CS698W: Game Theory and Collective Choice **Jul-Nov 2017**

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29.1 Recap

In the last lecture, we studied the VCG mechanism. We proved a claim that the payment for an agent that is not allocated any object in a combinatorial allocation is zero. We continue the analysis of VCG and make another claim.

29.2 VCG mechanism continued

Definition 29.1 (Individual Rationality) *A direct mechanism* $(f, \boldsymbol{p})$ *is* individually rational *(IR) if for every* $\theta \in \Theta$

$$v_i(f(\theta), \theta_i) - p_i(\theta) \geqslant 0, \ \forall i \in N.$$

This property ensures that by participating in the mechanism, no agent gets a negative utility. Hence a rational agent should volunterily participate in the game. We now show that in the setting we consider, VCG satisfies this condition.

Claim 29.2 *In the allocation of goods, VCG mechanism is individually rational.*

Proof: Recall the definitions of X and Y as follows.

$$X \in \arg\max_{a \in A} \sum_{i \in N} v_i(a, \theta_i), \quad Y \in \arg\max_{a \in A} \sum_{j \neq i} v_j(a, \theta_j).$$

The payoff of agent $i \in N$ according to VCG mechanism is given by

$$\begin{aligned} & v_i(X, \theta_i) - \sum_{j \neq i} v_j(Y, \theta_j) + \sum_{j \neq i} v_j(X, \theta_j) \\ &= \sum_{j \in N} v_j(X, \theta_j) - \sum_{j \neq i} v_j(Y, \theta_j) \\ &= \underbrace{(\sum_{j \in N} v_j(X, \theta_j) - \sum_{j \in N} v_j(Y, \theta_j))}_{\geqslant 0} + \underbrace{v_i(Y, \theta_i)}_{\geqslant 0} \geqslant 0 \end{aligned}$$

We get the second equality by adding and subtracting $v_i(Y, \theta_i)$. The first term in the last line is non-negative by definition of X and the second term is non-negative due to all allocations are of goods. This concludes the proof. ∎

29.3 Application Domain : Internet Advertising

Internet advertising is defined as the method of delivering promotional marketing messages called 'ads' to consumers when a user searches something on a search engine or visits a website. In this lecture, we will consider the methods of *efficiently* placing an ad and deciding the payment to the advertiser.

29.3.1 Advantages of Internet Advertising

Internet advertising is much better than the conventional advertising in newspapers/articles or radio because of the following advantages.

1. **User Data:** Using Internet ads, the advertiser can gain information about which set of buyers are interested in which set of ads. This enables the advertisers to target certain products to the interested users.

2. **Measurable Actions:** Classification of buyers is possible using Internet advertising. The buyers can be classified into a set of groups and be shown ads according to their interest.

3. **Low Latency:** Internet advertising allows the auctions to happen on the fly. The auction takes place just before the ad is to be shown. For example in search engines like Google, bidding takes place after an user enters a set of keywords to search and ads for the organizations that win the auction are shown. This also allows the advertisers to enter any time and enables automation of the whole process.

29.3.2 Types of Ads on Internet

1. **Sponsored Search Ads**
   In this type of ads, the advertisers bid on the keywords that are entered by the user while searching on a search engine. Some details of this method are as follows.

   - Some words can have multiple meanings based on the context. Like 'jaguar' can mean a car manufacturer or an animal. So to prevent the car ads when the animal jaguar is searched, stop words are used. Stop words are a set of words which if used with a particular word (here, jaguar), the ad (for car jaguar) is not presented.
   - One user can search for the same keyword multiple times. So to prevent activity of a malicious user, or to prevent draining of money from the advertiser, or to prevent irritating the user with same ad every time, a cap is set on the number of times an ad can be shown to a user.

2. **Contextual Ads**
   These type of ads are presented to a user based on the context of a page. For example, gmail can present certain type of ads based on the content of the email.

3. **Display Ads**
   It is the classical way of displaying the ads just like the banner ads in the newspapers.

29.3.3 Position Auction

This type of auction is used to sell multiple ad slots in a webpage. Let $N = \{1, 2, \ldots, n\}$ denote the set of bidders and $M = \{1, 2, \ldots, m\}$ be set of slots available to be auctioned. For simplicity we assume $m \geqslant n$, i.e., every advertiser gets a slot – the question is who gets which slot. Here among the set of slots, 1 represents the 'best' position and m represents the 'worst' position.

Advertiser value: It is the valuation received by the advertiser from her ad on the Internet. We make the following assumptions in this setting.

1. Only clicks generate the value to the bidders.
2. All the clicks are valued equally no matter which position it came from. This means that the role of position is only in the probability of being clicked. This assumption helps in decoupling the position effect from the value effect.

The expected value of agent i, when her ad is shown at position j is

$$v_{ij} = CTR_{ij} \cdot v_i.$$

Where CTR_{ij} is the *click through rate* (CTR) that denotes the probability that bidder i's ad gets clicked at position j. This quantity CTR_{ij} is again assumed to be decomposable into two terms: (1) the quality of the ad of the advertiser i, denoted by $CTR_i \in [0, 1]$, that reflects how good the ad is and (2) the effect of position on the CTR, denoted by $pos_j \in [0, 1]$. Therefore

$$CTR_{ij} = CTR_i \cdot pos_j, \quad CTR_i \in [0, 1], pos_j \in [0, 1].$$

Thereby the value of agent i when her ad is in position j is decomposable into the position effect and agent effect: $v_{ij} = pos_j \cdot (CTR_i \cdot v_i)$. Note that both pos_j and CTR_i are measurable and can be estimated by the auctioneer, but v_i is a private information (type) of the agent.

29.3.4 Deciding the Auction Mechanism

The auction mechanism decides the allocation of resources (e.g., the position at which an ad is displayed) and the payment to be made by each advertiser. The following are some standard methods of auctions.

1. **Early position auctions:** In this type of auction, the received bids are sorted in decreasing order of value and the positions are allotted according to the this order – the highest bidder receiving the first position and lowest bidder receiving last position – and they pay some fixed amount according to their position upfront. This type of bidding puts all risk on the advertiser – now an advertiser has to estimate the probability of getting a click since they are paying even before the ad is clicked.
2. **Pay per click model:** In this mechanism, the price charged to the advertiser is proportional to the number of clicks the users made on the ad. This mechanism puts all the risk on the website because a there could be an ad that is at one of the top positions but never gets clicked and hence does produce any revenue.
3. **Rank by expected revenue:** Modern sponsored search auctions uses a middle ground between these two extreme approaches. In this approach, the advertisers are ranked by product of *estimated* CTR and the bid of the advertiser. This mechanism shares the risk between the advertiser and the website.

   We use the following notations :

   - $eCTR_i$: the value of the click through rate of the Ad i estimated by the search engine
   - b_i: the amount that agent i is willing to pay if a click occours

- $x_i \in M$: the position assigned to agent i s.t. $x_i \neq x_k, i, k \in N$
- $x = (x_1, x_2, \ldots, x_n)$: allocation of ads to positions

Here we assume that the ads are allocated according to the decreasing order of their expected revenue, i.e., according to the decreasing order of $eCTR_i \cdot b_i$. Suppose the agent i reports his bid as b_i Then

$$\hat{v_i}(x) = pos_{x_i}(eCTR_i \cdot b_i)$$

is the reported value of the agent for the allocation x.

Allocation rule: Standard search engines and advertisement websites pick the allocation x^* such that it maximizes the sum of the valuations of the advertisers, i.e.,

$$x^* = \arg\max_x \sum_{i \in N} \hat{v_i}(x).$$

This method of allocation is exactly same as the VCG allocation.