

Decision Tree

- buys_comp = 'yes' : Y
buys_comp = 'no' : N

- Entropy for class labels :

$$\text{Info}(D) = I(9,5) = -\frac{9}{14} \log_2 \left(\frac{9}{14} \right) - \frac{5}{14} \log_2 \left(\frac{5}{14} \right) \\ = 0.940$$

- Info of each attribute :

Age	y_i	n_i	$I(y_i, n_i)$
≤ 30	2	3	0.971
30...40	4	0	0
> 40	3	2	0.971

$$\text{Info}_{\text{age}}(D) = \frac{5}{14} \times I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2) \\ = 0.694$$

Income	y_i	n_i	$I(y_i, n_i)$
high	2	2	1
medium	4	2	0.918
low	3	1	0.811

$$\text{Info}_{\text{income}}(D) = \frac{4}{14} I(2,2) + \frac{6}{14} I(4,2) + \frac{4}{14} I(3,1) \\ = 0.911$$

Student	y_i	n_i	$I(y_i, n_i)$
yes	6	1	0.592
no	3	4	0.985

$$\text{Info}_{\text{student}}(D) = \frac{1}{14} I(6, 1) + \frac{1}{14} I(3, 4) \\ = 0.789$$

Credit rating	y_i	n_i	$I(y_i, n_i)$
excellent	3	3	1
Fair	6	2	0.811

$$\text{Info}_{\text{cr}}(D) = \frac{6}{14} I(3, 3) + \frac{8}{14} I(6, 2) \\ = 0.892$$

$$\text{Gain}(\text{age}) = -\text{Info}_{\text{age}}(D) + \text{Info}(D) \\ = -0.694 + 0.940 = 0.246$$

$$\text{Gain}(\text{income}) = \text{Info}(D) - \text{Info}_{\text{income}}(D) \\ = 0.940 - 0.911 = 0.029$$

$$\text{Gain}(\text{student}) = \text{Info}(D) - \text{Info}_{\text{student}}(D) \\ = 0.940 - 0.789 = 0.151$$

$$\text{Gain}(\text{credit rating}) = \text{Info}(D) - \text{Info}_{\text{cr}}(D) \\ = 0.940 - 0.892 = 0.041$$

For age > 40

$$\begin{aligned} \text{Info}(D) &= I(3, 2) \\ &= -\frac{3}{5} \log_2\left(\frac{3}{5}\right) - \frac{2}{5} \log_2\left(\frac{2}{5}\right) \\ &= 0.971 \end{aligned}$$

Information of each attr.:

Income	y_i	n_i	$I(y_i, n_i)$
high	0	0	0
medium	1	1	1
low	2	1	0.918

$$\begin{aligned} \text{Info}_{\text{income}}(D) &= 0 + \frac{2}{5} I(1, 1) + \frac{3}{5} I(2, 1) \\ &= 0.951 \end{aligned}$$

Student	y_i	n_i	$I(y_i, n_i)$
yes	2	1	0.918
no	1	1	1

$$\begin{aligned} \text{Info}_{\text{student}}(D) &= \frac{3}{5} I(2, 1) + \frac{2}{5} I(1, 1) \\ &= 0.951 \end{aligned}$$

Cred. rating	y_i	n_i	$I(y_i, n_i)$
Fair	3	0	0
excellent	0	2	0

$$\text{Info}_{cr}(D) = \frac{3}{5} I(3,0) + \frac{2}{5} I(0,2)$$

$$= 0$$

$$\text{Gain}(\text{income}) = \text{Info}(D) - \text{Info}_{\text{income}}(D)$$

$$= 0.971 - 0.951$$

$$= 0.020$$

$$\text{Gain}(\text{student}) = \text{Info}(D) - \text{Info}_{\text{stud}}(D)$$

$$= 0.971 - 0.951$$

$$= 0.020$$

$$\text{Gain}(\text{cred. rating}) = \text{Info}(D) - \text{Info}_{cr}(D)$$

$$= 0.971 - 0$$

$$= 0.971$$

For age ≤ 30

$$\text{Info}(D) = I(2,3) = -\frac{2}{5} \log_2\left(\frac{2}{5}\right) - \frac{3}{5} \log_2\left(\frac{3}{5}\right)$$

$$= 0.971$$

Income	y_i	n_i	$I(y_i, n_i)$
h	0	2	0
m	1	1	1
l	1	0	0

$$\text{Info}_{\text{income}}(D) = \frac{2}{5} I(1,1)$$

$$= 0.40$$

Student	y_i	n_i	$I(y_i, n_i)$
yes	2	0	0
no	0	3	0

$$\text{Info}_{\text{student}}(D) = 0$$

cred. rating	y_i	n_i	$I(y_i, n_i)$
excellent	1	1	1
fair	1	2	0.918

$$\begin{aligned} \text{Info}_{\text{cr}}(D) &= \frac{2}{5} I(1, 1) + \frac{3}{5} I(1, 2) \\ &= 0.951 \end{aligned}$$

$$\text{Gain}(\text{income}) = \cancel{0.971} 0.971 - 0.4 = 0.571$$

$$\text{Gain}(\text{student}) = 0.971 - 0 = 0.971$$

$$\text{Gain}(\text{cred. rating}) = 0.971 - 0.951 = 0.020$$

Decision Tree:

