CS711A Assignment 2

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1 Question 1

1.1

$P_1\prime$	$\hat{P}_2$
b	c
a	a
c	b

The function is Strategy Proof and onto $\{a, b, c\}$ and also $f(P) \in \{P_1(1), P_2(1)\}$ s. t. $f(P_1, P_2) = a$.
We need to show that $f(P_1', P_2') = b$.
Going be contradiction, assume that $f(P_1', P_2') = a$. Consider a preference profile $(P_1', \hat{P}_2)$ as follows:
If $f((P_1', \hat{P}_2) = b$, then Player 2 manipulates from P_2' to $\hat{P}_2$ to change the outcome to a. Knowing it to be strategy proof, the value has to c.
For $(P_1', \hat{P}_2) \rightarrow (P_1, P_2)$, $D(c, P_1) = D(c, P_1') = \phi$, and $D(c, P_2) = D(c, \hat{P}_1) = \{a, b\}$.
Since $f(P_1', \hat{P}_2) = c$, we have $f(P_1, P_2) = c$, because the stategy proof social choice function is monotonic.

But since $f(P_1, P_2) = a$, we arrive at a contradiction. Hence, our assumption is false and value is b.

1.2

Supposing these preferences are generated from a single-peaked preference domain with the intrinsic ordering of the alternatives being $a < b < c$, it doesn't hold. It is because for $\hat{P}_2$, b cannot be the lowest preference since it makes it single peaked.
Since the conclusion is false, a mechanism that can have f(P1,P2) = a can be done using the Median Voter Social Choice Function (MVSCF). As every MVSCF is Strategy Proof and onto, f is the same.

Now consider, MV SCF selects the leftmost peak among the preference profiles. In our case, a is that corresponding peak.