CS711: Introduction to Game Theory and Mechanism Design	**Jul-Nov 2018**

Project: Assignment 2

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Question 1

a

Before solving the given problem, we prove:
Let P:$P_1(1) = a' \neq c' = P_2(1)$ and P':$P'_1(1) = b' \neq d' = P'_2(1)$. Then, $f(P) = a' \implies f(P') = c'$ if $c' \neq a', b'$ and $d' = b'$. Consider the given mentioned preference profile: Let $f(P) = a'$ and we assume that $f(P') = b'$.

P_1	P_2	P'_1	P'_2	P^*_1	P_2	P''_1	P''_2
a'	b'	c'	b'	c'	b'	c'	b'
-	-	-	-	a'	-	b'	c'
-	-	-	-	b'	-	-	-

Now we consider the transition from (P_1, P_2) to (P^*_1, P_2) via (P''_1, P''_2). The preference for b' has improved from P to P". Hence, by monotonicty, $f(P'') = b'$. If $f(P^*_1, P_2) = c'$, then by similar argument, we will have $f(P'') = c'$ which will give us a contradiction. Hence, $f(P^*_1, P_2) = b'$.
Now we consider the transition from (P'_1, P'_2) to (P^*_1, P_2). Here, if agent 1 reports P_1 in place of P^*_1, the outcome is a' which is more preferable to it than b'. This would imply that the given SCF is manipulable and hence, not strategyptoof. This results in a contradiction.
Therefore, our initial assumption was wrong $\implies f(P') = c'$. Thus proving, $f(P) = a' \implies f(P') = c'$. Approaching the problem, consider the preference profiles given as: Given, $f(P_1, P_2) = a$ and f is onto and

P_1	P_2	P'_1	P'_2	P^*_1	P^*_2
a	c	b	a	b	c
b	b	a	b	a	-
c	a	c	c	c	-

strategyproof. Therefore, we know that $f(P) \in \{P_1(1), P_2(1)\}$. Thus, $f(P'_1, P'_2) \in \{b, a\}$. Let, $f(P'_1, P'_2) = a$. Consider the transition from (P_1, P_2) to (P^*_1, P^*_2). Here, $a' = a', c' = b, b' = c, d' = c$. It satisfies the constraints of the above proof. Thus, $f(P^*) = b$ as $f(P) = a$.
Now consider the transition from (P'_1, P'_2) to (P^*_1, P^*_2). Here, with the reversal of agents we have $a' = a, b' = b, c' = c, d' = b$. It satisfies the constraints of the above proof. Hence, $f(P^*) = c$ as $f(P') = a$ which is a contradiction as $b \neq c$.
Therefore, $f(P') = b$.

b

The earlier conclusion does not hold in the this case.
The proof of the above question has unrestricted preferences. The proof uses the preference order: $c > a > b$ which will be not be possible in the single peaked preference case where the preliminary order is $a < b < c$,

and hence,it won't go through.
To obtain the given results, we design a median voter SCF with two agents. The intrinsic ordering of the preferences is $a < b < c$ Hence the number of phantom voters is 1.Phantom voters,$B = [a]$.
For preference profile,(P_1, P_2),$f(P) = median(B, P_i(1)) = median(a, a, c) = a$ and for (P'_1, P'_2),$f(P') = median(B, P'_i(1)) = median(a, a, b) = a$.

Question 2

We are given that number of projects,$|X| \geq 2$. Let total possible subsets of the projects be denoted by A. Then,$|A| = 2^{|X|} - 1 \geq 3$ (removing the empty subset). For Gibbard-Satterthwaite theorem to be applicable, the preferences must be unrestricted. Let us consider the case, when $S \neq T$. In this case either, S is preferred over T or S and T are indifferent. There is no scenario in which T is preferred over S. Thus, we have that the preferences are restricted. Therefore, the Gibbard-Satterthwaite theorem is not applicable in this scenario.

Question 3

Considering only the peak preferences of each voter. Let it be denoted by $P = (P_1(1), P_2(1)...P_i(1)...P_n(1))$ where n is the number of voter. Let the median voter be, $f(P) = a \in A$ where A is the set of all alternatives. Consider a group of agents, $K \subseteq N$,

1. Case 1: If $\exists i \in K$ where $P_i(1) = a$, then there is no profitable manipulation for the group. The agent whose peak is equal to median voter SCF will never be well-off unless the median voter is a as the function is single peaked preference.
2. Case 2: If the group contains agents which have peaks to the left of the median voter,ie, $\forall i \in K$,$P_i(1) < a$. In this case, if the agents report their peaks to the left, the median voter remains unchanged and there is no profitable manipulation. If any of them reports his peak to the right of a, then they shift the median s.t., $a < f(P'_K, P'_{-K})$. Then, $P_j(1) < a < P_j(1)$, for the person shifting his preference. Since, it is a single peaked preference case, the agent is worse off than it was before and has no profitable manipulation.
3. Case 3: $\forall i \in K$,$P_i(1) > a$, it follows on similar lines as above and the conclusion obtained is same.
4. Case 4: $\forall i \in K, P_i(1) \neq a$ and $\exists j, k \in K$ s.t. $P_j(1) < a$ and $P_k(1) > a$. In this case, if the group manages to shift the peak to the left, then Agent k is worse off than he was before else Agent j is worse off than he was before. Therefore, the whole group does not have profitable deviation.

Thus, the median voter is group strategy proof.