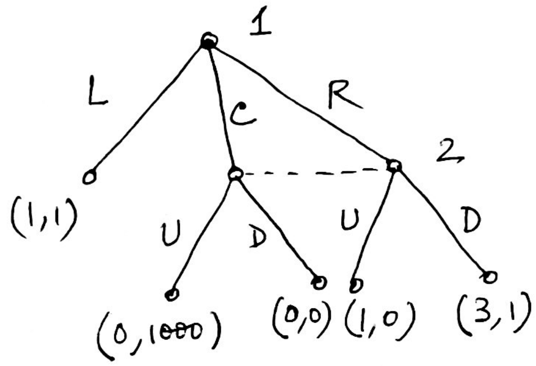
Lec 10

EFGs : NE does not capture credible threats

IIEFG : SPNE is not appropriate since player does not know which subgame he is playing.

'Optimal' could be defined if we knew the probability of the game arriving at the nodes of an information set.

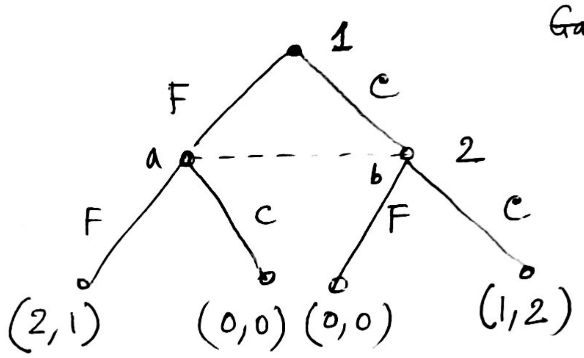


Player 2 doesn't know at which node the game is in his information set.

But R strictly dominates C, therefore 2 plays D.

Good scenario : cannot define equilibrium based on this.

~~Game choice~~ Football - Cricket game



If player 2 believes

$> \frac{2}{3}$ chance of staying in a → play F

$< \frac{2}{3}$ chance of staying in a → play C

$= \frac{2}{3}$ mix in any way.

" Equilibrium is tied to the belief of the players of the game."

Hence (belief, strategy) together should be treated while defining an equilibrium.

The new equilibrium concept makes this explicit.

~~Ga~~ Consider games only with perfect recall.

Information set of player i

$$I_i = \{ I_i^1, I_i^2, \dots, I_i^{k(i)} \}$$

Belief: In an IIEFG, the belief of player i is a map, $\mu_i : I_i^j \to [0,1]$ s.t. $\sum_{x \in I_i^j} \mu_i(x) = 1, \forall j$.

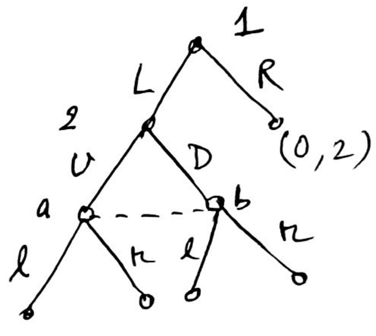
Given a strategy profile σ, we can compute the probability with which a node x is reached in G. denote this by $P_\sigma(x)$

Bayesian belief of player i: If the belief μ_i is derived from a mixed strategy profile σ using Bayes rule, i.e.,

$$\mu_i(x) = \frac{P_\sigma(x)}{\sum_{y \in I_i^j} P_\sigma(y)} \quad x \in I_i^j \quad \forall j = 1, \dots, k(i)$$

Then μ_i is Bayesian wrt σ.

$$\mu_i(a) = \frac{p_U \, p_L}{p_L p_U + p_L p_D} = p_U$$



$$\mu_i(b) = p_D \;\; \dots$$

perfect recall : mixed str. $\equiv$ behav. str.

Sequential Rationality

A strategy σ_i of player i at an information set I_i^j is sequentially rational given σ_{-i} and beliefs μ_i if $\forall \sigma_i'$

$$\sum_{x \in I_i^j} \mu_i(x)\, u_i(\sigma_i, \sigma_{-i} \mid x) \geq \sum_{x \in I_i^j} \mu_i(x)\, u_i(\sigma_i', \sigma_{-i} \mid x)$$

Examples from the previous games

Perfect Bayesian Equilibrium:

An assessment (σ, μ) is a PBE if for every player i

1. μ_i is Bayesian belief w.r.t σ
2. σ_i is sequentially rational given σ_{-i} and μ_i at every information set of i.

Theorem:
Every PBE is a Nash equilibrium.

Proof: Homework.

→ Football - Cricket game
Is $\left(\left(\frac{1}{2}, \frac{1}{2}\right), (q, 1-q)\right)$ a PBE ?