Assignment 2

Rudranil Bhowmik(18111055)
CS711 - Introduction to Game Theory and Mechanism Design

November 6, 2018

1 Question 1.

1.1 Part a

From the preference table by monotonicity we know that
$f(\acute{P_1}, \acute{P_2}) \in b, a.$
Now for contradiction lets assume
$f(\acute{P_1}, \acute{P_2}) = a$
Let us consider a different preference profile
$P'' = (\acute{P_1}, P_2)$
The preference profile table would be
:

P_1'	P_2
b	c
a	b
c	a

P_1' can be manipulated by changing the preference order to P_1 then f(P") will reduce to f(P)=a but as f is strategy-proof it contradicts the fact as strategy proof function cannot be manipulated.Thus it is a contradiction to fact f(P")=c(as assumed by the case above) and thus f(P")=b.
Let another preference profile P"' such that:

P_1	P_3
b	c
a	a
c	b

Here by the result given in question f(P_1, P_3) $\in \{b, c\}$.
Preference profile P"' follows monotonicity with preference profile P" as c is least preferred by P_1 in both case and similar goes with most preffered.
Thus $f(P''') = f(P'') = b$
Now for the main question:- From the previous iteration we know that f(P') $\in \{a, b\}$.

Assume f(P_1', P_2')=a.
P_2 can manipulate the result by changing its preference order to P_3 then f(P") will reduce to f(P)=b but as f is strategy-proof it contradicts the fact as strategy proof function cannot be manipulated.Thus it is a contradiction to fact f(P')=a and thus f(P')=b.

1.2 Part b

Now by question preferences are generated from a single-peaked preference domain. The earlier conclusion does not hold in this case as the order which assumed P_3 in part (a) which assumes a ordering b<a<c which is not possible in a single peaked preference domain and heathers the case invalid.
Now for proving f(P_1', P_2')=a // Lets use the median voting SCF with a phantom peak which is just on the left of a.In that case f(P')= a as we have peaks at left of a(phantom peak),a and b(greater than a) and thus a is the median peak and hence answer to this SCF.

2 Question 2

The set of alternatives is represented by $S : S \subseteq X$
Now if $|X| \geq 2$ then the number of $|A| \geq 3$
And S and T are two alternatives such that $S \subset T$ and $S, T \subseteq X$.
Now for S is indifferent to T if the highest preference of $S \cup T$ lies in $S \cap T$ this holds.
T P_i S- This will be possible when S doesn't contains highest priority element of S ∪ T and T contains the highest priority element of S ∪ T.
Now S P_i T can never arise as the inverse can't be true due to the assumption.

Hence the preference order where S is ranked higher than T never arise now this is a restriction of domain. Thus we cannot apply the Gibbard-Satterthwaite result here.

3 Question 3

To prove median voter SCF group strategy-proof.
For the question we have to prove that the social choice function f is group strategy-proof that is it cannot be manipulated by group of agents.
Here we consider only the peak preference of all agents and let $f(P) = a \in A$ A social choice function f is manipulable by a group of agents K ⊆ N if for some preference profile (P_K, P_K) there exists some preference profile P_K of agents in K .Here P=(p_1,p_2,p_3,...,p_i,....,p_n) and P_k profile of k_{th} agent.
Let us select a set of voters out of which some are peaked at point on left of median voter and a few peaked at point on right of median voter and rest peaked at median point itself:
1.Ones peaked at median position-They will not deviate as the outcome is what they wanted.
2.Ones peaked to left of median voter-If they try to deviate to left side then median voter doesn't changes.When they try to shift to right of median voter then only median will deviate.A group of agents can shift a median if they can shift their peak to the other side

of the median, and this will shift the outcome to the other side, which is not favourable according to them.
3.Subjects peaked to right of median voter-Similar case as the peaked to left side of median and thus will not try to manipulate.
Thus median voter SCF is group strategy proof.