Lec 8

Brother - Sister game has (several) PSNEs.

Theorem: Every finite perfect information EFG has a pure strategy NE.

Intuition: Since after every stage of a PI EFG, the player observes the action, there is no reason to mix strategies. Every equilibrium must be pure.

— will be obvious from the subgame perfection result.

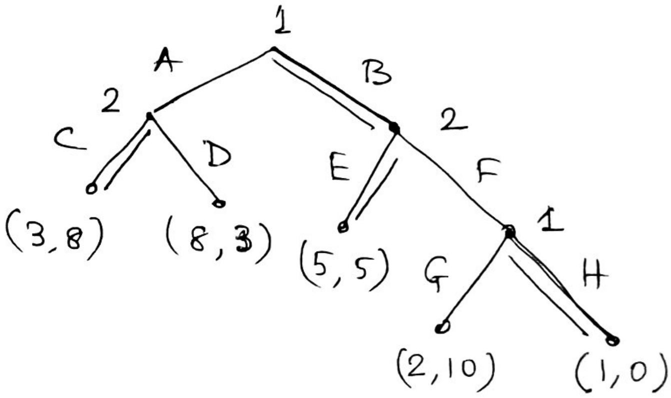
Examples of PI EFG: Chess, Tic-Tac-Toe, Bargaining

Zermelo (1913) showed using an argument similar to EFG that if both the players in chess are infinitely rational and intelligent, chess must be a very boring game.

NE is too weak a guarantee in an EFG.

(BH), (CE) is a NE of this game, But if 2 ever plays F, will 1 still stand with H commitment?

— Non-credible threat unlikely to happen.



SUBGAME: The EF game represented by the subtree at a node is called the subgame at that node.

Subgame perfect Nash equilibrium (SPNE)

— strategy profiles s such that for ~~any~~ all subgame G', $s|G'$ is a NE of G'.

. i.e. choose any subtree of the EFG, restriction of s to that subtree must be a NE in that reduced game.

Q: Is (0-2, NNY) (AH, CF) a SPNE?

A: No, since at the subgame denoted by history BF, H is not the best response of 1.

How to compute an SPNE?

Algorithm : Backward Induction

1. Start at the leaves of the greatest depth
2. for the player in the parent nodes find which action maximizes utility for that player.
3. retain that action and delete all edges, at that level translate the utilities to the parent node.
4. go up one level and repeat (1)
5. STOP if root is reached.

((AG), (CF)) is the SPNE of the previous game.

Is SPNE a PSNE of this game?

Appealing and natural, but is it practical?

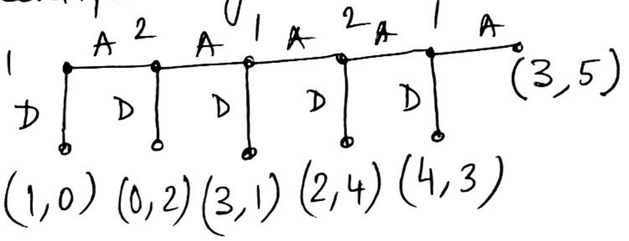
To find the SPNE the BI algorithm needs to traverse the whole tree of the game

EFG representation of chess has $\sim 10^{150}$ nodes.

Game softwares use heuristic pruning for computer players.

Criticisms of BI/SPNE :

Centipede game



SPNE: ((DDD), (DD))

Practical/Experimental : shows non-SPNE outcomes

Theoretical : If agent 1 plays A, the theory does not say anything about the outcome
– since the prediction itself says the state should not have been reached.

Expressive power of PI EFG

Weaker than NFG, since PI EFG $\rightarrow$ NFG

Can we represent the ~~Two~~ Neighboring Kingdoms' dilemma using PIEFG?

1 \ 2	A	D
A	5,5	0,6
D	6,0	1,1

PI EFG cannot represent a simultaneous move game.

Imperfect Information EFG

$$\langle N, A, \mathcal{H}, \mathcal{X}, P, (u_i)_{i\in N}, (I_i)_{i\in N} \rangle$$

$Z \subseteq \mathcal{H}$, terminal histories.

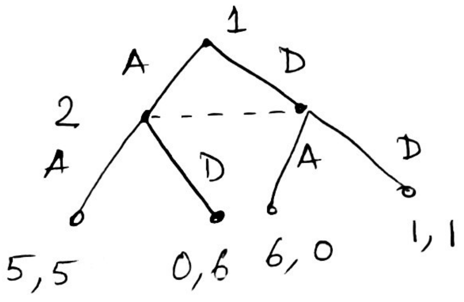
Information Set: I_i is a partition of $\{h \in \mathcal{H} \setminus Z : P(h) = i\}$

with the property that if $h, h' \in I_i$, then $\mathcal{X}(h) = \mathcal{X}(h')$. - action sets are same.

Information set is always non-empty, but can be singleton - PI EFG is also a II EFG with singleton information sets.

With this definition $\mathcal{X}$ can now be defined on information sets.

The simultaneous move game in II EFG representation



~~$I_1 = \{\{A\}, \{D\}\}$~~ $I_1 = \{\phi\}$

$I_2 = \{\{A\}, \{D\}\}$

Every NFG can now be represented in EFG.

Redundant representation - takes exponentially more space in ~~EFG~~ EFG representation.

(8-4)

However, II EFG is clearly a richer representation.

NFG $\rightarrow$ multiple II EFG representations

unique NFG $\leftarrow$ IIEFG } richer.