CS711: Introduction to Game Theory and Mechanism Design Jul-Nov 2018

Project: Assignment 2

Question 1 :
1.a:
Note that $f(P') \in \{b, a\}$. Assume for contradiction that $f(P') = a$. Consider another

P_1'	P_2
b	c
a	b
c	a

Table 1.1: consider new preference profile P''

preference profile $P'' = (P_1', P_2)$. So, $f(P'') \in \{b, c\}$. Since $f(P') = a$, $f(P'') = a$ else agent 2 will manipulate at P'' via P'. Since $f(P) = a$, Agent 1 will manipulate at P'' via P. This is a contradiction.
Since f is strategy-proof, it implies that $f(P_1', P_2') = b$.
1.b:
Suppose that these preferences are generated from a single-peaked preference domain with the intrinsic ordering of the alternatives being $a < b < c$.
Earlier conclusion does *not* hold in this case.
explanation: since from single-peaked preference domain, a is more preferred than b.
proving: $f(P') = a$, $f(P'') = b$, agent 1 will manipulate from $P'' toP$
$f(P'') = c$, agent 2 will manipulate from $P'' toP'$
implies $f(P') = a$

Question 2:
The set of alternatives is the set of all subsets of objects: $S : S \subseteq X$. If there are at least 2 projects then, the set of alternatives is at least 3. Now, consider two alternatives S and T such that $S \subsetneq T$. By definition of the preference ordering, any agent is either indifferent between S and T or likes T to S. Hence, the preference ordering where S is ranked higher than T can never arise. This is a restriction of the domain, and the Gibbard-Satterthwaite result *cannot* apply here.

Question 3:
Yes, the median voter SCF is group strategy-proof. We can prove this is similar to the proof that shows that the median voter SCF is strategy-proof.
A group of agents can shift a median if they can shift their peak to the other side of the median, and this will shift the outcome to the other side, which this agent will not like.