Lec 7

- MSNE : existence guaranteed but computationally hard.
- Another equilibrium concept : Correlated
- Nash eq. : every player picks strategies independently.
- If there were a trusted agent, who does the randomization and suggests the strategies to the individuals. The strategy will be called correlated strategy.
- It will be an equilibrium if for every player it becomes self enforcing, i.e., they find that ~~sug~~ following the suggestion is the best response for them.
- The ~~key~~ properties of the randomizing device - common knowled

Ex. 1:

	C	F
C	2,1	0,0
F	0,0	1,2

Ex. 2

	STOP	GO
STOP	0,0	1,2
GO	2,1	−10,−10

Trusted entity tosses a fair coin $(C,C),(F,F)$ w.p. $\frac{1}{2}$

Defn. A ~~strategy~~ correlated equilibrium $\pi : S \to [0,1]$ s.t. $\sum_{s \in S} \pi(s) = 1$, is a joint probability distribution over the strategy profiles s.t. $\forall i \in N$ and $\forall s_i \in S_i$

$$\sum_{\underline{s}_{-i} \in \underline{S}_{-i}} \pi(s_i, \underline{s}_{-i})\, u_i(s_i, \underline{s}_{-i}) \geq \sum_{\underline{s}_{-i} \in \underline{S}_{-i}} \pi(s_i, \underline{s}_{-i})\, u_i(s_i', \underline{s}_{-i}), \quad \forall s_i' \in S_i.$$

Explaining with Ex. 1 : If C has been suggested by the trusted entity

Expected payoff by listening to C: $\sum_{\underline{s}_{-1} \in \underline{S}_{-1}} p(\underline{s}_{-1} | C)\, u_1(C, \underline{s}_{-1})$

$$= \frac{1}{p(C)} \left[p(C,C)\, u_1(C,C) + \overset{0}{p(C,F)}\, u_1(C,F) \right]$$

$$= \frac{1}{1/2} \left[\frac{1}{2} \times 2 + \cancel{\tfrac{1}{2}}0 \times 0 \right] = 2$$

7-2 Play F: $\frac{1}{p(C)}\left[p(C,C)\,u_1(F,C) + p(C,F)\,u_1(F,F)\right]$

$$= \frac{1}{1/2}\left[\frac{1}{2}\times 0 + 0\times 0\right] = 0$$

Better to play C.

For ex. 2, if ~~prob~~ $\pi(S,S) = \pi(S,G) = \pi(G,S) = \frac{1}{3}$, $\pi(GG) = 0$

Then $\displaystyle\sum_{s_{-1}\in S_{-1}} p(s_{-1}\mid S)\,u_1(S,s_{-1}) = \frac{1}{p(S)}\sum_{s_{-1}\in\{S,G\}} p(S,s_{-1})\,u_1(S,s_{-1})$

$$= \frac{1}{2/3}\left[p(S,S)\,u_1(S,S) + p(S,G)\,u_1(S,G)\right] = \frac{3}{2}\times\frac{1}{3}\times 1 = \frac{1}{2}$$

$$\sum_{s_{-1}\in S_{-1}} p(s_{-1}\mid S)\,u_1(G,s_{-1}) = \frac{1}{2/3}\left[p(S,S)\,u_1(G,S) + p(S,G)\,u_1(G,G)\right]$$

$$= \frac{3}{2}\times\left[\frac{1}{3}\times 2 + \frac{1}{3}\times(-10)\right] = -4$$

Interpretation of correlated equilibrium

$$\sum_{s_{-i}\in S_{-i}} p(s_{-i}\mid \bar{s}_i)\,u_i(\bar{s}_i, s_{-i}) \geq \sum_{s_{-i}\in S_{-i}} p(s_{-i}\mid \bar{s}_i)\,u_i(s_i', s_{-i}) \quad \forall s_i'\in S_i,\ \forall i\in N$$

$$\Rightarrow \sum_{s_{-i}\in S_{-i}} p(\bar{s}_i, s_{-i})\,u_i(\bar{s}_i, s_{-i}) \geq \sum_{s_{-i}\in S_{-i}} p(\bar{s}_i, s_{-i})\,u_i(s_i', s_{-i})$$

Compute CE: $\forall s_i \in S_i$

$$\sum_{s_{-i}\in S_{-i}} \pi(s)\,u_i(s_i, s_{-i}) \geq \sum_{s_{-i}\in S_{-i}} \pi(s)\,u_i(s_i', s_{-i}) \quad \forall s_i'\in S_i,\ \forall i\in N$$

$n\,m^2$ inequalities

$\pi(s) \geq 0$ $\qquad$ m^n inequalities

$\displaystyle\sum_{s\in S}\pi(s) = 1$ $\qquad$ $\boxed{\text{Feasibility LP}}$

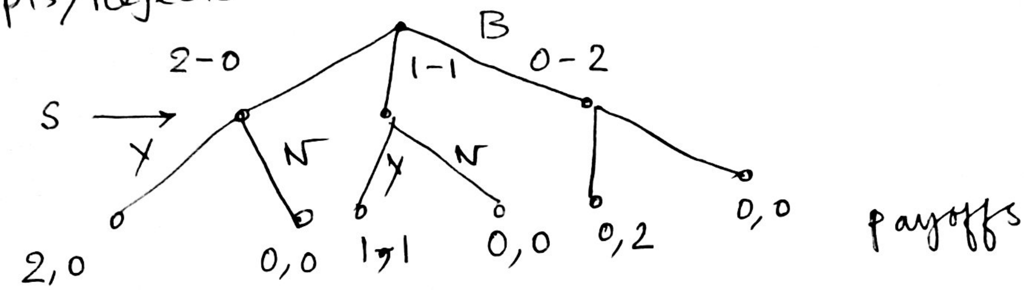
For MSNE, # of support profiles were 2^{mn}

$n\log_2 m$ vs mn ← exponentially larger.

⟨CE⟩ ⟨MSNE⟩

Extensive Form Games

Ex. Brother and Sister — two identical indivisible objects — 2 chocolates. Sequence: Brother divides, sister either accepts/rejects



A perfect information extensive form game (EFG) is represented by

$$\langle N, A, X, \mathcal{H}, P, (u_i)_{i\in N} \rangle$$

N : set of players

A : set of all possible actions

$\mathcal{H}$: set of "sequences of actions" (histories) satisfying

- empty sequence $\phi \in \mathcal{H}$
- If $h \in \mathcal{H}$ an initial subsequence/subhistory $h' \in \mathcal{H}$ — $h = (a^{(0)}, a^{(1)}, \ldots, a^{(T-1)})$
- A history is 'terminal' if
  - (a) it is infinite, OR
  - (b) $\nexists\, a^{(T)}$ s.t. $(a^{(0)}, a^{(1)}, \ldots, a^{(T-1)}, a^{(T)}) \in \mathcal{H}$.

— terminal history set is denoted by Z.

$X : \mathcal{H} \setminus Z \to 2^A$ ~~his~~ non-terminal history to action set mapping.

$P : \mathcal{H} \setminus Z \to N$ Player function

$u_i : Z \to \mathbb{R}$ utility of player i.

(7-4)

Explanation wrt the Brother-Sister game.

$N = \{B, S\}$, $A = \{2\text{-}0, 1\text{-}1, 0\text{-}2, Y, N\}$

$\mathcal{H} = \{\phi, (2\text{-}0), (1\text{-}1), (0\text{-}2), \underbrace{(2\text{-}0, Y), (2\text{-}0, N), (1\text{-}1, Y), (1\text{-}1, N), (0\text{-}2, Y), (0\text{-}2, N)}_{= Z \text{, terminal.}}\}$

$X(\phi) = \{2\text{-}0, 1\text{-}1, 0\text{-}2\}$, $X(2\text{-}0) = X(1\text{-}1) = X(0\text{-}2) = \{Y, N\}$

$P(\phi) = B$, $P(2\text{-}0) = P(1\text{-}1) = P(0\text{-}2) = S$

$u_1(2\text{-}0, Y) = 2$, $u_2(2\text{-}0, Y) = 0$

$\vdots$

Strategy set of agent i : complete contingency plan

$$S_i = \underset{\{h \in \mathcal{H} \setminus Z : P(h) = i\}}{\times} X(h)$$

$X(h)$ → available actions in that history

$\{h \in \mathcal{H} \setminus Z : P(h) = i\}$: histories where i is the player to play.

$S_1 = \{2\text{-}0, 1\text{-}1, 0\text{-}2\}$ Pl. 1 plays only in history ϕ

$S_2 = \{Y, N\} \times \{Y, N\} \times \{Y, N\}$ Pl. 2 plays in 3 histories

$= \{YYY, YYN, YNY, YNN, \ldots, NNN\}$

Given strategies one can transform this game into a NFG. and find NE.

	YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
2-0	2,0	2,0	2,0	2,0	0,0	0,0	0,0	0,0
1-1	1,1	1,1	0,0	0,0	1,1	1,1	0,0	0,0
0-2	0,2	0,0	0,2	0,0	0,2	0,0	0,2	0,0

But this representation is wasteful

- EFG is the ~~compact~~ succinct representation for certain games.
- Equilibrium ideas.