CS711: Introduction to Game Theory and Mechanism Design	**Jul-Nov 2018**

Project: Assignment 2

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Answer 1

(a)

Claim : If f is strategyproof then $f({P_1}', {P_2}') = b$.

Proof :

Let us assume

$${P_2}'' = c \succ a \succ b$$

Now since relative preference of a is more in ${P_2}''$ than in P_2, $f(P_1, P_2) = a$ and f is monotonic, we can say

$$f(P_1, {P_2}'') = a$$

Now we know that $f(P_1', {P_2}'') \in \{b, c\}$. Now if $f(P_1', {P_2}'') = c$, player 1 can use strategy P_1 instead of P_1' to win as player 1 always prefers a over c. This would imply that f is not strategyproof which is a contradiction. Hence $f(P_1', {P_2}'') \neq c$. Thus $f(P_1', {P_2}'') = b$. Now since relative preference of b is more in P_2' that in ${P_2}''$, $f(P_1', {P_2}'') = b$ and f is monotonic

$$f({P_1}', {P_2}') = b$$

Hence proved.

(b)

When preferences are generated from a single-peaked preference domain with the intrinsic ordering of the alternatives being $a < b < c$, the above conclusion **does not** hold.
This is because our assumed preference ${P_2}''$ cannot be generated from a single-peaked preference domain with given intrinsic ordering of a,b and c. Thus it can't be used as a basis for computing $f({P_1}', {P_2}')$.
We can have the following Median Voter SCF

$$f(P_a, P_b) = min(P_a(1), P_b(1))$$

where $P_a(1)$ and $P_b(1)$ are first preferences of player 1 & 2 in preference orders P_a & P_b respectively. f is both onto & strategyproof.
Hence $f(P_1, P_2) = a$ and $f(P_1', P_2') = a$.

Answer 2

The Gibbard-Satterthwaite result will not apply here

Explanation: Given $|X| \geq 2$. Thus size of set of alternatives ≥ 3. Let A and B be two alternatives such that $A \subset B$. In this situation either the agent is indifferent between A and B (when highest ranked project is in B as well as in A) or the agent prefers B (when when highest ranked project is in B but not in A). Thus there is no preference order in which the agent prefers A over B. This implies that there is a restriction in preference ordering. Thus Gibbard-Satterthwaite result will not apply here.

Answer 3

Yes, the median voter SCF group strategy-proof.

Explanation :

Let f be a k^{th} median SCF. Let set of players be denoted by N and let $K \subset N$. Let $P = \{P_1, P_1, .., P_n\}$ be the set of peak preferences of all players. Let the k^{th} median be a. Thus $f(P) = a$. When $P_i = a \; \forall i$ then the agents have no reason to manipulate f. Now let us assume that out of $k = |K|$ there are x agents whose peak preference $< a$ & remaining have their peak preferences $> a$ in the common order.

Case 1: $x = 0$ (or $x = k$) - When all the agents are on left side of a, they will have to move rightwards of a to change k^{th} median. Let final profile be P'. Now $f(P') > f(P)$. But all the agents who reported shifted peak preferences are now in worse condition as this is single-peaked domain model and they themselves prefer $f(P')$ less than a. So they will not manipulate f. Similarly for $x = k$, they will not manipulate f as that will again result in worse condition than before.

Case 2: $0 < x < k$ - To change the k^{th} median, at least one of the agents will have to shift his peak to opposite side of a. But as this will worsen his/her condition, the agent won't do it.As none of the agents will be ready to shift, they will not be able to manipulate f.

Hence proved.