1. (6 pts) Given the function $f(x) = 10x - 2x^2$ compute the slope of the secant line containing the points $(1, f(1))$ and $(2, f(2))$.

$f(2) = 10(2) - 2(2)^2 = 20 - 8 = 12$

$f(1) = 10(1) - 2(1)^2 = 8$

$\frac{f(a) - f(b)}{a - b} = m$

slope : $\frac{f(2) - f(1)}{2 - 1} = \frac{12 - 8}{2 - 1} = \frac{4}{1} = 4$

0

2. (12 pts) Determine any vertical asymptotes of the function. Justify your conclusion with an appropriate limit statement. Write the equation of each asymptote.

$$f(x) = \frac{x^2+16}{x^2-25}$$

$x^2-25=0$

$(x+5)(x-5)=0$

$x=\{-5,5\}$

Domain: $\{x \mid x \neq -5, 5\}$

Vertical Asymptotes: $x=-5$

$x=5$

$\lim_{x\to -5} \frac{x^2+16}{x^2-25}$

$= \lim_{x\to -5} \frac{x^2+16}{(x+5)(x-5)}$

= DNE due to vertical asymptote in which the one-sided limits approach $+\infty$ from the left and $-\infty$ from the right

x	y
-5.01	410.59
-5	undefined
-4.99	-409.41

$\lim_{x\to -5^-} f(x) = \infty$

$\lim_{x\to -5^+} f(x) = -\infty$

$\lim_{x\to -5} f(x) = $ DNE

$\lim_{x\to 5} \frac{x^2+16}{(x+5)(x-5)}$

= DNE due to vertical asymptote in which the one-sided limits approach $+\infty$ from the right and $-\infty$ from the left

x	y
5.01	410.59
5	undefined
4.99	-409.41

$\lim_{x\to 5^-} f(x) = -\infty$

$\lim_{x\to 5^+} f(x) = \infty$

$\lim_{x\to 5} f(x) = $ DNE

3. (6 pts) Determine the infinite limit. $\lim_{x\to -3^-} \frac{\sqrt{x+6}}{x^2-2x-15}$ $= \infty$

$\lim_{x\to -3^-} \frac{\sqrt{x+6}}{x^2-2x-15}$

$= \lim_{x\to -3^-} \frac{\sqrt{x+6}}{(x+3)(x-5)}$

$= \frac{\sqrt{3}}{0^-(-8)} = \infty$

together, they create a very small positive number (ex; 0.00000001)

therefore limit is positive ∞.

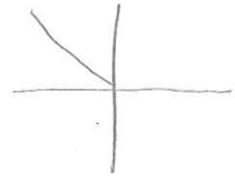
very small negative number (ex; -0.000001)

-0

4. Evaluate the limit, if it exists. (l'Hospital's rule is not permitted.)

$\frac{2}{3} + \frac{\sqrt{2}}{2} - 0$

(a) (7 pts) $\lim_{x\to 3}\left[\frac{2x}{\sqrt{45+12x}} + \sin\frac{\pi x}{4} - \ln(x-2)\right]$



$= \lim_{x\to 3}\frac{2x}{\sqrt{45+12x}} + \lim_{x\to 3}\sin\frac{\pi x}{4} - \lim_{x\to 3}\ln(x-2)$

$= \frac{2(3)}{\sqrt{45+12(3)}} + \sin\frac{3\pi}{4} - \ln 1 = \frac{2}{3} + \frac{\sqrt{2}}{2} - 0 = \frac{\sqrt{2}}{2} + \frac{2}{3}$ -1

Simplify

(b) (9 pts) $\lim_{x\to 3}\frac{\frac{1}{x+1}-\frac{1}{4}}{3-x}$

$\lim_{x\to 3}\frac{\left(\frac{1}{x+1}-\frac{1}{4}\right)}{3-x}\cdot\frac{4(x+1)}{4(x+1)} = \lim_{x\to 3}\frac{\frac{4\cancel{(x+1)}}{\cancel{x+1}} - \frac{\cancel{4}(x+1)}{\cancel{4}}}{\frac{4(x+1)}{3-x}}$

$= \lim_{x\to 3}\frac{4-(x+1)}{(3-x)(4(x+1))}$

$= \lim_{x\to 3}\frac{\cancel{3-x}}{(\cancel{3-x})(4x+4)}$

$= \lim_{x\to 3}\frac{1}{4x+4} = \frac{1}{12+4} = \frac{1}{16}$

(c) (9 pts) $\lim_{x\to 2}\frac{x-\sqrt{10-3x}}{x-2}$

$\lim_{x\to 2}\frac{x-\sqrt{10-3x}}{x-2}\cdot\frac{x+\sqrt{10-3x}}{x+\sqrt{10-3x}}$

$= \lim_{x\to 2}\frac{x^2-(10-3x)}{(x-2)(x+\sqrt{10-3x})}$

$= \lim_{x\to 2}\frac{x^2+3x-10}{(x-2)(x+\sqrt{10-3x})}$

$= \lim_{x\to 2}\frac{(x+5)\cancel{(x-2)}}{\cancel{(x-2)}(x+\sqrt{10-3x})}$

$= \lim_{x\to 2}\frac{x+5}{x+\sqrt{10-3x}} = \frac{2+5}{2+\sqrt{10-3(2)}} = \frac{7}{2+2} = \frac{7}{4}$

-1

5. (8 pts) Complete the epsilon-delta definition of limit.
Let f be a function defined on some open interval that contains the number a, except possibly at a itself. Then we say that the **limit of $f(x)$ as x approaches a is L**, and we write

$$\lim_{x \to a} f(x) = L$$

$\lim_{x \to a} f(x) = L$ is true if there is a value $\varepsilon > 0$ for every number $\delta > 0$ such that when $|x-a| < \delta$ then $|f(x) - L| < \varepsilon$

-2

6. (8 pts) Identify any discontinuities of the function. Justify your conclusion.

$$f(x) = \begin{cases} \sqrt{2x^2+1} & x \le -2 \\ x-2 & -2 < x < 2 \\ x^2-4 & x > 2 \end{cases}$$

to be continuos

- $f(a)$ must exist
- $\lim_{x \to a} f(x)$ must exist
- $\lim_{x \to a} f(x) = f(a)$

$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} \sqrt{2x^2+1} = \sqrt{9} = 3$

$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} x-2 = -2-2 = -4$

discontinuous at $x=-2$ since $\lim_{x \to -2} f(x)$ DNE because one-sided limits aren't the same

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x-2 = 2-2 = 0$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} x^2-4 = 2^2-4 = 4-4 = 0$

although one-sided limits are the same at $x=2$, and $\lim_{x \to 2} f(x)$ exists, there is no $f(2)$ contained in $f(x)$

$\therefore$ The function is discontinuous at $x=2$.

-2

7. (8 pts) For what value of the constant b (if any) is the function every where continuous.

$$f(x) = \begin{cases} b^2x^2 + 2bx & x < 3 \\ \cos \pi x & x \geq 3 \end{cases}$$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 4(9)(1)}}{2(9)} = \frac{-6}{18} = -\frac{1}{3}$

$\boxed{\text{function is continuous everywhere when } b = -\frac{1}{3}}$

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$

$= \lim_{x \to 3^-} b^2x^2 + 2bx = \lim_{x \to 3^+} \cos \pi x$

$= b^2(3)^2 + 2b(3) = \cos \pi(3)$

$= 9b^2 + 6b = \cos 3\pi$

$9b^2 + 6b = -1$

$9b^2 + 6b + 1 = 0$

$(3b + 1)(3b + 1) = 0$

8. Find the limit. Show your work

(a) (4 pts) $\lim\limits_{x \to \infty} e^{-2x}$

$\lim_{x \to \infty} \frac{1}{e^{2x}} = 0$

$\lim_{x \to \infty} e^{-2x} = 0$

(b) (4 pts) $\lim\limits_{x \to -\infty} \arctan x^2$

since always positive due to x^2

$\lim_{x \to -\infty} \arctan x^2 = \frac{\pi}{2}$

(c) (7 pts) $\lim\limits_{x \to \infty} \frac{\sqrt{6x^2 - 23x}}{7 - 2x}$

$= \lim_{x \to \infty} \frac{\sqrt{6x^2 - 23x}}{7 - 2x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim_{x \to \infty} \frac{\sqrt{(6x^2 - 23x)/x^2}}{\frac{7}{x} - 2}$

$= \lim_{x \to \infty} \frac{\sqrt{6 - \frac{23}{x}}}{\frac{7}{x} - 2} = \frac{\sqrt{6 - 0}}{0 - 2}$

$= -\frac{\sqrt{6}}{2}$

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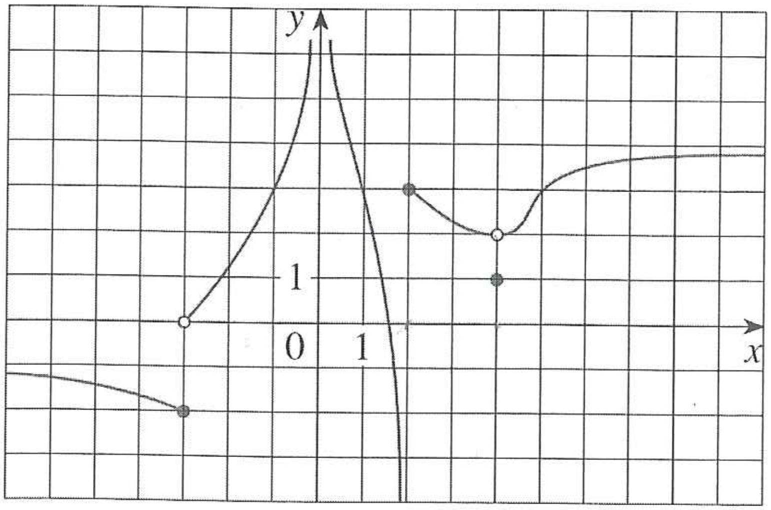
9. (12 pts) The graph of a function $f(x)$ is shown.



Answer the following:

(a) $\lim_{x \to 2^-} f(x)$ $= -\infty$

(b) $\lim_{x \to 4} f(x)$ $= 2$

(c) $\lim_{x \to -\infty} f(x)$ $= -1$

(d) $f(x)$ is left continuous at $x = -3$ (Write "left" or "right" in the blank)

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