Assignment 1

A1) Proof by induction

$\forall n \geq 1$

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

BestCase $(n=1)$

$LHS = 1$

$$RHS = \frac{1(1+1)}{2}$$

$LHS = RHS$

· Formula holds for $n=1$

· Assume for some $k \geq 1$

+ Induction hypothesis for k

$$1+2+3+\dots+k = \frac{k(k+1)}{2}$$

+ Induction hypothesis for $k+1$

$$1+2+3+\dots+k+k+1 = \frac{k(k+1)}{2} + k+1$$

Factor $(k+1)$

$$1+2+3+\dots+k+k+1 = (k+1)\left(\frac{k}{2}+1\right)$$

$$(k+1)\left(\frac{k+2}{2}\right) = \frac{(k+1)(k+2)}{2}$$

· Hence Mathematical induction holds true for all $n \geq 1$.

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A2] (a) $A = \{1,2,3,4\}$
$B = \{2,4,1,3\}$

Equal Sets → Every element of x is in y and vice versa.

$X = Y \iff (x \in X \Rightarrow x \in Y) \; \& \; (y \in Y \Rightarrow y \in X)$

Order and Repitition doesnt matter

$\{1,2\} = \{2,1\} = \{1,1,2\}$

Hence $A = B$

(b) Subset of set $A = \{1,3,5,7\}$
Every element of S is in A

n elements ⇒ 2^n subsets
including empty set ϕ & set itself

$n = 4$
$2^4 = 16$ subsets

$\{\phi\}, \{1\}, \{3\}, \{5\}, \{7\}, \{1,3\}, \{3,5\}, \{1,5\}, \{1,7\}, \{3,7\},$

$\{5,7\}, \{1,3,5\}, \{1,3,7\}, \{1,5,7\}, \{3,5,7\}, \{1,3,5,7\}$

(c) Write in set builder form
∴ $\{x \mid \text{property of } x\}$

$A = \{x \in Z \mid x \geq 1\}$

(d) $A = \{1, 3, 5, 7, 9, 11\}$

$B = \{1, 2, 3, 13\}$

$A - B = \{5, 7, 9, 11\}$

$B - A = \{2, 13\}$

(e) $A \cup (B \cup C)$ where $A = \{1, 3, 5\}$ $B = \{2, 4, 6\}$

$C = \{1, 5, 7\}$

Associative

$A \cup (B \cup C) = (A \cup B) \cup C$

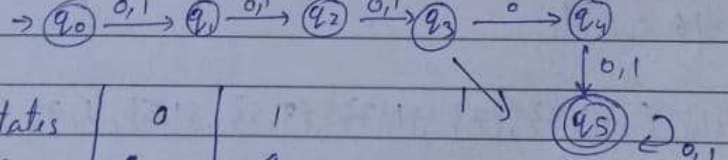
$(B \cup C) = \{2, 4, 6, 1, 5, 7\}$

$(A \cup B) = \{1, 2, 3, 4, 5, 6\}$

$A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6, 7\}$

A3] $L = \{w \mid w \in \{0,1\}^*$ & 4 th Symbol from the beginning of w is 0

1] DFA with 6 states, Single final State



States	0	1
q_0	q_0	q_1
q_1	q_2	q_1
q_2	q_3	q_d
q_3	q_9	q_d
q_4	q_4	q_4
q_5	q_5	q_5

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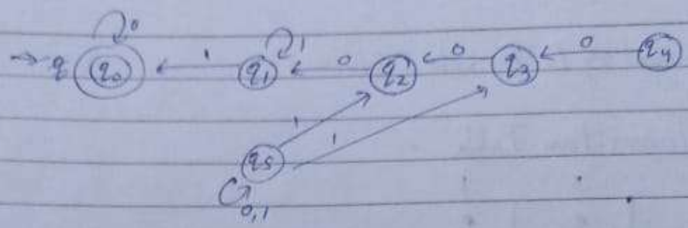


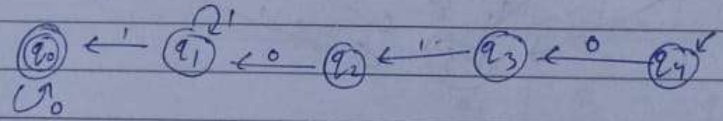
3] Yes, it is a valid finite automation because it has :-

- finite number of states
- Initial State q_1
- final State q_0

It is NFA because from q_5 there are 3 possible transitions on 1. In DFA, we have at most one transition from each symbol

4] S1 - Remove Unreachable States → q_5 (no incoming edge, only self loop

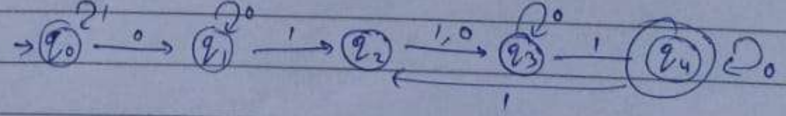
S2 - Diagram



A4] Given, string begins with 01
String ends with 10

$L = \{ \cancel{'10110', 010} \ (0,1)(1,0)^*(1,0) \}$

NFA



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Converting NFA into DFA

Transition Table :-

	0	1
q_0	q_1	q_0
q_1	q_1	q_2
q_2	q_3	q_3
q_3	q_3	q_4
q_4	q_4	q_2

DFA	0	1
q_0	$\{-\}$	$\{-\}$
q_1	$\{-\}$	q_2
q_2	$\{\}$	q_3
q_3	q_4	$\{\}$
q_4	$\{\}$	$\{\}$
$\{-\}$	$\{\}$	$\{\}$

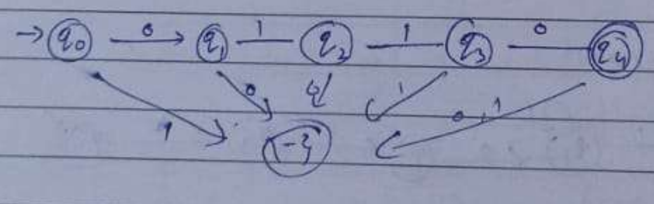
→ Transition Table

$Q = q_0, q_1, q_2, q_3, q_4$

$\Sigma = \{0, 1\}$

$F = q_4$

$q_0 = q_0$



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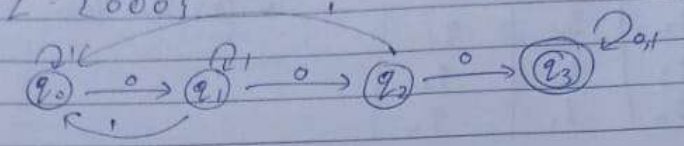
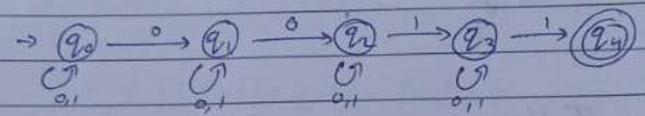
A5] DFA for language $\{0,1\}^*$ such that it contains '000' substring.

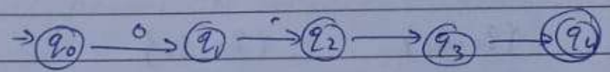
$L = \{000\}$

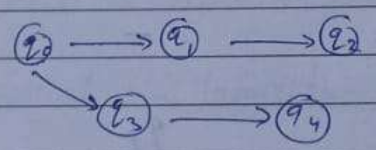


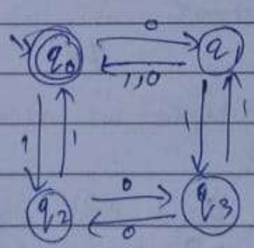
A6] $\Sigma = \{0,1\}$ and is with even no of zeroes & even no. of one's



DFA







Transition Table:

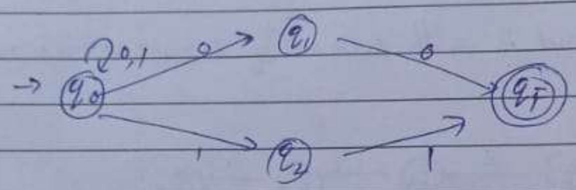
	0	1
q_0	q_1	q_2
q_1	q_0	q_3
q_2	q_3	q_0
q_3	q_2	q_1

Design DFA Divisible by 5 → There will be 5 states

	0	1
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

→ See this, then draw

A7]



NFA

Transition Table

	0	1
q_0	$\{q_0, q_1\}$	$\{q_0, q_2\}$
q_1	q_F	—
q_2	—	q_F
q_F	q_F	q_F

DFA Transition Table

	0	1
q_0	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_F\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_0, q_1, q_F\}$
$\{q_0, q_1, q_F\}$	$\{q_0, q_1, q_F\}$	$\{q_0, q_2, q_F\}$
$\{q_0, q_2, q_F\}$	$\{q_0, q_1, q_F\}$	$\{q_0, q_2, q_F\}$

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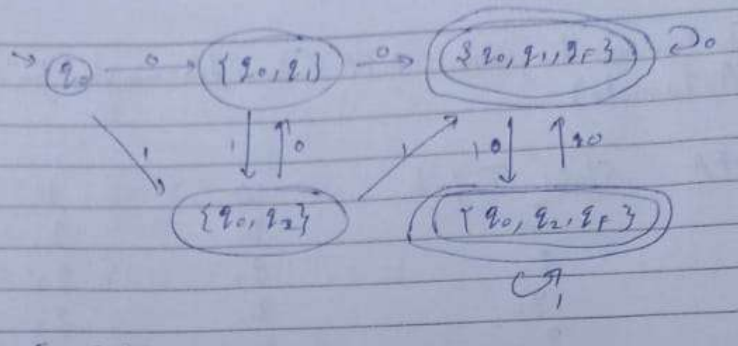


For DFA

$Q = q_0, \{q_0, q_1\}, \{q_0, q_2\}, \{q_0, q_1, q_F\}, \{q_0, q_2, q_F\}$

$\Sigma = \{0, 1\}$

$q_0 = q_0$

$F = \{q_0, q_1, q_F\}, \{q_0, q_2, q_F\}$

A8] S1 Remove the unreachable state

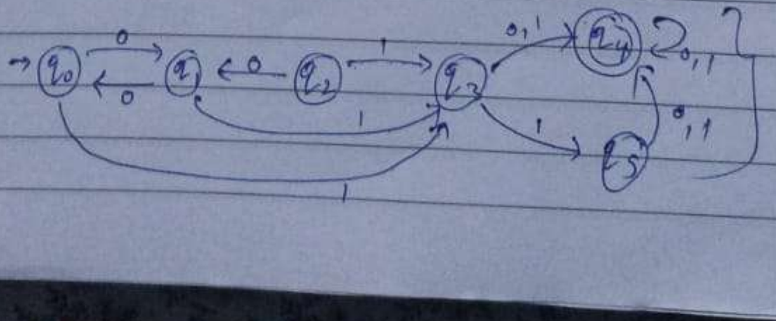
S2 Create the transition table of given DFA

S3 Create the transition table for non-final states

S4 Create the transition table for final states

S5 Remove the Duplicate rows from both the tables

S6 Redraw DFA with transition table formed after Combining final and non final states table.



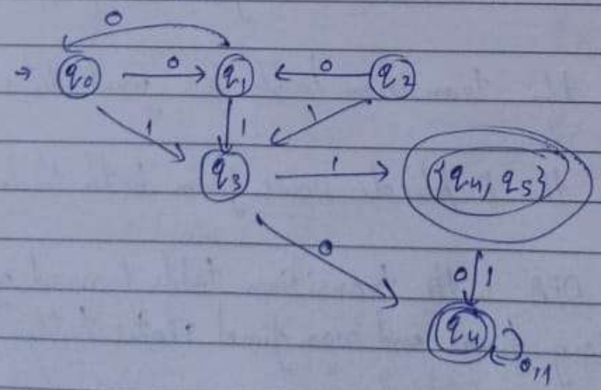
The Previous diagram is NFA (Non-deterministic finite Automata)

NFA

State/Input	0	1
q_0	q_1	q_3
q_1	q_0	q_3
q_2	q_1	q_3
q_3	q_4	$\{q_4, q_5\}$
q_4	q_4	q_4
q_5	q_4	q_4

DFA Transition Table

State/Input	0	1
$\rightarrow q_0$	q_1	q_3
q_1	q_0	q_3
q_3	q_4	$\{q_4, q_5\}$
q_4	q_4	q_4
$\{q_4, q_5\}$	$\{q_4\}$	q_4
q_2	q_1	q_3



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S1: Remove unreachable state
q_2 is unreachable, so we remove it.

S2: DFA Transition Table

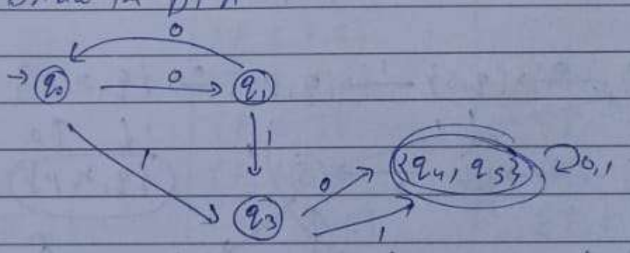
S3: Non-Final Transition Table

S4: Final State Transition Table

S5: Remove Duplicate rows

S6: Final Transition Table

S7: Draw the DFA



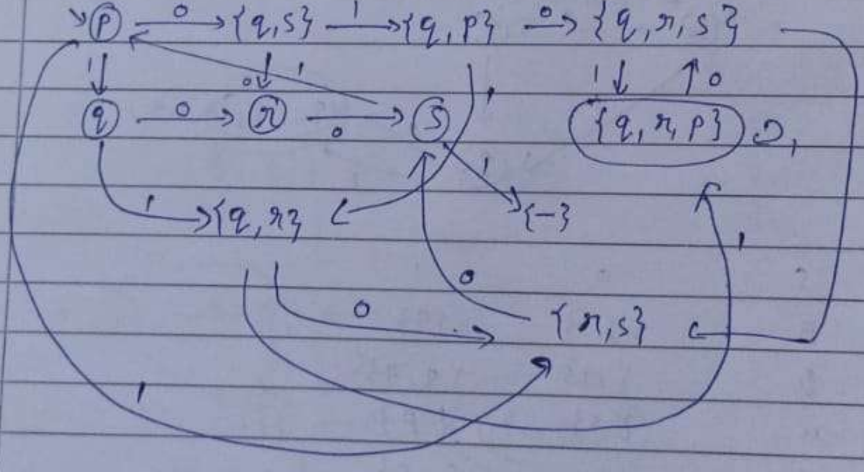
A10]

S	0	1
p	{q,s}	{q}
q	{r}	{q,r}
r	{s}	{p}
s	—	{p}

P.T.O

DFA Transition Table

S	0	1
p	{q, s}	{q}
q	{r}	{q, r}
{q, s}	{r}	{q, p}
r	{s}	{p}
{q, r}	{r, s}	{q, r, p}
{q, p}	{r, q, s}	{q, r}
s	{-}	{p}
{r, s}	{s}	{p}
{q, r, p}	{r, s, q}	{q, r, p}
{r, q s}	{s, r}	{p, q, r}
{-}	{-}	{-}

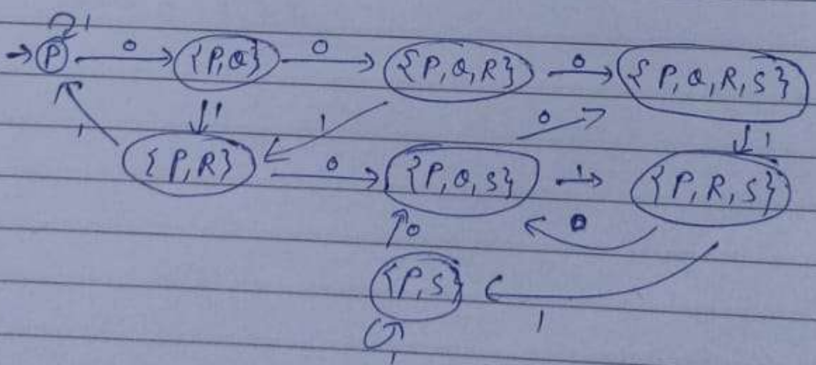


A1] Given NFA transition Table :-

S	0	1
P	{P, Q}	P
Q	R	R
R	S	—
S	S	S

DFA Transition Table

S	0	1
{P}	{P, Q}	{P}
{P, Q}	{P, Q, R}	{P, R}
{P, R}	{P, Q, S}	{P}
{P, Q, R}	{P, Q, R, S}	{P, R}
{P, Q, S}	{P, Q, R, S}	{P, R, S}
{P, R, S}	{P, Q, S}	{P, S}
{P, S}	{P, Q, S}	{P, S}
{P, Q, R, S}	{P, Q, R, S}	{P, R, S}



A13] $L = \{ "aa", "bb", \ldots \}$

$Q = q_0, q_1, q_2, q_3, q_4$

$\Sigma = \{a, b\}$

$q_0 = q_0$

$F = q_2, q_4$