Discussions:

1. Payment formula:

$$p_i(t_i, t_{-i}) = p_i(0, t_{-i}) + t_i f_i(t_i, t_{-i}) - \int_0^{t_i} f_i(x_i, t_{-i})\, dx_i$$

payment differs only by a constant

$$p_i(t_i, t_{-i}) - p_i(s_i, t_{-i}) = q_i(t_i, t_{-i}) - q_i(s_i, t_{-i})$$

holds for any payments p, q that make f DSIC

$$p_i(t_i, t_{-i}) = q_i(t_i, t_{-i}) + h_i(t_{-i})$$

property known as <u>revenue equivalence</u>.

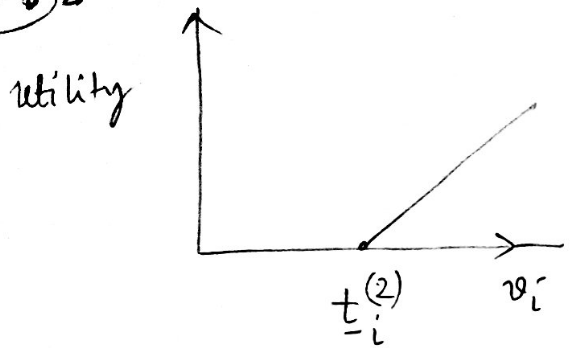
2. Difference with Roberts' result. Roberts' gives a functional form of the allocation function. While Myerson gives an implicit property. It is easier to answer other questions using a functional form rather than an implicit characterization.

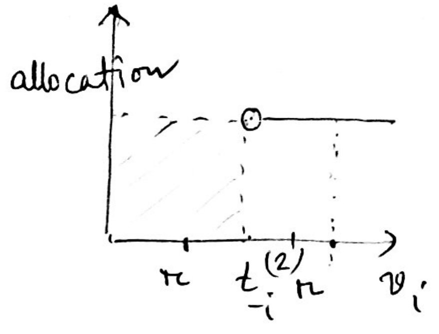
3. Corollary: An allocation rule is implementable iff it is non decreasing.

Examples:

1. Constant allocation rule — non-decreasing
   payment: constant (which can be zero)
2. Dictatorial: Give the object only to the dictator
   — non-decreasing, payment again constant/zero.
3. Second Price auction:
   allocation: efficient, give the object to the ~~most~~ agent who values the most. In case of ties, any allocation works. Recall the figure of utility and the allocation — subgradient at the breakpoint.

(34-2)





allocation is a subgradient of the utility. Clear when $v_i < t_{-i}^{(2)}$ and $v_i > t_{-i}^{(2)}$. When $v_i = t_{-i}^{(2)}$ any number between 0 to 1 is a subgradient. Hence the allocation can be anything. ~~Payment~~

Payment: Only one agent whose $v_i \geq t_{-i}^{(2)}$ and pays the integral $\left(p_i(0,t_{-i}) = 0\right)$ - which is $t_{-i}^{(2)}$ the second highest bid.

(4) Efficient allocation with a reserve price is also non-decreasing. In this allocation, if the highest value is below the reserve price, nobody gets the object. ~~The item goes~~ Otherwise, the item goes to the highest bidder.

Bidder i gets the item if $v_i > \max\{t_{-i}^{(2)}, r\}$

The payment is also $\max\{t_{-i}^{(2)}, r\}$.

(5) Not so common allocation rule: $N = \{1, 2\}$

$A = \{a_0, a_1, a_2\}$, a_0: object unsold

a_i: agent i gets the object, $i = 1, 2$.

Given a type profile $t = (t_1, t_2)$ the seller computes

$U(t) = \max\{2, t_1^2, t_2^3\}$

a_0 if $U(t) = 2$

a_1 if $U(t) = t_1^2$

a_2 if $U(t) = t_2^3$

This allocation rule is non-decreasing

Player 1 if $t_2^2 > \sqrt{\max\{2, t_2^3\}}$

wins

Player 2 if $t_3 > \sqrt[3]{\max\{2, t_1^2\}}$

Individual Rationality

Defn: A mechanism (f, p) is ex-post individually rational if $\forall t_i \in T_i \ \forall t_{-i} \in T_{-i} \ \forall i \in N$

$$t_i f_i(t_i, t_{-i}) - p_i(t_i, t_{-i}) \geq 0 .$$

Ex-post: Even after all agents have revealed their types, ~~ke~~ participating was weakly preferable.

Lemma: Suppose a mechanism (f, p) is DSIC. It is

(1) IR iff $\forall i \in N$ and $\forall t_{-i} \in T_{-i}$

$$p_i(0, t_{-i}) \leq 0 \qquad \text{—— (1)}$$

(2) A mechanism (f, p) is DSIC, ~~and~~ IR and satisfies no-subsidy, i.e., $p_i(t_i, t_{-i}) \geq 0 \ \forall t_i, \forall t_{-i}, i \in N$ iff $\forall i \in N$, $t_{-i} \in T_{-i}$

$$p_i(0, t_{-i}) = 0 . \qquad \text{—— (2)}$$

Comment: Revenue equivalence - exact.

Proof: Part 1: Suppose (f, p) is ~~DSIC and~~ IR, then

$$0 - p_i(0, t_{-i}) \geq 0 \Rightarrow p_i(0, t_{-i}) \leq 0$$

conversely, ~~te of~~ if $p_i(0, t_{-i}) \leq 0$, the payoff of agent i is

$$t_i f_i(t_i, t_{-i}) - p_i(t_i, t_{-i})$$

$$= \cancel{t_i f_i(t_i, t_{-i})} \underbrace{- p_i(0, t_{-i})}_{\geq 0} - \cancel{t_i f_i(t_i, t_{-i})} + \underbrace{\int_0^{t_i} f_i(x_i, t_{-i})\, dx_i}_{\geq 0}$$

$$\geq 0$$

(f, p) is IR.

Part 2: IR ~~rega~~ is equivalent to $p_i(0, t_{-i}) \leq 0$,

if $p_i(t_i, t_{-i}) \geq 0 \ \forall t_i, t_{-i} \ \forall i \Rightarrow p_i(0, t_{-i}) = 0 \ \forall t_{-i}, i \in N$

clearly $p_i(0, t_{-i}) = 0$ implies that (f, p) is IR and ~~non~~ no-subsidy.

(34-4)

Some more non-Vickrey auctions

(1) Consider a case where the mechanism is Groves but not Vickrey auction. The object goes to the highest bidder, but the payment is such that everyone is compensated some amount.

- highest and second highest bidders are compensated $\frac{1}{n}$ of the third highest ~~value~~ bid
- everyone else receives $\frac{1}{n}$ of the second highest bid

$$p_i(0, \underline{t}_{-i}) = -\frac{1}{n} \text{ second highest of } \{t_j, j \neq i\}$$

[for Vickrey auction, $p_i(0, \underline{t}_{-i}) = 0 \ \forall \underline{t}_{-i} \ \forall i \in N$]

WLOG, $t_1 > t_2 > \cdots > t_n$

agent 1 pays $= -\frac{1}{n} t_3 + t_1 - \int_0^{t_1} f_1(x, \underline{t}_{-1})\, dx$

$= -\frac{1}{n} t_3 + t_2$

2 pays $= -\frac{1}{n} t_3$

all others pay $= -\frac{1}{n} t_2$

Sum of payments $= -\frac{1}{n} t_3 + t_2 - \frac{1}{n} t_3 - \frac{n-2}{n} t_2$

$= \frac{2}{n}(t_2 - t_3) \xrightarrow{n \to \infty} 0$

asymptotically all surplus is redistributed.

Deterministic one.

(2) Allocate the object w.p. $(1-\frac{1}{n})$ to the highest bidder
$(\frac{1}{n})$ to the second highest bidder.

$$p_i(0, \underline{t}_{-i}) = -\frac{1}{n} \text{ second largest bid in } \{t_j, j \neq i\}$$

Not a Groves mechanism. [Green-Laffont]

1 pays $= -\frac{1}{n} t_3 + (1-\frac{1}{n}) t_1 - \frac{1}{n}(t_2 - t_3) - (1-\frac{1}{n})(t_1 - t_2)$

$= (1 - \frac{2}{n}) t_2$

2 pays $= -\frac{1}{n} t_3 + \frac{1}{n} \cdot t_2 - \frac{1}{n}(t_2 - t_3) = 0$

all else $= -\frac{1}{n} t_2$ total $= 0$.