1. (12 pts) Find the derivative. Do not simplify.

(a) $f(x) = \cos(7x)$

$f'(x) = -\sin(7x) \cdot 7$

$f'(x) = -7\sin(7x)$

(b) $f(x) = \sqrt{x^3 + 5x}$

$f'(x) = \frac{1}{2\sqrt{x^3+5x}} \cdot 3x^2+5$

$f'(x) = \frac{3x^2+5}{2\sqrt{x^3+5x}}$

0

2. (12 pts) Find the derivative. Do not simplify.

(a) $f(x) = e^{-(x+1)^2}$

$\ln f(x) = \ln e^{-(x+1)^2}$

$\ln f(x) = -(x+1)^2$

$\frac{f'(x)}{f(x)} = -2(x+1)$

$f'(x) = -2(x+1) \cdot e^{-(x+1)^2}$ ✓

$f'(x) = e^{-(x+1)^2} \cdot (-2(x+1))(1)$

$f'(x) = -2(x+1)e^{-(x+1)^2}$ ✓

(b) $f(x) = \arctan(1-2x^2)$

$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}\,dx$

$f'(x) = \frac{1}{1+(1-2x^2)^2} \cdot (-4x)$

$f'(x) = \frac{-4x}{1+(1-2x^2)^2}$ ✓

10

3. (10 pts) Find $\frac{dy}{dx}$ by implicit differentiation.

$\sec y = 3xy^2$

$\sec y \tan y \cdot y' = 3y^2 + 3x2y \cdot y'$

$y'(\sec y \tan y - 3x2y) = 3y^2$

$y' = \frac{3y^2}{\sec y \tan y - 3x2y}$

4. (10 pts) Use logarithmic differentiation or an equivalent method to find $\frac{dy}{dx}$.

$y = (4 + x^2)^{\tan x}$

$\ln y = \ln(4+x^2)^{\tan x}$

$\ln y = \tan x \cdot \ln(4+x^2)$

$\frac{y'}{y} = \sec^2 x \cdot \ln(4+x^2) + \tan x \cdot \frac{1}{4+x^2} \cdot 2x$

$\frac{y'}{y} = \sec^2 x \cdot \ln(4+x^2) + \frac{2x \tan x}{4+x^2}$

$y' = \left(\sec^2 x \cdot \ln(4+x^2) + \frac{2x \cdot \tan x}{4+x^2}\right)(4+x^2)^{\tan x}$

-0

5. (8 pts) If $y = \arcsin(3x-1)$, find the differential dy then evaluate at $x = \frac{1}{3}$ and $dx = 0.12$.

$dy = \frac{3}{\sqrt{1-(3x-1)^2}}\,dx$

$dy = \frac{3}{\sqrt{1-[3(\frac{1}{3})-1]^2}}(0.12)$

$dy = \frac{3}{\sqrt{1-(1-1)^2}}(0.12)$

$dy = \frac{3}{1}(0.12$

$\boxed{dy = 0.36}$ ✓

6. (16 pts) Find the limit.

(a) $\lim_{x\to 3} \frac{\sin(x-3)}{x^2-4x+3}$ $\quad \frac{\sin(3-3)}{3^2-4(3)+3} = \frac{0}{0}$

$\lim_{x\to 3} \frac{\cos(x-3)}{2x-4}$

$\lim_{x\to 3} \frac{\cos(3-3)}{2(3)-4} = \frac{1}{6-4} = \frac{1}{2}$ ✓

(b) $\lim_{x\to\infty} \frac{\ln(x^2)}{x}$ $\quad \frac{\infty}{\infty}$

$\lim_{x\to\infty} \frac{\frac{2x}{x^2}}{1}$

$\lim_{x\to\infty} \frac{2/x}{1}$

$\lim_{x\to\infty} \frac{2}{x} = \boxed{0}$ ✓

/0

7. (10 pts) A 30 foot long ladder is leaning against the wall of a house. The base of the ladder is pulled away from the wall at 2 feet per minute. How fast is the top of the ladder moving down the wall when the base of the ladder is 5 feet from the wall?

Label the figure with the variables used in your solution.

Give your result rounded to 2 decimal places in proper units.

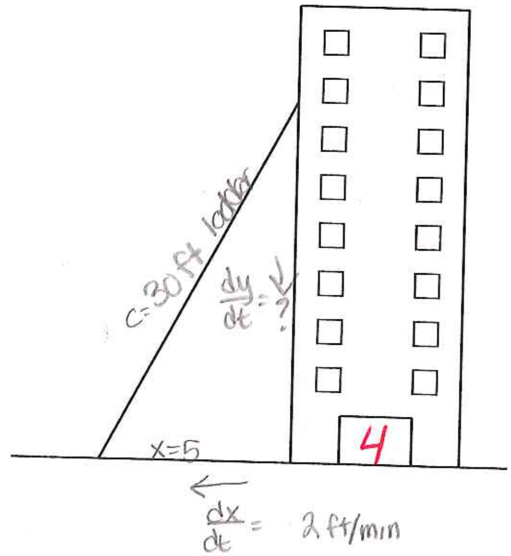
$x^2+y^2=c^2$

$5^2+y^2=30^2$

$25+y^2=900$

$y^2=875$

$y=5\sqrt{35}$



$2x\cdot\frac{dx}{dt}+2y\cdot\frac{dy}{dt}=0$

$x\cdot\frac{dx}{dt}=-y\cdot\frac{dy}{dt}$

$5\cdot 2=-5\sqrt{35}\,\frac{dy}{dt}$

$\frac{10}{5\sqrt{35}}=\frac{dy}{dt}$

$\frac{-2}{\sqrt{35}}=\frac{dy}{dt}$

$\frac{dy}{dt}=-0.34$ ft/min

100/100

-0

8. (12 pts) Sketch the graph of a function that satisfies the given conditions.

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The domain of f: $(-4,\infty)$,

$f(-3)=0$ $\qquad$ $f(2)=0$ $\qquad$ $f(0)=-3$ is the absolute minimum

$\lim_{x\to -4^+} f(x)=\infty$, $\qquad$ $\lim_{x\to\infty} f(x)=3$,

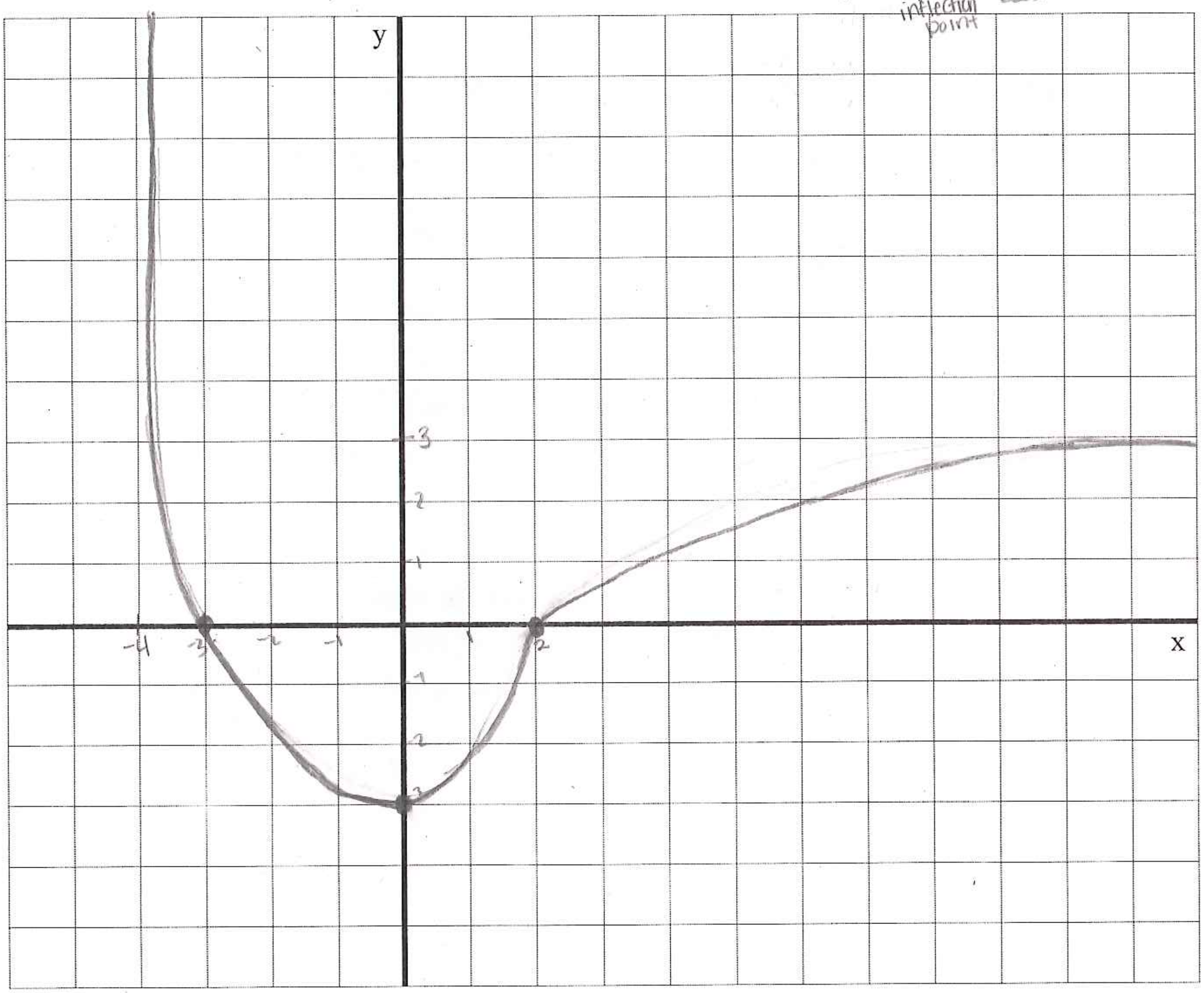
$f'(x)<0$ on $(-4,0)$ $\qquad$ $f'(x)>0$ on $(0,\infty)$

$f''(x)>0$ on $(-4,2)$ $\qquad$ $f''(x)<0$ on $(2,\infty)$

f' −4 0 +

f'' −4 + concave up 2 − concave down

inflection point



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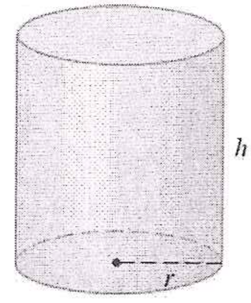
9. (10 pts) A closed cylindrical juice can is to have a volume of 2,000π cubic centimeters. Find the dimensions, r and h, that will minimize the surface area of the can (and hence the cost of the metal to manufacture the can).

$$V = \pi r^2 h$$
$$S = 2\pi r^2 + 2\pi rh$$

$V = 2000\pi \text{ cm}^3$



$2000\cancel{\pi} = \cancel{\pi} r^2 h$

$h = \frac{2000}{r^2}$ ✓

$S = 2\pi r^2 + 2\pi r h$

$S = 2\pi r^2 + 2\pi r \left(\frac{2000}{r^2}\right)$ ✓

$S = 2\pi r^2 + \frac{4000\pi}{r}$ ✓ $\quad r > 0$

$\frac{dS}{dr} = 4\pi r - \frac{4000\pi}{r^2}$ ✓

$\frac{dS}{dr} = \frac{4\pi r^3 - 4000\pi}{r^2}$

$\frac{dS}{dt} = 0 = \frac{4\pi r^3 - 4000\pi}{r^2}$

$4\pi r^3 - 4000\pi = 0$

$4\pi r^3 = 4000\pi$

$r^3 = \frac{4000\cancel{\pi}}{4\cancel{\pi}}$

$r^3 = 1000$

$r = 10$ ✓

$h = \frac{2000}{r^2}$

$h = \frac{2000}{10^2}$

$h = 20$

	−		+	
0		10		∞

therefore $r = 10$ is an absolute minimum ✓

$\boxed{\begin{array}{l} r = 10 \text{ cm} \\ h = 20 \text{ cm} \end{array} \text{ will be the dimensions that will minimize the Suface area of the can}}$ ✓

-0