CS711: Introduction to Game Theory and Mechanism Design

Assignment 1 – Semester 1, 2018-19
Computer Science and Engineering
Indian Institute of Technology Kanpur
Total Points: 30, Time: 2 days, ATTEMPT ALL QUESTIONS
Please submit your solutions as PDF files generated through LATEX. See the course webpage for LATEX tutorials. This is an author-blind submission. Do not put anything in the text of your solutions that indicates your identity. Please submit the solution PDF named as {roll number}.pdf, e.g., 1234567.pdf, which will be anonymized by the teaching staff. There is no need to write the question again in the solution PDF.

A general information: here are the values of certain variables that you will need to use in the solutions. **Write on top of your answerscript only the values of the variables x and y, and nothing more.** If the last two digits of your roll number are a and b respectively, i.e., 67 implies $a = 6, b = 7$, then, for the following questions:

$$x = \begin{cases} a & \text{if } a > 6 \\ a + 18 & \text{if } a \leqslant 6 \end{cases}$$

$$y = \begin{cases} b & \text{if } b > 9 \\ b + 9 & \text{if } b \leqslant 9 \end{cases}$$

1. Find all Nash Equilibria of this game (includes pure and mixed).

		Column Player	
		A	B
Row Player	A	$4, 0$	$2, 4$
	B	$0, x + y + 1$	$3 + x, y$

   Clearly state what the equilibrium strategies of the two players are.   **5 points.**

2. Consider the market share contest between two largest cloud service providers, Microsoft and Amazon. The price of every unit of cloud resource in the market with Microsoft (Firm 1) and Amazon (Firm 2) as the providers is

$$P(q_1, q_2) = (x + y) - q_1 - q_2$$

   where q_1 and q_2 is the amount of the resource produced by the two firms respectively. The amount of resource is a non-negative real number. The utility of Firm 1 is

$$u_1(q_1, q_2) = P(q_1, q_2) \cdot q_1 - q_1$$

and for Firm 2 is

$$u_2(q_1, q_2) = P(q_1, q_2) \cdot q_2 - \frac{x+y}{2} \cdot q_2$$

(a) Firms move simultaneously.

   (i) How much should each firm produce? **3 points.**

   (ii) What is each firm's utility? Show all steps for both the answers. **3 points.**

(b) Suppose in the same setting, Firm 1 moves first and decides its production of resource q_1, and then Firm 2 watches q_1 and decides how much resource q_2 to produce. The price in the market and the individual utilities have the same expressions as before.

   (i) How much should each firm produce in this changed scenario? **5 points.**

   (ii) What is each firm's utility? Show all steps for both the answers. **4 points.**

3. Three lions live in a jungle. They can hunt in the morning or evening, but not both. If a lion gets the morning time slot alone, it gets a payoff of x. If it gets the evening time slot alone, it gets a payoff of $x + y + 1$. If two lions hunt at the same time, both get zero.

   (i) Show that any strategy profile with two lions hunting in the morning and one in the evening is a pure-strategy Nash equilibrium. Show that any strategy profile with two lions hunting in the evening and one in the morning is a pure-strategy Nash equilibrium.

   (ii) Show that one mixed-strategy Nash equilibrium of the game is for a single lion to choose morning for sure, one lion to choose evening for sure, and the third lion to randomize however it likes.

   (iii) If all lions randomize in a symmetric mixed strategy equilibrium, with what probability do they hunt in the morning?

2+3+5 points.

Good Luck!