

1) Given a dynamic table (see section 17.4) that doubles in size when it needs more space. find the amortized runtime for inserting  $n$  elements.

a) Use the aggregate method.

Al) Given to find the amortized runtime for inserting  $n$  elements when it needs more space to double in size.

By using aggregate method we can calculate the total cost of (permo) performing a sequence by dividing with the number of operations.

let the table is 1 or be any constant size.

let first cost of insertion be 1, and second cost of insertion be 2. And third cost of insertion be 1.

we can assume fourth cost element inserting with a cost of 4.

Here we can observe that at even number index the cost is increasing by  $2^n$ .

The cost of insertion of table can be change with the change in size of the table. for  $n$  insertions in table the total cost can be calculated as.

$$1+2+1+4+1+8+1+16+\dots+1+2^n(\log(n)-1)$$

The resizing cost of table to its final size can be represented by  $2^n(\log(n)-1)$

By using aggregate method defination let divide the above by number of operations to get amortized cost per insertion.

$$O(\log n) - 1 = \frac{(1+2+1+4+1+8+1+16)}{\text{num}} = O(1)$$

This can be calculated by arithmetic and logarithmic operations.  $\therefore$  the amortized runtime is  $O(1)$ .



b) Use the accounting method.

A) By using accounting method to analyze the amortized runtime for inserting  $n$  elements into a dynamic table need to define potential function  
let table size initially be 1. so the size will become 1.

we have potential function as  $\frac{2T}{\text{num}} = \phi(r)$ .

first insert 1 element to the table. then the actual cost and potential cost be 1 & 2 respectively.

for 1 insertion amortized cost will be  $= 1 + 2 = 3$ .

then the total amortized cost for  $n$  insertions in the table be  $3n$  and per insertion the cost will be  $\frac{3n}{n} = 3 = O(3)$ .

$\therefore$  The amortized runtime for inserting  $n$  elements is  $O(1)$ .