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1. (6 pts) The position of a car is given by the values in the following table.

t (seconds)	0	1	2	3	4	5
s (meters)	0	18	35	74	110	175

Find the average velocity for the time period beginning when $t = 2$ and lasting 2 seconds.

$$\frac{\Delta y}{\Delta x} = \frac{110-35}{4-2} = \frac{75}{2} = \boxed{37.5 \text{ m/s}}$$

2. (10 pts) The graph of f is given below. Evaluate each limit if it is finite: otherwise write ∞, $-\infty$, or DNE

(a) $\lim_{x\to 0^-} f(x)$

$\lim_{x\to 0^-} f(x) = \boxed{\infty}$

(b) $\lim_{x\to -3^+} f(x)$

$\lim_{x\to -3^+} f(x) = \boxed{0}$

(c) $\lim_{x\to 4} f(x)$

$\lim_{x\to 4} f(x) = \boxed{2}$

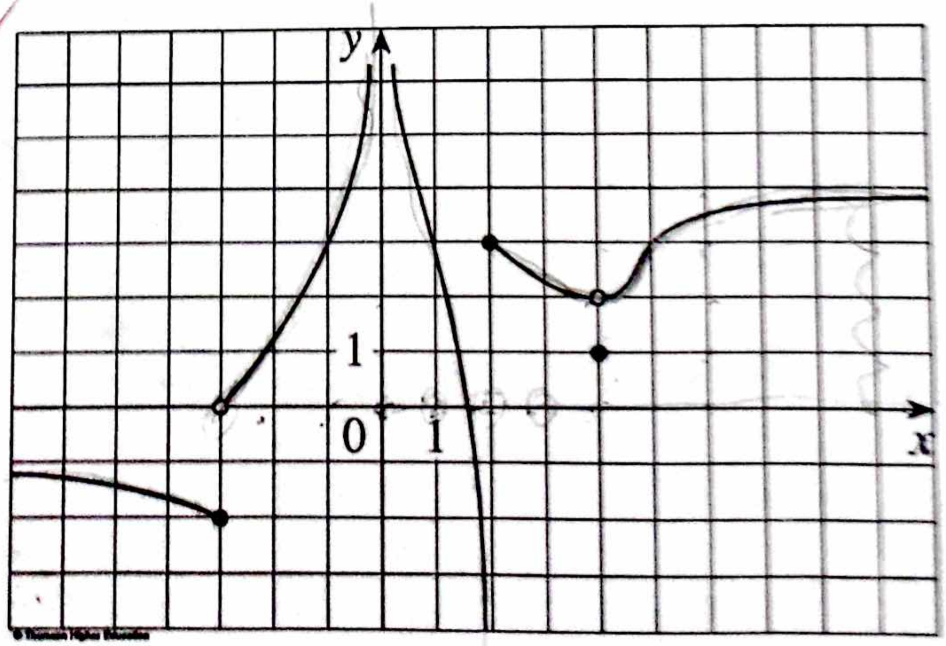
(d) $\lim_{x\to \infty} f(x)$

$\lim_{x\to \infty} f(x) = \boxed{4}$

y-value?

(e) $\lim_{x\to 2^-} f(x)$

$\lim_{x\to 2^-} f(x) = \boxed{-\infty}$



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3. (24 pts) Find the limit, give the exact value. (L'Hospital's rule is not permitted.)

(a) (6 pts) $\lim_{x\to -3}\left[\ln(x+4)-\cos\frac{\pi}{x}+\frac{\sqrt{2x+15}}{x^2-8}\right]$

* Distributive Property *

$\lim_{x\to -3}\left[\ln(x+4)-\cos\frac{\pi}{x}+\frac{\sqrt{2x+15}}{x^2-8}\right] = \left[\ln(-3+4)-\cos\frac{\pi}{-3}+\frac{\sqrt{2(-3)+1}}{(-3)^2-8}\right.$

$= \left[\ln(1)-\left(\frac{1}{2}\right)+\frac{3}{1}\right]$

$\lim_{x\to -3}\left[\ln(x+4)-\cos\frac{\pi}{x}+\frac{\sqrt{2x+15}}{x^2-8}\right] = \boxed{2.5}$ ✓

(b) (6 pts) $\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}$

$\lim_{x\to 2}\frac{x^2+x-6}{x^2-4}$

$\lim_{x\to 2}\frac{(x-2)(x+3)}{(x-2)(x+2)}$

$\lim_{x\to 2}\frac{(2+3)}{(2+2)} = \frac{5}{4}$

$\lim_{x\to 2}\frac{x^2+x-6}{x^2-4} = \boxed{\frac{5}{4}}$ ✓

-0

3. (Continued)

(c) (6pts) $\lim_{x\to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x}$

Combine one fraction numerator

$$\lim_{x\to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x} = \frac{\frac{\cancel{4}-x-\cancel{4}}{4(x+4)}}{x}$$

$$\lim_{x\to 0} \frac{-\cancel{x}}{4(x+4)} \cdot \frac{1}{\cancel{x}} = \frac{-1}{4(x+4)}$$

$$\lim_{x\to 0} \frac{-1}{4(0+4)} = \frac{-1}{16}$$

$$\lim_{x\to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x} = \boxed{\frac{-1}{16}}$$

(d) (6 pts) $\lim_{x\to\infty} \arctan\left(\frac{x^2-2x}{x}\right)$

$$\lim_{x\to\infty} \arctan\left(\frac{x^2-2x}{x}\right) = \frac{\cancel{x}\left(\frac{x^2}{x}-\frac{2\cancel{x}}{\cancel{x}}\right)}{\cancel{x}} = x-2$$

$$\lim_{x\to\infty} \arctan(x-2) = \lim_{x\to\infty} \arctan(\infty-2)$$

* Domain of arctan is $\mathbb{R}$ * $x \in \mathbb{R}$

$$\boxed{\lim_{x\to\infty} \arctan\left(\frac{x^2-2x}{x}\right) = \boxed{\frac{\pi}{2}}}$$

4. (8 pts) Complete the ε, δ definition of limit.

Let f be a function defined on some open interval that contains the number a, except possibly at a itself. We say that the **limit of** $f(x)$ **as** x **approaches** a **is** L, and we write

$\lim_{x \to a} f(x) = L$ if

for every Epsilon greater than zero, there exists a delta that's greater than zero, such that $|x-a|$ is less than delta if and only if $|f(x)-L|$ is less than Epsilon.

$\forall \epsilon > 0; \exists \delta > 0$ st $|x-a| < \delta \Rightarrow |f(x)-L| < \epsilon$

5. (10 pts) For what value(s) of $\underline{a}$ is the function f everywhere continuous?

$$f(x) = \begin{cases} a(x^2+2a) & x<0 \\ 4+a\sqrt{x+4} & x \geq 0 \end{cases}$$

$\lim_{x \to 0^-} a(x^2+2a) = a(0+2a)$

$\lim_{x \to 0^+} 4+a\sqrt{x+4} = 4+a\sqrt{0+4}$

To be continous everywhere the $\lim_{x \to 0^-}$ and $\lim_{x \to 0^+}$ must be equal

$a(2a) = 4 + a\sqrt{4}$

$2a^2 = 4 + 2a$

$0 = \frac{2a^2 - 2a - 4}{2} = a^2 - a - 2 = (a-2)(a+1)$

$a = 2$ and $a = -1$

-0

HA

6. (12 pts) Find all horizontal asymptotes of the function. Use appropriate limit statements to justify your conclusion.

$$f(x) = \frac{\sqrt{x^2+15}}{5x-20}$$

$\lim_{x \to a} f(x) = L$

L is on HA b/c
HA is y =

~~$5x - 20 = 0$~~

~~$x = \frac{20}{5} = 4$ *possible HA*~~

$$\lim_{x \to \infty} \frac{\sqrt{x^2+15}}{5x-20}$$

$$\lim_{x \to \infty} \frac{\sqrt{x^2\left(1+\frac{15}{x^2}\right)}}{x\left(5-\frac{20}{x}\right)} = \frac{|x|\sqrt{1+\cancel{\frac{15}{x^2}}}}{\cancel{x}\left(5-\cancel{\frac{20}{x}}\right)} = \frac{\cancel{x}\sqrt{1+0}}{\cancel{x}(5-0)}$$

$= \frac{1}{5}$ is a HA b/c the limit of $\frac{\sqrt{x^2+15}}{5x-20}$ at $x \to \infty$ is $\frac{1}{5}$

-4

$\lim_{x \to -\infty}$

$\lim_{x \to -\infty} f(x) = \boxed{-\frac{1}{5}}$

-4

7. (12 pts) Find an equation of the line tangent to the graph of $y = \sqrt{x-2}$ at the point with x-coordinate $x = 11$. Use the formula $m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$ to compute the slope. Express your answer in the form $y = mx + b$

7 work
$ Night!

$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\sqrt{x-2} - \sqrt{a-2}}{x - a}$

$a = 11$

$\lim_{x \to 11} \frac{\sqrt{x-2} - \sqrt{11-2}}{x - 11}$

$\lim_{x \to 11} = \frac{\sqrt{x-2} - 3}{x - 11} \cdot \frac{\sqrt{x-2} + 3}{\sqrt{x-2} + 3} = \frac{x - 2 - 9}{x - 11 (\sqrt{x-2} + 3)}$

$\lim_{x \to 11} = \frac{\cancel{(x-11)}}{\cancel{(x-11}(\sqrt{x-2} + 3)} = \frac{1}{6} = m$

$y = \sqrt{11-2}$

$y = 3$

$(11, 3)$ m

$y - 3 = \frac{1}{6}(x - 11)$

$\boxed{y = \frac{1}{6}x + \frac{7}{6}}$

8. (12 pts) Given the function $f(x) = 3x^2 - 1$, find the derivative of the function using the definition of derivative.

$f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{\left(3(x+h)^2 - 1\right) - \left(3x^2 - 1\right)}{h}$$

$$\lim_{h \to 0} \frac{\left(3(x^2 + 2xh + h^2) - 1\right) - \left(3x^2 - 1\right)}{h}$$

$$\lim_{h \to 0} \frac{\cancel{3x^2} + 6xh + h^2 \cancel{-1} \cancel{-3x^2} \cancel{+1}}{h}$$

$$\lim_{h \to 0} \frac{\cancel{h}(6x + h)}{\cancel{h}}$$

$$\lim_{h \to 0} 6x + h = 6x + 0 = 6x$$

$$f'(x) = \boxed{6x}$$

9. (6 pts) Circle any statement that is true for the function as shown.

(a) $f'(-4) < 0$

(b) $f'(4) = 0$

(c) f is differentiable at $x = 1$