CS711: Introduction to Game Theory and Mechanism Design
Assignment 2

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1 Problem 1

1.1 part a)

Given:

P1	P2	$P1^{'}$	$P2^{'}$
a	c	b	a
b	b	a	b
c	a	c	c

It is also given that f is onto. Now, let us f to be strategyproof and find f($P1^{'}$,$P2^{'}$). For this we will make two more preference profiles of player 1 and 2 as follows:

P11	P12	$P11^{'}$	$P12^{'}$
a	c	b	c
b	a	a	a
c	b	c	b

Here P12 = $P12^{'}$

Now we can see that D(a,(P1,P2)) $\subset$ D(a,(P11,P12)) so we can say that as f is strategyproof so it will be monotonic and so f(P1,P2) = a $\implies$ f(P11,P12) = a. Now, let us see f($P11^{'}$, $P12^{'}$) which would be either b or c (by using result ($\forall(P1, P2) f(P1, P2)\in$ P1(1),P2(1)). Let us take f($P11^{'}$, $P12^{'}$) = c. Player 1 prefers a over c in $P11^{'}$ so he can change his profile to P11 so that f(P11,P12) = f($P11$, $P12^{'}$) = a thus f does not remain strategyproof. Therefore, f($P11^{'}$, $P12^{'}$) = b. Now we can see that D(b,($P11^{'}$,$P2^{'}$)) $\subset$ D(a,($P1^{'}$,$P2^{'}$)) so we can say that as f is strategyproof so it will be monotonic and so f($P11^{'}$, $P12^{'}$) = b. $\implies$ f($P1^{'}$,$P2^{'}$) = b.

1.2 part b)

We cannot use method of part a as in single peaked preference we do not have P12 and $P12^{'}$ as valid.
We can use the left most peak among peak of agent SCF where f(P1,P2) = min(P_1(1),P_2(1))
from this we can see that f(P1,P2) = min(P1(1),P2(1)) = (a,c) so according to ordering $a < b < c$
we have f(P1,P2) = a
similarly we can see that f($P1^{'}$,$P2^{'}$) = min($P1^{'}$(1),$P2^{'}$(1)) = (b,a) so according to ordering $a < b < c$
we have f($P1^{'}$,$P2^{'}$) = a

2 Problem 2

According to Gibbard-Satterthwaite theorem, if $|A| > 3$ i.e the number of alternatives and if is onto and strategyproof then f is dictatorial. But we assume that all preference profile i.e. all possible order for each agent is possible. Otherwise it may not hold

Let us take a general agent i from all the agents. Now, agent i can have any preference ordering P_i over the projects. Let us look into ordering $P_i^{'}$ of subsets of projects of agent i.

Let us take two subsets of set of projects X, B and C. Now, we take $B = X$ and $C \subset X$ such that highest ranked project in C is not the highest ranked for the set X.

According to the question,B is preferred to C if the highest ranked project in B (according to P_i) is better than the highest ranked project in C - if these two projects are the same, then B and C are indifferent.

As $B = X$ and $C \subset X$ where highest ranked project in C is not the highest ranked for the set X so B$P_i^{'}$C or X$P_i^{'}$C. Thus the whole set is always more preferred to subsets where the highest ranked project is not present. Therefore, the $P_i^{'}$ or the preference over subsets of projects of agent i could not be the one where subset like C is strictly more preferred than X.

As shown above there is domain restriction on preference profile of agents on subset of projects therefore GS theorem will not be applied here.

then f is dictatorial.

3 Problem 3

We need to consider only the peak preferences of all the agents. So let us denote the preferences denoted only by their peaks, i.e., P = (P 1 (1), .., P i (1), .., P n (1)) and let f (P) = a A is the median of these peaks and the phantom peaks. Consider an agent i If P i (1) = a, then there is no reason for i to manipulate.Lecture 21: October 3, 2017 21-5 If P i (1) ¡ a, then if the agent shifts her preference to further left of a, the median will not change. If she manipulates to report her peak to further right of a, i.e. (P i , P i) 7 (P i , P i) s.t. a ¡ P i (1), this will imply that P i (1) ¡ a ¡ P i (1), and since P i is a single-peaked preference, by definition 21.4, a = f (P i , P i) P i f (P i , P i). Thus, i has no profitable manipulation. If a ¡ P i (1), again by exactly symmetrical arguments, i has no profitable manipulation. Hence, f is strategyproof.

We have show if median voter SCF is group strategy proof or not. For this, we consider the preference profiles of the agents but to see if median voter SCF is group strategy proof we only need to consider only the top preferred alternative of every agent. Thus, we define the preference profile in the following manner, P = (P_1, P_2,...,P_n) and $f(P) = a \in A$ is the median of these peaks and the phantom peaks. With the help of phantom peaks, we can make f(P) to be k^{th} smallest peak in all the peaks.

Let us consider a general group of agents $K \subseteq N$. To prove group strategy proof in this setup, we will take different case of group of agents and see if it is strategy proof with respect to them:

Case 1: If $\forall i \in K$ we have $P_i(1) = a$ then no group member would benefit from their true preference so it is strategy proof in this condition.

Case 2: If $\forall i \in K$ we have $P_i(1) < a$ then if any number of the group members shift their preferences to the left of a, then still the median would not change so that would not affect them. If some number of the group members shift their of preference to their right then let the new $f_i^{'}(1) > a$ for the new preference profile $P^{'}$. Now $P_i(1) < a < P^{'}(1)$, this will imply that $P_i(1) < a < P_i^{'}(1)$ and since P_i is a single-peaked preference, by definition, we get $f(P_i, P_{-i}) P_i f(P_i, P_{-i}^{'})$. Thus it will not be profitable for them. .

Case 3: If $\forall i \in K$ we have $P_i(1) > a$ then the argument for strategy proof are similar to the second case thus here also it is group strategy proof.

Case 4: If for some iK we have $P_i(1) > a$ and for some iK we have $P_i(1) < a$ and for rest $P_i(1) = a$. Now the let first subset where $P_i(1) > a$ be K^1 and for $P_i(1) < a$ be K^2 and third be K^3. If any two subset are non-empty in K then we will show that it will group strategy proof.

As shown in the first three cases, changing their preferences is not preferable for any subset group so now also it will not be preferable for all members of group. Thus, it is group strategy proof.