CS711: Introduction to Game Theory and Mechanism Design

Assignment 2

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1 Question 1

(a): in order to show $f(P_1^{'}, P_2^{'} = b)$

we can create more profile such as $(P_3, P_4) and (P_1, P_4)$

P_1	P_2	$P_1^{'}$	$P_2^{'}$	P_1	P_4	P_3	P_4
a	c	b	a	a	c	b	c
b	b	a	b	b	a	a	a
c	a	c	c	c	b	c	c

since if f is strategy proof then hence we can say that f is monotonic also by noting $f(P_1, P_2) = a$

therefore by moving from $(P_1, P_2) to (P_1, P_4)$ as f is monotonous, we can say that$f(P_1, P_4)$ should also be a

$f(P_3, P_4) = c$ is not possible as this contradicts the strategyproof condition given in the question as 1 can cheat and get a

therefore by using the data given in the question we can conclude that $(P_3, P_4) = b$

now if we move from (P_3, P_4) to $(P_1^{'}, P_2^{'})$ we can notice that choices for both is better and also knowing that $f(P_3, P_4) = b$ and since f in monotonous we can say that $f(P_1^{'}, P_2^{'}) = b$

(b): The earlier conclusion does not hold when the preference profiles are generated from single-peaked prefernce domain having intrensic ordering a¡b¡c because in our proof we generated $P_3 and P_4$ which we can not generate if there is single peekedness

now consider an SCF to compute $f(P_1^{'}, P_2^{'})$

Let R_1 be the 1st player and R_2 be the 2nd player and lets assume $R_1(1)$ gives us the first preference of player 1 and similarly $R_2(1)$ gives us the 1st preference of player 2

therefore,

$$f(R_1, R_2) = min[R_1(1), R_2(2)]$$

thus it forms Median Votes CHoice FUnction now the function being both onto and strategyproff we can conclude that if $R_1 = P_1^{'}$ and $R_2 = P_2^{'}$ then we have $f(P_1^{'}, P_2^{'}) = a$

2 Question 2

ACcording to the Gibbard Satterthwaite Theorem, if the social choice function for a set of alternatives A is onto and strategy proof then f is dictatorial

consider is S is equal to X and T is not equal to X , also its given in question that only if most high ranked project of S is preferred over most high ranked project of T then only S is preferred to T. hence, we can conclude that S will be as less preferred as much as T. hence we can say that its a restricted domain

hence, we cant apply Gibbard Satterthwaite here.

3 Question 3

yes, median voter scf is group strategy proof.

in order to prove this, Consider a function f, which will be our median voter social choice function. if we select set of all peaks and find the kth median from left, that what our function f will return .

now let the preferences of the group be given by P. therefore, $f(P) = a$

group of agents be k which is a subset of N. to prove median voter SCF is group strategy proof we need to proof it in 4 cases

case1: $P_i = aiK$. in this case as most preferred choice would be the SCF , any agent in the group k will not try to change anything,

case2: $P_i < aiK$ in this case if any agent in k group shifts his preference to left of a then nothing will happen but if agent changes to right then kth median will also move right but since preferences are single peeked then any agent in k wont have any beneficial change

case3: $P_i > aiK$ in this case if agent changes preference to right nothing will change but if shifts to left then median shifts leftwards , similar argument as case 2

case4: $P_i > a$ for some i in k and $P_i < a$ for others : in this case agents wont be able to help each other as if one gets beter then others will loose , hence agents wont be able to co operate

hence , median voter SCF is group strategy