CS425: Computer Networks

Midsem – Semester 2, 2019-20
Computer Science and Engineering, IIT Kanpur
Total Points: 40, Time: 2 hours
ATTEMPT ALL QUESTIONS

1. A packet-switched network does *store and forward* operation for each of its packets. This operation means that every switch in this network waits until the packet is completely received, inspects if the received packet has no error, and then transmits it to the next link. After it has received a packet, it can simultaneously do the checking and transmission while receiving another packet.

   Suppose that x bits of user data are to be transmitted over a k-hop path in a packet-switched network as a series of packets, each containing p data bits and h header bits, with $x >> p+h$. The bit rate of the lines is b bps and the propagation delay is negligible. Also assume that the time for inspection is negligible and all packets were received without error. What value of p minimizes the total delay? Show every step of your derivation. **10 points.**

   SOLUTION. *There can be multiple ways to find the total delay. But all should be equal to*

   $$\left(\frac{x}{p} - 1\right)\frac{p+h}{b} + k \cdot \frac{p+h}{b}.$$

   One way to get this delay is to consider the delay of the first packet. Each packet undergoes a transmission delay of $\frac{p+h}{b} = T_t$. *The first packet takes* kT_t *to reach the destination. In this time, the later packets have already reached certain part of the network. E.g., the second packet has covered* $k-1$ *hops, and it takes time* $(k+1)T_t$ *to reach the destination. So, for the last packet to reach the destination, it will take* $\left(\frac{x}{p} - 1\right)T_t$ *additional time after the first packet. The expression shows this two parts in additive manner.*

   Minimizing the delay w.r.t. p, *we get* $p^* = \sqrt{\frac{xh}{k-1}}$.

2. Consider the matrix representation of Hamming code as discussed in class. Suppose the following matrix is used as the generator instead of the standard Hamming (7,4) code.

   $$G = \left(\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right).$$

   Data bits are collected as chunks of 4 bits and are transmitted as $x^\top G$. Assuming that there can be at most 1 bit error in 7 transmitted bits, what data bits were transmitted if the received bits are (0110101)? Explain each step to reach your answer. How will your answer change if the received bits were (0110010)? **5 + 5 points.**

   SOLUTION. *The parity check matrix is given by*

   $$H = \left(\begin{array}{cccc|ccc} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right).$$

Hence by post-multiplying H with the received vector, we get

$$H \cdot (0110101)^\top = (001)^\top, H \cdot (0110010)^\top = (110)^\top.$$

Hence, the error was in the 7th and 4th bit respectively. The original messages were (0110) *and* (0111) *respectively.*

3. In a certain transmission scheme the data is transmitted as a matrix of bits with n rows and k columns (including check bits). The coding scheme used is horizontal and vertical parity for error detection – which means that each row and column satisfies the parity check (assume even parity, though it does not matter). Suppose that exactly 4 bits are inverted due to transmission errors.

   (a) Prove that the error will be undetected if and only if the bit errors form a rectangle.

   (b) Using the result above, derive an expression for the probability that the error will be undetected.

   5 + 5 points.

   SOLUTION. *Part (a) should argue that if they are not over a rectangle, then they will definitely be detected. This is rather easy to see but should be formally argued, maybe using a proof by contradiction.*

   The other direction, i.e., if it is on a rectangle, then it must be undetected is trivial.

   Given that (a) holds, the all possible ways 4 bits can get corrupted is $\binom{nk}{4}$. If the bottom-left point of the rectangle is at (i, j) then the number of rectangles that are possible on the right and above that point are $(k - i)(n - j)$. Hence, the total number of rectangles are $\sum_{i=1}^{k-1} \sum_{j=1}^{n-1} (k - i)(n - j)$. So, by the counting argument, the probability of being undetected is

$$\frac{\sum_{i=1}^{k-1} \sum_{j=1}^{n-1} (k - i)(n - j)}{\binom{nk}{4}} \text{ which is same as } \frac{\binom{n}{2}\binom{k}{2}}{\binom{nk}{4}}.$$

4. Suppose a R bps point-to-point is set up between ISRO's Earth control station located in Bengaluru and the Mars orbiter in the Mars Orbiter Mission (MOM). Assume spherical shapes for Earth and Mars and circular motion of the orbiter around Mars. The orbiter is orbiting in the same plane that connects the centers of these two planets and Bengaluru. Assume communication is possible only if the orbiter is in the line-of-sight (LOS) from the control station.

   The distance between the centers of the planets is D meters. The radii of Earth and Mars are R_1 and R_2 meters respectively. The orbiter is orbiting at a height h meters above Mars' ground. The speed of electromagnetic wave in air and space is c m/s.

   (a) Calculate the minimum and maximum round-trip-times (RTT) between the control station and the orbiter. **2 + 2 points.**

   (b) A camera on the orbiter takes pictures of Mars and sends to the Earth control station. What is the minimum and maximum time taken to send this picture to Earth if the size of the picture taken is L bits? **3 + 3 points.**

Show details of your calculations in each step.

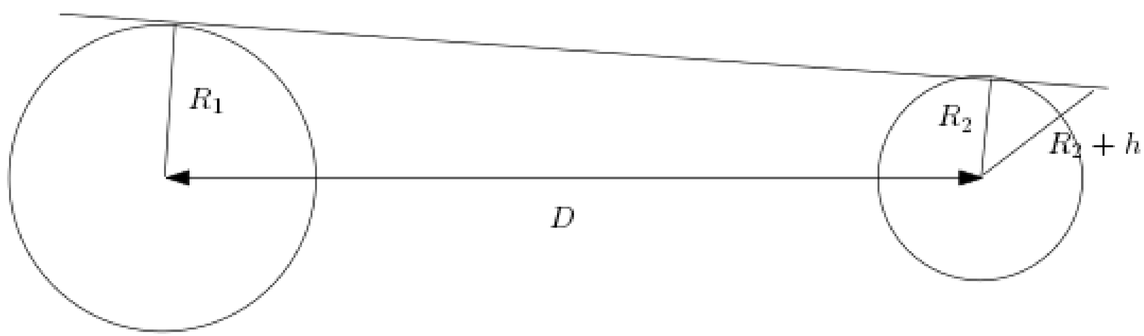
Solution. *The crucial part is to find the maximum and minimum distances. This happens when the orbiter is on the tangential line touching both the planets on the same side. The tangential line touching the planets on opposite sides is the other alternative, and it is shorter as explained later. Also, the following distances are same even if* $R_1 < R_2$ *(aside: in reality for Earth and Mars* $R_1 > R_2$*, but that doesn't impact the solution).*

$$d_{\min} = D - R_1 - R_2 - h$$
$$d_{\max} = \sqrt{D^2 - (R_1 - R_2)^2} + \sqrt{2R_2h + h^2}$$

Note: compute the other alternative (tangent touching the planets on different sides) for the maximum distance which turns out to be $\sqrt{D^2 - (R_1 + R_2)^2}$ *which is clearly smaller than* $\sqrt{D^2 - (R_1 - R_2)^2}$.

RTT: minimum = $2d_{\min}/c$*, maximum =* $2d_{\max}/c$.

Delay: minimum = $L/R + d_{\min}/c$*, maximum =* $L/R + d_{\max}/c$.



Good Luck!