CS711: Introduction to Game Theory and Mechanism Design	**Jul-Nov 2018**

Project: Assignment 2

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1 Solution 1

1.1 Part a

f(P_1,P_2)=a (given)
Consider the preference profile ($P_1^{'}$,$P_2^{''}$)

$P_1^{'}$	$P_2^{''}$
b	c
a	a
c	b

For any preference profile ($\bar{P}_1$, $\bar{P}_2$), f($\bar{P}_1$, $\bar{P}_2$) $\in$ {$\bar{P}_1(1)$, $\bar{P}_2(1)$}
Therefore for the above preference profile, f($P_1^{'}$,$P_2^{''}$) = {b,c}

But if f($P_1^{'}$,$P_2^{''}$) = c, then f(P_1,P_2) = c because of Monotonicity of f
(the dominated set of c is same in both the preference profiles (P_1,P_2) and ($P_1^{'}$,$P_2^{''}$))
Therefore f($P_1^{'}$,$P_2^{''}$) = b

Now consider the preference profile ($P_1^{'}$,$P_2^{'}$)

$P_1^{'}$	$P_2^{'}$
b	c
a	b
c	a

Since the dominated set of b in ($P_1^{'}$,$P_2^{''}$) is a subset of the dominated set of b in ($P_1^{'}$,$P_2^{'}$) and the social choice given by f($P_1^{'}$,$P_2^{''}$) is b,
therefore f($P_1^{'}$,$P_2^{'}$)=b (by Monotonicity of f)

Hence proved.

1.2 Part b

If the preferences are generated from a single-peaked preference domain with the intrinsic ordering of the alternatives being $a < b < c$, then the above conclusion does not hold.
The reason is that while going through the above conclusion, we used a preference profile $P_2^{''}$ in which c is preferred over a and a is preferred over b. Therefore, $P_2^{''}$ is not a single peaked preference profile. It has two peaks, each at a and c

Now that we are only allowed to have single peaked preferences, we cannot use the proof above.

To have f($P_1^{'}$, $P_2^{'}$) = a, we can use the following mechanism:

f(P) = $min_{i \in N}\{P_i(1)\}$
Where minimum is taken w.r.t. the order relation ¡. Hence the SCF picks the left-most peak among the peaks of the agents.

Therefore f(P_1, P_2) = a and f($P_1^{'}$, $P_2^{'}$) = a.

2 Solution 2

The preference domain in this case is the set of all the subsets of X, where X is the set of projects. This means that the preference domain is the power set of X.

Let x be the top ranked project in X according to the linear ordering P_i over X.
x can be a part of more than 1 subsets of X, since $|X| > 1$. Let S,T ⊆ X be two such sets. Now agent i is indifferent to the two subsets S and T according to the info given.

We will have to restrict our preference domain to remove indifferences. Gibbard-Satterthwaite theorem does not hold if indifferences are allowed among alternatives or if the preference domain is restricted.

3 Solution 3

Is the Median voter SCF group strategy proof?

A social choice function f is manipulable by a group of agents K ⊆ N if for some preference profile (P_K, P_K) there exists some preference profile $P_K^{'}$ of agents in K such that f($P_K^{'}$, P_{-K}) P_i f(P_K, P_{-K}) for all i ∈ K.

Let there be n players. $P_j(1)$ denotes the top preference of player j where j = 1,2,...,n

Assume that the median preference is held by player i and is given by $P_i(1)$ = a.
Let the group of agents trying to manipulate it be K.

For all those player j, whose $P_j(1)$ = a, then the player will not try to manipulate the outcome.

If all players whose $P_j(1)$ < a in common order of single peaked domain model form a group, and they

change their preference to an alternative b < a (in common order), then again the median will be a.
If they chose an alternative b > a, then since it is single peaked model and $P_j(1) < a < b$ therefore a is still preferred over b. Therefore they also won't be able to manipulate.

If all players whose $P_j(1) >$ a in common order of single peaked domain model form a group, and they change their preference to an alternative b > a, then again the median will be a.
If they chose an alternative b < a, then since it is single peaked model, and $b < a < P_j(1)$ therefore a is still preferred over b. Therefore they also won't be able to manipulate.

Now consider the case when the group K has some agents with $P_j(1) < a$ and some agents with $P_j(1) > a$. Now let them manipulate the result to an alternative b. if b < a, then the manipulation will reduce the payoff for those agents whose $P(1) > a$.
If b > a, then it will reduce the payoff for those agents whose $P(1) < a$.
Hence no such group can be formed that can manipulate the outcome.

Hence the Median Voter SCF is group strategy proof.