Lec 5

Announcements for scribing
- lec 2 due tonight
- lec 4 due tomorrow

, lec 02
send all .tex, .pdf and images (if any) in a folder - zip it and send.

Ref: Shoham, LB, Narahari: Game Theory and Mechanism Design.

General principle to ~~solve~~ find Nash equilibrium

- Enumerate the possible supports of $S_1 \times S_2 \times \cdots \times S_n$
- # of nonempty subsets of $S_i = 2^{|S_i|} - 1$

~~total~~ # of supports $= (2^{|S_1|} - 1) \times (2^{|S_2|} - 1) \times \cdots \times (2^{|S_n|} - 1)$

- For every support profile, use characterization theorem of Nash equilibrium.

~~Number~~ Equations to be solved:

$X_i \subseteq S_i$ is a support profile. $X_1 \times \cdots \times X_n$ - one support

$$W_i = \sum_{s_{-i} \in S_{-i}} \left(\prod_{j \neq i} \sigma_j(s_j) \right) u_i(s_i, s_{-i}) \quad \forall s_i \in X_i \quad \forall i \in N. \quad \text{--- (1)}$$

$u_i(s_i, \sigma_{-i})$ same for all $s_i \in \delta(\sigma_i)$

$$W_i \geq \sum_{s_{-i} \in S_{-i}} \left(\prod_{j \neq i} \sigma_j(s_j) \right) u_i(s_i, s_{-i}) \quad \forall s_i \in S_i \setminus X_i \quad \forall i \in N. \quad \text{--- (2)}$$

$$\sigma_i(s_i) > 0 \quad \forall s_i \in X_i, \ \forall i \in N \text{ --- (3)}, \quad \sigma_i(s_i) = 0 \quad \forall s_i \in S_i \setminus X_i, \ \forall i \in N \text{ --- (4)}$$

$$\sum_{s_i \in S_i} \sigma_i(s_i) = 1 \quad \forall i \in N \cdots (5)$$

$W_1 \cdots W_n$, $\sigma_i(s_i)\ \forall s_i \in S_i$ ($|S_i|$ variables) $\Big\}$ $n + \sum_{i \in N} |S_i|$ variables.

(1) $\rightarrow$ $n \times (|X_1| + \cdots + |X_n|)$ eqn.s

(2) $n \times (|S_1 \setminus X_1| + \cdots + |S_n \setminus X_n|)$ eqns

} may be non-linear.

(3) $n \times \sum_{i \in N} |X_i|$ (4) $n \sum_{i \in N} |S_i \setminus X_i|$ (5) n. for every support profile.

For 2 players this is a set of linear inequalities

— >2 players, non-linear,

Finding Nash equilibrium is an active area of research.

- Lemke-Howson Algorithm — see Shoham, Leyton-Brown.

NASH: Find a Nash equilibrium of a finite game.

Daskalakis, Goldberg, Papadimitriou (2009)

NASH is PPAD-complete.

<u>Theorem (Nash 1951)</u>: Every finite game has a (mixed) Nash equilibrium.