Game Theory & Mechanism Design (CS711), Autumn 2018
Indian Institute of Technology Kanpur
Homework Assignment Number 2

QUESTION 1

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Part a)

Preference Profiles							
P		P'		$\hat{P}$		P"	
P_1	P_2	P_1'	P_2'	$\hat{P}_1$	$\hat{P}_2$	P_1''	P_2''
a	c	b	a	b	c	b	c
b	b	a	b	a	b	a	a
c	a	c	c	c	a	c	b

Using the result that for any preference profile (P_1, P_2), (f(P_1, P_2)) $\in \{P_1(1), P_2(1)\}$, we can conclude that f$(P_1', P_2') \in \{b, a\}$

We know that f(P) = a.
We will prove by contradiction.
Let f(P') = a.

Now, let us construct $\hat{P}$ such that $\hat{P}_1 = P_1'$ and $\hat{P}_2 = P_2$
$f(\hat{P})$ is either b or c.

Let $f(\hat{P})$ = c
But we can see that the position of c is monotonically at least as good in P, as in $\hat{P}$. So, if $f(\hat{P})$ = c, then it follows that $f(P)$ = c as well. This is not true as we know $f(P)$ = a. Hence, this case is never possible.

So, $f(\hat{P})$ = b.

Now, let us construct P'' such that $P_1'' = \hat{P}_1$ and $P_2''(1)$ = c, $P_2''(2)$ = a, $P_2''(3)$ = b
As the top preferences in both the profiles $\hat{P}$ and P'' are same, and we know $f(\hat{P})$ = b, therefore $f(P'')$ = b as well.

But here, player 2 can manipulate the game by reporting a as his top preferred choice instead of c. This case has same top preferences as P' where we assumed the result was a. Thus, by manipulation, player 2 can get a which he prefers over the current output, b.

This violates the strategy proofness of the mechanism. Thus, our assumption must have been wrong. f(P') = a must be false.

f(P') = b.

The only thing left to prove is that, if we have two preference profiles with same top preference for both players in both profiles, result must be same.

Again by contradiction,
Suppose P, such that, $P_1(1) =$ a, $P_2(1) =$ b, $f(P) =$ a
And P', such that, $P'_1(1) =$ a, $P'_2(1) =$ b, and let $f(P') =$ b.
Construct $\hat{P}$, $\hat{P}_1(1) =$ a, $\hat{P}_2(1) =$ b, $\hat{P}_1(2) =$ b and $\hat{P}_2(2) =$ a. This is monotonic w.r.t to P', as position of b is atleast as good as that is P'. So, $f(\hat{P}) =$ b.
By following same argument starting from P, we can say that $f(\hat{P}) =$ a.
This is a contradiction, so $f(P')$ must be a.

Thus, the proof is complete.

Part b)
The above conclusion does not hold in the case if profiles are chosen from single peaked preference domain. This is because in the proof above, we defined profile P'' such that, for player 2, $P''_2(3) =$ b.
With the linear ordering, $a < b < c$, b cant be the last priority while maintaining single peak. i.e., this profile is not there single peaked domain.
The mechanism that will allow f(P') = a, is the median voter Social Choice Function, such that the left most peak is selected in each case.

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Gibbard-Satterthwaite result cannot be applied to this scenario.
For, Gibbard-Satterthwaite result to hold, the assumption we make is that all preference profiles are possible. So, in order for Gibbard-Satterthwaite result to be applicable here, all orderings of the subsets S $\subseteq$ X, should be possible.

But, here all ordering are not possible. A set's priority is determined by its highest priority member. So, all sets with same highest priority member will appear in succession in preference profiles.

Say, for 3 projects A,B and C. Then, the priority order $\{$ $(A) > (B) > (C) > (A, B)$... $\}$ is never possible as if (A) is top priority, than (A,B) will have higher priority than (B).

So, Gibbard-Satterthwaite result cannot be applied to this scenario.

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QUESTION 3

The median voter SCF group is strategy-proof.
We need to consider only the peak preferences of all players. The top preference of each player is denoted by :- P = $(P_1(1), P_1(1), ..., P_n(1))$
Let $f(P)$ = a is the median of these peaks and the phantom peaks.
Consider a group of agents K $\subseteq$ N. We consider all the possible cases :-

Case 1 : There is an agent i in the group, such that $P_1(i)$ = a.
Any change in the median peak, will result in a loss for this particular agent. So, such a group cannot manipulate the game.

Case 2 : For all agents i in the group, $P_1(i) <$ a.
For any of them to bring their peaks closer to the median peak, the median peak will have to reduce. i.e., median has to shift leftwards.
But for that to happen, some agent j with peak $>$ a will have to report its peak as less than a. But no such agent belongs to this set.
So, this set is also unable to manipulate for their gain.

Case 3 : For all agents i in the group, $P_1(i) >$ a.
By parallel argument as for case 2, this set is also unable to manipulate for their gain.

Case 4 : For some agents i in the group, $P_1(i) <$ a, and for some, $P_1(i) >$ a.
But in case, if some manipulated median peak is to the left of original, the agents with $P_1(i) >$ a will suffer.
If some manipulated median peak is to the right of original, the agents with $P_1(i) <$ a will suffer.
Such a set is also unable to manipulate for their gain.

Thus, the median voter SCF is group strategy-proof.