CS711: Introduction to Game Theory and Mechanism Design **6th November 2018**

Assignment 2

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Question 1

Part (a)

Let Q be a new strategy profile such that

Q_1	Q_2
b	c
a	a
c	b

Now, we show that $f(Q) = b$ by ruling out a and c

- *If* $f(Q) = a$: Not possible as is obvious from the description of Q above (only b or c can happen).
- *If* $f(Q) = c$: If player 1 unilaterally changes to Q_1' with order a, b, c with $Q_2' = Q_2$, the new strategy profile will have $f(Q') = a$, following monotonocity of f (following from the fact that f is SP) and noting the fact that $f(P) = a$ and $D(a, P_i) \subset D(a, Q_i')$ $\forall i$. Hence, the SP nature of f gets violated which is a contradiction.
- Thus, we are left with the only possibility that $f(Q) = b$.

Thus, $f(Q) = b$.
Now, arguing similarly as before, it is easy to observe that $D(b, P_i') \subset D(b, Q_i)$ $\forall i$ and we have already established that $f(Q) = b$ and f is monotone, thus making $f(P') = b$

Part (b)

No, the earlier proof will not go through because in Q_2, c, a, b can't be the preference of any player given the intrinsic ordering of $c > b > a$ (it will be multi-peaked, then).
Now, to make $f(P_1'.P_2') = a$, a suggested mechanism is to select the left-most peak amongst the peaks of both players. This mechanism can be easily verified to fulfill all the required conditons.

Question 2

We know that for the GS theorem to apply, the unrestricted nature of the domain is necessary. In this case, consider the domain of extension of P_i over the subsets to be Q_i. Now, let's say for a particular player i,

there are three subsets $q_1, q_2, q_3 \in Q_i$ with $q_1 = p_1, q_2 = p_2, q_3 = p_1, p_2$ where $p_1, p_2 \in P_i$ with the strict ordering $p_1 > p_2$ (assumed WLOG). Now, it is pretty obvious that an order of $q_1 > q_2 > q_3$ can't be realised in this case. In the other case with $p_2 > p_1$, we can similarly show that $q_1 > q_2 > q_3$ can't be realised here too. Thus, this choice of a pair of projects shows that the domain Q_i is restricted $\forall i$ and hence, GS theorem doesn't hold.

Question 3

Let the manipulators lie on both side of the peak. Obviously,this is not possible because the manipulators on the right will try to shift the peak towards them which will be a loss for those on the left and vice-versa. Hence, the collusion from both sides makes no sense.
Now, if all the manipulators are on the right, they can't shift the peak even an inch towards them (basic statistics and definition of median, which will require data on the other side to change as well). Similarly, if all of them are on the left, they can't shift the peak towards them either.
Thus, under no circumstances do we get $f(P'_k, P_{-k}) P_i f(P_k, P_{-k})$ and hence, the median voter SCF is group strategy proof.