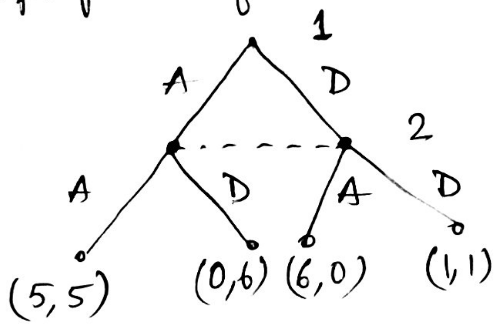
Expressive power of PIEFG is limited

Not every NFG can be represented in PIEFG

Imperfect Information EFG



1 \ 2	A	D
A	5,5	0,6
D	6,0	1,1

$$\langle N, A, \mathcal{H}, \mathcal{X}, P, (u_i)_{i\in N}, (I_i)_{i \in N} \rangle$$
$$(I_i) = \{I_i^1, \ldots, I_i^{k(i)}\}$$

Information set: I_i is a partition of $\{h \in \mathcal{H} \setminus Z : P(h) = i\}$ with the property that if $h, h' \in I_i^j$, then $\mathcal{X}(h) = \mathcal{X}(h')$.

- intuition of information set: player cannot distinguish between them, hence will have same actions.
- Information sets are always nonempty
- singleton information sets imply PIEFG.
- $\mathcal{X}$ can now be defined over information sets.

set of information sets of player i, $\mathbb{I}_i = \{I_i^1, \cdots, I_i^{k(i)}\}$

set of all information sets $\mathcal{I} = \cancel{\{\mathcal{I}_i : i \in N\}} = \bigcup_{i \in N} \mathbb{I}_i$

$\mathcal{X} : \mathcal{I} \to 2^A$

- ~~Player function also defined over info~~ sets

  ~~$P : \mathcal{I} \to N$~~

- Strategies of players: mapping from info sets to action sets

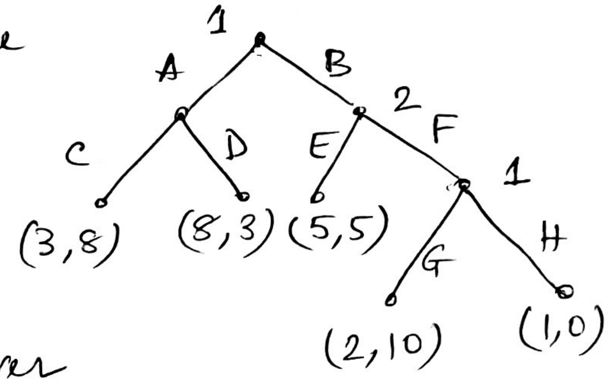
$$S_i = \bigtimes_{\tilde{I}_i \in \mathbb{I}_i} \mathcal{X}(\tilde{I}_i)$$

(9-2)

Representational Equivalence

- IIEFG is a richer representation than both NFG and PI EFG
- strategies can be richer in this game representation

game from last lecture



- Nash eq. talks about mixed strategies
- mixed strategies are probability distributions over the "complete contingency plans"

$0.6 \rightarrow (AG)$ and $0.4 \rightarrow (BH)$ - mixed st Pl 1

- However in IIEFG, we have the liberty of independently randomizing at every information set.

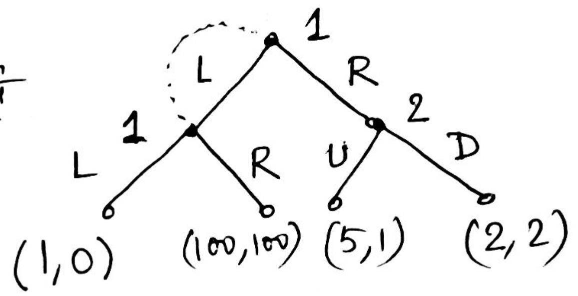
$0.3 : A$ at ϕ, $0.5 : G$ at (BF)

"behavioral strategies"

- Does mixed strategy look more general?

Ans: they're incomparable.

- IIEFG



NFG

	U	D
L	1,0	1,0
R	5,1	2,2

MSNE = WDSE : (R,D)

unique

Behavioral strategy:

$(p, 1-p)$ for (L,R) of player 1

D is a weakly dominant strategy of player 2.

Player 1's expected utility given player 2 always plays D : $p \cdot p \cdot 1 + p \cdot (1-p) \cdot 100 + (1-p) \cdot 2$

maximizing wrt p, $p = \frac{98}{198}$

Behavioral strategy equilibrium

$$\left(\left(\frac{98}{198}, \frac{100}{198} \right), (0,1) \right)$$

- What's wrong? Player 1 "forgets" what he played after playing L in ϕ.
- Class where mixed and behavioral strategies coincide :

Games of perfect recall :

[in words] at every opportunity to act, each player remembers perfectly what he did in previous rounds.

[formally] Player i has perfect recall in an IIEFG if for any two histories h and h' having in the same info set of i

$h = (v_0, a_0, v_1, a_1, \ldots, v_{n-1}, a_{n-1}, v_n)$ $\quad$ v_i's vertices

$h' = (v_0', a_0', v_1', a_1', \ldots, v_{m-1}', a_{m-1}', v_m')$ $\quad$ a_i's actions

it must be the case that

(1) $m = n$, length are same

(2) $\forall\ 0 \leq j \leq n-1$ $\cdot$ v_j and v_j' must be in the same information set of player i.

(3) $\forall\ 0 \leq j \leq n-1$ if $P(h_j) = i$ (h_j is the truncated history at length j from root)

$a_j = a_j'$ actions are same.

The game is of perfect recall if every player has perfect recall.

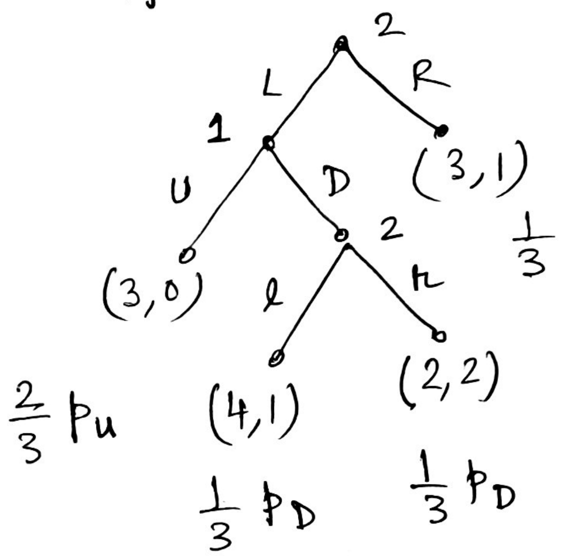
(9-4)

Observe: Every PIEFG is a game with perfect recall.

A behavioral strategy b_i and a mixed strategy σ_i are <u>outcome equivalent</u> if for every mixed strategy σ_{-i} of other players, the probability distributions induced over the terminal vertices by (b_i, σ_{-i}) and (σ_i, σ_{-i}) are the same.

<u>Theorem (Kuhn 1953)</u>: In a game with perfect recall, every mixed strategy is outcome equivalent to behavioral strategies.

- Pf. omitted, is constructive, illustrate with example.



$\sigma_2(Ll) = \sigma_2(Lr) = 1/3$

$\sigma_2(Rl) = \frac{1}{12}$, $\sigma_2(Rr) = \frac{1}{4}$

need an outcome equiv. behavioral str

mixed strategy of player 1

p_U and p_D

$b_2(L) = \frac{2}{3}$ $\quad b_2(R) = \frac{1}{3}$

$b_2(l) = \frac{1}{2}$ $\quad b_2(r) = \frac{1}{2}$

$b_2(R) = \sigma_2(R) = \sigma_2(Rl) + \sigma_2(Rr)$

$b_2(L) = \sigma_2(L) = \sigma_2(Ll) + \sigma_2(Lr)$

$b_2(L)\, p_U = \sigma_2(L)\, p_U$

$b_2(l)\, p_D\, \sigma_2(L) = \sigma_2(Ll)\, p_D$

$\Rightarrow b_2(l) = \dfrac{\sigma_2(Ll)}{\sigma_2(Ll) + \sigma_2(Lr)}$