CS-698W: Game Theory and Collective Choice **Jul-Nov 2017**

Lecture 10: 23 Aug, 2017

Lecturer: Swaprava Nath *Scribe(s): Asim Unmesh*

Disclaimer: *These notes aggregate content from several texts and have not been subjected to the usual scrutiny deserved by formal publications. If you find errors, please bring to the notice of the Instructor.*

10.1 Outcome Equivalence of Behavioral and Mixed Strategy

In the context of *Extensive Form Games* (EFG) *Behavioral Strategy* for an EFG is determined by a probability distribution over moves for each node the player is going to play at. Thus the player is randomizing at each node rather than playing a deterministic choice at each node.

A Behavioral Strategy b_i and a Mixed Strategy σ_i are **Outcome Equivalent** if for all σ_{-i} , the probability distribution induced over the terminal vertices are the same for (b_i,σ_{-i}) and (σ_i,σ_{-i}).

Theorem 10.1 (Kuhn 1953) *In Games with Perfect Recall every mixed strategy is Outcome Equivalent to behavioral strategies.*

10.2 Belief

We now define the set of all *Information Set* of player *i*.
$I_i = I_i^1, I_i^2, ..., I_i^{k(i)}$

In an Imperfect Information Extensive Form Game (IIEFG), the belief of player i is a map

$\mu_i^j : I_i^j \rightarrow [0,1]$

such that,

$$\sum_{x \epsilon I_i^j} \mu_i^j(x) = 1$$

10.3 Bayesian Belief of Player i

μ_i is Bayesian with respect to σ, if it is derived from mixed strategy profile σ using Bayes rule. That is:

$$\mu_i(x) = \frac{P_\sigma(x)}{\sum_{y \epsilon I_i^j} P_\sigma(y)} \qquad x \epsilon I_i^j \quad \forall j = 1, ..., k(i)$$

10.4 Sequential Rationality

A strategy σ_i of player i at an Information Set I_i^j is sequentially rational given σ_{-i} and beliefs μ_i if $\forall\ \sigma_i^{'}$

$$\sum_{x \epsilon I_i^j} \mu_i(x) U_i(\sigma_i, \sigma_{-i}|x) \geq \sum_{x \epsilon I_i^j} \mu_i(x) U_i(\sigma_i^{', \sigma_{-i}|x)}$$

10.5 Football Cricket Game example for Sequential Rationality

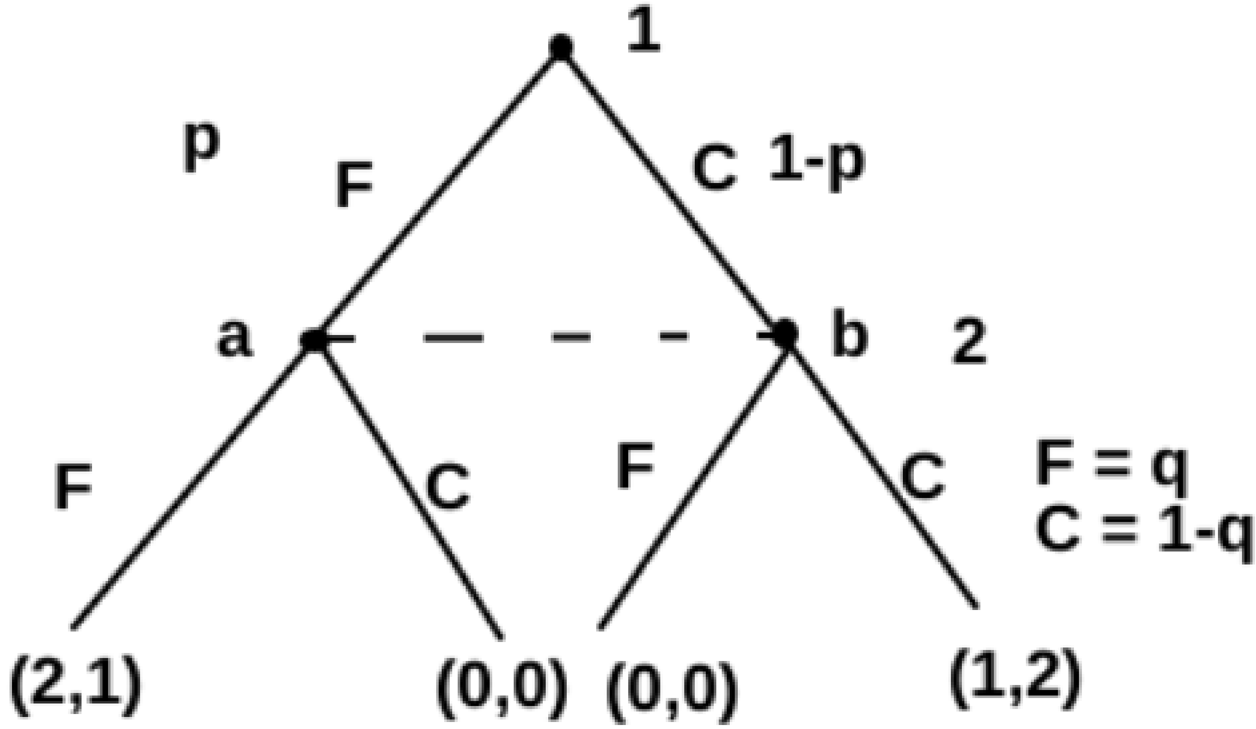

Figure 10.1: Football-Cricket Game for 2 players

In the above figure you can see Football Cricket game played between two players 1 and 2.
$\mu_2(a) = p = 0.5$
$\mu_2(b) = 1 - p = 0.5$

$\sum_{x \epsilon I_2^1} \mu_2(x) U_2(\sigma_1, \sigma_2 | x)$
$= 0.5[q.1 + (1\text{-}q).0] + 0.5[2 \times (1-q)]$
$= 0.5[2-q]$

Thus, to maximise his utility given his belief about the moves of player 1, which is (0.5,0.5), it will be sequentially rational for player 2 to keep q = 0, i.e., play football with zero probability.

So, σ *= ((0.5,0.5), (0,1))*

10.6 Perfect Bayesian Equilibirium(PBE)

An assessment (σ, μ) *is a PBE if for every player i*
1. μ_i *is Bayesian with respect to* σ*.*
2. σ_i *is Sequentially Rational given* σ_{-i} *and* μ_i *at every information set of i.*

Theorem 10.2 *Every Perfect Bayesian Equilibirium(PBE) is a Mixed Strategy Nash Equilibirium(MSNE).*