A2CS711A

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1 Question 1

1.1

The function in question is SP (Strategy Proof) and onto $\{a, b, c\}$.
It is also given that $f(P) \in \{P_1(1), P_2(1)\}$ s. t. $f(P_1, P_2) = a$.
To show that $f(P'_1, P'_2) = b$, we simply assume that $f(P'_1, P'_2) = a$, and consider a preference profile $(P'_1, \hat{P}_2)$ as follows:

$P_1\prime$	$\hat{P}_2$
b	c
a	a
c	b

If the function value at this profile is b, then Player 2 manipulates the profile from P'_2 to $\hat{P}_2$ to change the outcome to a. Since our function is already strategy proof, we have the function value at this profile to be c.

So if we go from $(P'_1, \hat{P}_2)$ to (P_1, P_2), we observe: $D(c, P_1) = D(c, P'_1) = \phi$, and also $D(c, P_2) = D(c, \hat{P}_1) = \{a, b\}$.
Since $f(P'_1, \hat{P}_2) = c$, we have $f(P_1, P_2) = c$, by monotonicity of our strategy proof SCF. But this is a contradiction to the actual value being a. Hence, the value of $f(P'_1, P'_2)$ is b.

1.2

No the earlier conclusion does not hold. Our proof was based on the profile $\hat{P}_2$. Due to the profiles being generated from a single preference domain $(a < b < c)$, b cannot be the lowest preference as in this profile, b in the lowest preference no longer makes our profile single peaked.
For our case, we can simply use the Median Voter SCF. As every MVSCF is Strategy proof and onto, our function would be the same.

We simply have to ensure that the outcome is a for both (P_1, P_2) and (P'_1, P'_2). For that to happen, MV SCF selects the leftmost peak among the preference profiles. SO whenever a is a peak as in our case, we are done.