Hw 4: Sarah Ward

7.1

$y = mx + b$

USUAL equation for isoclines (no mN_i term)

<u>for $F(N_1)$</u>

$$N_2 = -N_1 \cdot \frac{1}{\alpha_{12}} + \frac{1}{\alpha_{12}}$$

when $N_2 = 0$

$$-\frac{1}{\alpha_{12}} \cdot \alpha_{12}) - 1 = \boxed{1 = N_1}$$

<u>for $F(N_2)$</u>

$$N_2 = 1 - \alpha_{21} N_1$$

when $N_2 = 0$

$$N_1 = \frac{1}{\alpha_{21}}$$

solve for isoclines w/additional term

<u>for $F(N_1)$</u>

$$0 = r_1 N_1 (1 - N_1 - \alpha_{12} N_2) - mN_1$$

$$\frac{mN_1}{r_1 N_1} = 1 - N_1 - \alpha_{12} N_2$$

$$N_2 = \frac{-1}{\alpha_{12}} N_1 + \frac{1}{\alpha_{12}} - \frac{m}{\alpha_{12} r_1}$$

when $N_2 = 0$

$$\frac{1}{\alpha_{12}} N_1 = \frac{1}{\alpha_{12}} \left(\frac{m}{r_1} - 1 \right)$$

$$N_1 = \frac{m}{r_1} - 1$$

<u>For $F(N_2)$</u>

$$0 = r_2 N_2 (1 - N_2 - \alpha_{21} N_1) - mN_2$$

$$\frac{mN_2}{r_2 N_2} = 1 - N_2 - \alpha_{21} N_1$$

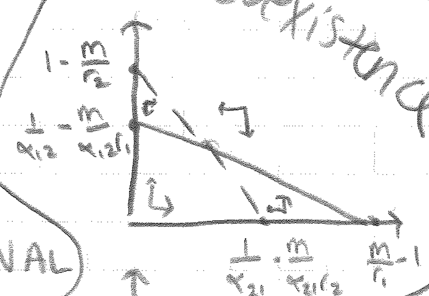
$$N_2 = -\alpha_{21} N_1 + 1 - \frac{m}{r_2}$$

when $N_2 = 0$

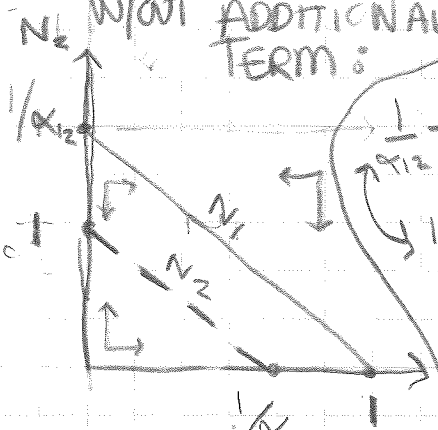
$$\alpha_{21} N_1 = 1 - \frac{m}{r_2}$$

$$N_1 = \frac{1}{\alpha_{21}} - \frac{m}{\alpha_{21} r_2}$$

Coexistence



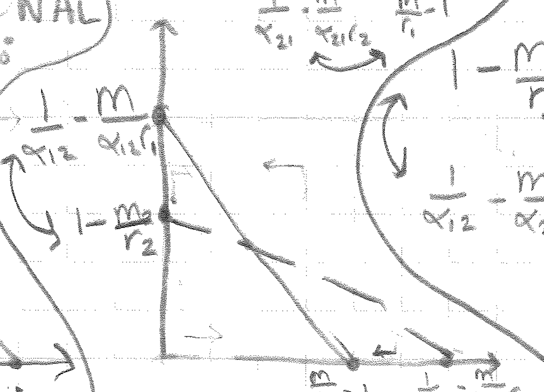
W/OUT ADDITIONAL TERM:



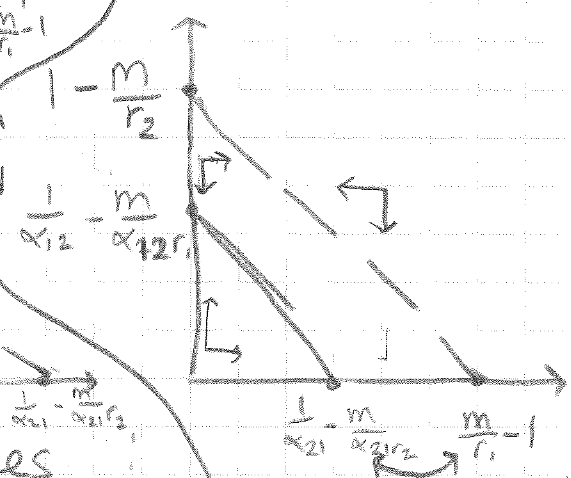
Coexistence

$$\alpha_{12} < \frac{1}{\alpha_{21}}$$

assuming $\alpha_{21} > 1$



If m makes either intercept switch then coexistence



If m makes both intercepts swap, then exclusion is by the other species

$\boxed{7.2}$

Species 1

$$\frac{dp_1}{dt} = m_1 p_1 (1 - p_1) - e p_1$$

7.19

Species 2

$$\frac{dp_2}{dt} = m_2 p_2 (1 - p_1 - p_2) - m_1 p_1 p_2 - e p_2$$

p_1 - fraction of space occupied by sp. 1
p_2 fraction of space occupied by sp 2
$m_{1/2}$ colonization rate
e extinction rate

a. solve for p_1

$$0 = m_1 p_1 (1 - p_1) - e p_1$$

so $\boxed{m_1 > e}$

$$0 = m_1 p_1 - m_1 p_1^2 - e p_1$$

$$0 = p_1 (m_1 - m_1 p_1 - e)$$

$\boxed{p_1 = 0}$ zero, $\quad 0 = m_1 - m_1 p_1 - e$

$$m_1 p_1 = m_1 - e$$

$$\boxed{p_1 = \frac{m_1 - e}{m_1}}$$

NON-zero

b. $0 = \frac{dp_2}{dt} = m_2 p_2 (1 - p_1 - p_2) - m_1 p_1 p_2 - e p_2$

$$= m_2 p_2 - p_1 m_2 p_2 - m_2 p_2^2 - m_1 p_1 p_2 - e p_2$$

$$p_2 (m_2 - p_1 m_2 - m_2 p_2 - m_1 p_1 - e)$$

$$m_2 - p_1 m_2 - m_2 p_2 - m_1 p_1 - e$$

$$p_2 m_2 = m_2 - p_1 m_2 - m_1 p_1 - e$$

$$p_2 = 1 - p_1 - \frac{m_1 p_1}{m_2} - \frac{e}{m_2}$$

zero $\boxed{p_2 = 0}$

$$1 - \left(\frac{m_1 - e}{m_1}\right) - \frac{m_1\left(\frac{m_1 - e}{m_1}\right)}{m_2} = \frac{e}{m_2}$$

$$= 1 - \frac{m_1 - e}{m_1} - \frac{(m_1 - e)}{m_2} - \frac{e}{m_2}$$

$$= 1 - \left(\frac{m_1 - e}{m_1}\right) - \left(\frac{m_1 - e}{m_2}\right) - \frac{e}{m_2}$$

for both species to survive:

$$\frac{m_1 - e}{m_1} < 1 \qquad \frac{e}{m_2} < 1$$

$$\frac{m_1 - e}{m_2} < 1$$

makes sense. colonization must be > extinction for both spp, and m_2 must be greather than m_1 - extinction to keep up w/ species I dominating

7.2 cont'd

basically: $m_1 < m_1 - e$ $m_2 < m_1 - e$ $m_2 > e$

c. SURVIVAL REQUIRES:

- $m_1 > e$, $\frac{m_1 - e}{m_1} < 1$, $\frac{m_1 - e}{m_2} < 1$, $\frac{e}{m_2} < 1$

so
$e < m_2 < m_1 - e$
$e < m_1 < m_1 - e$

if e slowly ↑ ...

when $\frac{dP_2}{dt} = 0 \rightarrow P_2 = 1 - \frac{m_1 - e}{m_1} - \frac{m_1 - e}{m_2} - \frac{e}{m_2}$ $\frac{dP_1}{dt} = 0 = P_1 = \frac{m_1 - e}{m_1}$

- P_2 would be eliminated first
- P_2 would have negative equilibrium @ highest rate of e

when $\frac{dP_2}{dt} = 0$

d. $P_2 = 1 - \frac{m_1 - e}{m_1} - \frac{m_1 - e}{m_2} - \frac{e}{m_2}$

if e slowly increases, the equilibrium of P_2 becomes negative

so as e ↑ P_2 @ $\frac{dP_2}{dt} = 0$ ↓

P_2 = fraction of space occupied by species 2

- makes sense because ~~the~~ ~~if~~ the extinction rate is too high, species 2 can't colonize fast enough to compensate and will die out

7.4

a. Competition:

LABORATORY → extrinsic factors controlled, looking for factors intrinsic to organism
field → extrinsic factors uncontrolled, as well as intrinsic factors uncontrolled

Field: Lotka-Volterra + some extrinsic
LAB: Lotka-Volterra only

7.4 cont'd

b. two species grown together, you are remaving extrinsic natural conditions. Its making an assumption that natural conditions like weather / resorces are a non-factor.

c. looking at overlap of resorces in assumed competitors.

↳ some experiments show that there is no effect of shared resorces on competition, others show that a shared resorce is a major factor in competition.
excluding all other factors is important to actually measure resorce competition

d. manipulating field populations:
- important to have controls
- controls cant be too close to manipulations to avoid them being affected
- replication
- deciding whether experiments actually work in actual populations

WRITE UP FOR R PROBLEM

- The conclusion from the initial experiment (t=1:20) would be that the two species coexist, however when the experiment continues to t = 1:100, the population growth of N_2 reaches a peak around 10 days + tapers off while N_1 continues to grow for another ~50 days before it tapers off.

Short term experiments should be interperetted as just a snapshot because the full story may not have played out yet. Long term studies show a better picture of the whole system.