Defn: Individual Rationality.

Claim 2: In the allocation of goods, VCG mechanism is individually rational, i.e., the payoff of every agent is non-negative.

Proof: The payoff of an agent (using earlier notation of X and Y) $X \in \underset{a \in A}{\operatorname{argmax}} \sum_{i \in N} v_i(a, \theta_i)$, $Y \in \underset{a \in A}{\operatorname{argmax}} \sum_{j \neq i} v_j(a, \theta_j)$

$$v_i(X, \theta_i) - \sum_{j \neq i} v_j(Y, \theta_j) + \sum_{j \neq i} v_j(X, \theta_j)$$

$$= \sum_{j \in N} v_j(X, \theta_j) - \sum_{j \neq i} v_j(Y, \theta_j) \quad \left[\text{add and subtract } v_i(Y, \theta_i)\right]$$

$$= \underbrace{\sum_{j \in N} v_j(X, \theta_j) - \sum_{j \in N} v_j(Y, \theta_j)}_{\geq 0 \text{ by defn. of } X} + \underbrace{v_i(Y, \theta_i)}_{\geq 0} \geq 0$$

Application Domain: Internet Advertising

Why is internet advertising successful?

(1) User data – advertiser can gather many information about the buyer and can target with specific products.

(2) Measurable actions – can classify buyers into categories and offer specific deals for them OR leave the customer if he is not interested in the products.

(3) Low latency: Real-time bidding and decision on showing ads are possible.

Types of ads on internet

(1) Sponsored search ad –
advertisers bid on the keywords entered by the users
- stopwords
- cap the number of times an ad is shown to the user (two purposes: stop draining money for uninterested users and malicious bidders)

(2) Contextual ads – depending on the content of the page or email.

(3) Display ads – Classical way of advertising
<Banner ads in newspapers>

Ad exchanges are facilitators between the advertisers and the search/website provider. It helps small businesses to customize the ads for certain objectives.

Position auctions:

Auctions to sell multiple ad positions on a page.

Let $N = \{1, \ldots, n\}$ be the set of advertisers

$M = \{1, \ldots, m\}$ set of slots

assume for simplicity $m \geq n$. – every ad is shown perhaps in different positions

1 being the best position

m " " worst position

Advertiser value

Assumptions:

① Clicks generate the value to the advertisers

② All clicks are valued equally – no matter which position they are displayed.

– the position only affects the chance of getting a click

These assumptions help decouple the value effect and position effect.

Agent i's expected value when her ad is shown at position $j \in M$.

$$v_{ij} = CTR_{ij} \cdot v_i$$

↑

Click-through-rate (CTR) of bidder i in position j : denotes the probability of getting a click.

$CTR_{ij} \in [0,1]$

Assumptions of CTR

decomposable into quality (CTR_i) and position effects (pos_j)

Note: this is an important assumption to make it separate effects, one is agent dependent, other is position dependent.

$$CTR_i \in [0,1] \quad , \quad pos_j \in [0,1]$$

$$CTR_{ij} = pos_j \cdot CTR_i$$

Hence the expected value of agent i is decomposable too

$$v_{ij} = pos_j \left(CTR_i \cdot v_i\right)$$

position effect is assumed to be decreasing with position. $pos_1 = 1$, $pos_j > pos_{j+1}$; $j = 1, \ldots, m-1$

$v_i \rightarrow$ private information of the bidder

pos_j and CTR_i are measurable

↑ over all ads in that position.

↑ over specific advertiser

Question: What mechanism (allocation and payment) to use for the position auctions?

(1) Early position auctions received bids for impression (for just showing the ad) by sorting them by bid-per-impression in decreasing order. (All risk on advertiser)

(2) Bids were on clicks — pay-per-click model ranked by bid-per-click. (All risk on the website)

(3) Today the standard approach is to rank advertisers by the product of estimated CTR and bid value — rank by "expected revenue".

Search engines estimate the CTR_i
- denote this as $eCTR_i$

Let the bid b_i : denotes the amount agent i is willing to pay if a click occurs

Ads are ranked by the decreasing order of $eCTR_i \cdot b_i$.

Let $x = (x_1, \ldots, x_n)$ denote an allocation of ads to positions, $x_i \in M$ denoting the position assigned to agent i, $x_i \neq x_k$, $\forall i \neq k$, $i, k \in N$.

Say agent i reports his bid as b_i

Then $\hat{v}_i(x) = pos_{x_i}(eCTR_i \cdot b_i)$

is the reported value of the agent for the allocation x.

The premise is exactly same as a VCG mechanism.

The winner determination problem

allocation x s.t. it maximizes the overall reported value of the agents, i.e.

Solves $\max_x \sum_{i \in N} \hat{v}_i(x)$

denote the x that maximizes with x^*.