CS698W: Game Theory and Collective Choice **Jul-Nov 2017**

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Disclaimer: *These notes aggregate content from several texts and have not been subjected to the usual scrutiny deserved by formal publications. If you find errors, please bring to the notice of the Instructor at swaprava@cse.iitk.ac.in.*

31.1 Recap

In the previous lecture, we discussed the advantages of VCG mechanism such as it is DSIC, never runs into deficit, never charges a losing agent and is individually rational for agents to partipate. It is the most used mechanism with money, but it is good to know the limitations of VCG.

31.2 Criticisms of VCG

31.2.1 Privacy

VCG mechanism forces the agents to report their valuations truthfully. If the interaction had to happen beyond one round, it may be preferable for the bidders to use a mechanism that uses the minimal information needed for the current round. Because, if all the agents know the valuations of other agents perfectly they can adjust their actions in the following rounds to achieve a higher utility. The privacy concern is also critical for the trust of a new auctioneer, since the auctioneer has the opportunity of introducing fake bidders to extract more money from a rich bidder.

31.2.2 Susceptibility to collusion

Consider an example with three agents and two possible allocations A and B as shown in Figure 31.1, by the VCG mechanism the player 1 and 2 should pay 150 and 50 respectively.

	A	B	Payment
1	200	0	150
2	100	0	50
3	0	250	0

Figure 31.1: p^{VCG} before manipulation

But, the players 1 and 2 together can change the amount of payment they have to make by manipulating their valuations, as shown in the Figure 31.2.

	A	B	Payment
1	250	0	100
2	150	0	0
3	0	250	0

Figure 31.2: p^{VCG} after manipulation

Therefore, VCG is not group strategy proof.

31.2.3 Not frugal

VCG mechanism does not charge an amount close to the seller's valuation/cost. It guarateed to bring revenue (since it is a no-deficit mechanism), but the payment could be very large. Consider an example of shortest path, routing a packet or delivery of an item as shown in the Figure 31.3.

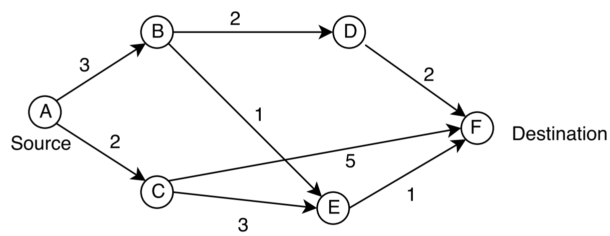

Figure 31.3: A network to deliver an item from A to F

Consider each edge as an agent. The efficient allocation for the shortest path is $A \to B \to E \to F$ with cost 4. Notice the effect of payment to $A \to B$ due to the cost of $A \to C$. If the cost of $A \to C$ is 2 then the payment of $A \to B$ is:

$$P^{AB} = \sum_{j \neq i} v_j(x^*_{-j}) - \sum_{i \in N} v_i(x^*) = 0 - (-1) + (-2) - 0 + (-3) - 0 = -4$$

Now, if cost of $A \to C$ becomes $x > 2$, then $p^{AB} = -(x+2)$ and $A \to B$ will get more money than before even though the cost for that edge remains the same. Clearly the payment is unbounded if x is unbounded.

31.2.4 Revenue Monotonicity Violated

A system maintains the revenue monotonicity, if the revenue increases with the increase in number of agents. But, the VCG mechanism violates the revenue monotonicity. Let us take an example given in the Figure 31.4.

	X	Y	p^{VCG}
1	0	90	0
2	100	0	90

Figure 31.4: The p^{VCG} before adding a third agent

But, if a new agent participates then the p^{VCG} for all the agent changes and the result is shown in Figure 31.5.

	X	Y	p^{VCG}
1	0	90	0
2	100	0	0
3	100	0	0

Figure 31.5: The p^{VCG} after adding the third agent

Hence both split or merging of agents in VCG has problems.

31.2.5 Not fully Budget Balanced

VCG mechanism does not always satisfies the balanced budget, almost always some surplus will be left. This surplus can not be redistributed as that will change the payoff of the agents and it can not be invested to things that can effect the payoffs. Therefore, it has to be destroyed and known as *money burning*.

31.3 Generalization of VCG

In view of the limitations of VCG mechanism, the class of mechanisms is needed to be expanded to take care of criticisms. This in particular considers the budget balance issue. VCG puts equal weightage for every agent which may be relaxed to yield mechanisms satisfying more desirable properties. In the following section, we consider a special class of allocation rules that subsumes the VCG allocation rule.

31.3.1 Affine maximizer allocation rule

Definition 31.1 (Affine Maximizer) *An allocation rule f is called* affine maximizer *(AM) if there exists weights $w_i \geq 0, \forall\ i \in N$, not all zero, and a function $\kappa : A \to \mathbb{R}$ such that*

$$f^{AM}(\theta) \in \arg\max_{a \in A} \left[\sum_{i \in N} w_i \theta_i(a) + \kappa(a) \right].$$

Observe that this is a superclass of VCG allocation rule. The question we will ask in this section is similar to the Gibbard-Satterthwaite kind of question in voting setup. In GS theorem, we allowed the preferences to be all possible total ordering, while here we will consider valuations that can take any arbitrary value. The question will be what class of allocation rules are implementable in such a domain.

Definition 31.2 (Independence of Non-influential Agents) *An affine maximizer rule with weights w_i, $i \in N$ and κ satisfies* independence of non-influential agents *(INA) if for all $i \in N$ with $w_i = 0$ we have:*

$$f(\theta_i, \theta_{-i}) = f(\theta_i^{'}, \theta_{-i}), \forall \theta_i, \theta_i^{'}, \theta_{-i}$$

The INA condition is a *tie-breaking condition.* If the affine maximizer returns a set of allocations instead of a single maximizer, this condition ensures that the ties are always broken consistently and is not influenced by an agent whose report is not accounted in the affine maximization problem. If a dictatorial rule has some value for two different alternatives and the tie is broken by the valuation of a non-dictatorial agent, then it violates INA.

Theorem 31.3 *An INA affine maximizer rule is implementable.*

Proof: Consider the payment

$$p_i^{AM} = \begin{cases} \frac{1}{w_i}\left[h_i(\theta_{-i}) - \left(\sum_{j \neq i} w_j \theta_j(f^{AM}(\theta)) + \kappa(f^{AM}(\theta))\right)\right] & \forall i : w_i > 0 \\ 0 & \forall i : w_i = 0 \end{cases} \tag{31.1}$$

The payoff for agent i, if $w_i > 0$, is given by

$$\begin{aligned}
&\theta_i(f^{AM}(\theta)) - p_i^{AM}(\theta_i, \theta_{-i}) \\
&= \frac{1}{w_i}\left[\sum_{j \in N} w_j \theta_j(f^{AM}(\theta)) + \kappa(f^{AM}(\theta)) - h_i(\theta_{-i})\right] \\
&\geqslant \frac{1}{w_i}\left[\sum_{j \in N} w_j \theta_j(f^{AM}(\theta_i^{'}, \theta_{-i})) + \kappa(f^{AM}(\theta_i^{'}, \theta_{-i})) - h_i(\theta_{-i})\right] \\
&= \theta_i(f^{AM}(\theta_i^{'}, \theta_{-i})) - \underbrace{\frac{1}{w_i}\left[h_i(\theta_{-i}) - \left(\sum_{j \neq i} w_j \theta_j(f^{AM}(\theta_i^{'}, \theta_{-i})) + \kappa(f^{AM}(\theta_i^{'}, \theta_{-i}))\right)\right]}_{=p_i^{AM}(\theta_i^{'}, \theta_{-i})}.
\end{aligned}$$

Therefore,

$$\theta_i(f^{AM}(\theta)) - p_i^{AM}(\theta_i, \theta_{-i}) \geq \theta_i(f^{AM}(\theta_i^{'}, \theta_{-i})) - p_i^{AM}(\theta_i^{'}, \theta_{-i}) \tag{31.2}$$

Hence agent $i : w_i > 0$ is truthful.

and for $i : w_i = 0$, due to INA,

$$f(\theta_i, \theta_{-i}) = f(\theta_i^{'}, \theta_{-i}), \forall \theta_i, \theta_i^{'}, \theta_{-i}$$

and payment is zero. So, the agent is weakly truthful. ■

Similar to GS theorem, we ask what if the valuations are *unrestricted*. An unrestricted valuation is the mapping $\theta_i : A \mapsto \mathbb{R}$ without any additional constraints. The answer is given in the following theorem which we present without proof.

Theorem 31.4 (Roberts 1979) *Let A be finite with $|A| \geq 3$. If the type space is unrestricted, then every onto and implementable allocation rule must be an affine maximizer.*

As before, restricting the space of valuations types gives us more mechanisms that are DSIC. For example, there are mechanisms other than VCG or Groves mechanisms that are truthful in auction settings.