CS711: Introduction to Game Theory and Mechanism Design 17111001 Aadil Hayat

Project: Assignment 2

2.1 Question 1

2.1.1 Unrestricted strict preferences

We can prove the required thing by inserting an intermediate step P'' between profile P and P'. P'' profile is just P with a and b alternatives interchanged.

Since f is an *onto* SCF $f(P_1'', P_2'') = b$ because P and P'' are essentially the same with just a and b replaced.

Using the fact that strategyproofness implies monotonicity we can say that if f is **strategyproof** then $f(P_1', P_2') = b$ because $D(b, P_i'') \subseteq D(b, P_i') \quad \forall i \in \{1, 2\}$ and $f(P_1'', P_2'') = b$.

2.1.2 Single-peaked preference domain

The earlier conclusion does not hold true in the case of intrinsic ordering among the alternatives because in this we can come up with a mechanism that is strategyproof and not give the same result as before. If the SCF f is defined as for preference profile $(\overline{P_1}, \overline{P_2})$, $f(\overline{P_1}, \overline{P_2}) = min(\overline{P_1}(1), \overline{P_2}(1))$.

Using this SCF we can get

$$f(P_1, P_2) = min(P_1(1), P_2(1)) = min(a, c) = a$$

$$f(P_1', P_2') = min(P_1'(1), P_2'(1)) = min(b, a) = a$$

This SCF f is strategyproof because only way an agent can change the outcome of the SCF is by choosing an alternative that is smaller than the other agent which would be less preferable because of the single-peaked preference domain.

2.2 Question 2

The Gibbard-Satterthwaite result does not apply here because of the following reasons:

- All the possible linear orderings are not available to be chosen by the agents because X will always among the top $|X|$ preferences for any agent.
- Depending on how we break the ties among the indifferent preferences of the agent it is possible that SCF is not onto because for example if the tie-breaker is based on the number of the elements in the set there would be intrinsic ordering among the set (which would make the problem partially domain-restricted also) then for some cases some subsets will never be the top most preference of any agent.

2.3 Question 3

For the single-peaked domain model with median voter SCF without the loss of generality we can conclude that for manipulating f group would only be formed by agents belonging to same side of the median because otherwise subgroups of the group would have conflicting objectives and any change in outcome would not strictly preferred by all the agents of the group.

Using the above conclusion we can say that for any group of agents $K \subseteq N$ the only way to change the outcome of the social choice function f is to misreport the peak of at least one of agent with value that belongs to the other side of the median (because otherwise misreported value would belong on the same side and the outcome of the SCF f will not change). But we can see that this change in outcome is strictly worse for all the agents in the group K as compared to real preference profile. Hence we can say that for some preference profile (P_K, P_{-K}) there does not exist a preference profile P'_K such that $f(P'_K, P_{-K})P_i f(P_K, P_{-K})$ for all $i \in K$. Therefore median voter SCF is group strategy-proof.