n(A) - n(A \cap B)$, $n(A \cap B) = 20$

Lecture 43:- Cardinality of union of two sets-part 2, time: 1:30

7) Which of the following pairs of sets are equal?

    A. X={x | x ∈ Z; $0 \leq x \leq 6$} and Y={x | x ∈ R; x < 7}
    B. X, the set of letters in "THYMIER" and Y, the set of letters in "THYMINE"
    C. A = {-2,-3}, B = {x:x is a Solution of $x^2 + 5x + 6 = 0$}
    D. A = {0}; B = {x: x is a positive integral root of the equation $x^2 - x - 42$}

Correct Answer: C

Solution:

Option A: Set X is {0,1,2,3,4,5,6} and set Y contains all the real numbers less than 7, therefore clearly we can say that these two sets are not equal.

Option B: set X has elements {T,H,Y,M,I,E,**R**} and set Y has elements {T,H,Y,M,I,**N**,E} so these two sets are not equal.

Option C: set A = {-2,-3} set B contains roots of the equation $x^2 + 5x + 6 = 0$ so set B = {-2,-3}, therefore set A and B are equal.

Option D: set B contains roots of equation $x^2 - x - 42$ i.e., B = {-6,7}, Hence set A is not equal to set B.

Lecture 61:- A Proof technique, time: 1:00

8) Suppose A = {1,2,3,4}. How many subsets of 2 distinct elements are possible?

    A. 4
    B. 6
    C. 5
    D. 7

Correct Answer: B

Solution:

Total number of subsets for any set with n elements are 2^n therefore here we have $2^4 = 16$ subsets.

Subsets of {1,2,34} are:

{{1},{2},{3},{4},{1,2},{2,3},{3,4},{4,1},{1,3},{2,4},{1,2,3},{2,3,4},{3,4,1},{4,1,2},{1,2,3,4},{ϕ}}

Here we can observe that the number of subsets with 2 distinct elements is 6.

Lecture 43:- Subsets

9) Simply $(A \cap B^c)^c \cup B$

    A. $A^c \cup B^c$
    B. B^c
    C. $A^c \cap B$
    D. $A^c \cup B$

Correct Answer: D

Solution:

$$
\begin{aligned}
(A \cap B^c)^c \cup B &= (A^c \cup (B^c)^c) \cup B \\
&= (A^c \cup B) \cup B \\
&= A^c \cup B \cup B \\
&= A^c \cup B
\end{aligned}
$$

Lecture 59:- Demorgan's law

10) The set O of odd positive integers less than or equal to 10 is _____.

    A. {1,2,3}
    B. {1,3,5,7,9}
    C. {1,2,5,7,9}
    D. {0,1,3,5,7,9}

Correct Answer : B

Solution: Odd numbers less than equal to 10 are 1, 3, 5, 7, 9, Therefore the set of all odd numbers less than equal to 10 is {1,3,5,7,9}