96

1. (4 pts) The position of a car in three second intervals is given in the following table. Find the average velocity over the time interval [12,15]. Give numerical results rounded to one decimal place with correct units.

4

t (seconds)	0	3	6	9	12	15
s (meters)	0	18.27	35.89	74.32	110.56	175.77

$$\frac{175.77 - 110.56}{15 - 12} = \frac{65.21}{3} \approx \boxed{21.7 \text{ m/sec}}$$ ✓

2. (10 pts) The graph of f is given below. Evaluate each limit if it is finite. Otherwise, write ∞, $-\infty$, or DNE.

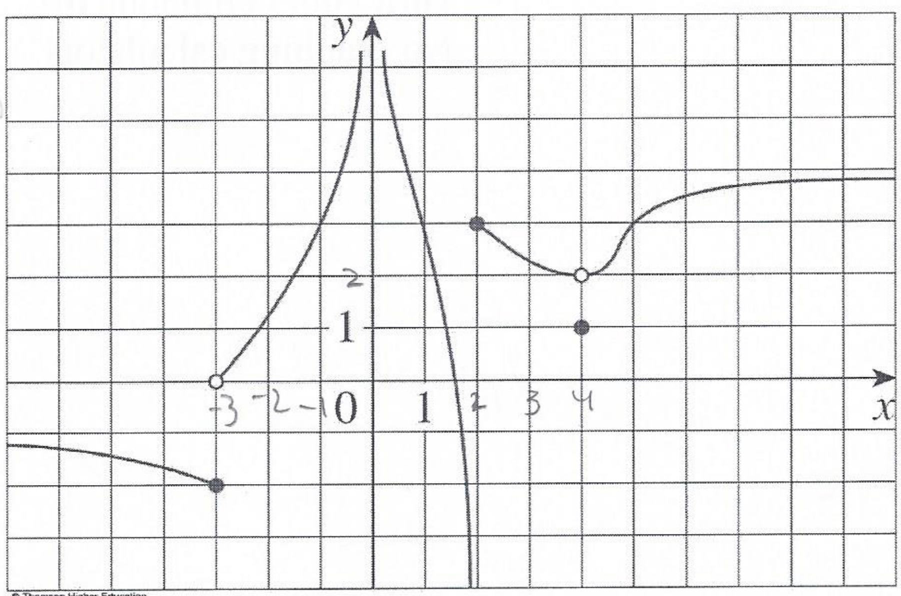
(a) $\lim_{x\to 0^-} f(x) = \boxed{\infty}$ ✓

(b) $\lim_{x\to -3^+} f(x) = \boxed{0}$ ✓

(c) $\lim_{x\to 4} f(x) = \boxed{2}$ ✓

(d) $\lim_{x\to \infty} f(x) = \boxed{4}$ ✓

(e) $\lim_{x\to 2^-} f(x) = \boxed{-\infty}$ ✓



-0

3. (17 pts) Find the limit. Give the exact value in its simplest form. (L'Hospital's rule is not permitted.)

(a) (4 pts) $\lim_{x\to -3}\left[\ln(2x+7)-\cos\left(\frac{\pi}{x}\right)+\frac{\sqrt{12x+45}}{x^2-8}+\sqrt{x^2}\right]$

$\ln\left(\lim_{x\to -3} 2x+7\right) - \cos\left(\lim_{x\to -3}\frac{\pi}{x}\right) + \lim_{x\to -3}\frac{\sqrt{12x+45}}{x^2-8} + \lim_{x\to -3}\sqrt{x^2}$

$= \ln(1) - \cos\left(-\frac{\pi}{3}\right) + \frac{\sqrt{9}}{1} + \sqrt{9}$ $\quad -1$

$= 0 - \left(+\frac{1}{2}\right) + 3 + 3 = \boxed{\cancel{\frac{9+\sqrt{3}}{2}}}$ $\quad 5.5$

(b) (5 pts) $\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}$

$\lim_{x\to 2}\frac{(x+3)\cancel{(x-2)}}{\cancel{(x-2)}(x+2)} = \lim_{x\to 2}\frac{x+3}{x+2} = \frac{(2)+3}{(2)+2} = \boxed{\frac{5}{4}}$

 -1

3. (Continued)

(c) (5 pts) $\lim_{x\to 0} \dfrac{\dfrac{1}{3-7x} - \dfrac{1}{3}}{x}$

$$\lim_{x\to 0} \frac{\frac{1}{3-7x}\cdot\frac{3}{3} - \frac{1}{3}\cdot\frac{3-7x}{3-7x}}{x} = \lim_{x\to 0} \frac{\frac{3-3+7x}{3(3-7x)}}{x}$$

$$= \lim_{x\to 0} \frac{7\cancel{x}}{3(3-7x)(\cancel{x})} = \lim_{x\to 0} \frac{7}{3(3-7x)} = \frac{7}{3(3-7(0))}$$

$$= \boxed{\frac{7}{9}}$$

(d) (3 pts) $\lim_{t\to\infty} \arctan\left(\dfrac{t^2-4}{t}\right)$

$$\lim_{t\to\infty} \arctan \frac{t^2-4}{t} = \arctan\left[t\left(\frac{t-\frac{4}{t}}{1}\right)\right]$$

$$= \arctan\left[\infty(\infty - 0)\right] = \arctan(\infty) = \boxed{\frac{\pi}{2}}$$

-0

4. (8 pts) Complete the ε,δ definition of limit.

Let f be a function defined on some open interval that contains the number a, except possibly at a itself. We say that the **limit of** $f(x)$ **as** x **approaches** a **is** L, and we write

$\lim_{x \to a} f(x) = L$ if and only if

Means that for every $\varepsilon > 0$ there is a $\delta > 0$

such that $0 < |x-a| < \delta$ then $|f(x) - L| < \varepsilon$

5. (8 pts) For what value(s) of a is the function f everywhere continuous?

$$f(x) = \begin{cases} \frac{2}{9}a^2x^2 + ax & x < 3 \\ \sin\left(\frac{\pi x}{2}\right) & x \geq 3 \end{cases}$$

* For f(x) to be continuous everywhere, $\lim_{x \to 3^-} f(x)$ and $\lim_{x \to 3^+} f(x)$ must exist, $f(3)$ must exist, and $\lim_{x \to 3} f(x) = f(3)$.

$$\frac{2}{9}a^2(3)^2 + 3a = \sin\left(\frac{(3)\pi}{2}\right)$$

$$\frac{18}{9}a^2 + 3a = -1$$

$$2a^2 + 3a + 1 = 0$$

$$(2a+1)(a+1) = 0$$

$a = -1$

$a = -\frac{1}{2}$

for f(x) to be continuous everywhere, $a = -1$ or $a = -1/2$

-0

6. (8 pts) Find all horizontal asymptotes of the function. Use appropriate limit statements to justify your conclusion and write the equation of any asymptote.

$*$ $f(x)=\dfrac{\sqrt{x^2+15}}{5x-20}$ Find where $\lim_{x\to\infty} f(x) = L$

and where $\lim_{x\to-\infty} f(x) = L$

$\sqrt{x^2} = |x| \begin{cases} x & \text{if } x>0 \\ -x & \text{if } x<0 \end{cases}$

$$\lim_{x\to\infty} \frac{\sqrt{x^2+15}}{5x-20} = \lim_{x\to\infty} \frac{\sqrt{\frac{x^2}{x^2}+\frac{15}{x^2}}}{\frac{5x}{\sqrt{x^2}}-\frac{20}{\sqrt{x^2}}} = \lim_{x\to\infty} \frac{\sqrt{1+\frac{15}{x^2}}}{\frac{5x}{x}-\frac{20}{x}}$$

$$= \lim_{x\to\infty} \frac{\sqrt{1+\frac{15}{x^2}}}{5-\frac{20}{x}} = \frac{\sqrt{1+\frac{15}{\infty}}}{5-\frac{20}{\infty}} = \frac{\sqrt{1+0}}{5-0}$$

$$= \frac{\sqrt{1}}{5} = \boxed{\frac{1}{5}}$$

$$\lim_{x\to-\infty} \frac{\sqrt{x^2+15}}{5x-20} = \lim_{x\to-\infty} \frac{-\sqrt{\frac{x^2}{x^2}+\frac{15}{x^2}}}{\frac{5x}{\sqrt{x^2}}-\frac{20}{\sqrt{x^2}}} = \lim_{x\to\infty} \frac{-\sqrt{1+\frac{15}{x^2}}}{\frac{5x}{-x}-\frac{20}{-x}}$$

$$= \lim_{x\to-\infty} \frac{\sqrt{1+\frac{15}{x^2}}}{-5+\frac{20}{x}} = \frac{\sqrt{1+\frac{5}{\infty}}}{-5-\frac{20}{\infty}} = \frac{\sqrt{1+0}}{-5+0}$$

$$= \frac{\sqrt{1}}{-5} = \boxed{-\frac{1}{5}}$$

Horizontal Asymptotes : $y=\frac{1}{5}$ and $y=-\frac{1}{5}$

-2

7. (11 pts) (a) (8 pts) Find the derivative of $f(x)=\sqrt{x+5}$ using the definition; $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$.

$\boxed{8}$

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{(x+h)+5}-\sqrt{x+5}}{h} \cdot \frac{\sqrt{(x+h)+5}+\sqrt{x+5}}{\sqrt{(x+h)+5}+\sqrt{x+5}}$$

$$= \lim_{h\to 0} \frac{\cancel{x}+h\cancel{+5}\cancel{-x}\cancel{-5}}{(h)\left(\sqrt{(x+h)+5}+\sqrt{x+5}\right)} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{(h)}\left(\sqrt{x+h+5}+\sqrt{x+5}\right)}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{x+h+5}+\sqrt{x+5}} = \frac{1}{\sqrt{x+0+5}+\sqrt{x+5}}$$

$$\boxed{f'(x) = \frac{1}{2\sqrt{x+5}}}$$

(b) (3 pts) Find an equation of the line tangent to f at $x=-1$

$$f'(-1) = \frac{1}{2\sqrt{(-1)+5}} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$f(1) = \sqrt{-1+5}$
$f(1) = \sqrt{4}$
$f(1) = 2$

$$y-2 = \frac{1}{4}(x+1)$$

$$\boxed{y = \frac{1}{4}x + \frac{9}{4}}$$

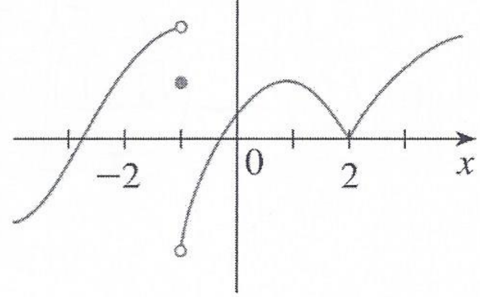
8. (8 pts) True/False. The graph of a function f is shown below. Circle T if the statement is true, otherwise circle F.

(a) T F $f'(-2) < 0$

(b) T F f is not differentiable at $x = 2$

(c) T F $f'(x) > 0$ for all x in the interval $(2,3)$

(d) T F $f'(-3) > f'(1.5)$



-0

9. (26 pts) Find the indicated derivative or value (exact in simplest form).

(a) (6 pts) $f(x)=5x^3-\dfrac{3}{x^2}+4\sqrt{x}+2^{\pi}$, $f'(1)$

$f(x)=5x^3-3x^{-2}+4x^{1/2}+2^{\pi}$

$f'(x)=15x^2+6x^{-3}+2x^{-1/2}+0$

$f'(x)=15x^2+\dfrac{6}{x^3}+\dfrac{2}{\sqrt{x}}$

$f'(1)=15(1)^2+\dfrac{6}{(1)^3}+\dfrac{2}{\sqrt{(1)}}$

$f'(1)=15+6+2$

$\boxed{f'(1)=23}$

(b) (6 pts) $y=(u^2-u)(6\sec u-u^{-4})$; $\dfrac{dy}{du}$ do not simplify.

$\boxed{f'(x)=(2u-1)(6\sec u-u^{-4})+(u^2-u)(6\sec u\tan u+4u^{-5})}$

(c) (7 pts) $\dfrac{d}{dx}\left[\dfrac{7-4x^5}{e^x+\tan x}\right]$ do not simplify.

$\boxed{f'(x)=\dfrac{(-20x^4)(e^x+\tan x)-(7-4x^5)(e^x+\sec^2 x)}{(e^x\tan x)^2}}$

(d) (7 pts) $f(x)=\cos(x)-\sin(x)$, $f''\left(\dfrac{\pi}{6}\right)$

$f'(x)=-\sin x-\cos x$

$f''(x)=-\cos x+\sin x$

$f''\left(\frac{\pi}{6}\right)=-\cos\left(\frac{\pi}{6}\right)+\sin\left(\frac{\pi}{6}\right)$

$f''\left(\frac{\pi}{6}\right)=-\frac{1}{2}+\frac{\sqrt{3}}{2}\longrightarrow -\left(\frac{\sqrt{3}}{2}\right)+\frac{1}{2}$

$\boxed{f''\left(\frac{\pi}{6}\right)=-\left(\frac{\sqrt{3}-1}{2}\right)}$ $\boxed{\frac{1-\sqrt{3}}{2}}$

-1

-1