Project: Assignment 2

Kumar Raj

Roll Number: 160350

2.1

(a)

P_1	P_2	P'_1	P'_2
a	c	b	a
b	b	a	b
c	a	c	c

We know that $f(P') \in \{b, a\}$.

Consider another preference profile $f(P'') = (P'_1,\ P_2)$.

Clearly, $f(P'') \in \{b, c\}$.

Assume for contradiction that $f(P'') =$ c.

Since $f(P') =$ c, so $f(P) =$ c also, by MONO, which is not possible as $f(P) =$ a.

Hence, $f(P'') =$ b.

Now, outcome for the profile:

P'_1	P'''_2
b	c
a	a
c	b

is 'b', as it can't be 'c' because otherwise $f(P) =$ c, again.

Hence, $f(P') =$ b because previous profile is dominated by this this profile. So, by MONO $f(P') =$ b.

(b)

In this case, the earlier conclusion doesn't hold. This is because the preference P' and P''' are not single peaked.

Define the new mechanism to be:
$f(P) =$ min($P_1(1)$, $P_2(1)$)

Then, $f(P'_1, P'_2) =$ a.

2.2

The set of alternatives is the set of all subsets of objects: $\{S$: $S \subseteq$X$\}$.

Let f be the SCF with profile over these alternatives. Also assume that f is strategy-proof.

Now, $|$X$|\geq 2$.

So, the set of alternatives is at least 3.

Now, consider two alternatives A_1 and A_2 such that $A_2 \subset$A$_1$.

Now, the way extension of the linear order, P_i is done, any agent either prefers A_1 over A_2 or is indifferent between the two alternatives. Hence there cannot be any preference ordering in which A_2 is more preferable to A_1. Hence a Pareto Efficient function can never outputs A_2. This leads to the restriction to the domain of the function. Hence we cannot apply the Gibbard-Satterthwaite result here as the function cannot be ONTO.

2.3

We need to consider only the peak preferences of all the agents. So let us denote the preferences denoted only by their peaks, i.e., $P = (P_1(1), .., P_i(1), .., P_n(1))$ and let $f(P) =$ a is the median of these peaks and the phantom peaks. Consider the group of agents K and i & j be any two agents of K. Let B be collection of phantom peaks.

1. If either $P_i =$ a or $P_j =$ a then the agent won't gain by misreporting his preferential order and the situation would be equivalent to manipulation of preference order by other candidates of the group.

2. If $P_i <$ a or $P_j <$ a then
(i) if any of the agents (say i) chooses profile P'_i such that $P'_i(1) < P_i(1)$ then the median of $(B,\ P')$ (where $P' = (P_1,\ P'_2,..\ ,P'_i,..\ ,P'_j,..\ ,P'_n)$) is going to remain the same as that of (B,(P'_j,P'_{-j})) (Since $P'_i(1) < P_i(1) <$ median(B, P)). So, $f(P_K,\ P_{-K}) = f(P'_j, P'_{-j})$ from which j can never get benefits. Same would be the outcome when both $P'_i(1) < P_i(1)$ & $P'_j(1) < P_j(1)$.
(ii) if both $P'_i(1) > P_i(1)$ and $P'_i(1) > P_i(1)$, the new median of $(B,\ P')$ can either remain a or increase in terms of the prescribed order of alternatives. If it remain a, they did not benefit and if it increases, they are further away from their true preferences, since $P_i(1) <$ a $< f(P'_K,\ P_{-K})$ now.

3. If both $P_i(1) >$ a and $P_j(1) >$ a then the argument would be similar as above.

4. If $P_i(1) <$ a and $P_j(1) >$ a then
(i) we can see $P'_i(1) < P_i(1)$ or $P'_j(1) > P_j(1)$, won't give better output as it won't alter the median that was without doing this(above argument)
(ii) Suppose $P'_i(1) > P_i(1)$ or $P'_j(1) < P_j(1)$ then the median $(B,\ (P'_K,\ P_{-K}))$ can possibly change in both directions. If it doesn't change, it does not benefit any agent in the group. If it decreases, it won't benefit i, as now $P_i(1) <$ a $< f(P'_K,\ P_{-K})$. Similarly, it does not benefit j if median increases. Thus manipulation benefits no one in the group.
(iii) Suppose $P'_i(1) < P_i(1)$ or $P'_j(1) > P_j(1)$ then again similar argument as above.

Hence the median voter SCF is group strategy-proof.