CS711A: Introduction to Game Theory and Mechanism Design

Assignment – 2

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1.(a)

Consider the following preferences P_1, P_2, P_3, P_4, P_5, P_6, P_1', P_2'. Where P_1, P_3, P_5 and P_1' are the preference profiles of player 1 and P_2, P_4, P_6 and P_2' are the preference profiles of player 2

P_1	P_2	P_3	P_4	P_5	P_6	P_1'	P_2'
a	c	a	c	b	c	b	a
b	b	b	a	a	a	a	b
c	a	c	b	c	b	c	c

Given, $f(P_1, P_2) = a$. We assume f is strategy proof. Therefore, f is monotonic.

$D(a, (P_1, P_2))$ is a subset of $D(a, (P_3, P_4))$. Since, f is monotonic we have $f(P_3, P_4) = a$.

For profile (P_5, P_6) , we have $f(P_5, P_6) = b$ or c. Assume, $f(P_5, P_6) = c$.

Player 1 prefers a over c i.e. $f(P_3, P_6)P_1f(P_5, P_6)$ .So , he can change his preference from P_5 to P_3.

This leads to a contradiction as f is strategy proof. Therefore, $f(P_5, P_6) = b$.

$D(b, (P_5, P_6))$ is a subset of $D(b, (P_1', P_2'))$. Therefore, $f(P_1', P_2') = b$ (Proved)

1.(b)

No, the earlier conclusion does not hold when the preference profiles are generated from single-peaked preference domain with the intrinsic ordering of the alternative as a<b<c. Our earlier proof used the preference (P_3, P_4) but P_3 is not a single preference profile and we cannot use it as a basis for computing $f(P_1', P_2')$.

We can use the following SCF to compute $f(P_1', P_2')$.

Let P_x denote 1st player and P_y denote 2nd player. $P_x(1)$ is the first preference of Player 1 and $P_y(1)$ is the first preference of player 2.

$$f(P_x, P_y) = \min\{P_x(1), P_y(2)\}$$

This is a Median Voter Social Choice Function. It both ONTO and strategy proof.

For $P_x = P_1$ and $P_y = P_2$ we have $f(P_1, P_2) = a$ (Given)

For $P_x = P_1'$ and $P_y = P_2'$ we have $f(P_1', P_2') = a$ (Required)

2.

The Gibbard Satterthwaite Theorem states that, for a set of Alternatives A, if $|A|\geq3$. If the Social Choice Function f (defined from a set of unrestricted preferences to A) is ONTO and strategy proof, then f is dictatorial.

Let S, T be a subset of X (set of projects).

According to Question, S is preferred over T if and only if the highest ranked project of S is preferred over the highest rank project of T. So, if S=X and T≠X. Then, S will be at least as preferable as T i.e. $P_i(1) = X$. Therefore, the set of preferences come from a restricted Domain.

Hence, the Gibbard Satterthwaite Theorem is not applicable here.

3.

Let f be a Median Voter Social Choice Function which selects the kth median from the left from the set of all peaks. Let the preference profile of the group be P. Let f(P) = a.

Let K be a subset of N. Let P_i be the preference profile of i^{th} player.

Case 1: - $P_i = a$ for all i in K. Therefore, they have no incentive to change.

Case 2:- $P_i < a$ for all i in K. So, if they report their preference to the left then it will not affect the k^{th} median. If they report their preferences to the right then k^{th} median will also move to the right. Let P' be the new reported profile. Therefore, $f(P)<f(P')$. But since all the preferences are single peaked then therefore, they will be worse off than what they originally were. So, they will not do so.

Case 3: - $P_i > a$ for all i in K. Following similar logic as Case 2 we can conclude that P_i will not manipulate their preferences.

Case 4: - $P_i>a$ for some i in K and $P_i<a$ for some i in K. These players cannot collaborate as it will make one better off at the expense of other.

Therefore, we can say that f is group strategy proof.

Proved.