CS711: Introduction to Game Theory and Mechanism Design **Jul-Nov 2018**

Project: Assignment 2

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1.1

1.1.1

Let $P = (P1, P2)$. Consider P'':

$P1''$	$P2''$
b	c
a	a
c	b

Since $\forall(\bar{P}_1, \bar{P}_2), f(\bar{P}_1, \bar{P}_2) \in \{\bar{P}_1(1), \bar{P}_2(1)\}$, b and c can be the only possible solutions for $f(P'')$.
Let c be the required solution. But $D(c, P)$ is same as $D(c, P'')$. So by monotonicity (strategyproofness implies monotonicity), $f(P)$ should equal c. But this is a contradiction as $f(P) = a$. Therefore $f(P'') = b$.
Now consider P':

$P1'$	$P2'$
b	a
a	b
c	c

Clearly $D(b, P'')$ is a subset of $D(b, P')$. So by monotonicity, $f(P') = b$ i.e. $f(P1', P2') = b$.

1.1.2

The earlier conclusion doesn't hold for the case of single peaked preference domain with $a < b < c$. This is because for such a case the preference order $P2'' = c > a > b$ is not possible.
We know that in single peaked preference domain, one of the peaks of some agent is taken as output. Since $f(P) = a$, this implies that the function f picks the leftmost peak as the output. In the case of P', the leftmost peak is again a. So the output of $f(P1', P2') = a$.

1.2

The Gibbard-Satterthwaite result doesn't apply here.
The Gibbard-Satterthwaite applies to a SCF only when the domain of that SCF is not restricted i.e. each

project can be an outcome of the SCF for some preference profile of the agents. I will show that this not the case with this SCF.
Consider two non-empty sets S and T such that S is a proper subset of T. Now for any agent, the project most preferred by him out of the available projects in the union of T and S can either be present in both S and T or only in T ($S \cup T = T$). So he will either prefer T or be indifferent towards both these subsets. It can never be the case that he prefers S over T and this holds for all the agents. Thus the domain of this SCF is restricted and we can't apply Gibbard-Satterthwaite in a restricted domain.

1.3

Yes, the median voter SCF is group strategy-proof.
Consider some group of agents A that want to manipulate the outcome in their favour. Therefore they will want to change the present outcome of the SCF. So the median voter, the one whose peak preference is taken as the outcome, will not be a part of any such groups as any other result will be lesser favourable to him and he won't want that. The cases that are left to handle are when the group A is made of only agents from right of median, agents from left of median or a mixture of these agents.

1.3.1 Group with agents with peaks on the right of median only

If such a group of agents want to shift the median, then at least one of them will need to report their preferences as left of the present median value because if they all report their value to the right of median, the median will not change as the number of peaks on the left and right of median still remain same. But if they report their peaks as left of the present median, the median will shift to the left and so will the outcome of the SCF. Clearly this new outcome is lesser favourable to the group of agents (case of single peaked preferences). So they have no other option but to report their true preference.

1.3.2 Group with agents with peaks on the left of median only

This is similar to the above case. Such a group can again only make the outcome lesser favourable by reporting their false preferences i.e. shifting the median to the right. So they won't do that.

1.3.3 Group with some agents with peaks on the right of median and some on the left

Such a group will never be formed as the only way that the people on one side can get a better outcome is by asking at least one person on the other side to report a value that will produce the outcome that is unfavourable for that person himself. So their will never be a consensus in such a group.

This shows that a group of agents can never manipulate median voter SCF i.e. median voter SCF is group strategy-proof.