CS711: Introduction to Game Theory and Mechanism Design **Jul-Nov 2018**

Project: Assignment 2

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Question 1.

1(a).

Given,
$f(P_1, P_2) = a$ follows unristricted strict preferences.

Proof:

Suppose $P_2'' = c \succ a \succ b$
We know that $f(P_1, P_2) = a$ and P_2'' has preference ordering $c \succ b \succ a$. Relative position of a in P_2'' has gone up in preference ordering than P_2 which implies that $f(P_1, P_2'') = a$. According to preference ordering of player 1 given in question we can say that $f(P_1', P_2'') \in \{b, c\}$. Let's assume $f(P_1', P_2'') = c$ and we know that P_1 prefer a over c in both cases P_1 and P_1' . Hence, If Player 2 choose profile P_2'' then player 1 will always report his strategy as P_1 rather than P_1' but we know that f is strategyproof. This is contradiction. So, $f(P_1', P_2'') = b$.

As, b is more prefered in P_2' than P_2'' and $f(P_1', P_2'') = b$. This implies $f(P_1, P_2') = b$.
Hence Proved.

1(b).

No, Earlier result will not be hold in single-peaked preference domain with intrinsic preference ordering $a \succ b \succ c$. Because P_2'' can not be generated through single peaked preference domain. Thus it can't be used as basis for computing $f(P_1', P_2')$.

Let's define the following Median Voter Social Function,

$$f(P_s, P_t) = min\{P_s(1), P_t(1)\}$$

Where P_s is player 1 and P_t is player 2, and $P_s(1), P_t(1)$ repesent first preference of player 1 and player 2 repectively. We can see that it is both Onto and Strategyproof function.

For $P_s = P_1$ and $P_t = P_2$, we have $f(P_1, P_2) = a$.
For $P_s = P_1'$ and $P_t = P_2'$ we will have $f(P_1', P_2') = a$.

Question 2.

No, Gibbard-Satterthwaite result will not be applicable here. Because the Gibbard Satterthwaite Theorem states that, for a set of Alternative A, if $|A| \geq 3$. If the social cohice function f(which is defined on unresricted preference to A) is ONTO and strategyproof, then f is dictatorial.
We know that, The set of alternatives is the set of all subset of object $\{S : S \subset X\}$. Since $|X| \geq 2$ so set of alternatives will be atleast 3. Now, Take two alternatives A and B such that $A \subset$ B. Now, There can be only two cases, Any agent is either indeifferent A between B or prefer B over A.

Case 1:
If higher ranked project is in A as $A \subset B$ implies this higher ranked project is also present in B, Then any agent will be indifferent between A and B.

Case 2:
If higher ranked project is in B, not in A. So, any agent will prefer B over A.

Hence, the preference ordering where B is ranked higher than A can never arise.So, this is a restricted domain, hence we can not apply Gibbard-Satterthwaite result here.

Question 3.

Yes, The group median voter SCF is group strategy-proof.

Proof:
Let f be a K^{th} Median Social Function which selects K^{th} median from left. Consider all the peak preferences of all group agents. Let's denote the preference of the group as P and $f(P) = a \in A$.
Given,
K is a subset of N and P_i is the peak peference od player i.

- When $P_i = a$ $\forall i$ in group.Then they have no reason to change their preferences.
- When $P_i < a$ $\forall i$ in group. If they shift their preference to further left, Then median will not change. If they report their preference to right of "a" then median will also shift to right. Now P' will be new profile which is more right from previous profile($f(p) < f(p')$). Because it is an single peaked model and they are moving further away from their peak. Hence, They will be get into worse condition than they were before. So, they will not change their reported preferences.
- When $P_i > a$ $\forall i$ in group. We will have same argument as above and can conclude that they will not manipulate their preferences.
- When some of $P_i > a$ and some $P_i < a$. Then agents that are left of a will not move to right as they will be in more worse condition then before and same hold for left one's. So, There will be no collaboration between players as collaborator will get reach into worse condition then before.

Hence, f is group strategy-proof.