Lec 3

Revisit example :

| 1 \ 2 | A | D |
|---|---|---|
| A | 5,5 | 0,6 |
| D | 6,0 | 1,1 |

Normal / Strategic form game

One shot

alternatives → outcomes

$a_i \to o_i$'s

$S_1 = S_2 = \{A, D\}$

$N = \{1, 2, \ldots, n\}$

$S_i$ = strategy set of player $i$

difference between action and strategy

$s_i \in S_i$    $s_i$ is a strategy of player $i$

$s_{-i}$ : strategy profile of all agents except ~~agen~~ player $i$

$(s_1, s_2, \ldots, s_n)$ : strategy profile

$= (s_i, s_{-i}) = s \quad \in S_1 \times S_2 \times \cdots \times S_n = S$

$u_i(s_1, s_2, \ldots, s_i, \ldots, s_n)$ : utility of ~~agent~~ player $i$

when player $j$ picks strategy $s_j$, $j = 1, \ldots, n$

$u_i : S \to \mathbb{R}$ (von-Neumann-Morgenstern utilities)

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Agent models

Concept #2 : Behavior of players

(1) Rationality : every agent picks ~~ae~~ strategies to maximize here utility.

(2) Intelligence : possess enough information about the game and able to find the best strategy for her.

(3-2)

# Concept #3: Common Knowledge

A fact is a common knowledge if

① All players~~s~~ know~~s~~ the fact

② All players~~s~~ know~~s~~ that all other players~~s~~ know~~s~~ ~~that player i knows the fact~~ the fact.

③ . . . .

Example: Island and 3 blue eyed people with a sage.

Assumption: Rationality is a common knowledge

The fact that all players are rational

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Dominant strategies, weakly

a strategy $s_i'$ is dominated by $s_i$ if $\forall s_{-i} \in S_{-i}$

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$$

and $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$ for some $s_{-i}$

a strategy $s_i'$ is strictly dominated by $s_i$ if $\forall s_{-i} \in S_{-i}$

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$$

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a strategy $s_i$ is (strictly/weakly) dominant strategy of Player i if $s_i$ strictly/weakly dominates all other strategies $s_i' \in S_i \setminus \{s_i\}$

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(Strict/weak) Dominant strategy equilibrium is a strategy profile $(s_i^*, s_{-i}^*)$ s.t. $\forall i \in N$ $s_i^*$ is a strictly/weakly dominant strategy. SDSE, WDSE

Explain with the previous example

Is SDSE/WDSE guaranteed to exist? – Example of Battle of Sexes.

Come to the world of weaker equilibrium guarantee.

Pure Strategy Nash equilibrium is a strategy profile $(s_i^*, s_{-i}^*)$ s.t.

$$\forall i \in N \quad u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*) \quad \forall s_i \in S_i$$