CS711: Introduction to Game Theory and Mechanism Design **Jul-Nov 2018**

Project: Assignment 2

Question 1

Part (a)

Consider $P_2'' = c \succ a \succ b$.
In P_2, $c \succ b \succ a$, but in P_2'', the ordering is $c \succ a \succ b$. So, the relative position of a has gone up in the preference ordering. Since $f(P_1, P_2) = a$, from monotonicity, it can be concluded that $f(P_1, P_2'') = a$.
It may be observed that $f(P_1', P_2'') \in \{b, c\}$. Say, $f(P_1', P_2'') = c$. It is evident that player 1 prefers a over c in both P_1 and P_1'. Hence, if player 2 chooses P_2'', player 1 can always report his strategy as P_1 instead of P_1' and get a to win (by $f(P_1, P_2'')$) instead of c (by $f(P_1', P_2'')$). But since f is strategy-proof, this cannot occur. Thus $f(P_1', P_2'') = b$.
In P_2'', $c \succ a \succ b$, whereas in P_2', $c \succ b \succ a$. Thus, the relative position of b has gone up in the preference ordering. Since $f(P_1', P_2'') = b$, from monotonicity, it can be concluded that $f(P_1', P_2') = b$. Hence proved.

Part (b)

No, the earlier conclusion does not hold true in this case.

The earlier proof does not go through because in getting to the fact that $f(P_1', P_2') = b$, we used a preference ordering for player 2, namely P_2'', which does not belong to a single-peaked preference domain with intrinsic ordering $a < b < c$. It has two peaks, namely a and c.

Let us choose the median voter social choice function for our new mechanism. Also, let us define this function in such a way that if there are multiple median candidates, then the left-most of them is chosen. Here, left-most is defined as per the intrinsic ordering. The median voter SCF is proved to be both ONTO and strategy-proof.

Thus, we have $f(P1, P2) = a$ (given) Note that this also follows the definition of our function.
For P_1' and P_2', medians are given by a and b. But since we are following the intrinsic ordering, we have $f(P_1', P_2') = a$ as well. Hence, proved.

Question 2

No, the Gibbard-Satterthwaite result will not apply here.
The set of alternatives is the set of all subsets of projects given by $\{S | S \subseteq X\}$. Since $|X| \geq 2$, set of alternatives is at least $2^2 - 1 = 3$. Consider two alternatives P and Q such that $P \subset Q$. According to the question, any agent is then indifferent between P and Q, or prefers Q to P. This is because if the higher ranked project is in P, then it is also in Q, hence the indifference. On the other hand if the higher ranked project is in $Q - P$, Q is definitely preferred. Thus, there cannot arise a preference order in which P is ranked above Q. Thus, there is a restriction in the preference ordering. As mentioned in the lectures, in a setting where preferences are restricted, Gibbard-Satterthwaite theorem may not hold. Hence, proved.

Question 3

Define :

- f : Median voter social choice function
- P_i : Preference profile of player i
- P : Preference profile of all players
- a : Outcome of P i.e. $f(P) = a$

Let R be a subset of all the players. Then several cases can arise:

- P_i has top preference a, $\forall i \in R$. None of the agents need to modify their choices as they are already satisfied.
- P_i has top preference that are less than a in the intrinsic ordering i.e. (lying left to a). So, if they report their preference to the left, it will not affect the median. The only way to change the median is to report their preferences to the right of the median. But since all preferences are single-peaked, all players in R are worse off than they were before, since they do not get to choose their top candidate.
- P_i has top preference that are more than a in the intrinsic ordering. The proof for this is same as above, since the condition is symmetric.
- P_i has top preference less than a for certain i, P_j has top preference more than a for certain j, such that $i, j \in R$. These players cannot help each other as it will simply make one group profit at the cost of the other.

Thus, any alteration leads to loss of the players. Hence, it is safe to say that f is strategy-proof.