Week 1

1) In how many ways the word '**PATHOGEN**' can be arranged such that letter **O** always comes to the left of **K**.

   A. 5040
   B. 40320
   C. 20160
   D. 10080

   Correct Answer : C

   Solution: Word '**PATHOGEN**' has all the distinct letters and there are 8! Ways to arrange them without any constraint. In half of the arrangements Letter **O** will be on the left of letter **K**. Thus there are $\frac{8!}{2} = \frac{40320}{2} = 20160$ ways to arrange the letters.
   Lecture 17: Problems on Permutations, time: 5:07

2) Rahul has 7 differently colored shirts, 5 different jeans, 2 pairs of shoes and 3 different caps. In how many ways can Rahul dress up before going to a party?

   A. 105
   B. 17
   C. 210
   D. 41

   Correct Answer : C

   Solution : By the rule of product total number of ways to dress up is $7 \times 5 \times 2 \times 3 = 210$
   Lecture 8: Problems on Rule of Sum and Rule of Product, time: 7:30

3) In how many ways can the word '**DOCUMENTATION**' be arranged so that all the consonants come together?

   A. $\frac{6!}{2!} \times \frac{13!}{2! \times 2! \times 2!}$
   B. $\frac{13!}{2! \times 2! \times 2!}$
   C. $\frac{7!}{2! \times 2!}$

D. $\frac{6!}{2!} \times \frac{7!}{2! \times 2!}$

Correct Answer : D

Solution: Condition consonants must come together. Here vowels are {O U E A I O} and consonants are {D C M N T T N}. Let's consider vowels as 1 unit and consonants as 1 unit. Number of ways to arrange vowels = 6!/2! and the number of ways to arrange consonants and 1 unit of vowels = 7C2 = 7!/2!×2! . Total number of ways in which we can make arrangements = $\frac{6!}{2!} \times \frac{7!}{2! \times 2!}$

Lecture 17: Problems on Permutations: 6:36

4) A question paper consists of 20 questions, having 2 parts. Each part consists of 10 questions. Students have to answer 12 questions in total, of which at least 4 questions should be from part 1 and at least 4 questions from part 2. In how many ways can the student select questions?

A. 44100
B. 9450
C. 123480
D. 40320

Correct Answer : C

Solution:

All possibilities to select 12 questions out of 20 questions, divided in 2 parts are:

Part 1	Part 2
4	8
5	7
6	6
7	5
8	4

Now,

$$4 \text{ from part } 1 \ \& \ 8 \text{ from part } 2 = \binom{10}{4} \times \binom{10}{8} = 210 \times 45 = 9450$$
$$5 \text{ from part } 1 \ \& \ 7 \text{ from part } 2 = \binom{10}{5} \times \binom{10}{7} = 252 \times 120 = 30240$$
$$6 \text{ from part } 1 \ \& \ 6 \text{ from part } 2 = \binom{10}{6} \times \binom{10}{6} = 210 \times 210 = 44100$$
$$7 \text{ from part } 1 \ \& \ 5 \text{ from part } 2 = \binom{10}{7} \times \binom{10}{5} = 120 \times 252 = 30240$$
$$8 \text{ from part } 1 \ \& \ 4 \text{ from part } 2 = \binom{10}{8} \times \binom{10}{4} = 45 \times 210 = 9450$$

Therefore, total number of ways in which the student can select questions is
$9450 + 30240 + 44100 + 30240 + 9450 = 123480$

Lecture 22: Problems on Combinations

5) What is the coefficient of the 6^{th} term in the expression $\left(\frac{5x^4}{13} - \frac{x^3}{2}\right)^7$

A. $-\frac{4375}{35152}$
B. $-\frac{525}{5408}$
C. $-\frac{35}{832}$
D. $-\frac{375}{2704}$

Correct Answer : B

Solution :

$$n = 7,\ r = 6,\ a = \frac{5x^4}{13},\ b = \ -\frac{x^3}{2}$$

$$T_r = \binom{n}{r-1} a^{n-r+1}\, b^{r-1}$$

$$T_6 = \binom{7}{6-1} a^{7-6+1}\, b^{6-1}$$

$$T_6 = \binom{7}{5} a^2\, b^5$$

$$T_6 = \frac{7!}{5! \cdot 2!} \cdot \left(\frac{25x^8}{169}\right) \cdot \left(-\frac{x^{15}}{32}\right)$$

$$T_6 = \ -\frac{525\, x^{23}}{5408}$$

Lecture 31: Problems on Binomial theorem

6) Which of the following is **NOT** an application of Catalan numbers?

   A. Counting the number of ways in which a polygon can be cut into triangles by connecting vertices by straight lines.
   B. Counting the number of ways in which a person can choose a red ball from a bag containing 10 black balls, 5 red balls and 15 blue balls.
   C. If *n* peoples are seated around a table, in how many ways can they shake their hands, such that nobody crosses their hand.
   D. Counting the number of expressions containing *n* pairs of parentheses.

Correct Answer : B

Solution : Choosing a ball out of n balls requires combinations.
Lecture 38: Examples of Catalan numbers

7) There are 2 vacant positions for the president and a vice president of a club. The post can be given to any of the 50 members of the club and a member can take more than one cabinet position. In how many ways can the 2 vacant positions be filled?

   A. 50^2
   B. ${}_{50}C_2$
   C. ${}_{50}P_2$
   D. $50!$

Correct Answer : A

Number of vacant posts available = 2
Number of members = 50
Since repetitions are allowed, the number of ways 2 vacant positions can be filled = 50^2

Lecture 17: Problems on Permutations

8) A bag contains five black balls, seven white balls, and nine red balls. The number of ways in which three balls can be drawn from the bag so that all the three drawn balls will have different colors is

A. 945
B. 63
C. 21
D. 315

Correct Answer : D

Number of black balls = 5
Number of white balls = 7
Number of red balls = 9
3 balls have been drawn from the bag. We want all three balls of different colors.
Since order doesn't matter in this case.
Total number of possible ways = $5 \times 7 \times 9 = 315$

Lecture 8: Problems on Rule of Sum and Rule of product

9) What is the total number of paths from (-3,-2) to (5,6) without crossing the line passing through (-3,-2) and (5,6) (i.e., without crossing the diagonal)?

A. 132
B. 1430
C. 429
D. 4862

Correct Answer : D

Solution : The grid between (-3,-2) and (5,6) is 8×8, therefore we have to travel 8 units right and 8 units up to reach point (5,6) from point (-3,-2).

total number of paths without crossing diagonal is:

$$
\begin{aligned}
C_n &= \binom{2n}{n} - \binom{2n}{n+1} \\
&= \binom{16}{8} - \binom{16}{9} \\
&= 48620 - 43758 \\
&= 4862
\end{aligned}
$$

Lecture 37: Catalan number part-4

10) Evaluate $\dfrac{{}_{20}\mathrm{C}_4 \times {}_{21}\mathrm{P}_2}{{}_{100}\mathrm{C}_{99}}$

A. 2034900
B. 20349
C. 244188
D. 24418800

Correct Answer : B

Solution:

$$\frac{{}_{20}\mathrm{C}_4 \times {}_{21}\mathrm{P}_2}{{}_{100}\mathrm{C}_{99}} = \frac{\frac{20!}{4!\times 16!} \times \frac{21!}{19!}}{\frac{100!}{99!\times 1!}} = \frac{4845 \times 420}{100} = \frac{2034900}{100} = 20349$$

Lecture 17, 22: Problems on combination, Problems on Permutations