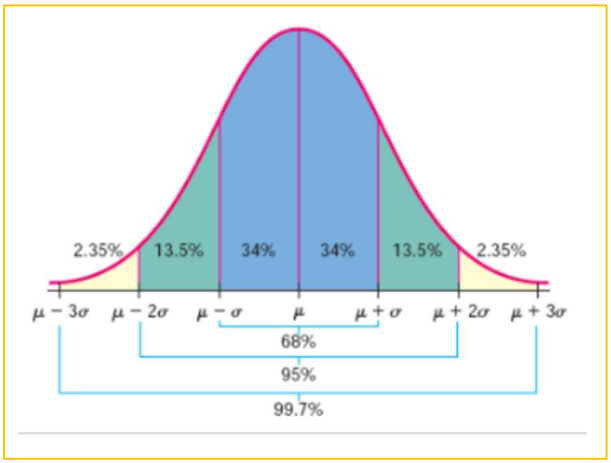
Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?



   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

Answer:
Given, μ = 45 minutes (mean)
σ = 8 minutes (standard deviation)
Committed delivery time = 1 hour = 60 minutes = x
Delay = 10 minutes
Actual x = x – delay = 60 – 10 = 50 minutes
z-score = (x - μ) / σ = (50 – 45) /8 = 5/8 → 0.625 → 0.7324
the probability that the service manager cannot meet his commitment = 1 – 0.7324 → 0.2676
i.e **26.76%**

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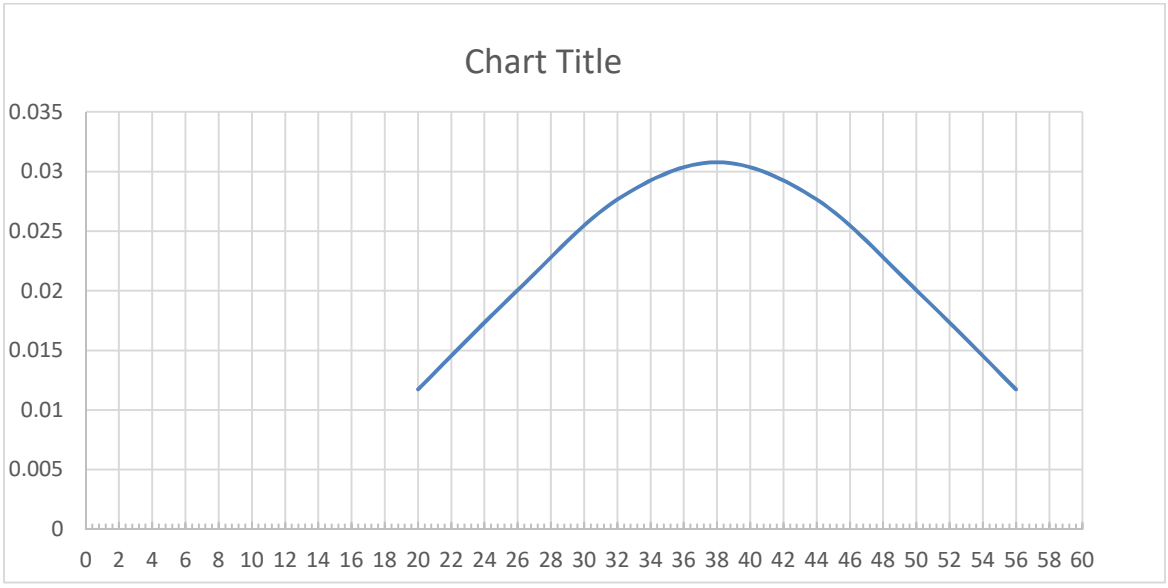
2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma = 6$. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer: Given, μ = 38 (mean) and σ = 6 (standard deviation)



In above normal distribution plot, 1 SD = 68% = 32 to 44

2 SD = 95% = 26 to 50

3SD = 99.7% = 20 to 56

A. This statement is **False** because,
   Employees elder than 44 = 50% – 34% (right side of mean) = 16% and
   Employees ages between 38 and 44 = right side of 1 SD (i.e. 50% of total 1 SD) = 34%
B. Employees under the age of 30
   $Z = (X-\mu)/\sigma = (30-38) / 6 = -8/6 = -1.33$ ➔ 0.09176 (by looking at z table)
   $P(X<=30) = 0.09176 * 400 = 36.704$
   Hence, this statement is **True.**

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3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Answer: iid = Independent and Identically Distributed

As both X1 and X2 variables are independent and identically distributed, X1 + X2 is normal with N(μ1+μ2,σ12+σ22). And 2X1 will just scale the normal distribution by 2 times

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4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

Answer: Probability of random variable taking value between a & b = 1 – 0.99 = 0.01→ 1 %

This 1% is distributed at both the extreme ends of the curve.

So, left point is at 0.005 and right point is at 0.995

Tail probability is calculated by **scipy.stats.norm.ppf(q , μ , σ) or scipy.stats.norm.interval(q , μ , σ)** with the help of python.

scipy.stats.norm.ppf(0.005,100,20) = 48.4834

scipy.stats.norm.ppf(0.995,100,20) = 151.5165 **or**

stats.norm.interval(0.99,100,20)= (48.4834, 151.5165)

Please refer python code in the attached jupyter notebook.

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5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5^{th} percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

Answer: A. Given → probability value for the annual profit of the company = 95%

$Profit_1 \sim N(5, 3^2)$ → μ1 , σ1 = (5,3)

$Profit_2 \sim N(7, 4^2)$ → μ2 , σ2 = (7,4)

➔ by using **scipy.stats.norm.ppf(q , μ , σ) or scipy.stats.norm.interval(q , μ , σ)** in python

For Profit1

stats.norm.interval(0.95,5*45,3) = (219.1201, 230.8798) **or**

scipy.stats.norm.ppf(0.025,5*45,3) = 219.1201

scipy.stats.norm.ppf(0.975,5*45,3) = 230.8798

For Profit2

stats.norm.interval(0.95,7*45,4) = (307.1601, 322.8398) **or**

scipy.stats.norm.ppf(0.025,7*45,4)= 307.1601

scipy.stats.norm.ppf(0.975,7*45,4)= 322.8398

So, a Rupee range for the annual profit of the company = Profit1 + Profit2

= [219.1201 + 307.1601, 230.8798+322.8398]

= **[526.2802, 553.7196]**

B.

5th percentile of profit (in Rupees) for the company = 5% of Profit1 + 5% of Profit2

= scipy.stats.norm.ppf(0.05,5*45,3) + scipy.stats.norm.ppf(0.05,7*45,4)

= 322.8398 + 308.4205

= 631.2603

So, 5th percentile of profit (in Rupees) for the company is **631.2603.**

C.

Division 2 with Profit2 ~ N(7, 42) has a larger probability of making a loss in a given year.