CS711: Introduction to Game Theory and Mechanism Design	**Jul-Nov 2018**

Project: Assignment 2

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1.1

P_1	P_2	P_1'	$\hat{P}_2$	P_1'	P_2'
a	c	b	c	b	a
b	b	a	a	a	b
c	a	c	b	c	c

(a) It is given that the winner for the preference profiles P_1, P_2 is a. We have to show that the winner for the preference profiles P_1', P_2' will be b if given that f is strategyproof.
Also, it is given that $f(P_1', \hat{P}_2) \in \{b, c\}$.
Let's assume that c is the winner of the $\{P_1', \hat{P}_2\}$. Now we see that the dominated set for c remains the same in both the profiles $(P_1, P_2)\&(P_1', \hat{P}_2)$, and it is given that f is monotonous, hence the winner of the (P_1, P_2) should be c. This is a contradiction!. Hence, $f(P_1', \hat{P}_2) = b$.
Now again if we compare $(P_1', \hat{P}_2)\&(P_1', P_2')$, we observe that b is dominated by a in both the preference profiles in a similar way and since f is monotonous, we get that $f(P_1', P_2') = b$.

(b) (i) No, the earlier conclusion does not hold in this case.

(ii) This is because the preference profile chosen by us in (a), i.e. (c a b) will be rendered invalid by following the intrinsic ordering of the alternatives being $a < b < c$. Because of this we won't be able to prove the fact that $f(P_1', P_2') = b$. To prove (a), the intrinsic ordering will have to be $b < a < c$ or $b < c < a$.

(iii) It is given that $f(P_1', P_2') = a$. Now, this is possible only if we pick up the left-most single peaked preference profile. On doing so, we'll have to use the intrinsic ordering of the alternatives to be $a < b < c$. Hence, the mechanism will be to pick the left-most single peaked preference from the given set of alternatives. So we will get $f(P_1', P_2') = a$.

1.2

Let $X = \{a, b, c\}$.
Let the ordering be defined by cP_ibP_ia.
Let $S = \{a, b, c\}$.
Let $T = \{a, b\}$.
Case 1:
Let's assume the highest ranked project in S is c and that in T is b. Clearly, in this case S is preferred over T.
Case 2:
Let's assume the highest ranked project in S is b and that in T is also b. Clearly, in this case S and T will

be indifferent.

So we observe that the orderings observed in case 1 and case 2 cannot be simultaneously admitted in the domain and hence the Gibbard-Satterthwaite result will **NOT** apply here.

1.3

Yes, the median voter SCF group is strategy-proof. We will prove this evaluating the following cases:
(Note: we will be only considering the preferences denoted by the peaks for brevity.)

- Case1: If all the k agents of the group lie on the left side of the median.
  In this case, there will no shift in the peak of the median vector as all the members lying on the left side will just have the impact same as before trying to manipulate the outcome.
- Case2: If all the k agents of the group lie on the right side of the median.
  Similar to the above case, again the agents won't be able to manipulate the outcome.
- Case3: After the failure of the above two options, the agents are only left with the option of shifting their peaks to the other side of the median. That is, the agents on the left will shift their peaks in the right and vice-versa. However, following this approach will just worsen the condition the agents on either side as this will simply result in shifting the peak in the side opposite to the preferred place. Hence, the agents have no incentive to go with this approach.
- Case4: If all k agents stick with peak same as the median, then there is clearly no incentive for any agent to manipulate.

So, after evaluating the above cases, we observe that the k agents group have no lucrative possibility of manipulating f. **Hence, f is group strategy-proof**