DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING, UCLA
ECE 239AS: COMPUTATIONAL IMAGING

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PSET 2 : PINHOLE CAMERAS

SL. NO.	TOPIC	MAX. POINTS	GRADED POINTS	REMARKS
1	Ray diagram	01		
2	Analysis	01		
3	Real pinhole camera	01		
4	Camera construction	02		
5	Verifying the camera	01		
6	Experimental analysis	01		
7	Fourier analysis- ideal	01		
8	Fourier analysis- real	01		
9	Analysis	01		
	Total	10		

1 Motivation

The simplest camera that one can think of is a pinhole camera. In such a camera, a tiny hole/orifice at the front acts as the aperture, through which the light enters. The back of the camera behaves as the screen on which the image is projected. Unlike conventional cameras that we are used to, a pinhole camera setup has no lens system for focusing the light rays incident on the system. In this problem set, we understand the theory behind pinhole cameras, as well as work towards building one.

2 Pinhole Camera Basics

Before building our camera, we look at some theory, to try and understand what we can expect from the camera. Let us consider a pinhole camera with the following top view:

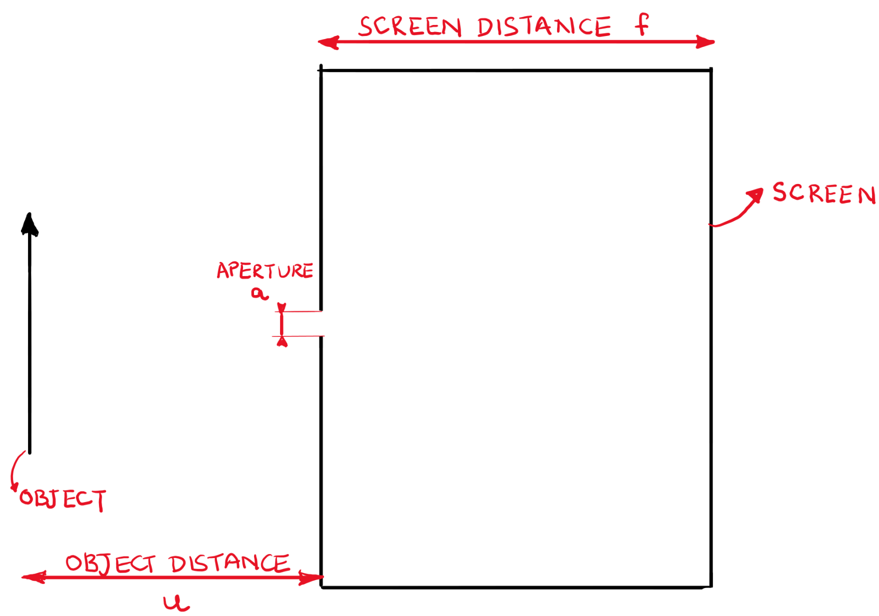

Figure 1: Top view of a simple pinhole camera.

Q1) Please draw the ray diagram for the image formation process in a pinhole camera. For this experiment, assume that the aperture a on the camera is sufficiently small (this is called an ideal pinhole camera). You can draw it by hand on a piece of paper, and add an image in the box below. Remember to mark out relevant dimensions in the diagram. (1 points)

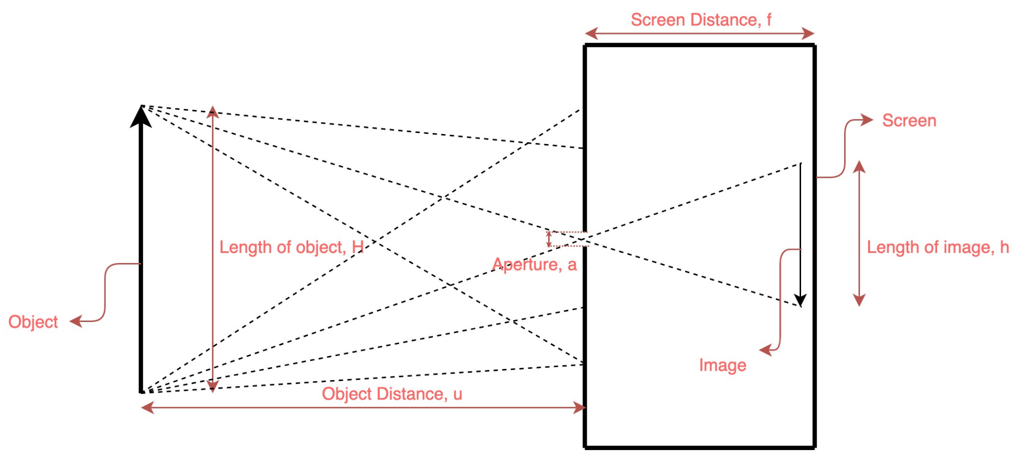

Figure 2: Ray diagram for ideal pinhole camera.

Q2) What is the size (length) of the image created by the camera? Also report the absolute value of the magnification factor of the camera, in terms of the dimensions in the ray diagram. Note that this is defined as $m = \frac{\text{length of the image}}{\text{length of the object}}$. How can this magnification factor be increased/decreased? (1 points)

$$h = \frac{Hf}{u} \tag{1}$$

where, h = size of image, H = size of object, f = screen distance, u = object distance.

$$m = f/u \tag{2}$$

where, m = magnification factor.
To increase/decrease m, increase focal length (screen distance) or get the camera closer to the object (decrease object distance) and vice versa.

Q3) Let us now look at what happens if the aperture has a finite size. Consider a finite aperture size of a, as shown in the ray diagram. From your understanding of pinhole cameras, what do you expect to happen to the image? No need to draw a ray diagram here- just a couple of sentences on how the image changes. (1 points)

The image will be blurry because a point on the object will be projected multiple times at multiple locations on the screen, causing the rays to overlap with each other.

3 Building a Pinhole Camera

Now that we understand how a pinhole camera works, the next step is to build a simple camera to see it in action!

Q1) Follow the steps here [1] for the 'without filters' pinhole camera (first part of the tutorial on the webpage) to create your own! For our experiments, we will build a camera with a small aperture, as well as a hole next to it so that you can use a camera (e.g. your cell phone camera) to capture the image formed. Note that while using the camera, the hole for it should be completely covered so that no light leaks into the setup.
Post an image of your camera setup in the box below. (2 points)

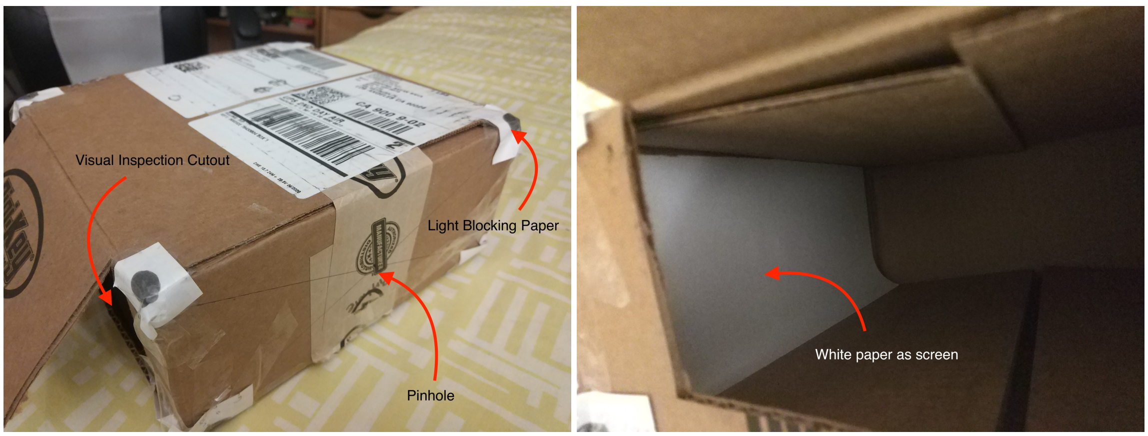

Figure 3: Illustration of DIY pinhole camera.

Q2) Add an image of a scene from your camera, as well as an image of the same scene from your pinhole camera in the box below. What are some of the properties of the image that you observe? (1 points)

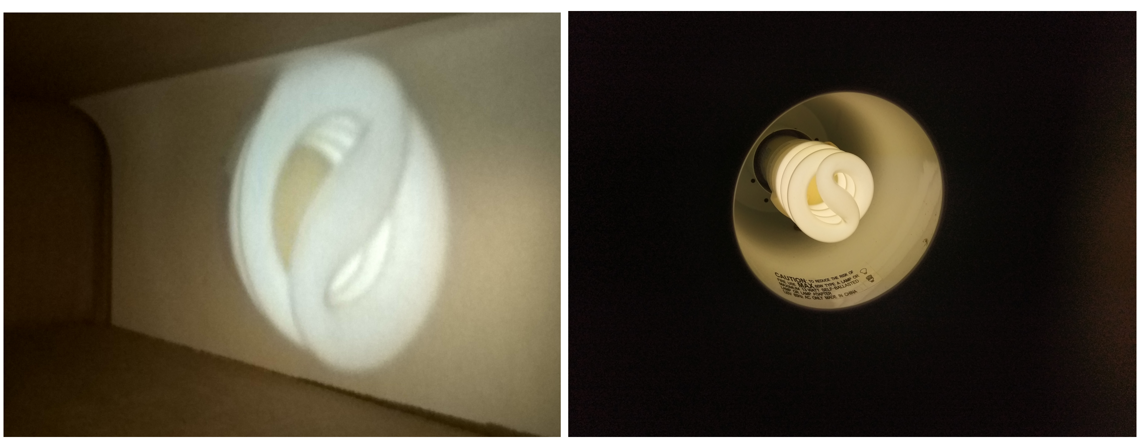

Figure 4: (Left) Pinhole camera image (Right) Smartphone camera image of same scene.

The pinhole camera image is inverted, dim and a little blurry. The camera works only with extremely bright objects.

Q3) Let us now try to experimentally verify our observations from Q3, Part 1. Seal the original pinhole with some tape (ensure that the tape does not let light through) and create a hole with a larger diameter next to it. Now, capture an image of the same scene as the previous part, with this new aperture, and post the image in the box below. How is this image different from the one with the smaller aperture? (1 points)

Figure 5: Pinhole camera image with larger aperture

The image from the pinhole camera with larger aperture is more blurry compared to the one with smaller aperture.

4 Frequency Analysis of Imaging Apertures

Having understood and built a pinhole camera, this part is dedicated to applying frequency analysis tools from the class to this system.

Q1) What is the transfer function (in the frequency domain) of the ideal pinhole camera system (infinitesimally small aperture)? (1 points)

Assuming no diffraction, the point spread function of an ideal pinhole camera is geometrically given by the cylindrical function (essentially 2D impulse response):

$$p[x,y] = \begin{cases} \beta & \sqrt{x^2+y^2} < a/2 \\ 0 & \sqrt{x^2+y^2} \geq a/2 \end{cases}$$

where, β = constant (assume to be 1), a = aperture (tends to 0). The transfer function is found by taking the Fourier transform of the point spread function, giving the Sombrero function:

$$H(\omega) = \frac{2J_1(\pi a \omega)}{\pi a \omega} \tag{3}$$

where, ω = angular frequency, J_1 = Bessel function of the first kind of order 0.

But, $a \to 0$, so equation (3) can be approximated as:

$$\lim_{a \to 0} H(\omega) = 0.5 \tag{4}$$

Q2) Now, assume the pinhole aperture is a square with a finite side length a. What is the transfer function of this system? Use an appropriate 2D coordinate system to make calculations simpler. (1 points)

Assuming no diffraction, the point spread function of a square pinhole with finite aperture is geometrically given by:

$$p[x,y] = \mathrm{sinc}^2(\frac{\pi a x}{f})\,\mathrm{sinc}^2(\frac{\pi a y}{f}) \tag{5}$$

where, f = focal length. Taking the Fourier transform of equation (5), we get the OTF as the two dimensional triangular function:

$$H(\omega_x, \omega_y) = \Lambda(\frac{\lambda \omega_x f}{a})\Lambda(\frac{\lambda \omega_y f}{a}) \tag{6}$$

where, $\Lambda(x) = 1 - |x|$ for $|x| < 1$ and 0 otherwise, λ = wavelength.

Q3) Which of the two apertures in the previous two questions will capture a sharper image? Please justify your answer numerically from the transfer functions you have calculated from both apertures? It may be helpful to read up on a concept known as "Modulation Transfer Function 50" (MTF50). (1 points)

The infinitesimally small circular aperture gives a sharper image than the square aperture. This is because as $\omega \to \infty$, $|H(\omega_i)|$ (which is the modulation transfer function) of the square aperture tends to 0 (from equation 6), suppressing high frequency components of the image and blurring it out. In case of the infinitesimally small circular aperture, $|H(\omega_i)|$ is constant (equation 4).

References

[1] [Online]. Available: https://alecs590.wordpress.com/assignment-5-pinhole-and-anti-pinhole/