Game Theory Assignment 2

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1 Question 1

1.1 Part a

The preference profiles can be constructed as follows:

$P_1 P_2$	$P_1^{‘} P_2^{‘}$	$P_1^{‘} \hat{P}_2$
a c	b a	b c
b b	a b	a a
c a	c c	c b

To prove: $f(P_1‘, P_2‘) = b$.
Proof by Contradiction.
Assume We have $f(P_1‘, P_2‘) = a$. Consider preference profile $(P_1‘, \hat{P}_2)$. Thus player 2 can manipulate from P_2 to $P_2‘$ since $a\hat{P}_2 b$. Thus, for the SCF to remain strategy proof, we must have $f(P_1‘, P_2‘) = b$.
Now consider the transition of profile $(P_1‘, \hat{P}_2) \rightarrow (P_1, P_2)$. Clearly th dominated set of c remains same for both the agents and hence, we must have $f(P_1, P_2) = c$. Thus, we have a contradiction.

1.2 Part b

If the preferences are generated from a single peaked preference domain with the intrinsic ordering of the alternatives being a¡b¡c, our earlier proof breaks down because any profile with b as its minimum is not admissible under the given restriction.
We construct our SCF in a way such that it selects the leftmost peak among the top two players's preference profiles.
It is easy to see that for chosen SCF we have: $f(P_1, P_2) = a$ and $f(P_1, P_2‘) = a$.
Now to prove that f is strategy-proof: Clearly, the player with leftmost peak has no reason to manipulate. The other player say p can manipulate by moving his peak to the left of the leftmost's person, but this is not profitable for p. Thus f is strategy proof.
Hence proved.

2 Question 2

We know that Gibbard-Satterthwaite Theorem can be applied only if the domain is unrestricted.
We will show that the domain in our case is unrestricted and hence Gibbard-Satterthwaite theorem can not be applied.
Proof: Consider two alternatives S and T such that $S \subset T$. Clearly all projects in S will be also present in T. Thus, any agent will be either indifferent between S and T or prefer T over S. Thus we can never have a preference ordering where S is ranked higher than T. Hence, our domain is restricted.
Hence Proved.

3 Question 3

Yes. The median vector SCF is group strategy proof.
Case 1: All agents have the same peak as median; in this case, no agent has any incentive to manipulate.
Case 2: Atleast one agent from the left of the median deviates to the right of the median. In this case, Since the preferences are single peaked, the new peak will lie further away from the original peak than the original median itself since the we go past the median to the other side.
Case 3: Atleast one agent from the right of the median deviates to the right of the median. Similar to case 2.
Thus in any of the cases, no agent has any reason to manipulate.
thus SCF is group strategy proof.