CS698W: Game Theory and Collective Choice	**Jul-Nov 2017**

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Revisit Example: Let's revisit the game of Neighbouring Kingdoms dilemma. This is a One-Shot Non-Cooperative game. Notice the Change in notation i.e Alternatives(a_i) $\rightarrow$ Outcomes(o_i) e.g. (A,A)= o_1, (A,D)=o_2, (D,A)=o_3, (D,D)=o_4.

1\2	A	D
A	(5,5)	(0,6)
D	(6,0)	(1,1)

3.1 Normal Form Representation

Games being played between n players in Normal/Strategic form is represented and dealt with using following notations.

$$N = \{1, 2, 3, \ldots, n\}$$

S_i = set of strategies of player i
$s_i \in S_i$: a strategy of player i
s_{-i} : strategy profile of all agents except player i
$\{s_1, s_2, \ldots, n\} = (s_i, s_{-i}) = s$: A strategy profile
Note that $s \in S_1 \times S_2 \times S_3 \ldots S_n = S$
$u_i(s_1, s_2, \ldots, s_n)$: utility of player i where s_j represents strategy picked by player j $\forall j = \{1, 2, \ldots, n\}$
Here Von-Neumann Morgenstern utility function is defined this way $u_i : S \rightarrow \mathbb{R}$

3.2 Behaviour of players

We assume following things about the behaviour of the players :

- Rationality : Every agent picks strategy to maximize her utility.
- Intelligence : Every agent possesses enough information about the game and is able to find the best strategy for her

Common Knowledge : A fact is known as a common knowledge if

1. All players know the fact.
2. All players know the fact that all other players know the fact.
3. All players know the fact that all other players know the fact that all others know the fact .... and so on.

Example: As an example of common knowledge we saw interesting case of 3 blue-eyed individuals and a sage. Here it is assumed that statement of the sage can't be questioned.He states that there is atleast one among the three with blue eyes and that the blue-eyed ones should leave the island. As a result of common knowledge, all three left the island after sufficient time.

Note : As per our assumptions, the fact that all players are rational, is a Common Knowledge.

3.3 Some important definitions

- **Strictly dominated strategy:** A strategy s_i' is strictly dominated by s_i if $\forall$ s$_{-i} \in S_{-i}$

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$$

- **Weakly dominated strategy:** A strategy s_i' is weakly dominated by s_i if $\forall$ s$_{-i} \in S_{-i}$

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$$

and $\exists$ s$_{-i}$ such that

$$u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$$

- **Strictly/Weakly dominant strategy:** A strategy s_i is strictly/weakly dominant strategy of player i if s_i strictly/weakly dominates all other $s_i' \in S_i \setminus \{s_i\}$

- **Strictly/Weakly dominant strategy equilibrium:** A strategy profile (s_i^*, s_{-i}^*) is an SDSE/WDSE if s_i^* is a SDS/WDS for i. $\forall i \in N$.

- **Pure strategy Nash equilibrium:** A strategy profile (s_i^*, s_{-i}^*) such that $\forall i \in N$ and $\forall s_i \in S_i$

$$u_i(s_i^*, s_{-i}^*) \geq u_i(s_i, s_{-i}^*)$$

Does every Game have a SDSE/WDSE? Can there be more than 1 Nash equilibrium? we will answer these questions through an example of another game.

1\2	C	F
C	(2,1)	(0,0)
F	(0,0)	(1,2)

Through this example it is clear that every game is not guaranteed to have a SDSE/WDSE. Also we can see that there exists 2 pure strategy Nash equilibrium in this game. However when there exists an SDSE, there is exactly one equilibrium.