**CS711: Introduction to Game Theory and Mechanism Design** **Jul-Nov 2018**

# Assignment 2

Rahul Kumar (150548)

**Question 1**

**(a)** Since the domain of preferences is unrestricted, we construct the preference profiles as follows:

| $P_1 P_2$ | $P_1^{'} P_2^{'}$ | $P_1^{'} \hat{P}_2$ |
|---|---|---|
| a c | b a | b c |
| b b | a b | a a |
| c a | c c | c b |

We will prove that $f(P_1^{'}, P_2^{'}) = b$ by contradiction. Let us suppose that $f(P_1^{'}, P_2^{'}) = a$ and f is strategyproof.
Then $f(P_1^{'}, \hat{P}_2) = c$
If this is not the case then player 2 can manipulate by reporting $P_2^{'}$ instead of $\hat{P}_2$ in $(P_1^{'}, \hat{P}_2)$ and the outcome will become $a$ which is more preferred than b.
Now if we consider the transition from $(P_1^{'}\hat{P}_2)$ to $(P_1 P_2)$ we see that $f(P_1 P_2)$ must be equal to $c$ (by monotonicity). But this is a contradiction.

**(b)** No the earlier conclusion does not hold in this case. The above proof used the construction of preference ordering like $c > a > b$. However,if the preferences are generated from a single peaked preference domain, then we can never have such a preference ordering for any player.

Now, let us design a mechanism such that $f(P_1^{'}, P_2^{'}) = a$ and it is strategyproof also. The SCF is such that it chooses that as the outcome from the top preferences which is the leftmost given the intrinsic ordering of the alternatives as $a < b < c$.
Now, we prove that this is strategyproof. The player who has the leftmost choice has no reason to manipulate. The other player can only manipulate by moving his peak further to the left of the first person's choice. But this will be worse for the second player as it will be further away from his original preference than the original outcome. So he wont manipulate.

**Question 2**

The set of alternatives is the set of all subsets of objects,i.e.
$\{S : S \subseteq X\}$. Now, according to question, there are atleast two projects in X. This means that the set of alternatives is at least 3(required for Gibbard-Sattherthwaite theorem) i.e. S has only project 1 or only project 2 or both projects. Now, consider two alternatives S and T such that
$S \subset T$
Now we see that any agent is either indifferent between S and T or likes T over S as T has all the projects that S has and may even have more preferred projects. Thus we can never have a preference ordering where S is ranked higher than T. This is a restriction of the domain and we cannot apply the Gibbard-Satterthwaite result here as we need unrestricted domain for applying it.

**Question 3**

Consider the case that the entire group has the same peak preference which is the median. In this case there is no point in deviating.

Now, consider the general case in which some peaks are to the left and some to the right of the median(all peaks on one side of the median is a special case of this situation). Then, in order to change the outcome, we need to change the median. Thus some of the members of the group will have to change their peaks by moving to the other side of the median by going past the median. To shift the median towards left, someone from the right must cross the median and set its new peak on the left of the median. Similarly, to shift the median towards right, someone from the left must cross the median and set it new peak on the right of the median. Since the preferences are single peaked, any new peak will lie further away from the original peak preference than the original median itself as the person went past the median and on the other side. Thus, even if some people from the group may benefit, the person who shifts his preference will clearly be at a loss. Thus it is group strategy proof.