5/10/21 CS21201 – Test-2 – DS Spadhi

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1. $a, b, c \in \mathbb{Z}^+ \cup \{0\}$

loop invariance,

$r = \lfloor \frac{a+b}{2} \rfloor$, $s = \lfloor \frac{b+c}{2} \rfloor$, $t = \lfloor \frac{c+a}{2} \rfloor$

$a = \lfloor \frac{a+b}{2} \rfloor$, $b = \lfloor \frac{b+c}{2} \rfloor$, $c = \lfloor \frac{c+a}{2} \rfloor$

After every loop

$\left(\frac{a+b}{2}\right) - 1 < r \leq \left(\frac{a+b}{2}\right)$ & $\left(\frac{b+c}{2}\right) - 1 < s \leq \frac{b+c}{2}$ $\left(\frac{c+a}{2}\right) - 1 < t \leq \frac{c+a}{2}$

so $r + s + t =$

$(a+b+c) - (3) < r + s + t \leq (a+b+c)$

$\Rightarrow$ After every loop

$\left(\frac{a+b}{2}\right) - 1 < a \leq \frac{a+b}{2}$, $\left(\frac{b+c}{2}\right) - 1 < b \leq \frac{b+c}{2}$,

$\left(\frac{c+a}{2}\right) - 1 < c \leq \frac{a+c}{2}$

if at a particular iteration st $a \neq b$ || $b \neq c$ || $c \neq a$.
no 2 is equal, then

case 1 (a = odd, b = odd, c = even) → 2 odd 1 even } permutations are not important
2 (a = even, b = even, c = odd) → 2 even 1 odd }
3 → all even
4 → all odd

Case 1: After iteration,
$a = \left(\frac{a+b}{2}\right)$, $b = \left(\frac{(b+c)-1}{2}\right)$, $c = \left(\frac{(a+c)-1}{2}\right)$

and $(r+s+t) = (a+b+c) - 1$

Case 2: After iteration $a = \frac{a+b}{2}$, $b = \frac{b+c-1}{2}$, $c = \frac{c+a-1}{2}$

$$r+s+t = (a+b+c)-(1)$$

Case 3: After iteration $a = \frac{a+b}{2}$, $b = \frac{b+c}{2}$, $c = \frac{c+a}{2}$

$$(r+s+t) = (a+b+c)$$

Case 4: same as case 3.

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if $a \neq b$ && $c \neq a$ && $b \neq c$.

then (assume) $a > b > c$

After iteration, integer division will stop.

~~4, 65, 1,2,3, ..., 2021~~

~~$(a,b,c,d) \Rightarrow 2024 \mid (a-b+c-d)$~~

~~x_1 — x_{65}~~

~~$\Rightarrow x_1 + x_2$~~

Suppose $a < b < c$ then without losing generality

$a \leq r < b$ & $a \leq t < c$ after every iteration

$b \leq s < c$

Initial range is $[a,b]$ & $[b,c]$ and the range changes to $[r,s]$ & $[s,t]$

where r,s is smaller range than a,b

s,t is smaller range than b,c

so eventually either $r=s$ or $s=t$ as range is reducing

after
every iteration.

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2. 65 distinct integers

— (1 — 2021)

$2021 \mid (a+c) - (b+d)$

Sol:- If 65 distinct intgers, chose 2 distinct integers without replacement, no of ways $^{65}C_2 = 2080$

Now for all such choices say a & b.

write all values of a+b.

2080 possible values of (a+b)

Divide all by (2021).

let remainders be $r_1, r_2, r_3, \ldots, r_{2080}$

$\Rightarrow$ number of pigeons = 2080

holes ; $1 < r \leq 2020$ = 2021 [no of possible remainders]

pigeons > holes, at least

2 remainders same.

$$\boxed{\begin{aligned} a+b &= 2021q + r \\ c+d &= 2021q' + r \end{aligned}}$$

so let those pairs be (a,b) & (c,d)

$a+b \equiv q \bmod (2021)$

$c+d \equiv q \bmod (2021)$

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By modulo arithmetic,

$$(a+b) - (c+d) \equiv 0 \bmod (2021)$$

or $(a+b)-(c+d)$ is divisible by (2021).

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3. ρ & σ are 2 binary relations on A. A composite relation $\rho \circ \sigma$ over A is defined as

$$\rho \circ \sigma = \{ (p, r) \mid \text{there exists some } q \in A \text{ such that } (p,q) \in \rho \text{ \& } (q,r) \in \sigma \}$$

(a) if $\rho \circ \sigma$ is an equivalence relation then,

if $(p,r) \in \rho \circ \sigma$ $(r \neq p)$ $\Rightarrow$ $(p,q) \in \rho$, $(q,r) \in \sigma$ for some $q \in A$. [by definition]

$\Rightarrow$

~~then, $(r,p) \in \rho \circ \sigma$ as~~

as ρ & σ are equivalent,

$(q,p) \in \rho$ and $(r,q) \in \sigma$

or $(r,p) \in \sigma \circ \rho$ [by definition]

only if $\sigma \circ \rho = \rho \circ \sigma$

will $(r,p) \in \rho \circ \sigma$ as well

Now, if $\rho \circ \sigma = \sigma \circ \rho$

implies.

if $(p,r) \in \rho \circ \sigma$ then $(p,r) \in \sigma \circ \rho$ as well.

so if $(p,r) \in \rho \circ \sigma$ then $(p,q) \in \rho$, $(q,r) \in \sigma$ for some $r \in A$

and $(p,r) \in \sigma \circ \rho$ so $(p,q') \in \sigma$, $(q',r) \in \rho$ for some $q' \in A$

then, if ρ & σ not equivalent,

(b) The inverse of τ over A is $\tau^{-1} = \{(q,p) \mid (p,q) \in \tau\}$

$(p, q \in A)$

$$(\rho \circ \sigma)^{-1} = \sigma^{-1} \circ \rho^{-1}$$

⇒ Solution:-

$(p, r) \in (\rho \circ \sigma)$

then $(p, q) \in \rho$
$(q, r) \in \sigma$ for some $q \in A$

$(\rho \circ \sigma)^{-1} = \{r, p \mid (p, r) \in (\rho \circ \sigma)\}$

$$\left[\begin{array}{l} \rho^{-1} = \{(p,q) \mid (q,p) \in \rho\} \\ \sigma^{-1} = \{(p,q) \mid (q,p) \in \sigma\} \end{array}\right]$$

⇒ if $(p, r) \in \rho \circ \sigma$ then $(r, p) \in (\rho \circ \sigma)^{-1}$

$(p, q) \in \rho$ then $(q, p) \in \rho^{-1}$

$(q, r) \in \sigma$ then $(r, q) \in \sigma^{-1}$

so if $(r, p) \in (\rho \circ \sigma)^{-1}$

$\Rightarrow$ $(q, p) \in \rho^{-1}$
&
$(r, q) \in \sigma^{-1}$

or $(r, q) \in \sigma^{-1}$
&
$(q, p) \in \rho^{-1}$

so $(r, p) \in \sigma^{-1} \circ \rho^{-1}$

therefore $(\rho \circ \sigma)^{-1} = \sigma^{-1} \circ \rho^{-1}$

→ $P(S)$ is power set of S.

$f : X \to Y$ $\quad g : P(A) \to P(B)$

$h : P(B) \to P(A)$

$g(A) = \{ b \mid \exists a \in A \text{ s.t. } f(a) = b \}$, and

$h(B) = \{ a \mid f(a) \in B \}$ for all

$A \subseteq X$ & $B \subseteq Y$. Prove

(i) f is injective iff $h(g(A)) = A$ for all $A \subseteq X$

⇒ ~~if f is injective ⇒ $f(g(A_1)) = f(g(A_2))$ ⇒ $A_1 = A_2$~~

⇒ if f is injective

then $g(A) = S$ all the images of f under A

now all elements in S correspond to unique values in set A.

and $(|S| = |A|)$ → that means there is a one to one correspondence b/w set S and set A.

⇒ $h(S) = \{ a \mid f(a) \in S \}$ → it gives set of pre-images of f under S → which are unique for all $b \in S$ and is equal to A itself as all images have a unique pre-image.

if $h(g(A)) = A$ then ~~for all $a \in A$~~ ~~$f(a) = B$~~ ~~belongs~~

then $g(A)$ is set of all images for all $a \in A$.

if $h(g(A)) = A \Rightarrow$ set of all pre-images for

images in g.

if f not injective, then for some

$A \subseteq X$ we can have $g(A)$ = set of images

but all images do not have a unique pre-image.

so for $h(g(A))$ we can exclude those pre-images

which are not giving unique images except 1,

still we satisfy $hg(A)$ so $h(g(A)) \subseteq A$ if $g(A)$ if

$A \subseteq X$ f is not injective. Therefore f has to

be injective

(b) f is surjective iff $g(h(B)) = B$ for all $B \subseteq Y$.

$\Rightarrow$ if $g(h(B)) = B$, then

$$h(B) = \{a \mid f(a) \in B\} = S$$

~~and for all~~ set of all pre-images of set B under

f.

$g(S)$ = set off all images of S under f.

if $g(S) = B$, then set of all images

are equal to the set of images which was

given as input to h and all the pre-

images it produced made a set a

images which was same as the

input image set.

that means the set of pre-images exaust all images in set B. else some elements in B may or may not have had a pre-image.

so f has to surjective as it true for all $B \subseteq Y$.

Now for

f is surjective, it implies,

all elements in Y have a unique pre-image in X.

now, if f is surjective,

then $h(B) \rightarrow$ all elements in B have a pre-image. if C

upon passing those pre-images of all images all formed. Since it is a function, no two ~~pre~~ images can be equal for a particular pre-image or a pre-image can't give rise to 2 images under f. $f \circ g(h(B)) = S = B$ as all images ~~for~~ for $h(B)$ are covered in B.