PROPOSED METHODOLOGY

Overview

In this section, an efficient and robust model using **Type-2 fuzzy logic** is introduced to segment a 3D brain MR image volume into its constituent tissue regions.

This chapter describes the underlying algorithm and the mathematical formula involved in the clustering algorithm.

The proposed algorithm involves calculating the global and local membership values for each of the voxels using entropy-based type-1 membership functions.

The primary type-2 fuzzy interval set is computed using the global membership values of the adjacent image volume of each of the voxels.

The secondary membership type-2 fuzzy interval set is computed using the local membership values of the adjacent image volume of each of the voxels.

Using these two-interval set, the final type-2 fuzzy interval set is calculated. The final type-2 fuzzy interval set is defuzzified and then normalized to get the final type-2 membership value of that particular voxel.

Once the global, local and normalized type-2 membership values are obtained for each of the voxels, the final membership values for the same is calculated. The final membership value of each of the voxel is calculated by regularizing the values of global, local and normalized type-2 membership values using suitable parameters, those are identified empirically.

Rest of the chapter explains the proposed algorithm in details.

Method

Due to noise, IIH and various other external factors like patient's movement during the imaging, coil inhomogeneity, etc., the intensity of the voxel distribution varies across the 3D brain MR image volume. This results in heterogenous and non-uniform visual texture and blurry edges.

As a result, there is an uncertainty in the process of predicting the class of each of the voxel. The uncertainty increases at the tissue or region boundaries.

To mitigate this uncertainty, we have introduced Type-2 Membership set to define the class uncertainty associated with each of the voxels.

Unlike, Type-1 Membership value which is a crisp membership value for each of the class, Type-2 interval set, represents a fuzzy set of possible values for each of the voxels.

As a result, Type-2 Membership values are a better representation of the class uncertainty associated with each of the voxel specially those are located at the edge of the tissue region.

Primarily, we have worked with three membership values:

1. **Global Type-1 Membership value** computed by an global entropy-based membership function which is calculated independently.

2. **Local Type-1 Membership value** computed by a spatially constrained likelihood-based entropy function which is calculated independently.

3. **Normalized Type-2 Fuzzy Membership value** computed using the Global Type-1 membership value and Local Type-1 Membership value.

The **global Type-1 membership value** represents the class uncertainty associated with a voxel's class in regard to the whole 3D image volume.

The **local Type-1 membership value** represents the class uncertainty associated with a voxel's class in regard to its immediate neighborhood image volume.

The **primary Type-2 fuzzy interval set** is concluded from the global membership values in the neighborhood for a particular voxel and the **secondary Type-2 fuzzy interval set** is concluded from the local membership values for a particular voxel.

Using the primary and secondary Type-2 fuzzy interval set, a final Type-2 fuzzy interval set is calculated.

For a particular voxel, this set represents a range of fuzzy values associated with a voxel's class uncertainty.

This interval set is then defuzzified and normalized to get the final **Normalized Type-2 Fuzzy membership value**.

The final membership function for a particular voxel is obtained by a weighted combination of these three membership functions to mitigate the tradeoff between the global, local and normalized Type-2 defuzzified membership value.

The 3D brain MR image volume is then segmented based on this final membership value for each of the voxels.

ALGORITHM

INPUT:

One **3D brain MR image volume A** having **height of X**, **width of Y** and **depth of Z** is provided.

C is a set of distinct clusters or tissue regions in which the volume has to be segmented.

OUTPUT:

The set of final cluster centers T, membership matrix G and segmented image volume $B_i, i = 1,2,3, \ldots, C$

STEPS:

1. Set the initial values of the parameters **G, H, I, P, Q, R M, and error E.**

   - **G, H** and **I** are the weighted parameters to calculate the new cluster centers and are subject to the constraint:
     (G + H + I) = 1 and $\mathbf{0 < G, H\ and\ I < 1.}$
   - The parameter **M** is the fuzzifier with a usual value $\boldsymbol{M \geq 1}$ and must be selected empirically.
   - **P, Q** and **R** are the regularizing parameters with usual values $\boldsymbol{P, Q\ and\ R \geq 1}$

2. Set the maximum number of iteration **N**.

3. Initialize the cluster centers $\boldsymbol{t_i^0}$. The cluster centers can be initialized based on the information available from the image histogram.

4. Using **FCM algorithm**, the initial global membership values and the initial local membership values for each of the voxels are initialized using the cluster centers calculated from the previous step using the equation given below:

$$\boldsymbol{\mu_{i_{jkl}}^0, u_{i_{jkl}}^0 = \frac{(a_{jkl} - t_i)^2}{\sum_{r=1}^{C}[(a_{jkl} - t_i)^2]^{\frac{1}{(M-1)}}}}$$

- a_{jkl} is the value of the intensity of the voxel at **j, k** and **l** coordinates.
- t_i is the intensity of the ***i****th* cluster center
- $\mu^{0}_{i_{jkl}}$ is the global membership value at the **0th** iteration of the ***i****th* cluster and for the voxel at **j, k** and **l** coordinates.
- $u^{0}_{i_{jkl}}$ is the local membership value in the ***0****th* iteration of the ***i****th* cluster and for the voxel at **j, k** and **l** coordinates.

5. Set iteration **n = 0**.

6. Repeat

a. The **global membership value** is calculated using the equation:

$$\mu^{(n+1)}_{i_{jkl}} = \frac{1}{\sum_{r=1}^{C} \frac{[d^{2}_{i_{jkl}} - ln(\mu^{(M)}_{i_{jkl}}) - 1]}{\left[([d^{2}_{r_{jkl}} - ln(\mu^{(M)}_{r_{jkl}}) - 1)^{\frac{1}{(M-1)}}\right]}}$$

- $d_{i_{jkl}}$ represents the Euclidean distance between the voxel a_{jkl} and centre of the i^{th} cluster t_i and is calculated using the following formula $d^{2}{}_{i_{jkl}} = \left|\left|a_{jkl} - t_i\right|\right|^{2} \ \forall j, k, l$

b. The **local membership value** is calculated using the equation:

$$u^{(n+1)}_{i_{jkl}} = \frac{1}{\sum_{r=1}^{C} \frac{[f^{-1}_{i_{jkl}} - \overline{d}^{2}_{i_{jkl}} - ln(u^{(M)}_{i_{jkl}}) - 1]}{[(f^{-1}_{i_{jkl}} - \overline{d}^{2}_{i_{jkl}} - ln(u^{(M)}_{i_{jkl}}) - 1)^{\frac{1}{(M-1)}}]}}$$

- $\overline{d}^{2}_{i_{jkl}}$ denotes the mean of the Euclidean distances between the neighbouring voxels of a_{jkl} and centre of the i^{th} cluster t_i and is calculated as $\overline{d}^{2}_{i_{jkl}} = \frac{1}{N} \sum_{x_{jkl} \in A_{jkl}} ||x_{jkl} - t_i||^{2}$.
- A_{jkl} is the set of neighbouring voxels of a_{jkl}.
- $f^{-1}_{i_{jkl}}$ is the likelihood or possibility measure of belongingness into the i^{th} cluster for the voxel a_{jkl} by using constrained local

neighbourhood information.

It is calculated as $f^{-1}_{i_{jkl}} = \frac{\sum_{x_{jkl} \in N_{jkl}} \left(u^{0}_{i_{jkl}} x_{jkl}\right)}{\sum_{x_{jkl} \in N_{jkl}} (x_{jkl})}$

c. The **primary Type-2 Fuzzy interval set** for each of the voxel is calculated using the following equation: $T_p^{(n+1)} = \{\mu_i \mid \mu_i \in A.\mu_{i_{jkl}}\}$
$A.\mu_{i_{jkl}}$ represents the global Membership values of the voxels in the neighborhood of the a_{jkl} voxel.

d. The **secondary Type-2 Fuzzy interval set** is calculated using the following equation: $T_s^{(n+1)} = \{u_i \mid u_i \in A.u_{i_{jkl}}\}$
$A.u_{i_{jkl}}$ represents the local Membership values of the voxels in the neighborhood of the a_{jkl} voxel.

e. The **final Type-2 Fuzzy interval set** is calculated using the following equation: $T_f^{(n+1)} = \{z_i = \mu_i u_i \mid \mu_i \in A.\mu_{i\ jkl}\ and\ u_i \in A.u_{i_{jkl}}\ \}$

f. The **Type-2 Fuzzy interval set** is de-fuzzifed using this equation:

$$\rho`_{i_{jkl}} = \frac{\sum_{z \in T_f^{(n+1)}} z}{\sum_{u \in T_s^{(n+1)}} u}$$

g. The **Normalized Type-2 Fuzzy membership value** is calculated using the following formula:

$$\rho_{\ i_{jkl}} = \frac{\rho`_{i_{jkl}}}{\sum_{r=1}^{C} (\rho`_{\mathrm{r}_{jkl}})}$$

h. The **new cluster center** is calculated as follows:

$$t_i^{(n+1)} = \frac{\sum_{j=1}^{Z}\sum_{k=1}^{Y}\sum_{l=1}^{X} [G\times\mu_{i_{jkl}}^{m} a_{jkl} + H\times u_{i_{jkl}}^{m} f^{-1}_{i_{jkl}} \bar{a} + \mathrm{I}\times\rho_{\mathrm{i_{jkl}}}^{\mathrm{m}} \mathrm{a_{jkl}}]}{\sum_{j=1}^{Z}\sum_{k=1}^{Y}\sum_{l=1}^{X} [G\times\mu_{i_{jkl}}^{m} + H\times u_{i_{jkl}}^{m} f^{-1}_{i_{jkl}} + \mathrm{I}\times\rho_{\mathrm{i_{jkl}}}^{\mathrm{m}}]}$$

i. The **error** in the current iteration is calculated using this formula:

$$\bar{\varepsilon} = \frac{\left\|t_1^{(n+1)} - t_1^{(n)}\right\| + \left\|t_2^{(n+1)} - t_2^{(n)}\right\| + \cdots + \left\|t_C^{(n+1)} - t_C^{(n)}\right\|}{C}$$

7. Until $\bar{\boldsymbol{\varepsilon}} <= \boldsymbol{E}$ ***or n*** $>= \boldsymbol{N}$

8. The **final Membership value** is calculated by the weighted regularization of the membership values obtained:

$$\boldsymbol{g}_{\boldsymbol{i}_{\boldsymbol{jkl}}} = \frac{(\boldsymbol{\mu}_{\boldsymbol{i}_{\boldsymbol{jkl}}})^{\boldsymbol{P}} (\boldsymbol{u}_{\boldsymbol{i}_{\boldsymbol{jkl}}})^{\boldsymbol{Q}} (\boldsymbol{\rho}_{\boldsymbol{i}_{\boldsymbol{jkl}}})^{\boldsymbol{R}}}{\sum_{\boldsymbol{r}=\boldsymbol{1}}^{\boldsymbol{C}}[(\boldsymbol{\mu}_{\boldsymbol{r}_{\boldsymbol{jkl}}})^{\boldsymbol{P}} (\boldsymbol{u}_{\boldsymbol{r}_{\boldsymbol{jkl}}})^{\boldsymbol{Q}} (\boldsymbol{\rho}_{\boldsymbol{r}_{\boldsymbol{jkl}}})^{\boldsymbol{R}}]}$$

9. Return the cluster centers $\boldsymbol{T} = \{\boldsymbol{t}_{\boldsymbol{1}}, \boldsymbol{t}_{\boldsymbol{2}}, \boldsymbol{t}_{\boldsymbol{3}}, \ldots, \boldsymbol{t}_{\boldsymbol{C}}\}$.

10. The membership matrix $\boldsymbol{G} = \{\boldsymbol{g}_{\boldsymbol{i}_{\boldsymbol{jkl}}}\} \; \forall \; \boldsymbol{j}, \boldsymbol{k}, \boldsymbol{l}$

11. Determine the clusters of the voxels $\boldsymbol{a}_{\boldsymbol{jkl}} \; \forall \; \boldsymbol{j}, \boldsymbol{k}, \boldsymbol{l}$ as follows and return the segmented 3D image volume $\boldsymbol{B}_{\boldsymbol{i}}, \forall \; \boldsymbol{i}$

$$\mathbf{cluster}\;(\mathbf{a}_{\mathbf{jkl}}) = \underset{\mathbf{i}}{\mathbf{arg\,max}} \{\mathbf{g}_{\mathbf{i}_{\mathbf{jkl}}}\} \; \mathbf{i} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots, \mathbf{C}$$

COMPLEXITY ANALYSIS

Time Complexity

Let us assume that $\boldsymbol{S}$ is a noisy image volume with size of $\boldsymbol{X} \times \boldsymbol{Y} \times \boldsymbol{Z}$ (height × width × depth) and is required to segment the image volume into C regions.

Each voxel is represented as a d-dimensional features vector.

The neighborhood image volume of each of the voxel is represented as $\boldsymbol{A_T}$.

Time complexity to calculate the global membership value

$\boldsymbol{M} = \boldsymbol{O}(\boldsymbol{dS}\mathrm{C})$ if d = 1, (We are considering only one feature of a voxel, that is the intensity. It is represented by a value in the range of [0,255])

then, $\boldsymbol{M} = \boldsymbol{O}(\boldsymbol{S}\mathrm{C})$

Time complexity to calculate the local membership value

Mean of each voxel: $\boldsymbol{P} = \boldsymbol{O}(\boldsymbol{A_T})$
Mean for all the voxels will be $\boldsymbol{P} = \boldsymbol{O}(\boldsymbol{SA_T})$

Likelihood of each of the voxel: $\boldsymbol{Q} = \boldsymbol{O}(\boldsymbol{A_T})$
Likelihood for all the voxels will be $\boldsymbol{Q} = \boldsymbol{O}(\boldsymbol{SA_T})$

Calculating the final value of the local membership value which also include the normalization process.
$\boldsymbol{U} = \boldsymbol{O}(\boldsymbol{SC} + \boldsymbol{2SA_T})$

Time complexity to calculate the Type-2 Fuzzy membership value

Primary and Secondary Type-2 interval set
$\boldsymbol{T_S} = \boldsymbol{O}(\boldsymbol{SA_T})$
Final Type-2 Fuzzy value $\boldsymbol{T_f} = \boldsymbol{O}(\boldsymbol{SA_T})$
Normalizing the Type-2 Fuzzy value $\boldsymbol{G} = \boldsymbol{O}(\boldsymbol{SC})$

Time complexity to calculate the Final weighted membership value

$\boldsymbol{F} = \boldsymbol{O}(\boldsymbol{SC})$

Total time complexity of the algorithm:

$$\boldsymbol{L = M + U + T_s + T_f + G + F}$$

$$\boldsymbol{L = O(SA_T + SC + 2SA_T + SA_T + SA_T + SC + SC) = O(5SA_T + 3SC)}$$

Space Complexity

Space complexity to store the intensity values of the voxels of the 3D image volume is $\boldsymbol{A = O(S)}$

Space complexity to store the cluster centers: $\boldsymbol{B = O(C)}$

Space complexity to calculate the Global Membership value

To calculate the global membership value, the global membership values of each of the voxels from the previous iteration and values from the current iteration have to stored. $\boldsymbol{M = O(SC + SC)}$

Space complexity to calculate the Local Membership value

To calculate the local membership value, the local membership value from the previous iteration and the values obtained from the current iteration have to be stored. $\boldsymbol{U = O(SC + SC)}$

Space complexity to calculate the Type-2 Membership value

To calculate the type-2 fuzzy membership value, two lists are required to store the neighborhood global and local membership values.

This would require a space complexity of $\boldsymbol{C = O(2A_T)}$**.**

We also have to store the values of the defuzzified Type-2 fuzzy membership values and the normalized Type-2 fuzzy membership values and this would require $\boldsymbol{D = O(SC + SC)}$**.**

At each of the iteration, the values of the Type-2 Fuzzy membership are calculated independently from the previous iteration. However, it depends on the values of the global and local membership function obtained in the current iteration.

Space complexity to calculate the Final Membership value

The global, local and Type-2 membership values are regularized to get the final membership value.

To store that value, we need a space complexity of $\boldsymbol{G} = \boldsymbol{O}(\boldsymbol{SC})$**.**

Final space complexity will be

$$\boldsymbol{F} = \boldsymbol{O}(\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{M} + \boldsymbol{U} + \boldsymbol{C} + \boldsymbol{D} + \boldsymbol{G})$$

$$\boldsymbol{F} = \boldsymbol{O}(\boldsymbol{S} + \boldsymbol{C} + \boldsymbol{2SC} + \boldsymbol{2SC} + \boldsymbol{2A_T} + \boldsymbol{2SC} + \boldsymbol{SC})$$

$$\boldsymbol{F} = \boldsymbol{O}(\boldsymbol{4SC} + \boldsymbol{S} + \boldsymbol{C} + \boldsymbol{2A_T})$$