CS711: Introduction to Game Theory and Mechanism Design **Jul-Nov 2018**

Project: Assignment 2

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Ans 1.

(a.)

Given,
A = {a,b,c} : Set of alternatives.
SCF (strategy proof and onto) f : f(P1,P2) = a

P1	P2	P1'	P2'
a	c	b	a
b	b	a	b
c	a	c	c

Table 1.1: Two preference profiles

Since, it's onto we can make any possible permutation of ordering over alternatives. We begin by constructing following two profiles.

$\hat{P}1$	$\hat{P}2$
b	c
a	a
c	b

Table 1.2: New preference profile

We know that outcome of f($\hat{P}1$,$\hat{P}2$) can be either b or c. let's say its 'c'.

Now consider a preference profile $\tilde{P} = \{\hat{P}1, P2\}$. Now outcome must be f($\tilde{P}$) = c, otherwise P2 will manipulate by switching to $\hat{P}2$. so $f(\hat{P}1, P2) = c$. But now with this result player 1 can manipulate by switching from $\hat{P}1$ to $P1$ which will give output as a which is preferred more by player 1.

Conclusion : So by contradiction $f(\hat{P}1, \hat{P}2) = b$.

Now outcome of f(P1',P2') must be b otherwise $\hat{P}2$ can manipulate to P2'

(b.)

Given single peaked domain,
Intrinsic order : $a < b < c$.

The earlier conclusion does not hold here because, the preference profiles that we created like $\hat{P}2$ is not possible due to property of single peaked functions(i.e restricted domain). so the contradiction we obtained earlier is now invalid.

Possible Mechanism:
One possible mechanism will a special case of median voter SCF where we select the minimum of all players peaks (.ie the left most). Let's denote this as f_m
So,
$f_m(P1, P2) = a$ and $f_m(P1', P2') = a$, since a is the minimum of all peaks.

Onto Check : the SCF f_m si onto since all players can choose any peak which satisfies unanimity property and so for its super set onto. so f_m si onto.

Strategy Proof Check : For any player to manipulate the player has to choose a peak lower than the minimum which is strictly less preferred by her as the current lowest (since this is a single peaked domain). So it's not manipulable and hence strategy proof.

This concludes the answer for this question.

Ans 2.

Given,
N : set of players
X : Set of Projects, $|X| \geq 2$.
A : Set of Alternatives (set of subsets of X), $|A| \geq 3$

Since it's given that two subsets of X can be indifferent between alternatives lets take a case such that:

$$S \subset P \subseteq X$$

where S and P are taken from set of alternatives. since S is a subset of P.
Now two things are possible for every player (since S is subset of P):

$$S \ I_i \ P$$

or,

$$P \ P_i \ S$$

where, I_i means indifference and P_i means strict preference.
This means that a preference profile where S is preferred over P (i.e: $S \ P_i \ P$) is never possible for any player which means that the domain does not allow SCF to create any constructive preference over all set of outcomes.

So it has a restricted domain and proof of Gibbard-Satterthwaite result (where we considered specific construction) cannot go through this type of domain and cannot be applied here.

Also note that this setting allows in-differences so we can create a non-dictatorial cases where the tie breaking criteria is fixed.

Ans 3.

Given,
A median voter SCF let's say we have phantom points defined to select the k'th value from left. We need to consider only the peak preferences of all the agents as P = (p1(1),p2(1),p3(1)....pi(1)...pn(1)) and f(P) = a.

Now we can define groups,

Case 1: Outcome is outside the range of Groups
Let G_l be some group on left of a, and G_r be some group on right of a. These groups can be empty as well.

Now G_l cannot manipulate the output unless some of the group member report peak further right of a. So median will either remain same or shift away from the group. i.e:

$$f(P_{G_l})P_if(P'_{G_l}), \forall\ i\epsilon G_l$$

So no profitable manipulation exist for any member of group.
Similarly and symmetrically we can argue for G_r that no profitable deviation exist for any member of group. i.e:

$$f(P_{G_r})P_if(P'_{G_r}), \forall\ i\epsilon G_r$$

So there does not exist any profitable deviation for any group members in this situation.

Case 2: Outcome is inside the range of Groups.

Observation : If members right of a are profited then the members on left will be hurt since this is a single peaked domain and so we conclude that members on right and left cannot be in same manipulable group.

Now again we can divide the range in two parts left and right group which essentially becomes case 1. So there is no profitable manipulation.

There is no profitable manipulation for any group.
Hence, proved Median Voter SCF is **group-strategy proof**.