Efficiency and Budget Balance

Uniqueness of Groves for Efficiency

$$f^{Eff}(t) \in \underset{a \in A}{\arg\max} \sum_{i=1}^{n} t_i(a)$$

<u>Theorem</u> (Green and Laffont (1979), Holmström (1979))
If the type space is "sufficiently" rich, every efficient and DSIC mechanism is a Groves mechanism.

Proof <u>sketch</u>: Two alternatives $A = \{a, b\}$

Welfares $\sum t_i(a)$ and $\sum t_i(b)$

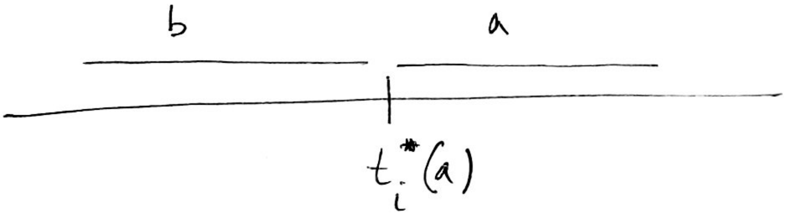
$\sum_{j \in N} t_j(a) \geq \sum_{j \in N} t_j(b)$, then a is chosen.

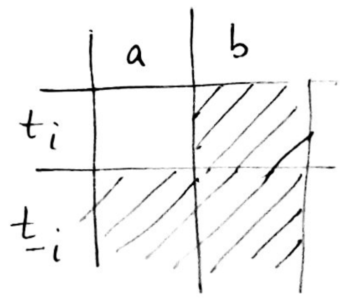
- fix the valuations of other agents to t_{-i}
- fix value of i at alternative b at $t_i(b)$.

$\exists$ some threshold $t_i^*(a)$ s.t.

$\forall\ t_i(a) \geq t_i^*(a)$ a is the outcome

$t_i(a) < t_i^*(a)$ b is the outcome





- Use DSIC for $t_i^*(a) + \epsilon = t_i(a)$, $\epsilon > 0$

$$t_i^*(a) + \epsilon - p_{ia} \geq t_i(b) - p_{ib} \quad \text{--- (1)}$$

(38-2)

similarly, $t_i'(a) = t_i^*(a) - \delta$, $\delta > 0$

$$t_i(b) - p_{ib} \geq t_i^*(a) - \delta - p_{ia} \quad \text{-- (2)}$$

ϵ, δ are arbitrary and therefore

$$t_i^*(a) - p_{ia} = t_i(b) - p_{ib}.$$

But $t_i^*(a)$ is the threshold of efficient outcome

$$t_i^*(a) + \sum_{j \neq i} t_j(a) = t_i(b) + \sum_{j \neq i} t_j(b)$$

$$\Rightarrow \quad p_{ia} - p_{ib} = \sum_{j \neq i} t_j(b) - \sum_{j \neq i} t_j(a)$$

hence the payment has to be $p_{ix} = h_i(t_{-i}) - \sum_{j \neq i} t_j(x)$

Theorem: (Greene & Laffont (1979))

No Groves mechanism is Budget Balanced.

$\nexists\, p_i^G$ s.t. $\sum_{i \in N} p_i^G(t) = 0 \quad \forall t \in T$.

PSketch: Two alternatives, $\{0, 1\}$ 0: The project is not undertaken
1: undertaken

~~$\emptyset$~~ in 0, all agents have zero value. Suppose $\exists h_i$ s.t. $\sum_{i \in N} p_i(t) = 0 \ \forall t$

All we need are three numbers w_1^+, w_1^-, w_2 to explain

let
$$w_1^+ + w_2 > 0 \quad \text{--- outcome} = 1$$
$$w_1^- + w_2 < 0 \quad \text{---- outcome} = 0$$

Groves payment at (w_1^+, w_2)

$$h_1(w_2) - w_2 + h_1(w_1^+) - w_1^+ = 0$$

at (w_1^-, w_2)
$$h_1(w_2) + h_1(w_1^-) = 0$$

$$\Rightarrow \quad w_2 = h_1(w_1^+) - h_1(w_1^-) - w_1^+$$ (RHS completely dependent on w_1)

Change w_2 slightly s.t. $w_1^+ + w_2 > 0$ and $w_1^- + w_2 < 0$

Then the equality cannot hold. □

Corollary: If the valuation space is sufficiently rich, no efficient mechanism can be both DSIC and BB.

Weakening DSIC for positive results

Allocation is still the efficient one

Payment is defined via a prior.

$$\delta_i(t_i) = \mathbb{E}_{\underline{t}_i | t_i} \sum_{j \neq i} t_j(a^*(t))$$

$$a^*(t) \in \underset{a \in A}{\arg\max} \sum_{i \in N} t_i(a)$$

Payment of this mechanism

$$p_i^{dAGVA}(t) = \frac{1}{n-1} \sum_{j \neq i} \delta_j(t_j) - \delta_i(t_i)$$

[d'Aspremont, Gerard-Varet (1979), Arrow (1979)]

This payment implements the efficient allocation rule in Bayes Nash equilibrium

$$\mathbb{E}_{\underline{t}_i | t_i}\left[t_i(a^*(t)) - p_i^{dAGVA}(t)\right]$$

$$= \mathbb{E}_{\underline{t}_i | t_i} \sum_{j \in N} t_j(a^*(t)) - \underbrace{\mathbb{E}_{\underline{t}_i | t_i}\left[\frac{1}{n-1}\sum_{j \neq i} \delta_j(t_j)\right]}_{\text{not a fn of } t_i}$$

$$\geq \mathbb{E}_{\underline{t}_i | t_i} \sum_{j \in N} t_j(a^*(t_i', \underline{t}_i)) - \cdots$$

$$\sum_{i\in N} p_i^{dAGVA}(t) = \frac{1}{n-1}\sum_{i\in N}\sum_{j\neq i}\delta_j(t_j) - \sum_{i\in N}\delta_i(t_i)$$

$$= \frac{n-1}{n-1}\cdot\sum_{j\in N}\delta_j(t_j) - \sum_{i\in N}\delta_i(t_i) = 0$$

Theorem: The dAGVA mechanism is Efficient, BIC and BB.

What about participation guarantee?

- The dAGVA does not satisfy IIR (interim individual rationality)

Theorem: [Myerson-Satterthwaite 1983]
In the bilateral trading problem, there is no mechanism that is BIC, efficient, IIR and Budget ~~Bdo~~ Balanced.

Summary