# Due-Job Clustering & Priority Scheduling — Stepwise Formulation

#### Purpose

Schedule due customers in a data-driven way that produces geographically tight routes while respecting plan cadence, leeway, skills/licensing, customer value, and operational capacity.

# Sets & Inputs (over planning horizon D)

- $\mathcal{J}$ : due jobs;  $\mathcal{Z}$ : zones;  $\mathcal{A}$ : areas;  $\mathcal{S}$ : service centers.
- For  $j \in \mathcal{J}$ : location  $(\operatorname{lat}_j, \operatorname{lon}_j)$ , zone z(j), area a(j), service center s(j).
- Due/leeway: hard due date  $D_j^{\text{due}}$ ; flexible window  $[D_j^{\text{min}}, D_j^{\text{max}}] \subseteq D$ .
- Cadence by plan:  $\operatorname{cad}(j) \in \{30, 60, 90, \dots\}$  days; last service date  $D_j^{\text{last}}$ .
- Time windows, duration estimate dur<sub>j</sub>, required skills/licenses  $\mathcal{L}_{j}$ .
- Business attributes: contract value  $V_j$ , payment likelihood  $p_j^{\text{pay}} \in [0, 1]$ , recent cancel/reschedule counts.
- History flags: last service was reservice? is Resvc\_j  $\in \{0,1\}.$
- Context: today  $D_{\text{now}}$ ; forecasted shift capacity per zone/day; urbanity class (affects radii).

#### **Derived Quantities**

• Cadence fit at day d:

$$U_2(j,d) = 1 - \min\left(\frac{\left|(d - D_j^{\text{last}}) - \text{cad}(j)\right|}{\tau_{\text{cad}}}, 1\right)$$

where  $\tau_{\rm cad}$  is a tolerance (e.g.,  $0.2 \times {\rm cad}(j)$ ).

• Window urgency at day d:

$$U_1(j,d) = \left(\operatorname{clip}\left(\frac{d - D_j^{\min}}{D_j^{\max} - D_j^{\min}}, 0, 1\right)\right)^{\gamma}, \quad \gamma > 1.$$

• Business value (normalized):

$$B(j) = 0.6 \min\left(\frac{V_j}{V_{90}}, 1\right) + 0.4 p_j^{\text{pay}}, \quad B_{\text{adj}}(j) = B(j) \cdot \left(1 - \text{clip}(w_c C_{30} + w_r R_{30}, 0, c_{\text{max}})\right)$$

with  $C_{30}$ ,  $R_{30}$  recent cancel/reschedule counts and weights  $w_c$ ,  $w_r$ .

• Quality boost:  $Q(j) = 1 + \beta_{resvc} \cdot isResvc_j$ .

# Local Density & Neighbor Graph

Let  $\mathcal{N}_r(j,d)$  be jobs within radius r km of j whose chosen day is d or whose leeway includes d.

$$N_d(j,d) = \operatorname{clip}\left(\frac{|\mathcal{N}_r(j,d)|}{N_{\text{ref}}}, 0, 1\right)$$

where  $N_{\text{ref}}$  is a typical route subcluster size (e.g., 8–10).

# Composite Priority Score

Weights  $(w_U, w_B, w_Q, w_N)$  default to (0.55, 0.25, 0.10, 0.10):

$$U(j,d) = 0.7 U_1(j,d) + 0.3 U_2(j,d), \qquad S(j,d) = w_U U(j,d) + w_B B_{\text{adj}}(j) + w_Q Q(j) + w_N N_d(j,d).$$

# Day Selection with Cluster-Aware Nudging

For each j:

- 1. Evaluate S(j,d) for all  $d \in [D_j^{\min}, D_j^{\max}]$  and set  $d^* = \arg\max_d S(j,d)$ .
- 2. Search nearby days d' with  $|d' d^*| \leq \Delta_{\text{max}}$  (e.g., 1–3 days) and accept d' if

$$N_d(j, d') - N_d(j, d^*) \ge \tau_N$$
 and  $S(j, d^*) - S(j, d') \le \varepsilon_S$ ,

then update  $d^* \leftarrow d'$ .

3. If capacity on  $d^*$  is saturated for zone z(j), consider next-best d by descending S(j,d) subject to the same rule.

# Geo Bucketing & Cluster Formation (per day and zone)

- 1. **Pre-bucket:** Partition jobs by geohash at precision yielding  $\sim 0.5-2$  km cells (urban) or 2-4 km (rural).
- 2. **Density merge:** Iteratively merge adjacent buckets if merged radius  $\leq r_{\text{max}}$  and size  $\leq C_{\text{max}}$ .
- 3. Sparse handling: Buckets with size  $< C_{\min}$  are parked for the "first-in-area" fallback (below) or are attached to the nearest dense cluster if distance  $\le r_{\text{attach}}$  and cluster constraints remain feasible.

# First-in-Area Fallback (anchor selection)

If a job is the first (no dense neighbors) for its area on day d:

$$\operatorname{Anchor}(j) := \begin{cases} \operatorname{zone} \ \operatorname{centroid} & \text{if close enough and zone non-remote,} \\ \operatorname{area} \ \operatorname{centroid} & \text{else if appropriate,} \\ \operatorname{service} \ \operatorname{center} & \text{if both centroids too far,} \\ \operatorname{customer} \ \operatorname{location} \ (j) & \text{for flagged remote pockets.} \end{cases}$$

Seed a new cluster at the chosen anchor, then attract nearby eligible jobs by day nudging (within  $\Delta_{\text{max}}$  and  $\varepsilon_S$ ).

#### Capacity Fit & Routing Handoff

- 1. For each cluster c, check  $\sum_{j \in c} \operatorname{dur}_j$  vs shift capacity for day d and zone z.
- 2. If oversubscribed, split c by k-means (k=2) respecting time windows and skill/licensing feasibility.
- 3. Build a routing request (e.g., VROOM) per cluster with:
  - Technician skill/license filters satisfying all  $\mathcal{L}_{i}$ ,
  - Time windows, depot/start location (team base or cluster centroid for meet-in-field),
  - Anti-waiting settings (small max wait penalties).

#### Parameters & Defaults

- Radii: r=1-2 km (urban), 3-6 km (rural). Merge cap  $C_{\text{max}}=12-14$ , min size  $C_{\text{min}}=3$ .
- Nudging:  $\Delta_{\text{max}}=2$  days,  $\tau_N=0.2$ ,  $\varepsilon_S=0.05$ .
- Weights:  $(w_U, w_B, w_Q, w_N) = (0.55, 0.25, 0.10, 0.10); \beta_{\text{resvc}} \in [0.1, 0.25].$

#### **KPIs & Outputs**

For each day and zone:

- Cluster set with centroids, radii, counts, and total duration.
- Priority-ordered job lists by cluster.
- Coverage vs capacity, average cluster radius, added travel vs alternative day, SPH, on-time %, wait minutes.

#### Pseudocode (high-level)

```
for d in planning_horizon:
pools = build_day_pools(d, skills/licensing, time windows)
for j in pools: choose d* via S(j,·), apply cluster-aware nudging
for z in zones:
   seeds = geohash_buckets(pools[z])
   clusters = density_merge(seeds, r_max, Cmax, Cmin)
   for sparse in parked(seeds, clusters):
     anchor = choose_anchor(z, area, service_center, sparse)
     seed_cluster(sparse, anchor); attract_neighbors_via_nudging()
   clusters = fit_to_capacity(clusters, shifts[z,d])
   routes = route_each_cluster(clusters, constraints)
   persist(routes, clusters, scores)
```

#### Notes

This formulation prioritizes urgency and cadence while opportunistically snapping jobs to denser days to thicken clusters, producing tighter routes without violating SLAs or business constraints.