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Q1.
a)

P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_1’	P_2’
a	c	a	b	a	c	b	c	b	a	b	a
b	b	c	c	b	a	a	a	a	c	a	b
c	a	b	a	c	b	c	b	c	b	c	c

$f(P_1,P_2)$= a (given)
$f(P_3,P_4)$=a (monotonicity) The position of a is getting weakly better from (P_1,P_2) to (P_3 P_4).
$f(P_5,P_6)$=a (monotonicity) The position of a is getting weakly better from (P_3 P_4) to (P_5 P_6)
$f(P_7,P_8)$ can only be b or c as $f(P_7\ P_8)$ can be either be $P_7(1)$ or $P_8(1)$.
But if $f(P_7\ P_8)$ =c then P_7 can manipulate to P5 and get A elected.
Hence, $f(P_7,P_8)$ =b.
$f(P_9,P_{10})$ =b (monotonicity)
The position of b is getting weakly better from (P_7 P_8) to (P_9 P_{10}).
f(P_1’,P_2’)=b (monotonicity)
The position of b is getting weakly better from (P_9 P_{10}) to (P_1’P_2’) .

b)
No. the earlier conclusion does not hold in this case as the profiles P_3,P_6 will not be allowed in single peaked preferences. There is restriction on the preferences.
<u>Mechanism such that f(P_1’P_2’)=a:</u>

$P_x(1)$	$P_y(1)$	$f(P_x, P_y)$
a	a	a
a	b	a
a	c	a
c	a	a
c	c	c
c	b	b
b	a	a
b	c	b
b	b	b

Since preferences are single peaked ,only 4 preference profiles P_1’ ,P_2’=P_1 , P_2 and P_4 are possible.

Showing that the above mechanism is strategyproof:

Case 1: $f(P_x, P_y)=c$ it is possible only when both $P_x(1)=P_y(1)=c$. Hence , none of them have any motivation to manipulate.

Case 2: $f(P_x, P_y)=a$ it is possible if $P_x(1)=a$ or $P_y(1)=a$ or $P_x(1)=P_y(1)=a$
If $P_x(1)=P_y(1)=a$, then none of them have any motivation to manipulate.
If $P_x(1)=a$ and $P_y(1)<>a$, then no matter what $P_y(1)$ is $f(P_x, P_y)=a$. Hence, none of them have any motivation to manipulate.
Similarly , we can argue for $P_x(1)<>a$ and $P_y(1)=a$.

Case 3: $f(P_x, P_y)=b$, it is possible only in the following cases:
1.) $P_x(1)=P_y(1)=b$ In this case, none of them have any motivation to manipulate.
2.) $P_x(1)=b$ and $P_y(1)=c$ .In this case, $P_y(2)=b$ (because of the restriction caused by single peaked preference). Therefore, none of them have any motivation to manipulate.
3.) $P_x(1)=c$ and $P_y(1)=b$
Similar to the above argument.

Q2. Let X be the set of projects.
Let S be the set of all the non-empty subsets of X .
Now, for any agent i with preference P_i over the projects , the preference over the elements of S would always have X (i.e. the subset having all the projects) as $P_i(1)$.
Hence, the preference profiles have a restriction and hence Gibbard-Satterthwaite result will not apply here.

Q3. We need to consider only the peak preferences of all the agents. So let us denote
$P = (P_1(1), .., P_i(1), .., P_n(1))$ and let $f(P) = a \in A$ is the median of these peaks where A is set of agents.
Consider a group of agents K , such that $K \subseteq A$.
Now if agents try to manipulate , then there are three cases possible:
Case 1: If $P_i(1) = a$ for all $i \in K$, then there is no reason for the agents to manipulate.
Case 2: If $P_i(1) < a$ for all $i \in K$,
Then if the agents shift their preference to any position left of a, the median will not change. If they shift their preference towards right of a, i.e. $(P_i, P_{-i}) \rightarrow (P_i', P_{-i})$ s.t. $a < P_i'(1)$ for all $i \in K$, since P_i is a single-peaked preference the median will move further towards right and they would be worse off.
Hence , they have no profitable manipulation.
Case 3: If $P_i(1) < a$ for all $i \in K$, then by arguments similar to case 2 , there is no profitable manipulation.
Note that only these three are the only cases possible as an agent with $P_i(1) < a$ will never collude with $P_i(1) < a$ as this collusion can not be beneficial for both at the same time.

Hence, f is group strategyproof.