Strategic Issues in DA algorithm

Consider the men-proposing version of this algorithm

The earlier example of a preference profile

P_{m_1}	P_{m_2}	P_{m_3}	P_{w_1}	P_{w_2}	P_{w_3}
w_2	w_1	w_1	m_1	m_3	m_1
w_1	w_3	w_2	m_3		
			m_2		

$\cancel{m_2}\ m_1$

$\cancel{m_1}\ m_2$

$\cancel{m_3}\ m_3$

Truthful: $m_1 \to w_2$ $\quad m_1 - w_2$ $\quad\mid\quad m_2 \to w_3 \quad m_2 - w_3$

$m_2 \to w_1$ $\quad m_3 - w_1$

$m_3 \nearrow$

Can anyone improve by a misreport of the preference

let w_1 report ~~$m_2 \succ m_1 \succ m_3$~~ $m_1 \succ m_2 \succ m_3$

$m_1 \to w_2$ $\quad m_1 - w_2$ $\quad\mid\quad m_3 \to w_2 \quad w_2 \times m_1 \quad w_2 - m_3$

$m_2 \to w_1$ $\quad m_2 - w_1$ $\quad\quad\quad\quad \checkmark\, m_3$

$m_3 \nearrow$ $\quad\quad\quad\quad\quad\quad\quad\quad m_1 \to w_1 \quad w_1 - m_1$

Theorem: The men (women)-proposing DA algorithm is strategyproof for men (women).

Q: Can there be a mechanism that is truthful for both?

A: NO.

Theorem: No stable matching algorithm can be strategyproof for both men and women.

Some open research directions in matching

- Fairness considerations — is there a stable match that is more egalitarian for both men and women
- Feasibility / multiple attributes — ~~the~~ the kidney exchange problem - it is a house allocation but not everyone can receive any kidney.
  - Similarly, if preferences are multidimensional, one preference for each attribute -
    - students to universities
    - advertisers to viewers
- Monetary transfers — classical quasi-linear setting
  - questions of revenue can be asked.

Strategic Network Formation

Networks are formed via connections between individuals. We ask for the incentives for individuals to form links - and reason for which sort of networks may result due to their strategic choices.

Game Theoretic model of network formation

- costs and benefits for agents associated with networks
- agents are the nodes, and they choose links
  - countries with trade relations
  - people choosing friends
  - researchers with research ~~on~~ collaborations
  - employees with companies
- Contrast individual and social choices.

Modeling choices for adding/forming / or deleting links

- consensus needed (undirected/directed)
- coordinate changes (network structures, influence)
- dynamic or static
- sophisticated agents - can compute the values
- can they compensate each other to form links?
- links adjustable in intensity.

Questions:

1. Which networks are likely to form?
2. Stable against perturbations?
3. Efficient from a global perspective? - ~~Government subsidies~~
4. How inefficient they are if not efficient?
5. Can intervention help improve efficiency?

Jackson-Wolinsky (1996) model of network formation

- $u_i(g)$: payoff to agent i if the network is g.

<u>Connections model</u> (JW 1996)

- $0 \leq \delta_{ij} \leq 1$ ... a benefit parameter for connection between i and j
- $0 \leq c_{ij}$ cost to i to maintain a link with j
- $l(i,j)$ - length of the shortest path between i and j

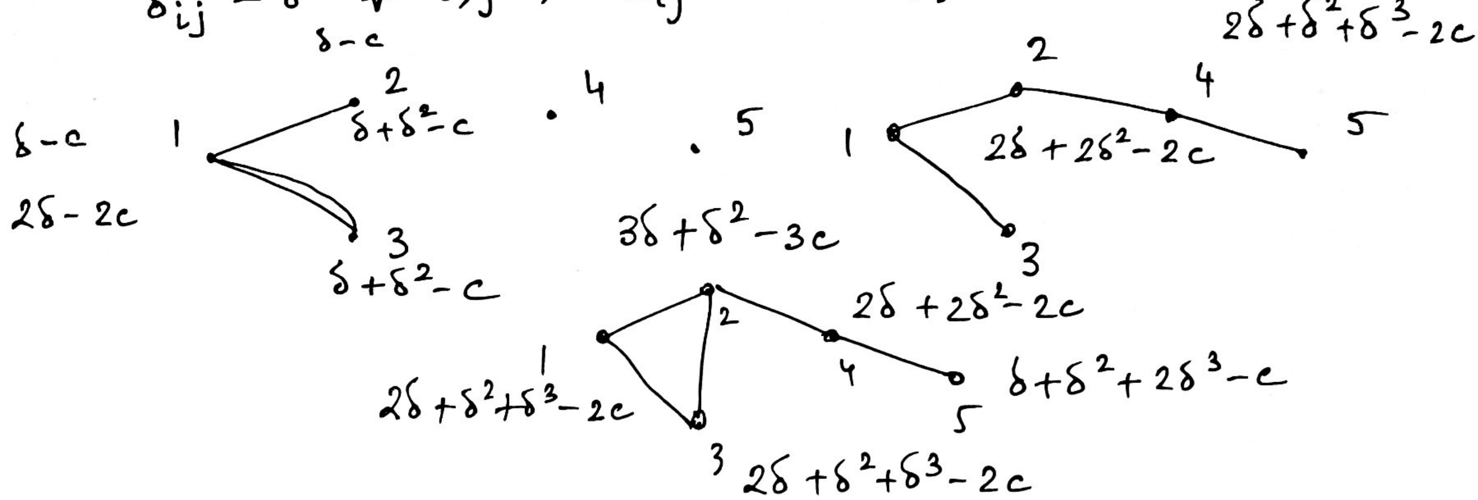
utility model:

$$u_i(g) = \sum_{j \in N \setminus \{i\}} \delta_{ij}^{l(i,j)} - \sum_{k \in N_i(g)} c_{ik}$$

(5-4)

Symmetric version

$\delta_{ij} = \delta \;\; \forall\, i,j \;, \quad c_{ij} = c \;\; \forall\, i,j$



Shortest path is considered as the contribution flow.

- What network will form?
- Is that optimal from the global perspective?

Modeling Incentives / Equilibrium

consensus is needed to form a link.

- Every agent announces her agents of choice and link forms if mutual agreements are received.
- Nash equilibrium: no agent can gain from unilateral deviation.

Not a good notion:

Both are NE but unsatisfactory since it says anything can happen

Any reasonable model should allow to form the link.

Other equilibrium notions have similar trouble

- The off-the-shelf concepts from non-cooperative game theory may not work.

Pairwise Stability

~~Current links~~

No single agent gain by deleting a link

No pair of agent gains by adding a link

Defn: A graph g is pairwise stable if

a) $\forall (ij) \in g \quad u_i(g) \geq u_i(g \setminus (ij))$ and

$$u_j(g) \geq u_j(g \setminus (ij))$$

b) $\forall (ij) \notin g \quad$ if $u_i(g + (ij)) > u_i(g)$

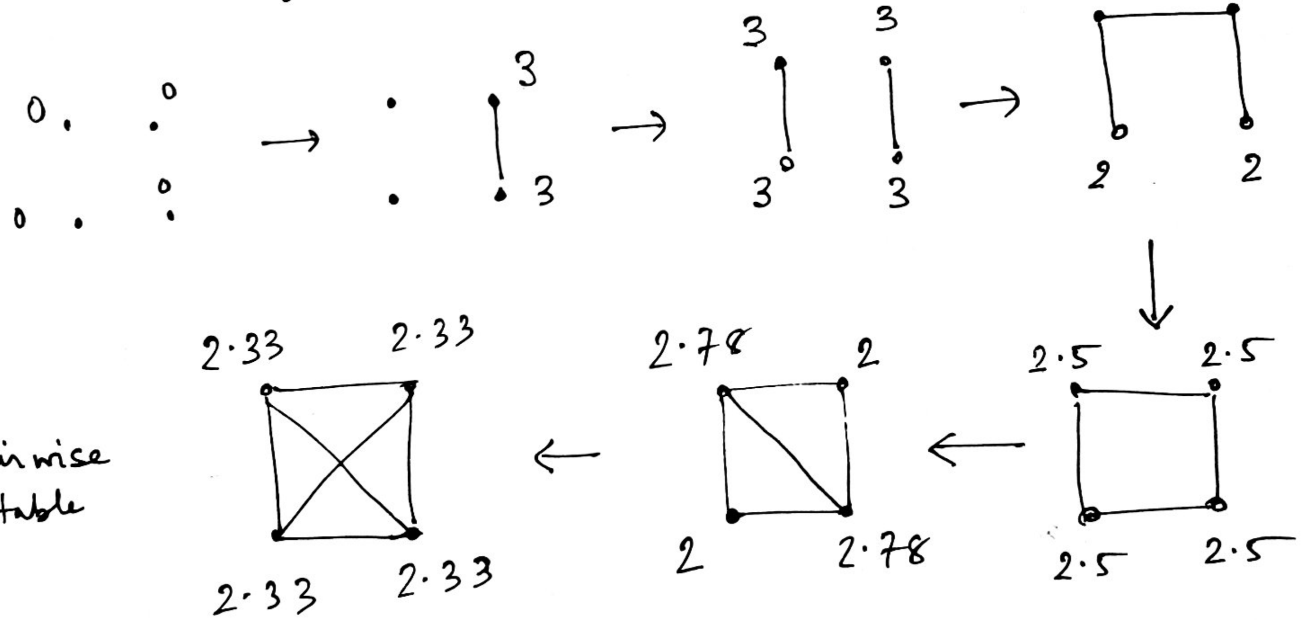
then $\quad u_j(g + (ij)) < u_j(g)$.

This is weak assumption – minimal to work with

Example: ①

pairwise stable.

②



pairwise stable

Note: The pairwise stable network gives worse payoff than ~~the~~ some unstable ones.

– Individual incentives drag the network from that optimal network.

Pareto efficiency

A network g is PE if $\nexists\, g'$ s.t.

$$u_i(g') \geq u_i(g) \;\forall i \in N \text{ and strict for some } j \in N.$$

Efficiency: $$g \in \underset{g \in \mathcal{G}}{\operatorname{argmax}} \sum_{i \in N} u_i(g).$$

utilitarian

Efficient $\Rightarrow$ PE. $\quad$!PE $\Rightarrow$!EFF.

Explanation using the previous example.

$\longrightarrow$ Back to connections model (δ, c) - symmetric version

~~Theorem~~: Consider efficient networks

Theorem: • When $c < \delta - \delta^2$ [low cost]

Classical market — complete network is uniquely efficient.

• when $\delta - \delta^2 < c < \delta + (n-2)\delta^2/2$ [medium cost]

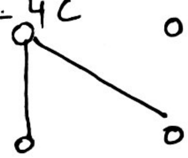
Amazon/ Flipkart — star networks with all agents are uniquely efficient.

• when $\delta + (n-2)\delta^2/2 < c$ [high cost]

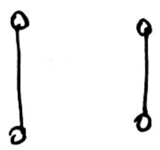
RIP trade platforms — empty network is uniquely efficient.

Why Stars?

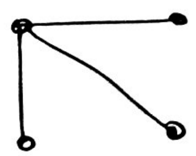
$4\delta + 2\delta^2 - 4c$



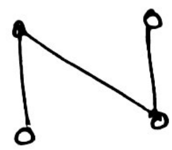
$4\delta - 4c$



With given number of links, stars are most efficient ways to connect individuals.



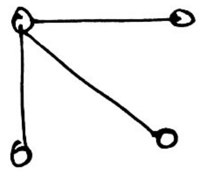
$6\delta + 6\delta^2 - 6c$



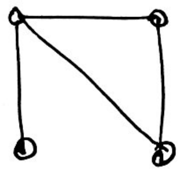
$6\delta + 4\delta^2 + 2\delta^3 - 6c$

indirect connections are longer

Star vs Complete



$6\delta + 6\delta^2 - 6c$



$8\delta + 4\delta^2 - \underline{8c}$

made some indirect connections direct - thereby more benefit ~~so~~ but increased the cost

When is the gain more than the cost

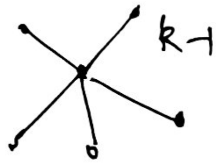
if $\delta - \delta^2 > c$, adding direct link is beneficial.

Proof: Part 1: If $i\,j$ are not directly connected then the benefit is at most δ^2. If they are connected, $\delta - c$ is the ~~cost~~ benefit but $\delta - c > \delta^2$ ~~hence~~ in the low cost region. Hence adding the edge is always beneficial. Others are weakly better.

Part 2: $c > \delta - \delta^2$: first show that the value of a component is maximum when the component is a star

- value of a star with k players is

$$2(k-1)(\delta - c) + (k-1)(k-2)\delta^2$$



- value of a network with k players and m links $(m \geq k-1)$ is at most

$$2m(\delta - c) + \left(2\cdot\binom{k}{2} - 2m\right)\delta^2$$

$\binom{k}{2} - m$ indirect links counted twice for each player

The difference between value of star and value of any other network is at least

$$2(m-(k-1))\left[\delta^2 - (\delta - c)\right] > 0 \quad \text{if } m > k-1$$

in a region $\delta^2 > \delta - c$

5-8

If $m = k-1$ and not a star, then some pair of nodes is at a distance of more than 2, so less value than a star:

- star is better.

Can two stars be better than one star?

Exercise: show that two stars with k and k' nodes give less ~~uti~~ welfare than one consolidated star.

~~Part 3:~~ Finally, whether to keep a star or empty look at a star of size n — if the total utility is > 0 then star is optimal

$$2(n-1)(\delta - c) + (n-1)(n-2)\delta^2 > 0$$

$$\Rightarrow \quad c < \delta + (n-2)\delta^2/2 \, .$$

else empty is better [gives cases 2 and 3] $\square$.

<u>Pairwise</u> stability

Low cost : $c < \delta - \delta^2$ - complete network is pairwise stable

medium/low cost : $\delta - \delta^2 < c < \delta$

- star is pairwise stable
- others are too

medium/high cost : $\delta < c < \delta + (n-2)\delta^2/2$

- star is not pairwise stable

high cost : $c > \delta + (n-2)\delta^2/2$

- empty is pairwise stable.