CS711: Introduction to Game Theory and Mechanism Design Jul-Nov 2018

Project: Assignment 2

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Question 1(a)

P_1	P_2	P_1'	P_2'
a	c	b	a
b	b	a	b
c	a	c	c

Given $f(P) = a$. As the SCF is both onto and strategy-proof, using the provided theorem $f(P') \in \{a, b\}$. Let us approach by contradiction and assume that $f(P') = a$.
Now take the preference profile:

P_1''	P_2''
b	c
a	a
c	b

Using the provided theorem $f(P'') \in \{b, c\}$. Let us assume b then player 2 can report P_2' and hence he will get a better payoff as $a > b$ in his preference order. Thus $f(P'') = c$. Now take the preference profile:

P_1'''	P_2'''
b	c
a	b
c	a

We know that $f(P'') = c$ and as dominated set of c is same for P'' and P''', $f(P''') = c$.
In this case player 1 misreports to be P_1 then he will get a better payoff as $a > c$ in his preference order.
Hence, there is a contradiction and $f(P') = b$

Question 1(b)

No, the earlier conclusion does not hold in the single-peaked preference domain. Because in the earlier proof we used the preference profile P_2'' but such preference profile is not possible in single-peaked preference domain as there are two peaks a, c in P_2''. Hence the earlier proof didn't go through.

Now, as the above conclusion is false, we can have a mechanism such that $f(P_1', P_2') = a$. As it is single-peaked preference domain we can take the minimum peak SCF. It is both onto and strategy-proof. So,

$$f(P) = min_{i \in N}\{P_i(1)\}$$

Here, minimum is with respect to the intrinsic ordering of the alternatives, which is $a < b < c$. Hence, $f(P_1, P_2) = a$ and $f(P_1', P_2') = a$.

Question 2

X is the set of all the projects. As the social choice function chooses a non-empty subset of projects, here the set of the choices/candidates of the players is the set of all the subsets of X i.e. the preferences are over the Power set of X.

As $|X| \geq 2$, there will be two sets $S, T \subset X$ such that $S \subset T$. Now from the definition of ranking we can see that, we can not have a scenario where S is strictly preferred over T. Because, if so then, the top ranked project in S P_i top ranked project in T. which is not possible as $S \subset T$. Hence there is a restriction on the domain of preferences over the power set of X.

Gibbard-Satterthwaite Theorem is applied when there is no restriction over the set of preferences and there is no case of indifference among the choices, which is not the case here. Hence, we can't apply Gibbard-Satterthwaite theorem in this scenario.

Question 3

We are considering the median voter Social Choice Function in the single peak model. We need to see if the median voter SCF is group strategy-proof.

Let us assume their are n players, i.e. the set of players is $\{1, 2, ..., n\}$ and their top preferences are $\{P_1(1), P_1(2), ..., P_i(1), ..., P_1(n)\}$. wlog assume that the median preference belongs to player i and $P_i(1) = a$. There is a group of agents $K \subseteq N$ who are trying to manipulate the result.

- All the players, j with $P_j(1) = a \notin K$ because if they are successful in manipulating, then the new preference b will be lower than a in their preference order.
- Case 1: $\forall i \in K \quad P_i(1) < a$. Now if they change their preference such that they still remain less than a then it is not going to change the median. If some of them report their top preference to be in the right of a. Now let $f(P_k^{`}, P_{-k}) = b > a$. For a $j \in K \quad P_j(1) < a < b$, as it is single peaked model so aP_jb. Hence no profitable manipulation.
- Case 2: $\forall i \in K \quad P_i(1) > a$. Now if they change their preference such that they still remain more than a then it is not going to change the median. If some of them report their top preference to be in the left of a. Now let $f(P_k^{`}, P_{-k}) = b < a$. For a $j \in K \quad b < a < P_j(1)$, as it is single peaked model so aP_jb. Hence no profitable manipulation.
- Case 3: $\exists \quad i, j \in K$ such that $\quad P_i(1) > a \quad$ and $\quad P_j(1) < a$. Now let us assume that after manipulating $f(P_k^{`}, P_{-k}) = b$. Take the case where $a < b$ then for $j \in K$ such that $P_j(1) < a$ this change is not beneficial as $P_j(1) < a < b$ hence, aP_jb. Now if $a > b$ then for $i \in K$ such that $P_i(1) > a$ this change is not beneficial as $P_i(1) > a > b$ hence, aP_ib.
  Hence, we can say that median voter SCF is group strategy-proof.