

Bit Manipulation

Count the no of set bits :

```
① int count (int n) {  
    int res = 0;  
    while (n > 0)  
    {  
        if ((n & 1) == 1)  
            res++;  
        n = n >> 1;  
    }
```

time complexity:

$O(\text{total bits in } n)$

② Brian - Kerningham Algorithm

time complexity = $O(\text{No of set bits})$

```
int countBits (int n) {  
    int res = 0;  
    while (n > 0) {
```

→ $n = n \& (n-1);$
res++;

At step, n

every s. .
 I am turning off.
 the last set bit. return res; }.

```

1 0 1 0 0 0
1 0 0 1 1 1
-----
1 0 0 0 0 0
  
```

→ Whenever one does
 a "n-1" bits
 then all the bits
 which are zero
 after the LSB word
 first bit set including
 that one converted
 to zero.

③ Lookup Table Based Approach

Θ(1) solution, but extra space
 required.

```

int table[256];
void initialize()
  
```

```

{ table[0] = 0;
  
```

```

    for (int i = 1; i < 256; i++)
        table[i] = (i & 1) + table[i/2];
  }
  
```

// 0 to 255 done.

```

int count (int n) {
    int res = table[n & 0xff];
    // ...
    return res;
  }
  
```

```

n = n >> 1;
res = res + table[n & 0x1];
n = n >> 2;
res += table[n & 0x3];
n = n >> 4;
res += table[n & 0x7];
return res;
}

```

→ Given an array of n numbers that has values in the range $[1 \dots n+1]$. Every number appears exactly once. Hence one number is missing. Find the missing number.

You do XOR of all the numbers in the array and then XOR it with (XOR of all the numbers in the range $[1 \dots n+1]$), you will get the missing number or the number that is single and every other number is appearing two times.

Finding two odd Appearing Numbers

→ Odd occurring elements are always 2.

XOR of all the nos.
↑
the 2 odd numbers.
XOR of 1st odd and 2nd occurring

```

void oddOccurring (int arr[], int n) {
    int XOR, res1, res2 = 0;

```

```

    for (int i = 0; i < n; i++)
        XOR = XOR ^ arr[i];

```

→ XOR of all the numbers

```

    int right-set-bit = XOR & ~ (XOR - 1);
    // Rightmost set bit.

```

```

    for (int i = 0; i < n; i++)

```

... set bit) != 0

$\Theta(n)$
Solution

```

    if ((arr[i] & right == 0))
        res1 = res1 ^ arr[i];
    else
        res2 = res2 ^ arr[i];
    }
    cout << res1 << " " << res2 << endl;
}

```

Generating Power Set using Bitwise Operators
All the Subsets

```

void printPowerSet (string str) {
    int n = str.length();
    int psize = pow(2, n);
    for (int counter = 0; counter < psize; counter++) {
        for (int i = 0; i < n; i++) {
            if (counter & (1 << i) != 0)
                print(str[i]);
        }
    }
}

```

$\Theta(2^n * n)$

Counter represents the 2^n subsets.
 Binary representation.

0	000
1	001
2	010
3	011
4	100
5	101

← We use the set bits in the binary representation to find the element.

2^0
 2^1
 \vdots
 2^n

110

of the ...

- The bitwise operations are found to be much faster and are some times used to improve the efficiency of a program.
- The left shift and right shift operators cannot be used with negative numbers.
- The & operator can be used to quickly check if a no is odd or even. The value of expression $(x \& 1)$ would be non-zero ^{only} if x is odd, otherwise the value would be zero.

Bit Manipulation Tricks

1) How to set a bit in the number 'num'?

$temp = (1 \ll n) \rightarrow \text{equivalent to } 2^n$
 $res = (num \mid temp)$
 \uparrow
 OR

'num'?

2) Unsetting a bit in the number num .

$$temp1 = 1 \ll n$$

$$temp = \sim temp1$$

$$res = (num \& temp)$$

3) Toggling a bit at n^{th} position:

$$temp = 1 \ll n$$

$$res = (num \oplus temp)$$

4) Divide by 2:

$$x = x \gg 1;$$

5) Multiplication by 2:

$$x = x \ll 1;$$

6) Find $\log_{base 2}$ of a 32 bit integer:

```
int log2 (int x) {
```

```
    int res = 0;
```

```
    while (x >> = 1)
```

```
        res++;
```

```
    return res;
```

```
}
```

7) Flipping the bits of a number:

in all bits set in a

Value = A number with ...
 given number.
 $res = value - number.$

8) Given a Number N , find the most significant set bit in the given number.

$$res = 2^{\lfloor \log_2(N) \rfloor}$$

For ex:

$$\begin{aligned} & \uparrow 10 \\ & 1010 \\ & \quad \rightarrow \text{MSB} \\ & \quad = 2^{\lfloor \log_2(10) \rfloor} \\ & \quad = 8 \end{aligned}$$

9) Given a Number N , the task is to find the XOR of all numbers from 1 to N .

$$N=1: 1 \rightarrow 1$$

$$N=2: 1 \wedge 2 \rightarrow 3$$

$$N=3: 1 \wedge 2 \wedge 3 \rightarrow 0$$

$$N=4: 1 \wedge 2 \wedge 3 \wedge 4 \rightarrow 4$$

$$N=5: 1 \wedge 2 \wedge 3 \wedge 4 \wedge 5 \rightarrow 1$$

$$N=6: 1 \wedge 2 \wedge 3 \wedge 4 \wedge 5 \wedge 6 \rightarrow 7$$

$$N=7: 1 \wedge 2 \wedge 3 \wedge 4 \wedge 5 \wedge 6 \wedge 7 \rightarrow 0$$

$$N=8: 1 \wedge 2 \wedge 3 \wedge 4 \wedge 5 \wedge 6 \wedge 7 \wedge 8 \rightarrow 8$$

Ans: 4

- 10 10 1
- 1) if $\text{rem} = 0$; then $\text{ans} = N$
 - 2) if $\text{rem} = 1$; then $\text{ans} = 1$
 - 3) if $\text{rem} = 2$; then $\text{ans} = N+1$
 - 4) if $\text{rem} = 3$; then $\text{ans} = 0$

10) MAXIMUM AND VALUE

Given an array $\text{arr}[]$ of N positive elements. The task is to find the max. AND value generated by any pair of the element from the array.

```
int checkBit ( int pattern , int arr[], int n)
{
    int count = 0;
    for (int i = 0; i < n; i++)
    {
        if ((pattern & arr[i]) == pattern)
            count++;
    }
    return count;
}
```

```
int maxAnd ( int arr[], int n) {
    int res = 0;
    int count;
    for (int bit = 31; bit >= 0; bit--)
```



```

for (int bit = 1; bit <= 31; bit++)
{
    count = checkBit(res | (1 << bit), arr, n)
    if (count >= 2)
        res = res | (1 << bit);
}

```

4	8	12	16
---	---	----	----

11) Count total set bits in all numbers from 1 to n.

