even number $\rightarrow$ $2m$

$\because (2m)^2 = 4m^2$

$\therefore$ (any even number)$^2$ = $4k$

$\rightarrow$ $\sqrt{2} \rightarrow$ irrational : proof.

Assume $\sqrt{2}$ is rational

$\therefore \sqrt{2} = \frac{p}{q}$ $(p, q \in \mathbb{Z}) \Rightarrow 2 = \frac{p^2}{q^2}$

$\because$ p & q are in its simplest form

$\therefore$ p & q are relatively prime, p & q can't be both multiple of 9.

From above, $p^2 = 2q^2$ (both even)

$p^2$ is even $\Rightarrow$ p is even

$\Rightarrow$ $p^2$ is a multiple of 4 $\Rightarrow 2q^2 = 4k$

$\Rightarrow$ $q^2$ is even $\Rightarrow$ q is even

But p & q can't be both multiples of 2.

$\therefore$ Our assumption was wrong.

$\Rightarrow$ $\sqrt{2}$ is irrational.

Hence Proved

$\boxed{\text{This is called LOGICAL DEDUCTION.}}$

## I. Statements

She is beautiful. $\rightarrow$ subjective, can't be considered true / false

New Delhi is the capital of India $\rightarrow$ can be only true or false

$\rightarrow$ statement / preposition.

What a beautiful evening! $\rightarrow$ not a statement

What time is it ?
How are you ?
How are you doing ?

} Questions

∴ both not statements.

Go to the gym
Don't eat junk.
Study well.

} Commands

Statements examples :

→ JK Rowling is the author of Harry Potter series. True
→ 3 + 2 = 2 False
→ Crow is not a bird. False

II. Negation

p : It rained at my location today.
q : Yesterday was a full moon day.

$\neg p$ : It did not rain at my location today. (p complement) → negation
$\neg q$ : Yesterday wasn't a full moon day.
→ not q

r : My weight is less than 70 kg.
$\neg r$ : My weight is not less than 70 kg.
or
$\neg r$ : My weight is greater than or equal to 70 kg.

Note :

P : I have a PhD degree.
$\neg P$ : I do not have a PhD degree.

| P | $\neg P$ |
|---|---|
| T | F |
| F | T |

→ Truth Table of NOT

$p : 2 \leq 3 \longrightarrow$ True (1)

$\neg p : 2 > 3 \longrightarrow$ False (0)

$q : 2+3 = 10 \longrightarrow$ False (0)

$\neg q : 2+3 \neq 10 \longrightarrow$ True (1)

## III OR Operator

He who knows not and knows not that he knows not is a fool, shun him.
He who knows not and knows that he knows not is simple, teach him.
He who knows and knows not that he knows is asleep, wake him.
He who knows and knows that he knows is an intellectual, follow him.

| Knowledgeable | Awareness of Knowledge | Action |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

→ Truth Table

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

### OR Operator for 3 Variables

| $p$ | $q$ | $r$ | $p \vee q \vee r$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

## IV. AND Operator

| p | q | p∧q |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| p | q | r | p∧q∧r |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

## V. Primitive & Compound Statements

New Delhi is the capital of India,
Paris is the capital of France, → Compound statement
Beijing is the capital of China,
Berlin is the capital of Germany.

The above compound ~~s~~ statement is False even if one of the primitive statements is false.

→ Primitive statement: 1 piece of information.
→ Compound statement: Pieces of information stitched together w/ OR or AND.

p : India is a nice place to live.
q : People in India are very friendly. → Primitive statements

$p \wedge q$: India is a nice ~~to~~ place to live and people here are very friendly.
→ Compound statement.

## VI. Problems Involving NOT, OR & AND Operators

1. $\neg p \wedge \neg q$

| p | q | ¬p | ¬q | ¬p ∧ ¬q |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |

2. $(p \wedge q) \vee \neg p$

| $p$ | $q$ | $\neg p$ | $p \wedge q$ | $(p \wedge q) \vee \neg p$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |

3. $(p \vee r) \wedge q$

| $p$ | $q$ | $r$ | $p \vee r$ | $(p \vee r) \wedge q$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

VII. Implication

p implies q

$p \rightarrow q$

| Q (p) | A (q) | M $(p \rightarrow q)$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Q $\rightarrow$ question is right or wrong
A $\rightarrow$ attempt (right or wrong)
M $\rightarrow$ marks probability

This is a typical $p \rightarrow q$ concept truth table.

* $a \rightarrow b$

| $a$ | $b$ | $a \rightarrow b$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$\longrightarrow$

→

| Rich | Happy | $R \rightarrow H$ | |
|---|---|---|---|
| 0 | 0 | 0 | (poor can be happy) |
| 0 | 1 | 0 | (poor might be unhappy) |
| 1 | 0 | 0 | (rich might be happy) |
| 1 | 1 | 1 | (rich might be unhappy) |

$\therefore \quad R \not\rightarrow H$

→

| Hungry | Eat | $H \rightarrow E$ | |
|---|---|---|---|
| 0 | 0 | 1 | not hungry, don't eat |
| 0 | 1 | 1 | not hungry, eat |
| 1 | 0 | 0 | hungry, don't eat → false |
| 1 | 1 | 1 | hungry, eat |

$\therefore$ $H \rightarrow E$ is true.

→ a : A number ends in zero

b : That number is even

| a | b | $a \rightarrow b$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

$\therefore$ $a \rightarrow b$ is True

→ p : Born in NY

q : Born in US

| p | q | $p \rightarrow q$ | |
|---|---|---|---|
| 0 | 0 | 1 | not impossible |
| 0 | 1 | 1 | not impossible |
| 1 | 0 | 0 | impossible |
| 1 | 1 | 1 | not impossible |

## VIII. Double Implication

$p$ implies $q$
&
$q$ implies $p$

$p$ necessary and sufficient condition $q$

$p$ if and only if $q$

→ Prime number → a number that doesn't have any divisors.
ex. 7, 11, 19, 53

a is not prime.
⇒ $a^2$ is not prime

$a^2$ is divisible by a
⇒ $a^2$ is not a prime number.

a is not prime ⇒ $a^2$ is not prime

→ a is even ⇒ $a^2$ is even

$a = 2m$
⇒ $a^2 = 4m^2$ ⟶ even

If a is odd ⇒ $a^2$ is odd
$a^2$ is even ⇒ a is even

∴ a is even

Hence, a is even ⟷ $a^2$ is even.

∵ $p \to q$ & $q \to p$, $p \leftrightarrow q$
$p$ & $q$ are equivalent.

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

(Truth table of $p \leftrightarrow q$)

Double implication

$p \to q$

$q \to p$
(converse of $p \to q$)

→ p : It rained

q : The weather is good.

$p \to q$ : When it rains, the weather is good.

Converse : When the weather is good, it implies that it has rained.

$q \to p$ : converse of $p \to q$.

$p \to q$

$\neg p \to \neg q$ is the <u>inverse</u> of $p \to q$

$\neg q \to \neg p$ is the <u>contraposition</u> of $p \to q$.

| p | q | ¬p | ¬q | ¬q → ¬p |
|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |

→ $p \to q$

IX. <u>XOR Operator</u>

<u>Truth Table:</u> Exclusive-OR (Ex-OR)

| a | b | $a \veebar b$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

<u>Truth Table for $p \veebar q \veebar r$</u>

| p | q | r | $p \veebar q$ | $p \veebar q \veebar r$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 |

$p \to \neg p$

| $p$ | $\neg p$ | $p \to \neg p$ |
|---|---|---|
| 0 | 1 | 1 |
| 1 | 0 | 0 |

$p \veebar (q \vee q p)$

| $p$ | $q$ | $(p \vee q)$ | $p \veebar (p \vee q)$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 |

$q \wedge (p \leftrightarrow q)$

| $p$ | $q$ | $p \leftrightarrow q$ | $q \wedge (p \leftrightarrow q)$ |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |

## X. Tautology & Contradiction

$p \to p \vee q$ (TAUTOLOGY) Always true

| $p$ | $q$ | $p \vee q$ | $p \to (p \vee q)$ |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 |

→ $p \to p \vee q$ : Show impossible, prove Tautology
$\quad 1 \quad\; 0$

→ $q \;\; p \;\; p \vee q$
$\quad 1 \;\; 1 \;\; 1$

$1 \not\to 0$ : $p \to q \vee p$ is alway true

## CONTRADICTION:

$\neg(p \to q) \wedge q$

| $p$ | $q$ | $p \to q$ | $\neg(p \to q)$ | $\neg(p \to q) \wedge q$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |

$\neg(p \to q) \wedge q$ is a CONTRADICTION.

→ For this expression to be 1,

$$\underbrace{\neg(\underset{0 \text{ or } 1}{p} \to \underset{1}{q})}_{\text{FALSE}} \wedge \underset{1}{q} \longrightarrow \text{always } 0.$$

## XI SAT Problem (Satisfiability Problem)

a b c d e f g h i j

Is there any assignment to the above given variables, which makes a boolean expression containing all of them TRUE?

$2^{10}$ possibilities, because there are $2^{10}$ binary numbers that are 10 digits long.

100 variables → $2^{100}$ variables.

Thus, the SAT problem is tough to solve.

## XII. Logical Equivalence

| $p$ | $q$ | $q \to p$ | $(\neg p \wedge (p \to q) \to \neg q)$ |
|---|---|---|---|
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 |

What does this signify?

→ They're logically the same

| $p$ | $q$ | $p \to q$ | $\neg p \vee q$ |
|---|---|---|---|
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 |

⟶ LOGICALLY EQUIVALENT

$p \to q$ equivalent to $\neg p \vee q$.

These are called equivalent boolean expressions.

→

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| $p$ | $q$ | $p \wedge q$ | $\neg p \wedge \neg q$ | $(p \wedge q) \vee (\neg p \wedge \neg q)$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 |

$\therefore \; p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q).$

→ $(p \wedge \neg q) \vee (\neg p \vee q)$

| $p$ | $q$ | $\neg p$ | $\neg q$ | $p \wedge \neg q$ | $\neg p \wedge q$ | $(p \wedge \neg q) \vee (\neg p \wedge q)$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 1 | 0 |

$\therefore \; (p \wedge \neg q) \vee (\neg p \wedge q) \equiv p \veebar q$

स्वप्निल

## XIII. Double Negation

It is not correct to not dress well on a dinner date.

→ double negation

→ I never said I don't like sweets.

$p$ : I like sweets.

$\neg p$ : I don't like sweets.

$\neg(\neg p)$ : It is not true that I don't like sweets.

↓

$p$ : I like sweets.

$\therefore \; \neg(\neg p) = p$

(DOUBLE NEGATION)

## XIV. Laws of Logic

1. Commutative Law (order doesn't matter)

$$p \vee q \equiv q \vee p$$

2. Associative Law (brackets don't matter - operator should be same)

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

3. Distributive Law ($\vee$ distributes over $\wedge$)

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

4. Idempotent Law:

$$p \vee p \equiv p$$
$$p \wedge p \equiv p$$

5. Identity Law:

$$p \wedge T \equiv p$$
$$p \vee F \equiv p$$

6. Inverse :

$$p \vee \neg p \equiv T$$
$$p \wedge \neg p \equiv F$$

7. Domination Law:

$$p \vee T \equiv T$$
$$p \wedge F \equiv F$$

8. Absorption Law :

$$p \vee (p \wedge q) \equiv p$$
$$p \wedge (p \vee q) \equiv p$$

—*—

→ $\neg (p \to q) \equiv \neg(\neg p \vee q)$ $\qquad \because p \to q \equiv \neg p \vee q$

$\equiv \neg(\neg p) \wedge \neg q$ $\qquad$ De Morgan's Law

$\equiv p \wedge \neg q$

→ $[(p \vee q) \wedge (p \vee \neg q)] \vee q \equiv [(p \vee q) \vee q] \wedge [(p \vee \neg q) \vee q]$ $\qquad$ Distributive Law

$\equiv (p \vee q \vee q) \wedge (p \vee \underbrace{\neg q \vee q}_{T})$ $\qquad$ Associative Law

$\equiv (p \vee q) \wedge (T)$

$\equiv p \vee q$

## XV. De Morgan's Laws

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

complement $\equiv$ negation

union $\equiv$ OR

intersection $\equiv$ AND

## XVI. Rules of Interference

PUZZLE

↓

Deduce & conclude

Statements → Conclusions

1) Statements are written one below the other.
2) Conclusion below horizontal line.
3) $p^1$ : True , $p^0$ : False
4) If $p^0$, $(\neg p)^1$

ex. $(\neg p)^1$, $(p \vee q)^1$

p : False

$p \vee q$ : True

$\longrightarrow \quad \begin{array}{l} (\neg p)^1 \\ (p \vee q)^1 \\ \hline \therefore q^1 \end{array}$

→ Ram, Michael → At least one of them ~~are~~ is intelligent.

$$\begin{array}{l} (p \vee q)^1 \\ (\neg q)^1 \\ \hline \therefore p^1 \end{array}$$

→ p: It is raining in Chandigarh.

q: The weather is very pleasant in Chandigarh today.

$$\begin{array}{l} (p \to q)^1 \\ p^1 \\ \hline \therefore q^1 \end{array} \qquad \begin{array}{l} q^1 \\ (p \to q)^1 \\ \hline \text{Can't conclude} \\ \therefore p^0 \end{array}$$

→

→ $p, q, r$

$(p^{0} \vee q)^{1}$
$((\neg p)^{1} \vee r)^{1}$
$(\neg r)^{1} \Rightarrow r^{0}$

---

$\therefore q^{1}$

→ $(p \vee q)^{1}$
$(q \vee r)^{1}$ $\Rightarrow$ cannot conclude anything

→ $(p \wedge q)^{1}$
$(q \to r)$

---

$\therefore r^{1}$

$(p \wedge q)^{1}$
$(q \to r)^{1}$
$(r \to s)$

---

$\therefore s^{1}$

Q. $p \to q^{0}$
$r^{1} \to (\neg q)^{1}$
$r^{1}$ ← given था

---

$\therefore p^{0}$
or $(\neg p)^{1}$

→ $(p^{1} \wedge q^{1})^{1}$
$(p^{1} \to r^{1} \wedge q^{1})^{1}$
$(r^{1} \to s^{0} \vee t^{1})^{1}$
$(\neg s)^{1}$

---

$\therefore t^{1}$

→ $p^{1}$
$(p^{1} \to q^{1})^{1}$
$(p^{1} \to (q^{1} \to r)^{1})^{1}$

---

$\therefore r^{1}$

→ p: Raj does Yoga regulary.
q: Raj's BP is normal

$(p \to q)^{1}$ ? $\therefore$ True.

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| 1 | 1 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 0 | 1 |

$p^{0}$ → can't say anything about q.
~~q~~ $p^{1} \to q^{1}$

$(p \to q)^{1}$
$p^{1}$

---

$\therefore q^{1}$

→ Show $(r \vee s)^1$

$\neg p^1 \vee s$

I $\neg t^0 \vee (s^0 \wedge r^0)$

~~$\neg t \vee (s \wedge r)$~~ → Assuming $(r \vee s)^0$

$\neg q^1 \vee r^0$

$q^0 \vee p^0 \vee t^1$

But $\neg t$ needs to be true ∴ Our assumption is false ⇒ $(r \vee s)^1$

→ Show: $t^1$

$\neg p^1 \wedge q^1$

$r^0 \rightarrow p^0$

$\neg r^1 \rightarrow s^1$

$\underline{s^1 \rightarrow t}$

∴ $t^1$

→ $p \rightarrow q$

$\underline{q \rightarrow r}$

∴ $p \rightarrow r$

$p^1 \rightarrow r^0$

$p^1 \rightarrow q^1$

$q^1 \rightarrow r^1$ → $r$ must be 1

∴ Assumption is wrong ⇒ $p \rightarrow r$

This is called LAW OF SYLLOGISM.

↓

$p \rightarrow q,\ q \rightarrow r$

$\Rightarrow p \rightarrow r$

—— x ——