Criticisms of VCG

① Privacy : reveal valuations truthfully

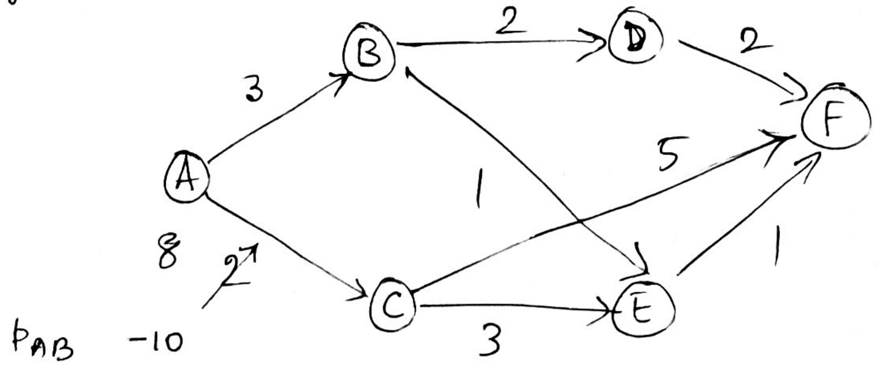
- if the interaction had to happen beyond one round, it may be preferable to use a mechanism that uses the minimal information needed for the current round and not have spillover effect.
- This also has a concern about the introduction of fake agents to increase the payment of the winner. Once it is known that the value was high enough, this fake agents may be the second losing bid ~~to~~ introduced by a malicious auctioneer.

② Susceptibility to Collusion

Players	A	B	Payment
1	200 (250)	0	150 (100)
2	100 (150)	0	50 (0)
3	0	250	0 (0)

③ Not Frugal : doesn't charge an amount close to the seller's valuation / cost. VCG is guaranteed to bring revenue / no deficit, but the payment could be very large.

Example of shortest path (routing a packet / an delivery item)
e.g. Amazon delivering your item



This is a cost setup - easy way to consider this is by negating all values and payments

Effect of payment to AB due to AE's cost

$$p_{AB} = 0 - (-1) + (-2) - 0 + (-3) - 0 = -4$$

(4) Revenue Monotonicity violated

revenue monotonicity: revenue weakly increases with number of agents.

	X	Y	Payment
1	0	90	0 → 0
2	100	0	90 → 0
3	100	0	0

Split or merging of agents is problematic.

(5) Not fully Budget Balanced.

almost always some surplus will be left.
Cannot be ~~re-allo~~ redistributed as that will change their payoffs. Cannot be invested to things that can affect the payoffs — e.g. to charity if someone cares for it etc.
It has to be destroyed — known as money burning.

Caution: this does not mean VCG is useless. Rather it is the most-used mechanism with money — but needs to be carefully applied and good to know its limitations.

Generalization of VCG

Need to expand the class of mechanism to take care of multiple limitations

- Budget Balance issue
- Equal weightage for every agent may not be required
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Affine Maximizer allocation rule

- Superclass of VCG mechanisms / efficient allocations – therefore hope to satisfy more properties.
- Can ask a Gibbard - Satterthwaite kind of question in the quasi-linear setting with public goods

$$f^{AM}(\theta) \in \underset{a \in A}{\operatorname{argmax}} \left(\sum_{i \in N} w_i \, \theta_i(a) + K(a) \right)$$

where $w_i \geq 0 \ \forall i \in N$ and not all zero

$K : A \to \mathbb{R}$ is any arbitrary function.

Special cases:

1. Efficients :
2. Dictatorial

w_i's are different $\Rightarrow$ not anonymous

K's are " for different a $\Rightarrow$ gives emphasis on certain allocations than others.

Definition: An AM rule f^{AM} with weights $w_i, i \in N$ and K satisfies independence of non-influential agents (INA) if for all $i \in N$ with $w_i = 0$ we have that $f(\theta_i, \theta_{-i}) = f(\theta_i', \theta_{-i}), \forall \theta_i, \theta_i' \ \forall \theta_{-i}$.

Tie-breaking requirement.

If a dictatorial rule has same value for two different alternatives and the tie is broken by the valuation of a non-dictatorial agent – it violates INA.

Theorem: An INA affine maximizer rule is implementable.

Note: Without INA, an affine max may not be truthful / implementable. E.g., pick the worst alternative of the $w_i = 0$ agent.

Proof: Consider the payment

$$p_i^{AM}(\theta_i, \underline{\theta}_{-i}) = \begin{cases} \frac{1}{w_i}\left[h_i(\underline{\theta}_{-i}) - \left(\sum_{j \neq i} w_j \theta_j(f^{AM}(\theta)) + \kappa(f^{AM}(\theta))\right)\right] & \forall i : w_i > 0 \\ 0 & \forall i : w_i = 0 \end{cases}$$

Payoff of agent i if $w_i > 0$

$$\theta_i(f^{AM}(\theta)) - p_i^{AM}(\theta_i, \underline{\theta}_{-i})$$

$$= \frac{1}{w_i}\left[\sum_{j \in N} w_j \theta_j(f^{AM}(\theta)) + \kappa(f^{AM}(\theta)) - h_i(\underline{\theta}_{-i})\right]$$

$$\geq \frac{1}{w_i}\left[\sum_{j \in N} w_j \theta_j(f^{AM}(\theta_i', \underline{\theta}_{-i})) + \kappa(f^{AM}(\theta_i', \underline{\theta}_{-i}))\right] - h_i(\underline{\theta}_{-i})$$

$$= \theta_i(f^{AM}(\theta_i', \underline{\theta}_{-i})) - \underbrace{\frac{1}{w_i}\left[h_i(\underline{\theta}_{-i}) - \left(\sum_{j \neq i} w_j \theta_j(f^{AM}(\theta_i', \underline{\theta}_{-i})) + \kappa(f^{AM}(\theta_i', \underline{\theta}_{-i}))\right)\right]}_{p_i^{AM}(\theta_i', \underline{\theta}_{-i})}$$

for $i : w_i = 0$

$$f^{AM}(\theta_i, \underline{\theta}_{-i}) = f^{AM}(\theta_i', \underline{\theta}_{-i}) \quad \forall \theta_i', \theta_i, \forall \underline{\theta}_{-i}$$

and payment is zero, so the agent is weakly truthfull.

Similar to GS theorem, we ask what if the valuations are unrestricted.

$\theta_i : A \to \mathbb{R}$ Θ_i contains all such valuation functions, no restriction imposed.

- A little ~~[illegible]~~ difficult assumption for real scenarios, but this gives a benchmark of what is possible to be DSIC in the QL domain.

With this unrestricted space of valuations, we can characterize the class of DSIC mechanisms

<u>Theorem</u> [Roberts' 1979 ~~theorem~~]

Let A be finite with $|A| \geq 3$. If the type space is unrestricted, then every onto and implementable allocation rule must be an affine maximizer.

Proof skipped.

As before, restricting the space of valuations/types gives ~~[illegible]~~ no more mechanisms that are DSIC.

For example, there are more than VCG/ ~~gs~~ Groves mechanisms that are truthful in auction settings.