CS711: Introduction to Game Theory and Mechanism Design **Jul-Nov 2018**

Project: Assignment 2

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1 Solution to 1^{st} Question

Part a

Given f is an onto SCF with $f(P_1, P_2) = a$. And for any preference profile $(\bar{P}_1, \bar{P}_2)$, $f(\bar{P}_1, \bar{P}_2) \in \{\bar{P}_1(1), \bar{P}_2(1)\}$. We have to prove that if f is strategyproof then $f(P_1', P_2') = b$.

I will assume that $f(P_1', P_2') = a$ and prove that this is a wrong assumption, which implies that $f(P_1', P_2') = b$. Assuming f is strategy-proof, which implies that either of the agents are not worse off by being truthful about their preferences.

Consider the preference profile (P_1', P_2'). We assumed that $f(P_1', P_2') = a$. Now consider the transition of Agent 1's preference from P_1' to P_1. As we know that $f(P_1, P_2') \in \{P_1(1), P_2'(1)\} = a$ (also unanimous), which is the Agent 1's most preferred alternative. Thus, clearly Agent 1 is better off by manipulating his true preferences, which is not the case if f is strategy-proof. Hence my initial assumption is wrong.

Thus, $f(P_1', P_2') = b$. Because, $f(P_1', P_2') \in \{P_1'(1), P_2'(1)\} = \{a, b\}$ and $f(P_1', P_2') \neq a$.

Part b

- No,the earlier conclusion of "If f is strategy-proof and *onto*, then $f(P_1', P_2') = b$" doesn't hold in this case.
- When the preferences are generated from single peaked preference domain, there is a domain restriction. So, for a preference profile $(\bar{P}_1, \bar{P}_2)$, $f(\bar{P}_1, \bar{P}_2) \in \{\bar{P}_1(1), \bar{P}_2(1)\}$ doesn't not hold. So, there is a chance for output of preference profile (P_1', P_2') resulting in alternative c (which is not the case in above question). And when $f(P_1', P_2')$ is assumed to be c, there is no contradiction for it. So, we can't conclude that $f(P_1', P_2') = b$ when f is strategy-proof and onto and preferences are generated from single peaked preference domain.
- Given the SCF is *onto*, there $\exists P$ for which the outcome of SCF is a. But, I don't think it can be possible for the given preference profile (P_1', P_2'), outcome would be a. When $f(P_1', P_2') = a$, it contradicts with strategy-proofness of the SCF.

2 Solution to 2^{nd} Question

The set of all possible subsets of X is the set of available alternatives: $\{A : A \subseteq X\}$. Let us assume that there are two alternatives A and B, such that $A \subset B$. From the preference ordering mentioned in the question, agent i prefers B to A, $B \succeq A$ (BP_iA) $\forall i$. So, there can be no preference ordering where A can be placed (ranked) higher than B, which is a domain restriction. Hence Gibbard-Satterthwaite result can't be applied here.

3 Solution to 3^{rd} Question

Yes, Median Voter SCF is group strategy-proof. There are only 2 types of possible coalition in this question.

- All the agents in the group have their peaks to the left of the Median agent's peak:

  The only way the group of agents($K \subseteq N$) can shift the median (towards them) and be as close to it as possible is by shifting their peaks (preferences) to the right (other side) of the median voter's peak. But, doing this shifts median to further right from their true peaks, which is not ideal for the agents. Hence, they are worse off from manipulating their true preferences. So, the $K \subseteq N$ group of agents don't manipulate in this case.

- All the agents in the group have their peaks to the right of the Median agent's peak:

  Following the same logic, group of agents($K \subseteq N$) can shift the median (towards them) and be as close to it as possible by shifting their peaks (preferences) to the left (other side) of the median voter's peak. But, doing this shifts median to further left from their true peaks, which is not ideal for the agents. Hence, they are worse off from manipulating their true preferences. So, the $K \subseteq N$ group of agents don't manipulate in this case also.

There is no possibility of agents distributed on both sides of the median voter forming a coalition or group. Because, agents on the left side prefer median shifting towards left and agents on the right side prefer median shifting towards right, which can't happen simultaneously. Also median agent himself is never better off by including in such coalition and manipulating his true preferences as his peak preference is already winning.

Hence, a group of agents $K \subseteq N$ are never better off manipulating their true preferences. This proves Median Voter SCF is group strategy-proof.

(Proof of median voter scf is strategy-proof in the scribed notes is taken as inspiration for this proof)