Project: Assignment 2

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Ques No. 1(a)

P_1	P_2	P'_1	P'_2
a	c	b	a
b	b	a	b
c	a	c	c

We know $f(P') \in \{a, b\}$.

Let the preference profile $f(P")$ = (P'$_1, P_2$).

We can clearly observe $f(P") \in \{b, c\}$.

Now, let us make the assumption that $f(P')$ = c.

But since $f(P')$ = c, we get $f(P")$ = c. Since $f(P')$ = c, we get $f(P)$ = c by Monotonicity, which is not possible as $f(P)$ = a. Hence, $f(P")$ = b.

Now, consider the given profile:

P'_1	P'''_2
b	c
a	a
c	b

The outcome of the above profile cant be 'c' because then $f(P)$ = c.

So it is 'b'. As the latter profile is dominated by the former, by mono we can say $f(P') =$ b.

Ques No. 1(b)

We know that P' and P''' are not single peaked. So, the earlier conclusion doesnot hold. Defining the new mechanism , $f(P) = \min \{ \ P_1(1) \ , \ P_2(1) \ \}$. Then, $f(P_1',P_2') =$ a.

Ques No. 2

We can infer that the preference set is the power set of X, where X is the set of projects and the prefeerence domain is the subset of all the subset of X. Let the top ranked project in X be x_1, according to the linear ordering P_i over X.

$\mid X \mid > 1 \implies$ x_1 can be a part of more than 1 subsets of X.

Let X_1 and $X_2 \subseteq$ X be 2 such subsets .

Now according to the given info,X_1 and X_2 are indifferent to agent 'i'. We will have to restrict our preference domain to remove indifferences. Gibbard-Satterthwaite theorem does not hold if indifferences are allowed among alternatives or if the preference domain is restricted.

Ques No. 3

Let there be N players and $P_t(1)$ be the top preference of the 't' player where t $\in \{1, 2, ..., N-1, N\}$.

Lets make an assumption that the median preference is held by player 't' and is given by $P_t(1) = $ a. Let K be the group of manipulating agents. The player 't' will not try to manipulate the outcome $\forall$ t s.t. $P_t(1) = $ a.
If all the players, s.t. $P_t(1) <$ a, in a common order of single peaked domain model form a group, and they alternate to b $>$ a then as $P_t(1) <$ a $<$ and being single peaked , a is still preffered over b. Even if they alternate to b $<$ a (in a common order), then the median is still a. Therefore they cant manipulate.
If all the players, s.t. $P_t(1) >$ a,in a common order of single peaked domain model form a group, and they alternate to b $<$ a, then being single peaked and b $<$ a $< P_t(1)$ therefore a is preffered over b. And if they alternate to b $>$ a the again the median is a.
Now let K has some agents with $P_t(1) <$ a and some agents with $P_t(1) >$ a. Let them manipulate s.t. the outcome is b. If b $>$ a, then it will decrease the utility for those agents with $P_t(1) <$ a. If b $<$ a, then manipulation will lead to decrease in the utility for those with $P_t(1) >$ a . We can hence conclude that no such group can be formed which can manipulate the outcome. Hence the Median Social Choice Function is a group stratergy proof.