PORTFOLIO OPTIMIZATION

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Introduction

People across the world are interested in investing in the stocks to increase their wealth and it is estimated to be over 1.2 billion people doing so. While investing, there are many decisions that individual needs to take like what stocks to buy, portfolio optimization, which shares to sell and when to sell. Here I have tried to explore portfolio optimization and linear programming concepts together to make such decision.

The problem statement in front of us is to decide which stocks to be sold from existing investment to generate minimum cashflow immediately maintaining the best return or (optimal portfolio) for future under given constraints.

Data Set

To get the real time experience the historical prices for eight stocks is obtained from yahoofinance.com. The stock prices are obtained on monthly basis for three years from April 2015 till April 2017. When we download the prices, we are get Opening, closing, Adjusted Closing, High and Low price of each day. For our case I have considered the opening price of the stock. Following is the list of eight stocks that are considered for our analysis. We are given that we have equal quantity of each stocks in our portfolio.

Table 1 List of Stocks

Yahoo!
General Electric
Microsoft
Bank of America
JPMorgan Chase
Cisco Systems, Inc
Intel
Pfizer

During trade of stocks various costs are involved like that of brokerage, transaction cost, taxes etc. For this problem, we have considered capital gain tax and transaction cost for calculations.

Capital gain tax: The tax amount calculated on the capital gain i.e. the difference between the revenue generated after selling the stocks and the price at which stocks were bought. If loss is accounted on selling the stocks no tax is charged.

Transaction cost: The cost is calculated as a percentage of total revenue dealt with for a given transaction

Part A: Linear Programming

We have 150 stocks of eight companies and we are given their purchase price, current price, and estimated price of next year. We are in urgent need of 10,000 USD and for that we decide to liquidate our stocks in such a manner that we have maximum estimated value of portfolio next year. Moreover, we have considered capital gain tax as 30% and transaction cost as 1% of the revenue involved in the transaction. Also, for the case of simplicity we have assumed no short selling or buying is involved i.e. we can only sale the number of shares we have and not more than that.

Table 2 Stocks: Purchase Price, Current Price and Estimated future price

Number	Stock	Number of Shares	Price Purchased Last Year	Current Price	Next Year Price Estimate
1	Yahoo!	150	44.45	36.54	46.50
2	General Electric	150	24.72	31.50	29.76
3	Microsoft	150	40.60	55.05	65.81
4	Bank of America	150	15.42	13.47	23.65
5	JPMorgan Chase	150	60.41	59.02	87.99
6	Cisco Systems, Inc	150	27.31	28.30	33.70
7	Intel	150	31.13	32.27	36.19
8	Pfizer	150	34.84	29.44	34.11

The above problem can be addressed as linear Integer programming and can be solved using Excel solver with the help of simplex Algorithm. Problem formulation is as below.

Let x1, x2 …. x8 denote the decision variable where

Table 3 List of Decision Variable

x1	No. of Yahoo! Stocks to be sold
x2	No. of General Electric Stocks to be sold
x3	No. of Microsoft Stocks to be sold
x4	No. of Bank of America Stocks to be sold
x5	No. of JPMorgan Chase Stocks to be sold
x6	No. of Cisco Systems, Inc Stocks to be sold
x7	No. of Intel Stocks to be sold
x8	No. of Pfizer Stocks to be sold

Problem formulation

Net cash flow generated will be given by sum of Revenue generated by selling stock – capital gain tax – transaction cost

For example

Capital gain tax: (x1*current price of Yahoo!) – (x1* Price at which Yahoo! Was bought) * 30%

If this value is negative (if price of Yahoo stock dropped in a year) then capital gain tax is considered as 0

Transaction cost: (x1*current price of Yahoo!*1%)

Cash flow = (x1*current price of Yahoo!) – capital gain tax – transaction cost

Total Cash flows = Sum of all the cash flows for each stock

Objective Function

$$\textbf{Max} = \sum_{xi=1}^{xi=8}(\mathbf{150} - \boldsymbol{xi}) * \boldsymbol{current\ price\ of\ stock\ xi}$$

Constraints

- Currently we have a constraint of generating minimum of 10,000 USD in cash flow
- Number of stocks sold (xi) cannot be less than 0
- Number of stocks sold (xi) cannot be greater than 150
- Xi being number of stocks needs to be integer

On solving the above formulated problem using Excel solver we get following output.

Table 4 Part A List of Stocks to be sold

No. of Yahoo! Stocks to be sold	0
No. of General Electric Stocks to be sold	150
No. of Microsoft Stocks to be sold	0
No. of Bank of America Stocks to be sold	0

No. of JPMorgan Chase Stocks to be sold	0
No. of Cisco Systems, Inc Stocks to be sold	0
No. of Intel Stocks to be sold	145
No. of Pfizer Stocks to be sold	36

Also, the portfolio Value that we will have next year will be 42,717 USD

Portfolio Value	**$42,717**

Result

Thus, by selling 331 shares total cash flow of 10,005 USD is generated in such a way that the estimated value of portfolio next year will be maximum at 42,717 USD.

Table 5 Part A: Summary of Investment

	Total Shares	Portfolio Value	Remarks
Year of Purchase	**1200**	**$41,832**	
Current Year	**1200**	**$42,839**	Before selling
Current Year	**869**	**$32,375**	After selling
Next Year	**869**	**$42,717**	Estimated value

Part B: Portfolio Optimization

For the given data of stocks, we would like to optimize our portfolio and thereby find percentage of each stock that should be kept for deciding with respect to part A. Portfolio will be optimized for the maximum return condition once and minimum standard deviation condition other time to cater risk lover and risk averse nature of Investor in decision making. Portfolio Optimization is nonlinear optimization problem and is solved using Excel Solver with ***GRG Non-Linear Algorithm***.

Following steps are followed to formulate the problem

- Natural Log is used to calculate Returns of stock
- Excess return is calculated by subtracting the mean return on stocks
- Variance-Covariance Matrix is formed using the Excess return table
- Once the portfolio is optimized and weights are calculated for maximum return
- Next the portfolio is optimized and weights are calculated for minimum return

Results

On optimizing it is found that if our investor is risk lover he should be interested in max return on future portfolio and thus consider not selling all his stock Pfizer. This will be used as constraint in Part C for Risk Lover decision maker. If the investor is Risk Averse in nature he will not like much variance in his portfolio and hence will try to minimize the standard deviation of Portfolio. On doing so we find that risk Averse should not keep any stock of Yahoo! And Bank of America in his portfolio. This will be used as an additional constraint for decision making in part C.

Table 6 Part B: Weights of Stock for Optimized Portfolio

% of Stock	**Max Return**	**Min Std. Dev**
PF. Return	**8.84**	8.24
PF.SD	0.06	**0.04**
Yahoo!	0	**0.00**
General Electric	0	0.30
Microsoft	0	0.01
Bank of America	0	**0.00**
JPMorgan Chase	0	0.24
Cisco Systems, Inc	0	0.01
Intel	0	0.23
Pfizer	**1.00**	0.22
	Risk Lover	**Risk Averse**

Part C: Linear Optimization with Portfolio Optimization Results' constraints

Risk Lover: We define risk lover investor as the one who is more willing to take risk and is inclined towards maximum return on portfolio. From Part B, we get that for maximum return on portfolio we need to stay 100% invested in Pfizer and thus we include this as an additional constraint in Part A to get the maximum value of Portfolio.

Risk Averse: We define risk averse investor as the one who hates risk and always tries to minimize the variance or standard deviation of the portfolio that is minimize risk. From part B, we infer that for such an investor the optimal portfolio will not have stocks of Bank of America and Yahoo and try to use these as additional constraints and get maximum value of portfolio for such investor.

Results and Conclusions

From the results below we can see how differently one will end up with the portfolio depending on different levels of risks a person is willing to take. Estimated Portfolio Value for Risk Averse is lower compared to Risk lover which is due to different exposure to risks. Moreover, we see the diversification with optimization reduces the estimated portfolio value when compared to Part A and this is due to accounting for variance in portfolio and not just the return.

Table 7 Part C: Summary of Portfolio Value

Investor Type	Portfolio Value	Risk (Std. dev of portfolio)
Risk Lover	**$42,686**	0.06
Risk Averse	**$40,485**	0.04

Table 8 Part C: Summary of Stock percentage

Investor Type --->	Risk Lover	Risk Averse
No. of Yahoo! Stocks to be sold	0	150
No. of General Electric Stocks to be sold	150	89
No. of Microsoft Stocks to be sold	0	0
No. of Bank of America Stocks to be sold	0	150
No. of JPMorgan Chase Stocks to be sold	0	0
No. of Cisco Systems, Inc Stocks to be sold	32	0
No. of Intel Stocks to be sold	150	0
No. of Pfizer Stocks to be sold	0	0

References

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