Dynamic Learning-based Mechanism Design for Dependent Valued Exchange Economies

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Outline of Talk

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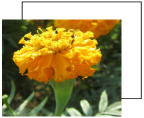
Image Annotating with Strategic Annotators

Image Owner



Ann. 1 **Ann. 2** ... **Ann. n**

Image Annotating with Strategic Annotators

Image Owner

Workload

Private Information

Known: Optimal control problem

Known only to the agents: Mechanism design problem

Efficiency **Efficiency** **Efficiency**

Ann. 1 **Ann. 2** ... **Ann. n**

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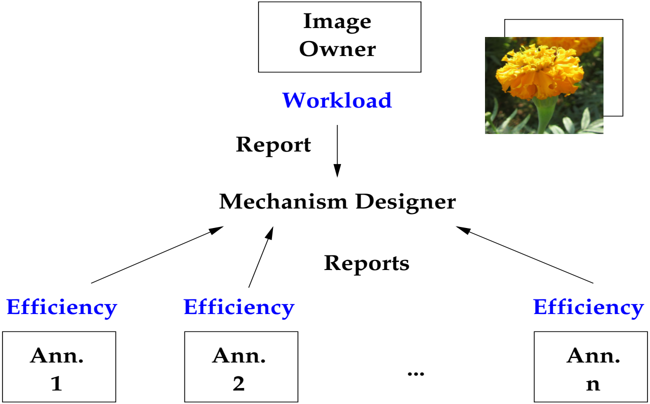

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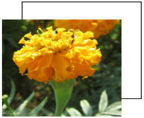

Mechanism Design Problem

Decisions	Guarantees
Allocation Rule	Truthfulness
Payment Rule	Voluntary Participation

Ann. 1 Ann. 2 ... Ann. n

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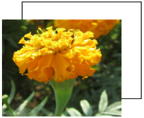

Problem Category:

Dynamic Mechanism Design

Dependent Values

Exchange Economy

Ann. 1	**Ann. 2**	...	**Ann. n**

Dynamic MD in Dependent-valued Exchange Economies: This talk

Outline of Talk

1. Motivation
   - Image Annotating: An Example
2. The Problem Formulation
   - The Model and Notation
   - Mechanism Design: A Quick Review
   - Related Work
3. Main Results
   - The Generalized Dynamic Pivot Mechanism
   - Main Theorem
4. Summary and Future Work

Outline of Talk

The Model

- Set of agents, $N = \{0, 1, \ldots, n\}$, 0 denotes the image owner
- At each time t,
  - Agent i observes type (workload or efficiency) $\theta_{i,t} \in \Theta_i$
  - Agent i reports $\hat{\theta}_{i,t}$
- Value function of agent i, $v_i : A \times \Theta \to \mathbb{R}$ (dependent values)
  - contrast with independent values: $v_i : A \times \Theta_i \to \mathbb{R}$
- Allocation function a_t selects agents given the reports
- Payment p_i transferred to agent i
- Discount factor δ for infinite horizon
- *Quasi-linear* utilities: $u_i = v_i + p_i$
- **Assumptions**
  - Dynamic types and fixed population
  - Types evolve in a Markov process
  - Type transitions independent across agents, i.e.,

$$F(\theta_{t+1}|a_t, \theta_t) = \prod_{i \in N} F_i(\theta_{i,t+1}|a_t, \theta_{i,t}) \; \forall \; t.$$

Outline of Talk

Mechanism Design: A Quick Review

- Game Theory: Analysis of strategic interaction between decision making agents (players)
- Mechanism Design: Reverse question of Game Theory
  - Method of designing the rules of a game
  - Goal is to achieve a certain outcome in the game
  - Does so in the presence of multiple *self-interested* agents
  - Agents have *private information* which is not available to the game designer
- For quasi-linear setting, outcome splits into *allocation* and *payment*
- **Desirable properties:** Truthfulness, Voluntary Participation, Efficiency, Budget Balance

Mechanism Design: Some Definitions

- The social welfare is defined as,

$$
\begin{aligned}
& W(\theta_t) \\
&= \max_{\pi_t} \mathbb{E}_{\pi_t,\theta_t}\left[\sum_{s=t}^{\infty} \delta^{s-t} \sum_{i\in N} v_i(a_s,\theta_s)\right] \\
&= \max_{a_t} \mathbb{E}_{a_t,\theta_t}\left[\sum_{i\in N} v_i(a_t,\theta_t) + \delta \mathbb{E}_{\theta_{t+1}|a_t,\theta_t} W(\theta_{t+1})\right]
\end{aligned}
$$

- Efficient Allocation (EFF): maximizes the social welfare
- Within Period Ex-post Incentive Compatibility (EPIC): Given the current type of an agent, reporting true type maximizes utility if all other agents report their types truthfully in all stages of the game
- Within Period Ex-post Individual Rationality (EPIR): Under same conditions as above, truthful report of an agent guarantees non-negative payoff at each stage of the game

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Where does this work fit in?

Valuations	STATIC	DYNAMIC
Independent	**VCG Mechanism** (Vickery, 1961; Clarke, 1971; Groves, 1973)	**Dynamic Pivot Mechanism** (Bergemann and Välimäki, 2010) (Cavallo et al., 2006)
Dependent	**Generalized VCG** (Mezzetti, 2004)	

- VCG guarantees
  - DSIC (stronger than EPIC), EFF, under certain conditions EPIR
- GVCG guarantees
  - EPIC, EFF, under certain conditions EPIR
- DPM guarantees
  - EPIC, EFF, EPIR, in non-exchange economies, budget balanced

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Dependent	**Generalized VCG** (Mezzetti, 2004)	**Generalized Dynamic Pivot Mechanism**

- VCG guarantees
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- GVCG guarantees
  - EPIC, EFF, under certain conditions EPIR
- DPM guarantees
  - EPIC, EFF, EPIR, in non-exchange economies, budget balanced
- GDPM guarantees
  - EPIC, EFF, EPIR, but requires more reports from agents than DPM

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Outline of Talk

Setting: Dependent-valued Exchange Economies

- If values are dependent, *Efficiency* and *Truthfulness* cannot be guaranteed with single stage mechanisms[1]
  - Without imposing any voluntary participation constraint
- Need to split the decisions of allocation and payment [2]

[1] P. Jehiel and B. Moldovanu. Efficient Design with Interdependent Valuations. *Econometrica*, (69):1237–1259, 2001.

[2] Claudio Mezzetti. Mechanism Design with Interdependent Valuations: Efficiency. *Econometrica*, 2004.

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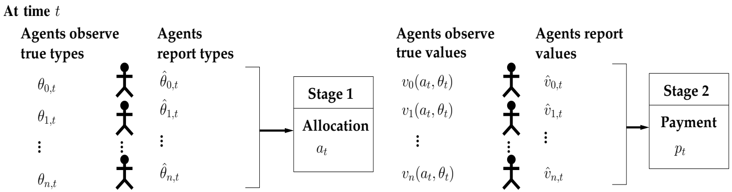

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The Generalized Dynamic Pivot Mechanism (GDPM)

- The task is to design the allocation and payments on the reported types and values
- The allocation maximizes the social welfare given true reports,

$$a^*(\hat{\theta}_t) \in \arg\max_{a_t} \mathbb{E}_{a_t,\hat{\theta}_t}\left[\sum_{i \in N} v_i(a_t, \hat{\theta}_t) + \delta \mathbb{E}_{\theta_{t+1}|a_t,\hat{\theta}_t} W(\theta_{t+1})\right]$$

- The payment to agent i at t is given by,

$$p_i^*(\hat{\theta}_t, \hat{v}_t) = \sum_{j \neq i} \hat{v}_{j,t} + \delta \mathbb{E}_{\theta_{t+1}|a^*(\hat{\theta}_t),\hat{\theta}_t} W_{-i}(\theta_{t+1}) - W_{-i}(\hat{\theta}_t)$$

Outline of Talk

Main Theorem

Theorem

GDPM is efficient, within period ex-post incentive compatible, and within period ex-post individually rational.

Proof ingredients:

- Utility of each agent is maximized at their true types (EPIC)
- At the true types, allocation is EFF by design
- Utility of each agent is non-negative at their true types (EPIR)

Outline of Talk

Summary and Future Work

Summary:

- Looked into the setting of exchange economies with dependent values
- Identified the key differences with the existing mechanisms
- Proposed a mechanism for this new paradigm

Future work:

- Design of mechanisms which also satisfy budget balance in this setting
- Insist on *consistent payments*, i.e., buyers pay and sellers get paid in each round
- Agents learn their types over time
- Study revenue properties of these mechanisms
- Study the dynamic population model with the dependent value structure
- Study the (im)possibility results for this setting

Thanks for your attention !!

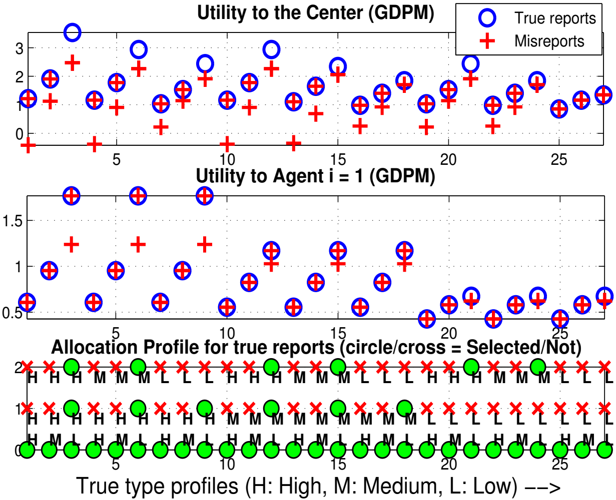
Simulation Setting

- 3 players: 1 Center (Image owner), 2 Annotators
- $\theta_{i,t} \in \{1, 0.75, 0.5\}$ corresponding to the levels $\{H, M, L\}$, for all agents
- Value structure:

$$
\begin{aligned}
v_0(a_t, \theta_t) &= \left(\frac{k_1}{\theta_{0,t}} \sum_{i \in a_t, i \neq 0} \theta_{i,t} - k_2 \right) \mathbf{1}_{0 \in a_t}; \\
v_j(a_t, \theta_t) &= -k_3 \theta_{j,t}^2 \mathbf{1}_{j \in a_t}, \qquad j = 1, 2; \\
& \quad k_i > 0, i = 1, 2, 3.
\end{aligned}
$$

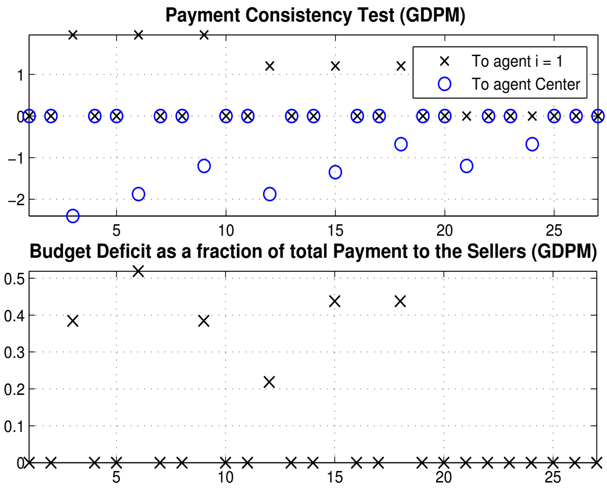
Simulation Results

Truthfulness:



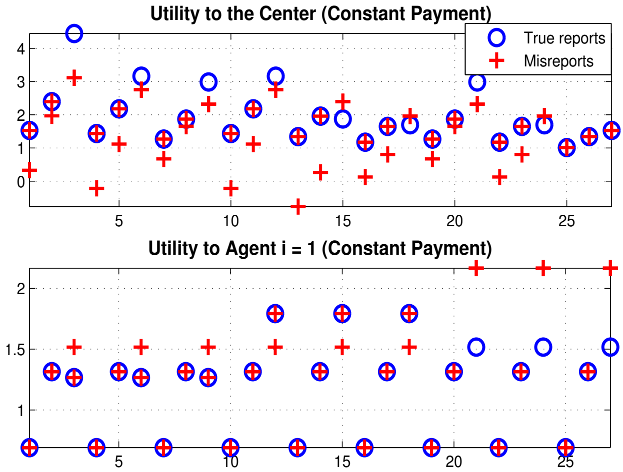
Simulation Results (Contd.)

Payment Consistency and Budget Properties:



Simulation Results (Contd.)

Comparison with a Naïve Mechanism (CONST):



Simulation Summary

	EFF	EPIC	EPIR	PC	BB
GDPM	✓	✓	✓	×	×
CONST	×	×	×	✓	✓

- All of these properties may not be simultaneously satisfiable

Complexity of GDPM

- For non-strategic setting, the types are *completely* or *partially* observable
- The problem: finding an optimal policy for a Markov decision process (MDP)
- Complexity: polynomial in the size of state-space[3]
- For GDPM: We need to solve $|N| + 1$ MDPs.
  - 1 for computing the allocation
  - $|N|$ for computing payment of each agent
- Complexity: polynomial in the number of agents and state-space
- This complexity is the same as dynamic pivot mechanism (Bergemann and Välimäki, 2010)

[3] Yinyu Ye. A New Complexity Result on Solving the Markov Decision Problem. Math. Oper. Res., 30:733-749, August 2005