CMO Assignment 0

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Question 1

You are given access to a function $f : \mathbb{R} \to \mathbb{R}$ through an oracle $\mathcal{O}$. When queried with your five-digit SR number and $x^{(t)} \in \mathbb{R}$, $\mathcal{O}$ will return a tuple

$$\left(f(x^{(t)}), f'(x^{(t)})\right),$$

where

$$f'(x^{(t)}) = \left.\frac{df}{dx}\right|_{x^{(t)}}$$

is the derivative of f at $x^{(t)}$. Find the solution to the optimization problem

$$x^* = \underset{x \in \mathbb{R}}{\operatorname{argmin}} f(x)$$

using any computational technique.

0.1 Technique Used

The oracle provides both the function value $f(x)$ and its derivative $f'(x)$ at any input x. Since the oracle returns the derivative at each point, we assume the function is differentiable there, which allows us to use gradient descent effectively.
We started with an initial guess $x_0 = 0$. At each step, we updated x by moving opposite to the slope (derivative) at that point:

- If the slope $f'(x_t)$ is positive, we decrease x to move towards the minimum.
- If the slope is negative, we increase x to move towards the minimum.

Formally, the update rule is:

$$x_{t+1} = x_t - \eta\, f'(x_t),$$

where η is the *step size*, controlling how big each update is.
We repeated this process until either the change in x was very small:

$$|x_{t+1} - x_t| < \epsilon,$$

where ϵ is the *stopping threshold*, or until a maximum number of iterations was reached.

We use a stopping threshold because in practice the change in x may never become exactly zero. This small value ϵ defines when the updates are close enough to consider the algorithm converged.
Generally, without second derivative information, confirming global optimality is not guaranteed.Hence, the solution found is a local minimum, which suffices given the problem constraints.If possible, we could try running gradient descent from multiple different starting points and pick the best solution found to better approximate the global minimum.

After the process ended x* and f(x*) were found as:

$$x^* = 0.1996290463091759, \quad f(x^*) = 0.12341052565163615$$

0.2 Ques 1 Code output:

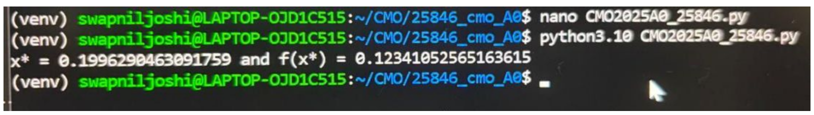

Figure 1: Output for x* and f(x*)

References

[1] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[2] W. Rudin, *Principles of Mathematical Analysis*, 3rd Edition, McGraw-Hill, 1976.