03/09/15.

DISCRETE STRUCTURES (CS21001)

Department of Computer Science and Engineering,

Indian Institute of Technology, Kharagpur

MID SEMESTER EXAMINATION

Marks – 60 **Time – 2hrs**

Instructions:

1. **Answer all four questions.**
2. **All parts of a single question should be solved at the same place on the answer script. Parts done separately shall not be evaluated.**
3. **No clarifications! Answer making suitable assumptions wherever necessary.**
4. **Write the proofs stepwise and strictly not in prose. Proofs written as prose shall not be evaluated.**

Question 1:

(a) Prove that a binary relation is irreflexive, transitive and antisymmetric if and only if it is transitive and asymmetric. **[5]**

(b) Let $k \in N$, $S = \{1, 2, \ldots, k\}$, and $A = P(S) \setminus \{\emptyset\}$, where $P(S)$ denotes the power set of S, and $\emptyset$ denotes the empty set. In other words, the set A comprises all non-empty subsets of $\{1, 2, \ldots, k\}$. For each $a \in A$ denote by $min(a)$ the smallest element of a (notice that here a is a set). Define a relation σ on A as follows: $a \, \sigma \, b$ if and only if either $a = b$ or $min(a) < min(b)$. Prove that σ is a partial order on A. Justify if σ is also a total order on A? **[5]**

(c) Let $S = \{x, y, z\}$ and $P(S)$ denote the power set of S. Let R be subset relation on $P(S)$. Show that the partial order defined by R is isomorphic to D_{42} which is the partial order of the set of divisors of 42 under the divisibility relationship. Indicate the mappings. **[4]**

Question 2:

(a) Let a_n be the value returned by the following C function upon input n. **[5]**

```
unsigned int a ( unsigned int n )
{
    unsigned int sum, i;

    if (n == 0) return 1;
    sum = 0;
    for (i=0; i<n; ++i) sum += a(i);
    return sum;
}
```

(i) Derive a recurrence relation for the sequence a_n, $n \geq 0$.

(ii) Show by induction on n that $a_n = 2^{n-1}$ for all $n \geq 1$.

(b) $a, b, c, d \in N$. Both a, b are divisible by both c, d (written, $c|a$, $c|b$ and $d|a$, $d|b$). Also, c does not divide d. Someone gives the following proof that cd divides both a and b.

Proof. Since $d|a$ and $d|b$ there exist x, y such that: $a = xd$ and $b = yd$ (1)

Also, c divides both a, b so we have $c|dx$ and $c|dy$. But since c does not divide d it must be that $c|x$ and $c|y$ that is $x = uc$ and $y = vc$ for some $u, v \in N$. Substituting for x and y in (1) we get $a = ucd$ and $b = vcd$. So, cd divides both a, b. [5]

(i) Is the above proof correct? If not where and what precisely is the error?

(ii) Can you produce a counter-example?

(c) Formally argue that the sum of two irrational numbers can be either rational or irrational. You need to prove any intermediate argument you use. [5]

Question 3:

(a) Find out the number of integers between 1 and 10, 000 that are divisible by any of the integers 2, 3, 5 and 7. [5]

(b) Seven points lie inside a hexagon of side length 1. Show that two of the points are at distance at most 1 apart. [3]

(c) A party is attended by $n \geq 2$ people. Prove that there will always be two people in attendance who have the same number of friends at the party. (Assume that the relation "is a friend of" is symmetric, that is, if x is a friend of y then y is a friend of x.) [7]

Question 4:

(a) Solve the recurrence relation: $a_n = 9a_{n-1} - 26a_{n-2} + 24a_{n-3}$ for $n > 2$ with $a_0 = 0$; $a_1 = 1$; $a_2 = 10$ to obtain an expression for a_n. [7]

(b) Let $ABCDEF$ be a regular hexagon of side length 1, and O be the center of the hexagon. In addition to the sides of the hexagon, line segments are drawn from O to each vertex, making a total of twelve unit line segments. Formulate a recurrence to count the number of paths that start at O and terminate at O. Solve the recurrence using suitable initial conditions. [9]