Games : Strategic interaction between decision making agents that are rational and intelligent.

Game Theory : Study of such interactions

Example : Prisoner's dilemma

Agent perspective

Predictive guarantees

1 \ 2	~~Confess~~ Cooperate	Defect
C	−2, −2	−10, 0
D	0, −10	−5, −5

Mechanism Design : Designer's perspective

Prescriptive guarantees

Example : Two women claiming to be the mother of a child − comes to the king to decide.

The women perfectly know the truth − the king does not. Can he design a "game" such that its outcome is the desired one − true mother gets the child.

Solution given in the mythology : King orders to cut the child and give to each of them.

Real mother withdraws her claim.

King awards her the child.

That is an example of a mechanism.

Flaw : The mechanism does not commit to its declared principle of operation.

The mechanism will not work a second time.

(1-2)

A mechanism must be committed to its rules and must be reproducible.

Proposal 2: If every woman was given an option to pay for the baby.

Rule: Highest bidder wins and pays the losing bid.

Assuming that the real mother's value for the child is much higher than the value of the false mother

- the child goes to its rightful mother and at a small payment.
- But this is reproducible and committed to its rules.

Mechanism: Rules of a game such that desired outcomes result in the equilibrium of the game.

Formal model:

$N = \{1, 2, \dots, n\}$ set of agents/players

X = set of outcomes

θ_i : private information of agent i, also called type.

Θ_i : set of all possible types of i.

$u_i : X \times \Theta_i \to \mathbb{R}$ utility of agent i

Examples:

(1) Voters satisfaction with candidates denoted by θ_i. ~~Say~~ for agent 1, value of 1 when a is elected is $u_1(a, \theta_1)$ which is more than $u_1(b, \theta_1)$ and $u_1(c, \theta_1)$, $u_1(a, \theta_1) > u_1(b, \theta_1) > u_1(c, \theta_1)$

If type changes, the utility changes too

$u_1(c, \theta_1') > u_1(b, \theta_1') > u_1(a, \theta_1')$

② Single object allocation (indivisible)

outcome $x \in X$ is a tuple (a, p)

a is the allocation and p is the payment vector.

θ_i denotes agent i's value for the object.

$$a = (a_1, \dots, a_n), \quad p = (p_1, \dots, p_n)$$

$$a_i \in \{0,1\}, \quad \sum_{i=1}^{n} a_i \leq 1$$

$$u_i((a,p), \theta_i) = a_i \theta_i - p_i$$

The objective of a mechanism is captured via a social choice function (SCF).

$$f : \Theta_1 \times \Theta_2 \times \dots \times \Theta_n \to X$$

The planner/social decision maker wants to take this decision — given a type profile take a certain outcome — but he does not know the true types — private to the agents. Therefore a mechanism is needed to uncover the types truthfully — we call that a mechanism <u>implements</u> a social choice function.

Back to the King-Baby example. The types are the true mother's identity and the goal is to <u>assign the baby to the true mother</u>, but (θ_1, θ_2) are unknown to the king — hence he can ~~take ad~~ use the mechanism second price auction to implement this SCF.

(1-4)

Mechanism

Defn: A mechanism is a collection of message spaces and a decision rule, ~~...~~ $\langle M_1, M_2, \dots, M_n, g\rangle$

- M_i is the message space for agent i.
- $g : M_1 \times M_2 \times \dots \times M_n \to X$

A mechanism is called direct when $M_i = \Theta_i$, $g = f$.

Defn: In a mechanism $\langle M, g\rangle$, a message m_i is weakly dominant for agent i at θ_i if

$$u_i(g(m_i, m_{-i}), \theta_i) \geq u_i(g(m_i', m_{-i}), \theta_i) \quad \forall m_i' \in M_i, \ \forall m_{-i} \in M_{-i}$$

Defn: A SCF $f : \Theta \to X$ is implemented in dominant strategies by $\langle M, g\rangle$ if

(1) $\exists$ message mappings $m_i : \Theta_i \to M_i$ s.t. $m_i(\theta_i)$ is a dominant strategy for agent i at θ_i, $\forall \theta_i \in \Theta_i$ $\forall i \in N$

(2) $g(m_i(\theta_i), m_{-i}(\theta_{-i})) = f(\theta_i, \theta_{-i})$, $\forall \theta \in \Theta$.

f is dominant strategy implementable by $\langle M, g\rangle$.

Defn: A direct mechanism is strategyproof (or dominant strategy incentive compatible, DSIC) if

$$u_i(f(\theta_i, \theta_{-i}), \theta_i) \geq u_i(f(\theta_i', \theta_{-i}), \theta_i) \quad \forall \theta_i, \theta_i' \in \Theta_i, \ \forall \theta_{-i} \in \Theta_{-i}, \ \forall i \in N.$$

Revelation principle: If f is DSI, then f is DSIC.