Tutorial-4

Q1 $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$a = 3$, $b = 2$, $f(n) = n^2$

$a \& b$ are constant $\& f(n)$ is function

Master's Th. is applicable

$c = \log_b a$

$= \log_2 3 = 1.58$

$n^c = n^{1.58}$ which is $< n^2$

Case 3 is applicable

$\therefore T(n) = \Theta(n^2)$

Q2 $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$a = 4$, $b = 2$, $f(n) = n^2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2 \Rightarrow n^c = f(n)$

Case 2 $\quad T(n) = \Theta(n^2 \log n)$

Q3 $T(n) = T\left(\frac{n}{2}\right) + 2^n$

$a = 1$, $b = 2$, $f(n) = 2^n$

$c = \log_b a = \log_2 1 = 0 \Rightarrow n^c = n^0 = 1$

$f(n) > n^c$

Case 3

$T(n) = \Theta(2^n)$

Q4 $T(n) = 16T\left(\frac{n}{4}\right) + n$

$a = 16 \qquad b = 4 \qquad f(n) = 4$

$c = \log_b a = \log_4 16 = 2$

$n^c > f(n)$

Case1 $T(n) = \theta(n^2)$

Q6 $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

$a = 2, b = 2, f(n) = n\log n$
$c = \log_b a = \log_2 2 = 1$
$n^c = n$
Case 3 is applied
$T(n) = \theta(n\log n)$

Q7 $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$

$a = 2, b = 2 \quad f(n) = n/\log n$
$c = \log_2 2 = 1$
$n^c = n$
non polynomial diff b/w n^c & $f(n)$
∴ Master Th. not applicable

Q8 $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2, b = 4, f(n) = n^{0.51}$
$c = \log_b a = \log_4 2 = 0.5$
$n^c = n^{0.5}$
$f(n) > n^c$
Case 3 is applicable
$T(n) = \theta(n^{0.51})$

Q9 $T(n) = 0.5\,T\left(\frac{n}{2}\right) + \frac{1}{n}$

$a < 1$ ∴ Master Th not applicable

Q10 $T(n) = 16T\left(\dfrac{n}{4}\right) + n!$

$a = 16, \quad b = 4 \quad f(n) = n!$

$c = \log_4 16 = 2$

$n^c = n^2$

$f(n) > n^c \quad case\ 3$

$T(n) = \theta(n!)$

Q11 $T(n) = 4T\left(\dfrac{n}{2}\right) + \log n$

$a = 4, \quad b = 2, \quad f(n) = \log n$

$c = \log_2 4 = 2$

$n^c = n^2$

$n^c > f(n)$

$case\ 1$

$T(n) = \theta(n^2)$

Q12 $T(n) = \sqrt{n}\ T\left(\dfrac{n}{2}\right) + \log n$

a is not constant so Master Th. not applicable

Q13 $T(n) = 3T\left(\dfrac{n}{2}\right) + n$

$a = 3, \quad b = 2, \quad f(n) = n$

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.58} > f(n)$

$case\ 1$

$T(n) = \theta(n^{1.58})$

Q14 $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

$a = 3, \quad b = 3 \quad f(n) = \sqrt{n}$

$c = 1$

$n^c = n > \sqrt{n}$

case 1 $\quad T(n) = \theta(n)$

Q15 $T(n) = 4T\left(\frac{n}{2}\right) + cn$

$a = 4, \quad b = 2, \quad f(n) = cn$

$n^c = n^2 > f(n)$

case 1

$T(n) = O(n^2)$

Q16 $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

$a = 3, \quad b = 4 \quad f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.78$

$n^c = n^{0.78} < f(n)$

Case 3

$T(n) = \theta(n\log n)$

Q17 $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$a = 3, \quad b = 3, \quad f(n) = \frac{n}{2}$

$c = 1$

$n^c = n > f(n)$

case 1 : $T(n) = \theta(n)$

Q18 $T(n) = 6T\left(\dfrac{n}{3}\right) + n^2 \log n$

$a = 6$, $b = 3$, $f(n) = n^2 \log n$

$c = \log_3 6 = 1.63$

$n^c = n^{1.63} < f(n)$

Case 3: $T(n) = \Theta(n^2 \log n)$

Q19 $T(n) = 4T\left(\dfrac{n}{2}\right) + n \log n$

$c = \log_2 4 = 2$

$n^c = n^2 \quad f(n) = n \log n$

$n^c > f(n)$

Case 1 $\therefore T(n) = \Theta(n^2)$

Q20 $T(n) = 64 \, T\left(\dfrac{n}{8}\right) + n^2 \log n$

$a = 64$

$b = 8$

$c = \log_8 64 = 2$

$n^c = n^2 < f(n)$

Case 3 $\therefore T(n) = \Theta(n^2 \log n)$

Q21 $T(n) = 7 \, T\left(\dfrac{n}{3}\right) + n^2$

$a = 7$, $b = 3$, $f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77} < f(n)$

Case 3 $\therefore T(n) = \Theta(n^2)$

Q22 $T(n) = T\left(\dfrac{n}{2}\right) + n(2 - \cos n)$

$\therefore f(n)$ is not regular function

so, Master Th not applicable