Assignment 2

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Q1

a)

Note that strategyproofness also implies monotonicity. Therefore f is also monotone($MONO$). Now consider the following strategy profiles.

P_1	P_2	P_1	$P_2^{''}$	$P_1^{'}$	$P_2^{''}$	$P_1^{'}$	$P_2^{'}$
a	c	a	c	b	c	b	a
b	b	b	a	a	a	a	b
c	a	c	b	c	b	c	c

We know that $f(P_1, P_2^{''}) \in \{a, c\}$. Consider the transition from (P_1, P_2) to $(P_1, P_2^{''})$, position of 'a' weakly improves. Thus using monotonicity of f , we conclude $f(P_1, P_2^{''}) = a$.

Consider the profile $(P_1^{'}, P_2^{''})$. $f(P_1^{'}, P_2^{''}) \in \{b, c\}$. If $f(P_1^{'}, P_2^{''}) = c$, agent 1 can manipulate by reporting P_1 in place of $P_1^{'}$, because $f(P_1, P_2^{''}) = a$, which is more preferred by agent 1. Since f is strategy-proof, it implies that $f(P_1^{'}, P_2^{''}) = b$.

Note that $f(P_1^{'}, P_2^{'}) \in \{b, a\}$. Consider the transition from $(P_1^{'}, P_2^{''})$ to $(P_1^{'}, P_2^{'})$, position of 'b' weakly improves. Thus using monotonicity of f , we conclude $f(P_1^{'}, P_2^{'}) = b$.

Hence proved.

b)

No, the earlier conclusion does not hold in this case.
The strategy $P_2^{''}$ is invalid in the case of single peaked preferences, because this strategy has 2 peaks namely, a and c. Therefore our proof does not go through.
A left-most peak selecting median voter scf is such a mechanism. All median voter $SCFs$ are strategy-proof and $ONTO$, and also since we select the left-most peak our scf satisfies $f(P_1, P_2) = a$ and $f(P_1^{'}, P_2^{'}) = a$.

Q2

The set of alternatives in this case are the subsets of projects. Now considers two subsets P and Q such that P is a subset of Q *i.e.* $P \subset Q$. Since all projects of P are also present in Q, the highest ranked project in P will either be same as highest ranked project of Q or worse (lower ranked). Therefore an agent will always either prefer Q over P or would be indifferent between the two of them. Therefore, there cannot be a case in which P is preferred more than Q. Thus, this is a restriction of domain and hence we cannot apply the Gibbard-Satterthwaite result here.

Q3

We need to consider only the peak preferences of all the agents. Let $f(\mathrm{P})$ = a is the median of these peaks(of agents) and the phantom peaks.
Now, Let us consider that a group of K agents want to manipulate the function f. Let K_1 of them(call them subgroup 1) have a peak on the left and K_2(call subgroup 2) have their peak on the right of a. If anyone from subgroup 1 want to change the median they have to shift their preferences to the right, because shifting their preferences to the left does not change the median. Similarly, if anyone from subgroup 2 want to change the median they have to shift their preferences to the left.
Let L people from subgroup 1 shift(wrongly report) their preference to the right and R people from subgroup 2 shift their preference to the left.
Case 1. L=R In this case the median does not change since equal number of people from both sides are shifting their preference to the other side.
Case 2. L>R In this case the new median shifts to the right of a. Note that for the L agents of subgroup 1 this new median is even less preferable, because their original preferences lied to the left of a. Thus, this is not a profitable deviation.
Case 2. L<R In this case the new median shifts to the left of a. Note that for the R agents of subgroup 2 this new median is even less preferable, because their original preferences lied to the right of a. Thus, this is not a profitable deviation.
Thus, manipulation is never better off for the whole group and hence, the median voter SCF is group strategy-proof.