Week 2

I. Examples, Definition & Notations

A set is a collection of objects
↳ {a list of things}

Sets can be finite or infinite.

→ We use capital letters to denote the set
→ Small letters to denote elements of set.

$S = \{a, b, c, d\}$

→ a belongs to S

$a \in S$

$z \notin S$

$A = \{1, 2, 3, 4, \dots, 100\}$

$\equiv A = \{x : 1 \leq x \leq 100, x \text{ is an integer}\}$ → very handy
↳ set builder notation

II. Set - Problems

$\{1, 2, 3, 4\} = \{x \mid x < 5, x \text{ is a +ve integer}\}$

$\{1, 2, 3, \dots 10\} = \{x \mid x < 11, x \text{ is a +ve integer}\}$

→ A set is a collection of objects having some properties in common, but this may not always the case.
↳ unordered

→ Order of elements doesn't matter. Sets are considered equal if they've same elements.
→ In a set, repeated elements are only counted once.
→ Number of elements in a set or size of a set is called cardinality of the set

III. Subsets

$S = \{a, b, c, d, e, f\}$

$T = \{a, c, e, f\}$

T is called a subset of S.

$T \subseteq S$.

Empty set : ϕ

$\phi = \{\}$

Singleton : Set w/ one element

Q. Is ϕ an element of $\{\phi\}$?

True

Q. $\{\phi\} \in \{\{\phi\}, \phi\}$?

True

Q. $A = \{1, \{1\}\}$ Does $\{1\} \in A$?

Yes

Q. $\{\{1\}\} \subseteq A$?

Yes

Q. $\phi \in \{0\}$?

No.

Empty set is a subset of every set.

Q. $\{\{\phi\}\} \subset \{\{\phi\}, \phi\}$?

True

Q. $2 \in \{\{2\}, \{2, \{2\}\}\}$?

False

Q. $\phi \subset \{0\}$?

True , $\phi \notin \{0\}$

→ $S = \{N, Z, Q, R\}$

natural no.s, integers, rational no.s, real nos

cardinality of $S = 4$

→ $\{a, \{a, \{b\}\}\} \longrightarrow$ cardinality $= 2$

→ $\{x \mid x \in Z, x^2 = 2\} \longrightarrow$ cardinality $= 0$

→ $\{\phi\} \rightarrow$ cardinality $= 1$

IV. Union and Intersection of Sets

Intersection: Set of those elements which are common to both sets of whose intersection is being taken.

$A \cap B \rightarrow$ A intersection B

Union: Set of those elements which are either in set A or in set B.

$A \cup B \rightarrow$ A union B

$x \in A \cup B$ if $x \in A$ or $x \in B$

$y \in A \cap B$ if $y \in A$ and $x \in B$

→ $N = \{1, 2, 3, 4, \ldots\}$

$A = \{2, 4, 8, \ldots\}$

$B = \{1, 3, 5, \ldots\}$

$A \cap B = \{\}$

$\boxed{\text{If } A \cap B = \phi \text{, then A and B are disjoint sets}}$

$A \cup B = N$

→ $A = \{\{1, 2, 3\}, \{1, 2, 4\}, 1\}$

$B = \{1, 2, 3\}$

$A \cup B = \{\{1, 2, 3\}, \{1, 2, 4\}, 1, 2, 3\}$

Q. $A \subset B$. What can you say about $A \cup B$?

$A \cup B = B$

$A \cap B = A$

Examples w/ 3 Sets:

$A = \{1, 2, 3, 4, 5, 6\}$, $B = \{1, 3, 5, 7, 9\}$

$C = \{5, 6, 7, 8, 9\}$

$A \cap B \cap C = \{5\}$

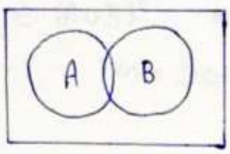
$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A \cup (B \cap C) = ?$

$B \cap C = \{5, 7, 9\}$

$A \cup (B \cap C) = \{1, 2, 3, 4, 5, 6, 7, 9\}$

V. Cardinality of Union of Sets



low_key

$|A \cup B| = |A| + |A| - |A \cap B|$

Q. In a classroom of 100 students, 30 of them play cricket, 80 like playing football. How many of them like both?

sol $100 = 30 + 80 - \cancel{110} (A \cap B)$

$\Rightarrow A \cap B = 10$

$\Rightarrow$ 10 students like playing both cricket and football.

For 3 sets,

$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

VI. Power Set

Power set = set of all subsets of a given set.

$\rightarrow$ If $|A| = n$, then, all possible subsets of A = cardinality of power set of $A = 2^n$

If $|A| = n$, then

no. of subsets w/ r elements $= {}^nC_r = \binom{n}{r}$

$\therefore$ For 5 elements,

no. of subsets $= \binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5}$

$= 2^5 = 32$

Q. Cardinality of power set of $\{a, b\}$?

$= 2^2 = 4$ elements

If power set is P, then

no. of elements in power set of P $= |P(P(S))| = 2^4 = 16$

VII. Complement of a Set & De-Morgan's Laws

Consider a class of 100 people.

U = all students $= \{a_1, a_2, a_3, \dots, a_{100}\}$

$P \subseteq U,\ P = \{a_1, a_2, a_3, \dots, a_{60}\}$

$P^c = U - P = \{a_{61}, a_{62}, a_{63}, \dots a_{100}\}$

→ $P \cap P^c = \phi$

→ $P \cup P^c = U$

Compliment of a set is always w/ respect to a bigger set, of which the given set is a subset of.

De Morgan's Law:

$(A \cup B)^c = A^c \cap B^c$

$(A \cap B)^c = A^c \cup B^c$

De Morgan's law states the above statements.

→ If a set is contained in the other set and the other set is also contained in the given set, then these two sets are equal / same.

Q. Show that: if $x \in (A \cup B)^c$, then

$x \in A^c \cap B^c$

$x \in (A \cup B)^c \Rightarrow x \notin A \cup B$

$x \notin A$ & $x \notin B$

$\Rightarrow x \in A^c$ & $x \in B^c$

$\Rightarrow x \in A^c \cup B^c$ $\qquad \Rightarrow (A \cup B)^c \subset (A^c \cap B^c)$

Hence Proved

Q. Show that: if $y \in A^c \cap B^c$, then $y \in (A \cup B)^c$

$y \in A^c \cap B^c \Rightarrow y \in A^c$ & $y \in B^c$

$\Rightarrow y \notin A$ & $y \notin B$

$\Rightarrow y \notin (A \cup B)$

$\Rightarrow y \in (A \cup B)^c \Rightarrow (A^c \cap B^c) \subset (A \cup B)^c$

Hence Proved

Thus from above two problems,

$(A \cup B)^c = A^c \cap B^c$

Application of De Morgan's Law:

$(A \cup (B \cap C)^c)^c = A^c \cap (B \cap C)$

VIII. Set Difference:

$A - B$ = A with commonalities b/w A and B removed.

$\hookrightarrow$ difference b/w two sets.

$A = \{1, 2, 3, 4, 5, 6\}$

$B = \{10, 5, 30, 6, 100\}$

$A - B = \{1, 2, 3, 4\}$

→ $A - B$ need not be equal to $B - A$.

→ They're actually disjoint sets.

$(A - B) \cap (B - A) = \phi$

$$\boxed{A - B = A \cap B^c}$$

Proof:

$x \in A - B$

$\Rightarrow x \in A, x \notin B$

$\Rightarrow x \in A, x \in B^c$

$\Rightarrow x \in A \cap B^c \Rightarrow A - B \subset A \cap B^c$

$x \in A \cap B^c$

$\Rightarrow x \in A \ \& \ x \in B^c$

$\Rightarrow x \in A \ \& \ x \notin B$

$\Rightarrow x \in A - B \quad \Rightarrow A \cap B^c \subset A - B$

Hence, $A - B = A \cap B^c$.

(Proved)

IX. Symmetric Difference

$$S \triangle C = (S - C) \cup (C - S)$$

Those elements exclusively in S and exclusively in C and never in the intersection.

Try to show that: $S \triangle C = (S - C) \cup (C - S) = (S \cup C) - (S \cap C)$.

Set Theory :

1. Sets
2. Union & intersection of sets
3. Complement of a set
4. Power set
5. De Morgan's law
6. Symmetric Difference
7. History

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