**CS711: Introduction to Game Theory and Mechanism Design** **Jul-Nov 2018**

Project: Assignment 2

This lecture's notes illustrate some uses of various LaTeX macros. Take a look at this and imitate.

# 2.1

## 2.1.1 Part a

Consider the preference profile $(P'_1, P_2)$:

| $P'_1$ | $P_2$ |
|---|---|
| b | c |
| a | b |
| c | a |

We argue that $f(P'_1, P_2)$ is $b$. Reason: Instead, if it was $c$ (the only other possible alternative), then Player 1 can misreport his preference profile as $P_1$, and have $a$ become the winner, whom he prefers more than $c$.

Now, given that Player 1 is reporting $P'_1$, we know for sure that no matter how Player 2 misreports his profile, he cannot do better than getting $b$ elected as winner. In other words, no matter what preference profile Player 2 reports, $c$ (Player 2's top preference), will not be chosen as a winner.

Now, consider a new preference profile $(P'_1, P^*_2)$.

| $P'_1$ | $P^*_2$ |
|---|---|
| b | c |
| a | a |
| c | b |

It follows from the previous statement that $f(P'_1, P^*_2)$ is not equal to c. Neither is it equal to $a$, because of the result that for any preference profile $(\bar{P}_1, \bar{P}_2)$, $f(\bar{P}_1, \bar{P}_2) \in \bar{P}_1(1), \bar{P}_2(1)$. Therefore, $(P'_1, P^*_2) = b$.

Note that the position of $b$ weakly improves when we transition from $(P'_1, P^*_2)$ to $(P'_1, P'_2)$. We now use the fact that strategyproofness implies monotonocity, and assert that $(P'_1, P'_2) = b$.

## 2.1.2 Part b

- The earlier conclusion does not hold in this case.

- The earlier proof does not go through, because one of the preference profiles we used for Player 2, namely $P^*_2$, is not a single peaked preference profile.

- One mechanism through which we can have $f(P'_1, P'_2) = a$ is simply through the median voter SCF. Since we know that $f(P_1, P_2) = a$, if we construct the single peaked profiles we note that the median voter peak must appear to the left of $a$. This will enable $f(P'_1, P'_2)$ to be equal to $a$.

## 2.2

The Gibbard-Satterthwaite result only applies to unrestricted preference profiles. However, in this scenario, we have a restriction imposed on the preference profiles. The elements in the preference profiles are drawn from the power set (set of all subsets) of set X. Therefore every element is a subset of set X. The restriction imposed here is that suppose there are two subsets of $X$, say $S$ and $T$, and if $S \subset T$, then there arise two possibilities: either Agent i is indifferent to $S$ and $T$, or he prefers $T$ over $S$. Therefore, we cannot construct any preference profile where $S$ is preferred over $T$.

Because we have shown that a restriction exists over the possible preference profiles, we cannot apply the Gibbard-Satterthwaite result here.

## 2.3

This situation can be divided into the following cases:

### 2.3.1 Case 1

All the peaks of agents belonging in set $K$ lie to one side of the median.

WLOG, assume all the peaks of agents in $K$ lie to the right of the median. Now, if the agents in $K$ misreport by reporting their peaks to the further right, then this won't change the result at all. If they misreport by reporting their peaks to the left of the median, then this will result in the median being shifted to the left, which is less desirable to the agents in $K$, compared to the original result. Similarly, the same result can be shown if all the peaks of agents in $K$ lie to the left of the median.

### 2.3.2 Case 2

The median lie somewhere in between the peaks of the agents in $K$.

In this case, if an agent (or a subset of agents) in $K$, having peaks to one side, decide to misreport by reporting their peaks to be on the other side of the median, then the median will shift to the other side, which is obviously a less desirable result to those peaks. This shift will be beneficial to agents having peaks on the other side, but the condition specifies that the final result must be more desirable for *every* agent in $K$, which is not satisfied.

In consideration of these two cases, we can claim that the median voter SCF is indeed group strategy proof.