CS711: Introduction to Game Theory and Mechanism Design **Dhananjay Singh**

Project: Assignment 2

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Question 1

Part (a)

P_1	P_2	$P_1^{'}$	$P_2^{'}$	Q_1	Q_2
a	c	b	a	a	c
b	b	a	b	c	a
c	a	c	c	b	b

$P = (P_1, P_2)$
Let us now consider another preference profile $Q^{'} = (P_1, Q_2)$. Then,

$$f(Q^{'}) = a \text{ , through monotonicity on P}$$

$P^{'} = (P_1^{'}, P_2^{'})$. We know that,

$$f(P_1^{'}, P_2^{'}) \in \{b, a\}. \tag{1.1}$$

Let us assume that $f(P_1^{'}, P_2^{'}) = a$

Let us now consider another preference profile $P^{"} = (P_1^{'}, Q_2)$. Then,

$$f(P^{"}) \in \{b, c\} \tag{1.2}$$

Since $f(P^{'}) = a$, we must have $f(P^{"}) = c$ otherwise agent 2 will have an incentive to manipulate to $P_2^{'}$ at $P^{"}$.

Also, we must have $f(P^{"}) = b$ otherwise agent 1 will have an incentive to manipulate to P_1 at $P^{"}$,as $f(Q^{'}) = a$.

We can see that we have attained a contradiction in the above two arguments and hence our assumption that $f(P^{'}) = a$ must be incorrect. Therefore we have from equation (1.1) that $f(P^{'}) = b$

Part (b)

If the preferences are generated from a single-peaked preference domain with the intrinsic ordering of the alternatives being a < b < c, then the above conclusion does not hold.

In going to the preference profile we used ordering Q_2 in part (a) which is not single peaked.Therefore the above solution does not go through.

To have $f(P_1^{'}, P_2^{'}) = a$, we can use the following mechanism:

$$f(P) = \min_{i \in N} P_i(1)$$

where minimum is taken w.r.t. the order relation $<$. Hence the Social choice function picks the left-most peak among the peaks of the agents.
Therefore $f(P_1^{'}, P_2^{'}) = a$.

Question 2

Given
(1)Preference ordering P_i is linear over the set of projects X ,i.e., in-differences between alternatives are not allowed.
(2)S,T $\subseteq$ X
(3)$| X |> 2$

Since the number of projects is greater than or equal to 2, we must have at least 3 sets of alternatives.

Without loss of generality, let us take some S and T such that,

$$S \subset T, i.e., S \subseteq T and S \neq T$$

In this particular case, the highest ranked project in S (according to P_i) can either be equal to the highest ranked project in T or be worse off. It can never be better than the highest ranked project in T. Therefore, we will never have the case when an agent prefers S over T, which in turn implies that there will be no preference ordering where S is preferred over T. Quite clearly, the domain of preferences gets restricted.

We need to remember that Gibbard-Satterthwaite apply only when we have unrestricted preferences domain. It is not applicable if the domain is restricted. Therefore, in the given situation, Gibbard-satterthwaite result is not applicable.

Question 3

Group Strategy-Proof- A social choice function f is manipulable by a group of agents K $\subseteq$ N if for some preference profile $(P_K, P_K\ )$ there exists some preference profile P_K of agents in K such that $f(P_K, P_K) P_i f(P_K, P_K)$ for all i $\in K$.

Let us denote the preferences only by their peaks.

$P = (P_1(1), .., P_i(1), .., P_n(1))$ and let f(P) $= a \in A$ is the median of these peaks and the phantom peaks

Let the player that holds median preference be person λ,i.e.,$P_\lambda(1) = a$. Now all players k,such that $p_k(1)$ will have no incentive to deviate and hence, will never be a part of the group that is trying to manipulate their strategy.

Let us first consider the possibilities of types of groups formed.
Type A All people in the group are such that $P_i(1) <$ a in the common order.
Type B All people in the group are such that $P_i(1) >$ a in the common order.
Type C Some people($\neq 0$)are such that $P_i(1) >$ a and some people($\neq 0$) are such that $P_i(1) <$ a.

Note that these are the only three possible types of groups possible.Let us now consider each case turn by turn.

Type A If all players report their preference to be some b_i <a where b_i denotes the manipulated choice of player i, f(P) = a and manipulation is not able to change the outcome.
If all players report their preference to be some b_i >a, then they will be able to change the median to some c, such that a<c. But since the preferences are single peaked it is even less preferred for this group of manipulators than outcome a.
Thus their is no profitable manipulation for Group Type A.
Similar argument holds for manipulators of Group Type B.

Type C Let this group be able to manipulate the outcome to some b, such that b<a. Then the members k of the group who had $p_k(1)$ >a will now be worse off and they will refrain from such manipulation.

Let us now consider that this group be able to manipulate the outcome to some b, such that b>a. Then the members k of the group who had $p_k(1)$ <a will now be worse off and they will refrain from such manipulation.

Hence we can see that no such group of type C will ever be formed.

Hence we have shown that no group type has a profitable manipulation possible. Therefore median voter SCF is Group Strategy Proof.