Lecture 27: 13th October2017

Lecturer: Swaprava Nath *Scribe(s): Sudhir Kumar*

27.1 Recap

In the last lecture we looked into uniform rule SCF and proved that uniform rule SCF is PE,ANON and SP and also found out that if a SCF is PE, ANON and SP then it is uniform rule SCF. We also looked into Mechanism of transfer and Quasi Linear Preferences. A SCF can be defined as,

$$F = (f, \underline{\mathrm{p}})$$

$$F : \Theta \longrightarrow X$$

$$whereXis(a, \pi)$$

allocation function:

$$f : \theta \longrightarrow A$$

payment function:

$$p_i : \theta R$$

27.2 Example of Allocation function

- Constant Rule: Where allocaton funtion is constant for all θ.

$$f^c(\theta) = a, a\epsilon A$$

- Dictatorial Rule: In this rule a player dictatorialy choose best outcome for himself.

$$f^d(\theta)\epsilon \operatorname*{argmax}_{a\epsilon A} v_d(a, \theta_d), \exists d\epsilon N$$

- Allocatively efficient rule/ Utilitarian rule:

$$f(\theta)\epsilon \operatorname*{argmax}_{a\epsilon A} \sum_{i\epsilon N} v_i(a, \theta_i)$$

- Weighted efficient rule:

$$f(\theta)\epsilon \operatorname*{argmax}_{a\epsilon A} \sum_{i\epsilon N} w_i v_i(a, \theta_i), w_i \geq 0(whereallw_iisnotzero)$$

- Max-Min/Egalitarian/Ravtsian allocation rule:

$$f^r(\theta)\ \epsilon\ \operatorname*{argmax}_{a\epsilon A} \min_{i\epsilon N} v_i(a, \theta_i)$$

- Affine maximizer rule:

$$f_{AM}\ \epsilon\ \operatorname*{argmax}_{a\ \epsilon\ A}[\sum_{i\ \epsilon N} \lambda_i v_i(a, \theta_i) + \kappa(a)], where\lambda_i \geq 0(notallzero)$$

27.3 Examples of payment rules

- Weak budget balanced/ no-deficit/ feasible

$$\sum_{i\ \epsilon\ N} p_i(\theta) \geq 0, \forall \theta \epsilon \Theta$$

- No subsidy

$$p_i(\theta) \geq 0, \forall i \epsilon N and \forall \theta \epsilon \Theta$$

- Budget balanced

$$\sum_{i \epsilon N} p_i(\theta) = 0$$

We are more interested in finding an allocation that is "truthful" even if there is no restriction on payments.

27.4 Incentive compatibility

$$\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_n)$$

$$v_i(f(\hat{\theta}), \theta_i) - p_i(\theta)$$

(payoff for agent i)

Definition: A direct mechanism $(f, \underline{p})$ is domination strategy incentive compatible(DSIC) if,

$$\forall i \epsilon N, \forall \theta_{-i} \epsilon \Theta_{-i} and \forall \theta_i, \hat{\theta}_i \epsilon \Theta_i$$

$$v_i(f(\theta_i, \theta_{-i}), \theta_i) - p(\theta_i, \theta_{-i}) \geq v_i(f(\hat{\theta}_i, \theta_{-i}), \theta_i) - p(\hat{\theta}_i, \theta_{-i})$$

Note: Reporting types truthfully id WDSE. if the above mention condition holds then,$\underline{p}$ implements f in dominant strategy.

Example: Consider,

$$N = 1, 2, \theta_1 = \theta_2 = \theta^H, \theta^L$$

$$f : \theta_1 \times \theta_2 \longrightarrow A$$

is allocation rule. If $\underline{p}$ implements f in dominant strategy, then For Player 1:

$$v_i(f(\theta_H, \theta_2), \theta_H) - p(\theta_H, \theta_2) \geq v_i(f(\hat{\theta}_L, \theta_2), \theta_H) - p(\hat{\theta}_L, \theta_2),\ \forall \theta_2 \epsilon \Theta_2$$

$$v_i(f(\theta_L, \theta_2), \theta_L) - p(\theta_L, \theta_2) \geq v_i(f(\hat{\theta}_H, \theta_2), \theta_L) - p(\hat{\theta}_H, \theta_2),\ \forall \theta_2 \epsilon \Theta_2$$

Similarly for player 2,

$$v_i(f(\theta_H, \theta_1), \theta_H) - p(\theta_H, \theta_1) \geq v_i(f(\hat{\theta}_L, \theta_1), \theta_H) - p(\hat{\theta}_L, \theta_1),\ \forall \theta_1 \epsilon \Theta_1$$

$$v_i(f(\theta_H, \theta_1), \theta_H) - p(\theta_H, \theta_1) \geq v_i(f(\hat{\theta}_L, \theta_1), \theta_H) - p(\hat{\theta}_L, \theta_1),\ \forall \theta_1 \epsilon \Theta_1$$

27.5 Impact of DSIC on payments

1. Change in payment if we add new function in payments of agents which will not depend on type of agent i (θ_i)it will depend only on θ_{-i} Consider $(f, \underline{p})$ is DSIC then

$$v_i(f(\theta_i, \theta_{-i}), \theta_i) - p(\theta_i, \theta_{-i}) \geq v_i(f(\hat{\theta}_i, \theta_{-i}), \theta_i) - p(\hat{\theta}_i, \theta_{-i})$$

$$\forall i \epsilon N, \forall \theta_{-i} \epsilon \Theta_{-i} and \forall \theta_i, \hat{\theta}_i \epsilon \Theta_i$$

now consider,

$$q(\theta_i, \theta_{-i}) = p(\theta_i, \theta_{-i}) + h(\theta_{-i})$$

Question: $(f, \underline{q})$ Is this DSIC?
Answer: Yes it will be DSIC, consider

$$v_i(f(\theta_i, \theta_{-i}), \theta_i) - q(\theta_i, \theta_i)$$

$$v_i(f(\theta_i, \theta_{-i}), \theta_i) - p(\theta_i, \theta_i) - h(\theta_{-i}) \geq v_i(f(\theta_i, \theta_{-i}), \theta_i) - p(\hat{\theta}_i, \theta_i) - h(\theta_{-i})$$

Since $(f, \underline{p})$ is DSIC so the above equation holds.

2. If allocation is same of different types of agent i. Let two types of agent i will be $(\theta_i, \theta_{-i}) and (\hat{\theta}_i, \theta_{-i})$

$$f(\theta_i, \theta_{-i}) = (\hat{\theta}_i, \theta_{-i}) = a(say)$$

Let us assume that $(f, \underline{p})$ is DSIC, then
when agent i type is θ_i

$$v_i(f(\theta_i, \theta_i), \theta_i) - p(\theta_i, \theta_{-i}) \geq v_i(\hat{\theta}_i, \theta_i), \theta_i) - p(\hat{\theta}_i, \theta_{-i})$$

$$p_i(\hat{\theta}_i, \theta_{-i}) \geq p_i(\theta_i, \theta_i)$$

when agent i true type is $\hat{\theta}_i$,

$$v_i(f(\hat{\theta}_i, \theta_i), \hat{\theta}_i) - p(\hat{\theta}_i, \theta_{-i}) \geq v_i(\theta_i, \theta_i), \hat{\theta}_i) - p(\theta_i, \theta_{-i})$$

$$p_i(\theta_i, \theta_{-i}) \geq p_i(\hat{\theta}_i, \theta_i)$$

from above two conclusions we can see that,

$$p_i(\theta_i, \theta_{-i}) = p_i(\hat{\theta}_i, \theta_{-i})$$

so, If you cannot change allocation we cannot change the payments vector.

References

[CW87] D. Coppersmith and S. Winograd, "Matrix multiplication via arithmetic progressions," *Proceedings of the 19th ACM Symposium on Theory of Computing*, 1987, pp. 1–6.