Week 5

1) Which of the following is(are) true for the given function?

$$f: R \rightarrow R$$
$$f(x) = x^2 + 2$$

R is a set of real numbers.

A. f is not injective
B. f is bijective
C. f is surjective
D. f is not surjective

Correct Answer: A, D

Solution: A function f is injective if $f(x_1) = f(x_2)$ implies $x_1 = x_2$, i.e. no two elements in the domain will have the same image. f is called surjective if for any element in the co-domain there is a pre-image in the domain, i.e. for any y in the co-domain, there exists an x in the domain such that $f(x)$ = y. A function f is said to be bijective if it is both injective and surjective. Since $f(x) = x_2 + 2$, we have $f(-1) = 3 = f(1)$. Hence, f is not injective. Now, the co-domain of the function is given as R. Now if f is surjective then the codomain and the range should be the same, which means every element in the codomain should have a preimage. Now let us try to find a preimage for 1 (observe that $1 \in$ R, as the codomain of the function is given as R). To find the preimage of 1, we have to find an element a from the domain for which $f(a) = 1$, i.e. $a^2 + 2 = 1$, i.e. $a^2 = -1$. Now we know that the square of any real number cannot be negative. Hence there cannot exist any real number a (in the domain) for which $f(a) = 1$. Hence 1 has no preimage. So codomain and range are not the same. Hence f is not surjective. Also, $1 \in$ R. Let x be such that $x \in$ R, and $f(x) = 1$. As the function is neither injective nor surjective, therefore it is not bijective.

Lecture 187: Examples of bijection

2) Consider the following table:

Name of students	Height (in cm)
Rohit Amit Krishna Atharva Nitin	168 165 169 170 165

We can think of this as a function f from the set of students to the set of integers between 160 and 170. Now pick out the correct statement from the following.

A. f is onto but not one-to-one.
B. f is bijective.
C. f is one to one but not onto.
D. f is neither one to one nor onto.

Correct Answer: D

Solution:
The codomain for the function is nothing but the set of integers between 50 and 70.

A. There is no student in the list whose Height is 162, but 162 is in the codomain. Hence the function is not onto. The range of the function is $\{165,168,169,170\}$, which is not the same as the codomain of the function.
B. A function is called bijective if it is both one to one and onto. The given function is neither one-to-one nor onto, hence it is not bijective.
C. Amit and Nitin have the same Height. Hence the function is not one-to-one.
D. From the above explanation clearly the function is neither one-to-one nor onto.

Lecture 167: Definition of function part - 1

3) Let $f : R \to R$ such that $f(x) = \frac{x}{2} + 7$

A. f is not a function
B. f is bijective
C. f is injective
D. f is surjective

Correct Answer : B

Solution: $f: R \to R$, $f(x) = 2x + 1$ is bijective since for each y there is a unique x $2y - 7$ such that $f(x) = y$. In general $f: R \to R$, $f(x) = ax + b$ (where a is non-zero) is a bijection. Each real number y is obtained from (or paired with) the real number $x = 2y - 7$.

Lecture 187: Examples of Bijection

4) If a function is defined as $f(x) = 2x + 14$ then the value of $f^{-1}(25)$ is

   A. 3
   B. 5
   C. 39
   D. 25

Correct Answer: B

Solution: $f(x) = 2x+14$, $f^{-1}(25) = x$, $f(x) = 25$ thus $25 = 2x+14$, $x = 5$ and $f^{-1}(25) = 5$ and $f(5) = 25$

Lecture 205: Inverse functions

5) Set C has cardinality p and a total of 5040 bijective functions. What is the value of p^2?

   A. 144
   B. 25
   C. 81
   D. 49

Correct Answer: D

Solution: The number of bijective functions = $m!$
Lecture 196: Number of bijections

6) find the domain and range of the following real-valued function. $f(x) = \sqrt{3 - x}$

(Note: $\sqrt{\ }$ denotes the positive square root)

A. domain=$\{x \in \mathrm{R} \mid x \leq 3\}$
range=$\{x \in \mathrm{R} \mid x \leq 0\}$

B. domain=$\{x \in \mathrm{R} \mid x \neq 3\}$
range=$\{x \in \mathrm{R} | x \geq 3\}$

C. domain=$\{x \in \mathrm{R} \mid x \geq 3\}$
range=$\{x \in \mathrm{R} \mid x \geq 0\}$

D. domain=$\{x \in \mathrm{R} \mid x \leq 3\}$
range=$\{x \in \mathrm{R} \mid x \geq 0\}$

Correct Answer: D

Solution: The set of real numbers R includes all rational and irrational numbers. $\sqrt{a}$ is real-valued if $a \geq 0$. If f has to be real-valued, then

$$3 - x \geq 0$$
$$\Rightarrow 3 \geq x$$

Hence, the domain of the function *f* is $\{x \in \mathrm{R} \mid x \leq 3\}$. Since $\sqrt{\ }$ denotes the positive square root (as given in the question statement), the range of function *f* is nothing but all the positive real numbers, i.e. $\{x \in \mathrm{R} \mid x \geq 0\}$.

Lecture 173: one one function example-1

7) If *f* and *g* are function from *R* to *R* and $f(x) = 3x^2 + x - 13$ and $g(x) = \frac{20}{3x+8}$ then $fog(12)$ is.

A. $-\frac{267}{49}$

B. $\frac{20}{1301}$

C. $-\frac{1443}{121}$

D. $\frac{5}{11}$

Correct Answer: C

Solution: $g(12) = \frac{5}{11}$, $f(g(x)) = f(\frac{5}{11}) = -\frac{1443}{121}$

Lecture 203: Example of composition of functions part-2

8) Let us define a function f: $\boldsymbol{Z} \to \boldsymbol{Z}$ as follows,

$$f(x) = \begin{cases} \frac{x}{2} & \text{if } x \text{ is even} \\ 0 & \text{if } x \text{ is odd} \end{cases}$$

$\boldsymbol{Z}$ is a set of integers.

A. onto but not one-to-one.
B. one-to-one but not onto.
C. one-to-one and onto.
D. neither one-to-one nor onto.

Correct Answer: A

Solution: It is clear that $f(1) = f(3) = f(-1) = 0$ as $f(x) = 0$, for each odd integer x. So f is not one-to-one. Moreover, for any integer y, $f(2y) = y$ as $2y$ is always an even integer. Therefore, f is onto. Hence f is onto but not one to one.

Lecture 180: Introduction to onto function part-1

9) The relation R is defined as $R = \{(x,y) : x, y \in \boldsymbol{N}, x + y = 5\}$ then the range is?

A. {2,4}
B. {2,3,4}
C. {2,3}
D. {1,2,3,4}

Correct Answer: D

Solution: Pair of value (x,y) for x + y = 5. $R = \{(1,4),(2,3),(3,2),(4,1)\}$ range is the value set D so range {1,2,3,4}

Lecture 173: One one function example-1

10) Let A be set with cardinality n and set B with cardinality m, there are a total of 3024 one to one functions from A to B, what are the values of n and m respectively?

A. 4 and 8
B. 4 and 9
C. 6 and 7
D. 7 and 6

Correct Answer: B

Solution: If set A has n elements and set B has m elements, $m \geq n$, then the number of injective functions or one-to-one functions is given by $\frac{m!}{(m-n)!}$ i.e., ${}_{m}\mathrm{P}_{n}$.

Lecture 194: Number of One-one functions-part-3