CS711: Introduction to Game Theory and Mechanism Design **Jul-Nov 2018**

Project: Assignment 2

Name : Madhukant **Roll No. :** 150368

Question 1

part(a)

P_1 P_2	P_1'' P_2''	P_1' P_2''
a c	b c	b a
b b	a a	a b
c a	c b	c c

Given SCF f is strategyproof we have to show $f(P_1', P_2') = b$

Consider a preference profile $P'' = (P_1'', P_2'')$ as shown above.

Since , $f(P_1'', P_2'') \in \{b, c\}$ (given in question)

Assume $f(P_1'', P_2'') = c$

However in preference profile (P_1, P_2), c has same dominated set as the dominated set in $f(P_1'', P_2'')$
Since f is Strategyproof $\implies$ f is Monotone Thus, if $f(P_1'', P_2'') = c$ then $f(P_1, P_2) = c$ as they have same dominated set.
which is not true as $f(P_1, P_2) = c$ is given. Hence, $f(P_1'', P_2'') \neq c$

Thus, $f(P_1'', P_2'') = b$

Now, dominating set of b in P' is same as dominated set of b in P''. Hence, $f(P_1', P_2') = b$ by Monotonicity

part(b)

Given intrinsic order $a < b < c$

Given $f(P_1, P_2) = a$ that means SCF is picking left most peak

Now, since SCF is picking leftmost peak, and $f(P_1', P_2') = min(P_1'(1), P_2'(1)) \implies f(P_1', P_2') = a$. Hence, earlier conclusion does not hold in this case.

Reason is that we choosed (P_1'', P_2'') as preference profile in which preference of P_2 is c, a, b but since $a < b < c$ thus this can't be a preference as preference can be single peaked only, here we have preference which is not a valid single peak preference.

Mechanism in which $f(P_1', P_2') = a$ can be - choosing left most peak as winner. Since, $a < b < c$. Therefore, if one has given highest preference of a then it will be chosen as winner as a is the left most element in all preferences. Hence $f(P_1', P_2') = a$ as $f(P_1', P_2') = min(P_1'(1), P_2'(1))$

Question 2

Let X is set of projects $X = \{x_1, x_, ..., x_n\}$

And, SCF $f : X \to \mathcal{P}(X)$

when Agent i evaluates subsets according to given manner in question , we can convert linear ordering of her preference P_i into linear order of of preference over $\mathcal{P}(X)$.

Example of how a preference profile of player in X can be converted to preference profile in $\mathcal{P}(X)$ as -

Let the $X = \{x_1, x_, ..., x_n\}$ be the set of projects. Total no. of non-empty subsets of X = $2^n - 1$

Wlog, Let agent j has preferencce profile $x_1 > x_2 > ... > x_n$. Now this can be converted to a corresponding preference in $\mathcal{P}as-$
$\{x_1\}, \{x_1, x_2\}, \{x_1, x_2, x_3\}...\{x_1, x_2, ...x_n\}, \{x_2\}, \{x_2, x_3\}...\{x_n\}$

i.e. $\{x_1\} = \{x_1, x_2\} = \{x_1, x_2, x_3\}...\{x_1, x_2, ...x_n\} > \{x_2\} = \{x_2, x_3\}...\{x_n\}$
Here $set_1 = set_2 \implies set_1 I_j set_2$ and $set_1 > set_2 \implies set_1 P_j set_2$

Now we have to check whether Gibbard-Satterthwaite(GS) theorem can be applied or not.

For GS to hold one condition of it is that all preference profile should be possible i.e. set of preferences should not be restricted. But in our setting we can find an preference setting which can't be a preference of any agent.
Lets, Consider an ordering a player k -
$...\{x_3\}, \{x_1, x_2\}, \{x_1, x_3\}...$

since $\{x_3\}$ is preferred or indifferent to player k than $\{x_1, x_2\}$ that means maximum preferred project in $\{x_3\}$ which is x_3 is more preferred than maximum preferred project in $\{x_1, x_2\}$. Let Wlog x_1 is more preferred than x_2 i.e. x_3 is more preferred than x_1

but in $\{x_1, x_2\}, \{x_1, x_3\}$ maximum preferred project in $\{x_1, x_2\}$ which is x_1 is more preferrable than maximum preferred project in $\{x_1, x_3\}$ which is x_3 which is a contradiction as just above we showed player k prefers

x_3 over x_1 hence, this can't be a valid preference in any preference of preference domain.
Thus, preferences with these ordering can't exist. Hence, all preference profiles are not possible in this setting thus Gibbard-Satterthwaite theorem can't be applied here as it require preference domain to be unrestricted.

Question 3

Suppose K agents wants to manipulate median voter SCF.
Now, Let median voter SCF picks median of peaks and assign it as winner.

Out of the K agents who wants to manipulate the median voter SCF, all can be on left side, right side of median voter in preference line or some on left and some on right.

Case 1 : all agents lie on right side of median voter

to manipulate SCF f a non-empty subset should move from right side of median to left side of median to change the median. as shifting peaks within RHS of median won't change the median and hence utility will be same.

In this case median will be shifted to left of the original median voter and hence the winner will be on left. As all of them have decreasing utility on left of their peak thus making winner to left peak will decrease their utility further than what they have in original median. which will not be acceptable for them as their utility is decreasing by this.

Case 2: all agents lie on left side of median voter

To, change the median of preference a non-empty subset of $P' \subseteq K$ should move from left to right of median. Using similar logic as in previous case, going right will shift the median to right and hence further decrease their current utility as their utility decrease on going right. Hence, they can't manipulate the SCF f when all are on left side of current median.

Case 3: K_1 agents have peak toward left of median and K_2 agent have peak toward right of median where $0 < K_1 < K$ and $0 < K_2 < K$ and $K_1 + K_2 = K$.

Now, K_1 people shifting their peaks within themselves won't effect the median and similarly for K_2 people on right. Thus, few people have to go from left of median to right of median and few from right to left. Let P_1' goes from left of median to right of median and P_2' goes from right of median to left of median.

If $P_1' = P_2'$ then median won't change and hence, utility for them remain same. Thus, to manipulate the SCF f, $P_1' \neq P_2'$ Hence, effectively a subset of people goes from right to left or from left to right.

From above two cases we have proved that if a subset of player change their peak from left to right or right to left then they can't increase their utility and hence can't manipulate the SCF f.
thus, shifting of uneven people from left to right and right to left will decrease their utility as effectively a subset P'' will change their peak from one side to another which will decrease their individual utility (proved in above cases).

Thus by any means a group of player can't manipulate the median voter SCF. Hence , median voter SCF is **group strategy-proof**