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Tutorial-2

Q1

```
fun(int n)
{ int j=1, i=0
  while (i<n)
  { i = i+j
    j++ }
}
```

$\frac{k(k+1)}{2} = n$

$k^2 = n \Rightarrow k = \sqrt{n}$ $\quad T.C = O(\sqrt{n})$

Q2

$T(0) = 0$

$T(1) = 0$

$T(n) = T(n-1) + T(n-2) + 1$

let $T(n-1) = T(n-2)$

$T(n) = 2T(n-1) + 1$

Using back subs

$T(n) = 2 \cdot 2(T(n-2) + 1) + 1$

$= 4(T(n-2)) + 3$

$T(n-2) = 2T(n-3) + 1$

$= 2(2(2(T(n-3)+1)+1)+1$

$= 8T(n-3) + 7$

$T(n) = 2^k T(n-k) + 2^k - 1$

$T(0) = 0$

$n - k = 0 \Rightarrow n = k$

$T(n) = 2^n T(n-n) + 2^n - 1$

$= 2^n + 2^n$

$T(n) = TC = O(2^n)$

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Q3 log(log n)

```
fun(int n)
{ for (int i = n; i >= 2; pow(i, i/2))
    { some O(1)
    }
}
```

n log n

```
for (int i = 1; i <= n; i++)
  for (int j = 1; j <= n; j = j * 2)
    { for some O(1)
    }
}
```

n^3

```
for (int i = 1; i < n; i++)
for (int j = 1; j < n; j++)
for (int k = 1; k < n; k++)
  some O(1)
```

Q4 $T(n) = T(\frac{n}{4}) + T(\frac{n}{2}) + cn^2$

assum $T(\frac{n}{2}) >= T(\frac{n}{4})$

$T(n) = 2T(\frac{n}{2}) + cn^2$

$c = \log_b a$

$c = \log_2 2 = 1$

$n^c < f(n)$

$Tc = O(n^2)$

Q5

i	j
1	n
2	n/2
3	n/3
:	:
n	n/n

T.C = O(n log n)

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Q6 $i = 2, 2^k, (2^k)^k, (2^k)^k \ldots k$ — $2^{k \log k (\log n)}$

$2^{k \log k (\log n)} = n$

$T.C = O(\log(\log n))$

Q7 $T(n) = T\left(\frac{99}{100} n\right) + \frac{n}{100}$

$T(1) = 0$

Putting $n = \frac{99}{100} n$

$T\left(\frac{99}{100} n\right) = T\left(\left(\frac{99}{100}\right)^2 n\right) + \frac{99}{(100)^2} n$

$T(n) = T\left(\frac{99^k}{100} n\right) + \frac{(99)^{k-1}}{(100)^k} n$

$\left(\frac{99}{100}\right)^k n = 1$

$n = \left(\frac{100}{99}\right)^k$

$k = \log_{\frac{100}{99}} n$

$TC = \Theta(n \log n)$ Ans

Q8 a) $1 < \log(\log n) < \log n < \sqrt{n} < \log(n!) < n \log n < n^2 < 2^n < 2^{2n} < 4^n < n!$

b) $1 < \log(\log n) < \sqrt{\log n} < \log(n) < 2n < 4n < 2(2^n) < \log(2n) < 2 \log n < n < \log(n!) < n \log n < n!$

c) $96 < \log_2 n = \log_8 n < n \log_6 n = n \log_2(n) < 5n < 8n^2 < 7n^3 < 8^{2n}$