How to maximize the revenue earned by the auctioneer?

maximize w.r.t. What knowledge of the auctioneer? — The common prior distribution over the types.

Accordingly, The notions of incentive competibility and individual reationality have to change.

Bayesian Incentive Compatibility

 $T_i = [0,b_i]$, common prior G over $T = \overset{\infty}{X} T_i - g$ denotes the density $G_i(\underline{s}_i|s_i)$ is the conditional distribution over \underline{s}_i , given i's type is \underline{s}_i . similarly $g_{-i}(\underline{s}_i|s_i)$ is derived via Bayes rule from g.

Every mechanism $(f, p_1, ..., p_n)$ induces an expected allocation and payment trule $(x, \underline{\pi})$

$$X_{i}(S_{i}|t_{i}) = \int_{S_{i}} f_{i}(S_{i},S_{i}) g_{-i}(S_{i}|t_{i}) dS_{i}$$

reported true allocation common prior two levels of expectation

expected payment

$$\pi_{i}(A_{i}|t_{i}) = \int_{\mathcal{L}_{i} \in \mathcal{I}_{i}} p_{i}(A_{i},A_{i}) g_{i}(A_{i}|t_{i}) dA_{i}$$

Expected utility of agent i $t_i \, \alpha_i \, (t_i | t_i) - \pi_i (t_i | t_i)$

Defin: A mechanism (f, p) in Bayesian incentive compatible (BIC) if $\forall i \in \mathbb{N}$, $\forall s_i, t_i \in T_i$ $t_i \propto_i (t_i | t_i) - \pi_i (t_i | t_i) \gg t_i \propto_i (s_i | t_i) - \pi_i (s_i | t_i)$.

Similarly, f is Bayesian implementable if $\exists \not \models s.t. (f, \not \models)$ is BIC

Independence and Characterization of BIC mechanisms

Assume that the priors are independent, i.e., agent i's value is drawn from a distribution G_i (density g_i) independently from other agents.

$$G(A_1, A_2, ..., A_m) = \underset{i \in N}{\text{TT}} G_i(A_i)$$

$$G(A_i | t_i) = \underset{j \neq i}{\text{TT}} G_j(A_j)$$

We will use the shorthand $\alpha(t_i) = \alpha(t_i|t_i)$

Defin: An allocation rule is non-decreasing in expectation (NDE) if $\forall i \in \mathbb{N}$, $\forall s_i, t_i \in T_i$ with $s_i < t_i$ we have $\alpha_i(s_i) \leq \alpha_i(t_i)$.

Note: The nules that are non-decreasing (defined before) are always NDF. But there can be more nules that are NDF.

Characterization of BIC rules

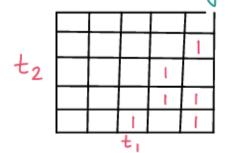
Theorem: A mechanism (f, p) in the independent prior setting in BIC if in NDE, and

2)
$$p_i$$
 satisfies $\pi_i(t_i) = \pi_i(0) + t_i \alpha_i(t_i) - \int_0^{t_i} \alpha_i(z) dz$
 $\forall t_i \in T_i, \forall i \in N.$

Remark: Bayesian version of the earlier theorem

Proof: in similar lines as before [exercise]

An allocation rule may be NDE but not non-decreasing.



all 5 types are equally likely $\alpha_1(t_1)$ and $\alpha_2(t_2)$ are monotone but $f(t_1,t_2)$ is not.

As we are in the Bayesian setting now, we can define an analog of individual nationality

Defn: A mechanism (f, p) is interim individually reational (IIR) if for every bilder $i \in N$, we have

$$t_i \alpha_i(t_i) - \pi_i(t_i) > 0 \quad \forall t_i \in T_i$$

Lemma: A mechanism (f, +) is BIC and IIR iff

(f in NDE,

2) p_i satisfies $\pi_i(t_i) = \pi_i(0) + t_i \, \alpha_i(t_i) - \int_0^{t_i} \alpha_i(z) \, dz$ $\forall t_i \in T_i, \forall i \in N.$

3 $\forall i \in N$, $\pi_i(o) \leq 0$.

Proof sketch: 1) and 2) uniquely identify a BIC mechanism. So, The proof requires to show that IIR along with 1) and 2) are equivalent to 3)

 (\Rightarrow) apply IIR at $t_i=0$ on (2) and get $\pi_i(0) \leq 0$

 $(\Leftarrow) \quad t_i \propto_i (t_i) - \pi_i(t_i) = -\pi_i(0) + \int_0^{t_i} \alpha_i(s_i) ds_i \geqslant 0 \quad \forall \quad \pi_i(0) \leqslant 0$