CS711: Introduction to Game Theory and Mechanism Design **Jul-Nov 2018**

Project: Assignment 2

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Answer 1

Part A
Construct two Preference Profile $P_{new1} = (P_1^{'}\ , P_2\ )$ and $P_{new2} = (P_1^{'}, \hat{P}_2)$

P_{new1}		P_{new2}	
$P_1^{'}$	P_2	$P_1^{'}$	$\hat{P}_2$
b	c	b	c
a	b	a	a
c	a	c	b

f is strategyproof and onto $\Rightarrow$ f is Pareto Efficient. Because in P_{new1} a is always dominated by b and f is Pareto Efficient, $f(P_1^{'}, P_2) \neq$ a.
Assume that $f(P_{new1})$ is c. Now player 1 in P_{new1} can manipulate its preference and report it as P_1 to get $f(P_1, P_2) =$ a as outcome, which it prefers more than c. But this is contradiction to the fact that f is strategy-proof.$\therefore$ the assumption that $f(P_{new1})$ is c is false.
Since $f(P_{new1}) \neq$ a and $f(P_{new1}) \neq$ c, and there are only three alternatives, $f(P_{new1}) =$ b.

We know that $f(P_{new2}) = f(P_1^{'}, \hat{P}_2) \in \{P_1^{'}(1), \hat{P_2(1)}\} = \{b, c\}$. Assume $f(P_{new2}) =$ c. $D(c, P_{new2}) \subseteq D(c, P_{new1})$. Since f is monotonus, $f(P_{new1}) =$ c but this is contradiction to the fact that $f(P_{new1}) =$ b. So our assumption that $f(P_{new2})$ is c is false. Since $f(P_{new2}) \in \{b, c\}$ and it is not c, it is b.

$f(P') \neq$ c because f is Pareto Efficient and c is dominated by a in all preferences. Assume $f(P') =$ a. In this case, player 2 in P_{new2} can manipulate its preferences to P_2' and get $f(P_1', P_2') =$ a as outcome which it prefers more than b. But this is contradiction to the fact that f is strategy-proof. $\therefore$ the assumption that $f(P') =$ a is wrong. Since $f(P') \neq$ c and $f(P') \neq$ a, $f(P') =$ b.

Part B
The earlier conclusion does not hold in this case because the construction of $P_{new2}(2) = \{c > a > b\}$ is not possible in the given single peaked preference domain with intrinsic ordering of alternatives as $\{a < b < c\}$. We can use a median voter social choice function g having phantom peak to the left of alternative a. Median voter function is strategy-proof and onto. So it can be taken as a possible choice for f. $g(P) =$ a ($\because$ a is the median). It satisfies the given output for preference profile P. $g(P') =$ a ($\because$ a is the median).

Answer 2

Consider two different subsets of X, Y and Z such that Y $\subseteq$ Z. There can be two cases.
First case - Highest rank project according to Agent i is present in both Y and Z. In this case Agent i is indefferent between Y and Z
Second Case - Highest rank project according to agent i is present in Z but not Y. In this case Z P_i Y
But there can never be a scenario in which Y P_i Z because if highest rank project according to i is present in Y and we know that Y $\subseteq$ Z, so it will also be present in Z. $\therefore$ i will be indifferent between them. This restricts the domain of preferences.
Gibbard-Satterthwaite Theorem assumes unrestriced preferences for its proof. $\therefore$ Gibbard-Satterthwaite Theorem is not applicable here.

Answer 3

There can be four types of groups in this case.

1) Group consists of all agents having peak at median. The output of f (median) is their most preferred alternative. Therefore they won't misreport.

2) Group consists of agents having peaks to left of median or at median. Agents having peak at median won't misreport. If rest of the agents misreport their preferences to the left of median, median will not change. If they misreport their preferences to the right of the median, median will shift right and they will prefer the new outcome (new median) less (by definition of single peaked preference). Therefore they are better off by reporting their true preferences.

3) Group consists of agents to the right of the median or at median. Agents having peak at median won't misreport. If rest of the agents misreport their peaks to the right of median, median will not change. If they misreport their peaks to the left of median, median will shift left and they will prefer the new outcome (new median) less (by definition of single peaked preference). Therefore they are better off reporting their true preferences.

4) Group consists of agents having peaks to left,right or at median point.Agents having peak at median won't misreport. Rest of the agents can manipulate their preferences to shift median to either left or right. If median shifts to left, agents in group who were earlier to the right of median, will prefer current outcome (new median) less than earlier outcome (by definition of single peaked preference). Therefore agents to the right of median won't report wrong preferences in this case. It now reduces to case 2.
If median shifts to right, agents in group who were earlier to the left of the median, will prefer current outcome (new median) less than earlier outcome (by definition of single peaked preference). Therefore agents to the left of median won't report wrong preferences in this case. It now reeduces to case 3.

Since no agent in any kind of group can be better off by misreporting their preferences, median voter SCF is group strategy-proof.