160740 Tanay Batra	**6th Nov 2018**

Assignment 2

Question 1 - Solution

Given that there are two agents with three alternatives {a,b,c}. Also it is given that f is onto SCF with f(P1,P2)=a.

We have to show that is f is strategyproof f(P1',P2')=b.

Acc. to Lemma 20.4 (Ref. notes shared on course website), for preference profile (P1,P2), f(P1,P2) belongs to {$P_1(1)$,$P_2(1)$}

Consider (P_1',P_2") where P_2" have the order (c a b)^T
Now, f(P_1',P_2")={b,c}
Here, we will assume that f(P_1',P_2")=c
Now consider the profile f(P_1',P_2) as f is MONO we can write that f(P_1',P_2)=c
Next we have that f(P1,P2)=a and f(P_1',P_2)=c, and a P1 c, Hence f(P1,P2) P1 f(P1',P2)
This implies that f is manupulable which contradicts the fact that f is strategy proof
So, our asuumption is wrong, then it results that f(P_1',P_2")= b

Now we have that (P_1',P_2') and (P_1',P_2") differs in Prefernce order of agent 2 only. Since, f(P_1',P_2") =b therefore by monotonicity we have that f(P_1',P_2') =b

Hence Proved

Part(b)

If the preferences are generated from a single-peaked preference domain with the intrinsic ordering of the alternatives being a ¡ b ¡ c. then the earlier conclusion does not holds as P_2" is not allowed in this domain (violation of single peaked domain we can't have b at bottom)
Now we have to provide a mechanism such that f(P_1',P_2') = a

Take a median voter SCF we can find a B such that f(P')=median(B,peaks(P)) [equation 1]for all profiles.
Let us take B as 'a' (beacuse asked in question) then we have f(P) belongs to {a,b,c} such that every outcome is possible.
(We have P1',P2',P1"=(c b a),P2"=(b c a))
Note: f is strategy proof as its a median voter SCF also onto
f(P1',P2')=b; (median B,P1'(1),P2'(1))
f(P2',P1")= a; (median B,P2'(1),P1"(1))
f(P1",P2")=c; (median B,P1"(1),P2"(1))

Question 2 - Solution

It is assumed that all preference profile i.e. all possible order for each agent is possible for **Gibbard-Satterthwaite** to hold. In a setting where these preferences are restricted, Gibbard-Satterthwaite theorem may not hold.

In the question it says that that for any pair of subsets of projects if the highest ranked project in one is better than the highest ranked project in other then the First subset is preferred over the second one. According to question, subsets are the alternatives like for set of project [a,b] we have [{a},{b},{a,b}] as alternatives.

Let's take two subsets of X say A and B such that one is the strict subset of other i.e. $A \subset B$.
In question its given that $|X| \geq 2$ where we can have 3 alternatives. Suppose we are taking 3 projects (we can have 5 alternatives from this) such that
subset A is [{a},{b},{a,b}] and B is [{a},{b},{a,b},{c,b},{a,c},{a,b,c}] if we have a preference $a \succ b$ then the highest ranked project from A would be 'a' (here we can show that the alternative {a,b} from subset A will also give its highest ranked project as 'a' and thus overall we have 'a'). Since we have not assumed the preference position for c the highest ranked project from B would be either 'c' or 'a'.
It would be 'c' if assumed that 'c' is preferred over 'a' otherwise the highest ranked project from B also would be 'a'.
This means that agent will prefer subset B over A or they are indifferent i.e. $B \succeq A$. But A would not be preferred over B (even weakly) , which suggests that all possible orders are not possible (as $A \succ B$ will never happen). Hence due to the restriction in domain Gibbard-Satterthwaite theorem does not hold.

Question 3 - Solution

Question - Is the median voter SCF group strategy-proof?

To prove if the median voter SCF a group strategy-proof we have to show that the social choice function can not be manipulated by any group of agents. Group of agents here can be seen as if some agents form a coalition and try to manipulate the SCF.
We have proved that median voter SCF is a strategy-proof, the proof that it is also a group strategy-proof is similar. We will show that there is no incentive for the group of agents to manipulate. Proof :
Since, the social choice function f:Sn is a median voter social choice function therefore there exist some B such that f(P) is median of B, P1(1),...,Pn(1) for all preference profiles.

let's take b as median
suppose agent i have Pi(1)=f(P)=b then the agent has no incentive to manipulate

If the agent i's peak is on to the left of b Now the only way now for agents/group of agents to change to change the outcome is by changing the median (b) so when the agents change their peak to right of the current median, the median can be changed. In that case the outcome would be the one which is further right of the peak or we can say that the agent gets worse-off as he is having an higher amount of utility at present. Thus there is no incentive for agents/group of agents to manipulate.

Similarly for any agent on the right of current median follows the above criterion to not manipulate.

Deviating from current peaks the agents or the group of agents would get worse-off.

This implies that the median voter SCF is group-strategy proof.