CS711: Introduction to Game Theory and Mechanism Design **Jul-Nov 2018**

Project: Assignment 2

1.1 Two Agent Three Alternatives

(a)

Consider a profile

$$P = c \succ a \succ b$$

so

$$f(P_1, P) = a$$

because in P a is preferred more than in P_2, $f(P_1, P_2) = a$ and f is monotonic.

Now,

$$f(P_1', P) \in \{b, c\}$$

because $f(\bar{P}_1, \bar{P}_2) \in \{\bar{P}_1(1), \bar{P}_2(1)\}$

If $f(P_1', P) = c$, then player 1 can change profile form P_1' to P_1 to change outcome to a as he prefers a over c. But it means f is not strategyproof which is a contradiction.

So $f(P_1', P) = b$. hence $f(P_1', P_2') = b$ as preference of b is more in P_2' than in P

(b)

When we have a single-peaked preference domain with the intrinsic ordering being $a < b < c$ considered profile P cannot be generated. Thus above proof does not hold and we cannot conclude anything.

A mechanism can have SCF

$$f(P_1, P_2) = min(P_1(1), P_2(1))$$

Clearly f is both onto and strategy proof and $f(P_1', P_2') = a$.

1.2 Set of Projects

Let us consider two set of projects A and B such that A is proper subset of B and $|B| > 1$. Also let p be the highest ranked project in B.

Now, either $p \in A$ or $p \notin A$.

If $p \in A$ then p will be highest in A (proof: $A \subsetneq B$ so any other higher would mean p not highest in B which is a contradiction). So agent will consider A and B indifferent.

If $p \notin A$ then agent will prefer B as all other projects in A have lesser rank by same contradiction.

So there will be no preference order where A is ranked higher than B. This is a case of restricted domain and hence Gibbard-Satterthwaite result cannot be applied.

1.3 Group strategy-proof

If the Group of agents want to manipulate the outcome they have to change the median. If any person in group can wishes to change median they can only do by changing his/her peak to the other side of median. All person in group have peak to one side of the median. So doing that will only shift the median farther away from the peak preferred outcome of every person in group. So none in group can manipulate the result in their favour. Thus f is group strategy-proof.