CS-698W: Game Theory and Collective Choice	**Jul-Nov 2017**
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9.1 Imperfect-information extensive-form games

The PIEFG is not able to represent the simultaneous move games like neighboring kingdoms dilemma. Hence there is a need to move for a more general representation.

Definition 9.1 ***Imperfect-information extensive form game:*** *An imperfect-information extensive form game is a tuple $(N, A, H, Z, \chi, P, (u_i)_{i\in N}, (I_i)_{i\in N})$, where: I_i is a partition* [1] *of $\{h \in H/Z : P(h) = i\}$ with the property that if $h, h' \in I_i^j$, then $\chi(h) = \chi(h')$ or in words* ***Information Set*** *I_i^j of a player is a set of players decision nodes which are indistinguishable to him.*

Set I_i for every player i, is collection of Information set I_i^j, at every level where player i is a player.

$$I_i = \{I_i^1, I_i^2, ..., I_i^{k(i)}\}$$

Note : Information set is always non-empty but it can be singleton. When information set i.e I_i^j is singleton, it implies that there is no ambiguity at which node we are at in given information set. Such cases are referred as *Perfect information extensive form game.*

By *Definition 9.1*, we can in fact define χ function over Information sets I_i^j, rather than defining them over histories h,h' as $h, h' \in I_i^j$ and $\chi(h) = \chi(h')$, therefore

$$\chi(h) = \chi(h') = \chi(I_i^j)$$

9.1.1 Strategy set of player i

Definition 9.2 *Consider Neighboring Kingdoms Dilemma, in IIEFG representation, here, when player 2 is playing he doesn't know at which node (a or b), he is actually present at. Thus his action here is independent of the node he is present at. So, we define Strategy Set for player i (S_i) as the cross product of the action set in every different Information Set, where player i is playing.*

$$S_i = \prod_{I_i^j \in I_i} \chi(I_i^j)$$

[1] Partition is disjoint set of subsets s.t union of those subsets is whole set.

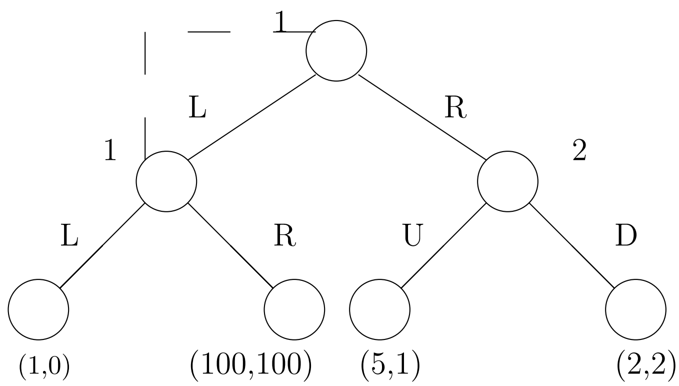

Figure 9.1: Neighbouring Kingdom's Dilemma

9.1.2 Representational equivalence

From *Definition 9.2*, we can infer that when player 2 is playing, he can play A or he can play D. Thus player 2 have 2 different pure strategy, rather than 4 which he would have, if we had all Information set as singleton. *So, Imperfect Information Extensive Form Game is richer representation than NFG and PIEFG.*

9.2 Behavioural Strategy versus Mixed Strategy

Definition 9.3 *While a mixed strategy assigns a probability distribution over pure strategies, a behavioural strategy assigns at each information set a probability distribution over the set of possible actions.*

In other words, while in mixed strategy probability distribution happens over whole contingency plan, but in behavioural strategy we independently randomize our action at each level.

Let's consider following in IIEFG representation of a particular game. Discontinuous line between node a and node b, suggests that player 1 is forgetful of what he plays at level 1. As player 1 forgets what he played at level 1, thus his action set contains only $\{L, R\}$ instead of $\{LL, LR, R\}$.

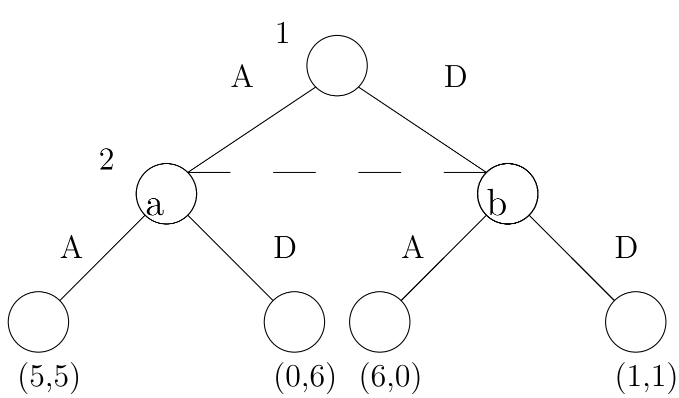

Figure 9.2: IIEFG representation of a game

9.2.1 Mixed strategy

Looking at Normal form game representation, we can conclude that for player 1 *R dominates L*, and for player 2 *D weakly dominates U*. Thus game have **weakly dominant strategy equilibrium** (R,D). So, game has unique MSNE ((0,1),(0,1)).

9.2.2 Behavioural strategy

In behavioural strategy, we randomize our action at each turn. From IIEFG representation we can assume that player 2 is rational enough to play D, as it is always more favourable for him.
So probability distribution for player 2 is (0:U, 1:D), let probability distribution for player 1 be (p:L, (1-p):R).
Thus expected payoff for player 1 is,

$$p^2 \times 1 + p(1-p) \times 100 + (1-p) \times 2$$

which is maximum at p=$\frac{98}{198}$. So behavioural strategy equilibrium for this game is (($\frac{98}{198}$, $\frac{100}{198}$), (0,1)).

We can see difference in MSNE and BSE, this is due to games with imperfect recall, but when we consider games with perfect recall, two strategies are equivalent. We will see this in upcoming lectures.

9.3 Games with Perfect Recall

Let us consider type of games where at every opportunity of a player to act, he remembers exactly what he did at his every previous turn to play. Such games are called *Games with Perfect Recall.*

Definition 9.4 *Player i has perfect recall in IIEFG, if for any two histories* $h, h' \in I_i^j$ *where* $h = (v_0, a_0, v_1, a_1, ..., v_{m-1}, a_{m-1}, v_m)$ *and* $h' = (v'_0, a'_0, v'_1, a'_1, ..., v'_{n-1}, a'_{n-1}, v'_n)$ *where* $v_i, v'_i \in$ *node and* $a_i, a'_i \in$ *action at particular node, then*

1. *m=n*
2. *at every level i.e* $\forall j \; 0 \leq j \leq (m-1)$, v_j, v'_j *must be in same information set of player i.*
3. $\forall j \; 0 \leq j \leq (m-1)$, ***if*** *P($h_j$)=i* ***then*** $a_j = a'_j$, *where* h_j *is h truncated at level j.*

Game with Perfect Recall *is a game where all players have perfect recall.*

References

[CW87] M. Jackson, K. Leyton- Brown and Y. Shoham, Game Theory Lecture 4-8 ”Imperfect Information Extensive Form: Definition, Strategies”