Week 4

1) Which of the following matrices represent a reflexive relation? (multiple select question)

A. $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Correct Answer: C, D, E

Solution: A reflexive relation must have all diagonal entries as 1
Lecture 131: Reflexive relation-matrix representation

2) A = {srijit, akash, abhi} and B = {shraddha, sanchita}
Which of the following subsets belong to A × B?

A. {(srijit, sanchita), (abhi, shraddha), (akash, sanchita), (srijit, shraddha)}
B. {(abhi, shraddha), (akash, shraddha), (sanchita, srijit), (abhi, sanchita)}
C. {(akash, akash), (akash, shraddha), (srijit, sanchita), (abhi, shrijit)}
D. {(shrijit, shraddha), (shraddha, shraddha), (shraddha, sanchita), (abhi, shrijit)}

Correct Answer: A

Solution: $A \times B = \{(x,y) \mid x \in A, y \in B\}$
A × B = {(srijit, shraddha),(srijit, sanchita),(akash, shraddha),(akash, sanchita),(abhi, shraddha),(abhi, sanchita)}
Lecture 124: Set representation of a relation

3) What is the total number of reflexive relations of the set {5,7,13,15}?

   A. 256
   B. 14
   C. 64
   D. 4096

   Correct Answer: D

   Solution: total number of reflexive relations is $2^{n^2-n} = 2^{4^2-4} = 4096$
   Lecture 132: Number of reflexive relations

4) S = {1,2,3,4,5}. A relation R on set S is defined as $R = \{(b,a) \mid 0 \leq -a + b \leq 3\}$
   What is the cardinality of set R?

   A. 25
   B. 8
   C. 14
   D. 12

   Correct Answer: C

   Solution: a,b ∈ S and $0 \leq -a + b \leq 3$
   R = {(1,1),(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(2,5),(3,3),(3,4),(3,5),(4,4),(4,5),(5,5)}
   Therefore, the cardinality of R is 14.
   Lecture 126: Examples of relations

5) Let *R* be a relation on a collection of sets defined as follows,

$$R = \{(A,B) \mid A \subseteq B\}$$

Which of the following statement(s) is/are correct? (multiple select question)

A. R is reflexive and transitive
B. R is symmetric
C. R is anti-symmetric
D. R is reflexive but not transitive

Correct Answer: A, C

Solution:
Given $R = \{(A, B) \mid A \subseteq B\}$

A. For any set A, we have $A \subseteq A$. Hence $(A, A) \in R$. Hence R is reflexive. Moreover, for three sets A, B and C if $(A, B) \in R$ and $(B, C) \in R$, then from the given relation, we have $A \subseteq B \subseteq C$. Clearly $A \subseteq C$, and form the definition of R we have $(A, C) \in$ R. Hence R is transitive.
B. Let $A = \{1, 2\}$ and $B = \{1, 2, 3, 4\}$. Clearly $A \subseteq B$, but $B \nsubseteq A$. Hence $(A, B) \in R$ does not imply $(B, A) \in R$. Hence R is not symmetric.
C. If $A \subseteq B$, and $B \subseteq A$, then $A = A$. Hence if $(A, B) \in R$ and $A \neq B$, then $(B, A) \notin R$. Therefore R is anti-symmetric.
D. As R is transitive, the last option is not correct.

Lecture 129, 133, 140: Reflexive relations, symmetric relations, transitive relations.

6) Let a relation R be defined as $R = \{(A, B) \mid$ Both A and B live in the same city$\}$. Pick out the correct statement(s).

A. R is anti-symmetric
B. R is reflexive
C. R is transitive
D. R is symmetric

Correct Answer: B, C, D

Solution:
Given $R = \{(A, B) \mid$ Both A and B live in the same city $\}$.

A. "*A and B live in the same city*" is the same as "*B and A live in the same city*". Hence $(A, B) \in R$, implies $(B, A) \in R$. Hence it is symmetric, it is not anti-symmetric.
B. Clearly $(A, A) \in R$, for all A. Hence R is reflexive.

C. Suppose $(A, B) \in R$ and $(B, C) \in R$. Hence A and B live in the same city, and B and C also live in the same city. Therefore A and C live in the same city, and from the definition of R, it is clear that $(A, C) \in R$. Hence R is transitive.
D. As proved in option A. R is symmetric.

7) Which of the following is an equivalence relation?

A. $R = \{(a,b) \mid \text{both } a \text{ and } b \text{ are even non-zero integers and } \frac{a}{b} \text{ is an integer}\}$
B. $R = \{(x,y) \mid y - x = 0\}$
C. R=$\{(1,2),(2,3),(3,4),(4,5),(5,6)\}$
D. R=$\{(a,b) \mid a \le b^3\}$

Correct Answer: B

Solution:

A. A relation R on a set A is said to be reflexive if $(a, a) \in R$ for all $a \in A$. R is called symmetric if $(a, b) \in R$ implies $(b, a) \in R$, and R is called transitive if (a, b) and (b, c) is in R implies $(a, c) \in R$. If a relation R is reflexive, symmetric and transitive, then it is called an equivalence relation. For any non-zero even integer a, $\frac{a}{a} = 1$ is an integer. Hence, $(a,a) \in R$, which implies that R is reflexive. Now, let $a = 4$, and $b = 2$. Then, $\frac{a}{b} = \frac{4}{2} = 2$ is an integer. Hence, $(a, b) \in R$. But $\frac{b}{a} = \frac{2}{4} = \frac{1}{2}$ is not an integer. Therefore, $(b, a) \notin R$. It follows that R is not symmetric. Let $(a, b) \in R$ and $(b, c) \in R$. That is, both $\frac{a}{b}$ and $\frac{b}{c}$ are integers. Hence, their product $\frac{a}{b} \cdot \frac{b}{c} = \frac{a}{c}$ is also an integer. It follows that $(a, c) \in R$. Therefore, R is transitive. Although R is reflexive and transitive but not symmetric, it is not an equivalence relation.
B. $y - x = 0$ is the equation of a line, it can be written as $y = x$, therefore $(x,x) \in R$ and by the above explanation, R is an equivalence relation.
C. Similar explanation as option A.
D. Similar explanation as option A.

Lecture 153: Equivalence relations

8) Suppose the cardinality of a set A is 4 and the cardinality of a set B is 3, what are the cardinalities of the cartesian product A × B and the power set of A × B?

A. 7 and 128
B. 12 and 144
C. 12 and 4096
D. 7 and 49

Correct Answer: C

Solution: Let the cardinality of set A be n(A) and the cardinality of set B be n(B). Then, the cardinality of the cartesian product of (A × B) is n(A × B) = n(A) × n(B) = 4 × 3 = 12. If a set A has cardinality n, then the cardinality of the power set of A is 2^n. It follows that the cardinality of the power set of (A × B) is 2^{12} = 4096. Hence, option C is correct.
Lecture 123: Cartesian product

9) Which of the following collection of subsets is a partition of $A = \{1,2,3,4,5\}$

A. $\{1,2,3\}, \{2,3,4,5\}$
B. $\{4\}\{2\}\{3\}\{1,5\}\{2,3\}$
C. $\{1,5\}, \{2,3\}, \{4,5\}$
D. $\{1,2\}\{5\}\{3,4\}$

Correct Answer: D

Solution: Sets in option D are mutually disjoint.
Lecture 156: Partitions-part 2

10) Let A be a set with cardinality n, and B be a set with cardinality m. There are a total of 64 symmetric relations on A, and 216 anti-symmetric relations on B. What is $n \cdot m$?

A. 9
B. 3
C. 6
D. 12

Correct Answer: A

Solution: total number of symmetric relations is $2^{\frac{n^2+n}{2}}$ and the total number of anti-symmetric relations is $2^n \cdot 3^{\frac{n^2-n}{2}}$ therefore, $n = 3$, $m = 3$, $n \cdot m = 9$

Lecture 145: Number of Antisymmetric relations.