Module-4
Semiconductor Physics

Calculation of Carrier concentration in an intrinsic semiconductor:

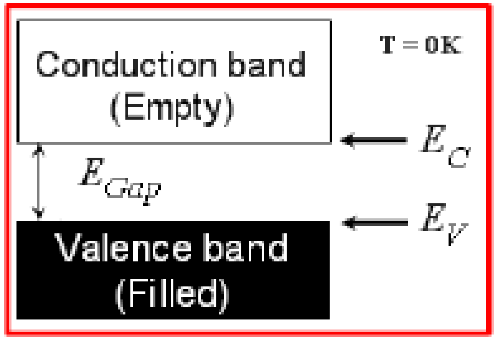
a) Carrier concentration in Conduction band:

Let ‘dn’ be the number of electrons available between energy states ‘E’ and ‘E+dE’ in the conduction band.

dn= Z(E) F(E)dE, where Z(E) dE is the density of states in the energy interval E and E+dE and F(E) is the electron occupancy probability.

If E_c is the energy corresponding to the bottom of the conduction band then the number of electrons in the conduction band is given by

$$n = \int_{E_c}^{\infty} Z(E)F(E)dE$$



where the density of states is given by

Since the electron is moving in a periodic potential, 'm' should be the effective mass.
Since the energy E starts at E_c

$$Z(E)dE = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2}\left(E - E_c\right)^{1/2} dE$$

The Fermi – Dirac distribution function, gives probability of an electron occupying an energy state 'E'

$$F(E) = \frac{1}{1 + exp\left(\frac{E - E_f}{k_B T}\right)}$$

where E_f is the Fermi level.

If

$$(E - E_f) >> k_B T \qquad F(E) \cong exp\left(\frac{E_f - E}{k_B T}\right)$$

$$\therefore n = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2}\int_{E_c}^{\infty}\left(E - E_c\right)^{1/2}\exp\left(\frac{E_f - E}{k_B T}\right)dE$$

$$= \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2}\exp\left({E_f}/{k_B T}\right)\int_{E_c}^{\infty}\left(E - E_c\right)^{1/2}\exp\left(\frac{-E}{k_B T}\right)dE$$

To solve this integral we put

$$E - E_c = x$$

$dE = dx$

$$\therefore E = E_c + x;$$

$$\therefore n = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2} \exp\left(E_f \big/ k_B T\right) \int_0^\infty x^{1/2} \exp\left(\frac{-E_c - x}{k_B T}\right) dx$$

Using Gamma function

'the integral part is equal to $(k_B T)^{3/2} \dfrac{\pi^{1/2}}{2}$

$$= \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2} \exp\left(\frac{E_f - E_c}{k_B T}\right) \int_0^\infty x^{1/2} e^{\frac{-x}{k_B T}} dx$$

$$\therefore n = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2} \exp\left(\frac{E_f - E_c}{k_B T}\right) (k_B T)^{3/2} \frac{\pi^{1/2}}{2}$$

$$= 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} exp\left(-\frac{E_c - E_f}{k_B T}\right)$$

$$= N_c \, exp\left[-\frac{(E_c - E_f)}{k_B T}\right]$$

where N_c is a pseudo constant.

Concentration of holes in the valence band of an intrinsic semiconductor:

Let dp be the number of holes in the energy interval E and E+ dE in the valence band.

$$dp = Z(E)[1-F(E)]dE$$

$1-F(E)$ gives the probability that an electron will be missing from an energy state E (i.e) a hole is created.

$$1-F(E) = 1-\frac{1}{1+exp\left(\frac{E-E_f}{k_BT}\right)}$$

for $(E-E_f) >> k_BT$

$$[1-F(E)] \cong exp\left(\frac{E-E_f}{k_BT}\right)$$

$$Z(E)dE = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} E^{\frac{1}{2}} dE$$

Since E_V is the energy of the top of the valence band

$$Z(E)dE = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2}\left(E_V - E\right)^{1/2} dE$$

The number of holes (per unit volume) in the valence band is given by

$$P = \int_{-\infty}^{E_V} \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2}\left(E_V - E\right)^{1/2} \exp\left(\frac{E - E_f}{k_B T}\right) dE$$

$$= \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} \exp\left(\frac{-E_f}{k_B T}\right) \int_{-\infty}^{E_V} \left(E_V - E\right)^{1/2} \exp\left(\frac{E}{k_B T}\right) dE$$

To solve this integral; put $E_V - E = x$

or $E = E_V - x$; $dE = -dx$.

$$\therefore P = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} \exp\left(\frac{-E_f}{k_B T}\right) \int_{\infty}^{0} x^{1/2} \exp\left(\frac{E_V - x}{k_B T}\right)(-dx)$$

$$= \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} \exp\left(\frac{E_V - E_f}{k_B T}\right)\int_0^\infty x^{1/2} \exp\left(\frac{-x}{k_B T}\right)dx$$

$$= \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} \exp\left(\frac{E_V - E_f}{k_B T}\right)\left(k_B T\right)^{3/2} \frac{\pi^{1/2}}{2}$$

$$= 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2} exp\left[\frac{-\left(E_f - E_V\right)}{k_B T}\right]$$

$$= N_V \; exp \; \left[\frac{-\left(E_f - E_V\right)}{k_B T}\right]$$

Where

$$N_V = 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2}$$

is a pseudo constant.

Fermi level and the intrinsic carrier concentration:

Since n = p for intrinsic semiconductors,

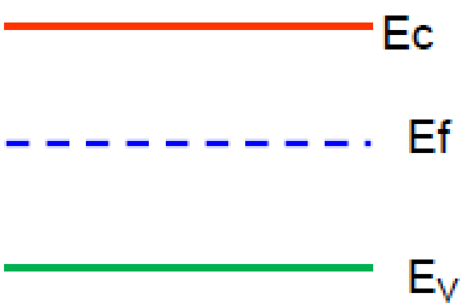
$$2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} \exp\left[\frac{E_f - E_C}{k_B T}\right] = 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2} exp\left[\frac{E_V - E_f}{k_B T}\right]$$

$$\Rightarrow \frac{exp\left(\dfrac{E_f - E_C}{k_B T}\right)}{exp\left(\dfrac{E_V - E_f}{k_B T}\right)} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

$$\Rightarrow exp\left[\frac{2E_f - (E_C + E_V)}{k_B T}\right] = \left(\frac{m_h^*}{m_e^*}\right)^{3/2}$$

Taking log on both sides and simplifying

$$E_f = \frac{E_C + E_V}{2} + \frac{3k_B T}{4} log\left(\frac{m_h^*}{m_e^*}\right)$$



If

$m_h^* = m_e^*$

or T=0°K

$$E_f = \frac{E_C + E_V}{2}$$

In Intrinsic semiconductors

$$n = p = n_i$$

where n_i is the intrinsic carrier concentration

$$\therefore n_i^{\,2} = np \longrightarrow \text{Mass action law}$$

$$\therefore n^2_i = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2} \exp\left[\frac{E_f - E_C}{k_B T}\right] \bullet 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)^{3/2} exp\left[\frac{E_V - E_f}{k_B T}\right]$$

$$= 4\left(\frac{2\pi k_B T}{h^2}\right)^3 \left(m_e^* m_h^*\right)^{3/2} exp\left(\frac{E_V - E_C}{k_B T}\right)$$

$$= 4\left(\frac{2\pi k_B T}{h^2}\right)^3 \left(m_e^* m_h^*\right)^{3/2} exp\left(\frac{-E_g}{k_B T}\right)$$

Where $E_C - E_V = E_g$ the forbidden energy gap.

$$\therefore n_i = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2} \left(m_e^* m_h^*\right)^{3/4} exp\left(\frac{-E_g}{2k_B T}\right)$$

$$\therefore n_i = 2\left(\frac{2\pi k_B T}{h^2}\right)^{3/2}\left(m_e^* m_h^*\right)^{3/4} exp\left(\frac{-E_g}{2k_B T}\right)$$

Conclusion:

Carrier Concentration in intrinsic semiconductors

Depends

1. Strongly on Temperature (T)
2. Exponentially on Energy Gap (E_g)

Independent of

3. Fermi Energy level (E_f)

Factor of 2 represents formation of 2 charge carriers

Limitation of Intrinsic semiconductor

- Low conductivity
  - Conductivity is 10^7 less then that of copper
- Conductivity depends on Temperature
  - Increases exponentially with temperature
- No Control on Conductivity

Extrinsic Semiconductor

Extrinsic Semiconductor

- Selective doping of impure atoms, known as dopants, into intrinsic semiconductors to make their conductivity more depend on applied voltage than temperature
- These dopants can be pentavalent from Group V or trivanlent from Group-III element. Typical doping levels ranges from $10^{20}$to 10^{27} atoms/m^3
  – Pentavalent atoms , also known as donor atom, give excess electrons in the semiconductors and these type of extrinsic semiconductors are known as ***n-type semiconductor***

  Example of pentavalent dopant: P, As, Sb

  – Trivalent atoms , also know as acceptor , give excess holes in the semiconductor and these type of extrinsic semiconductors are known as ***p-type semiconductor***

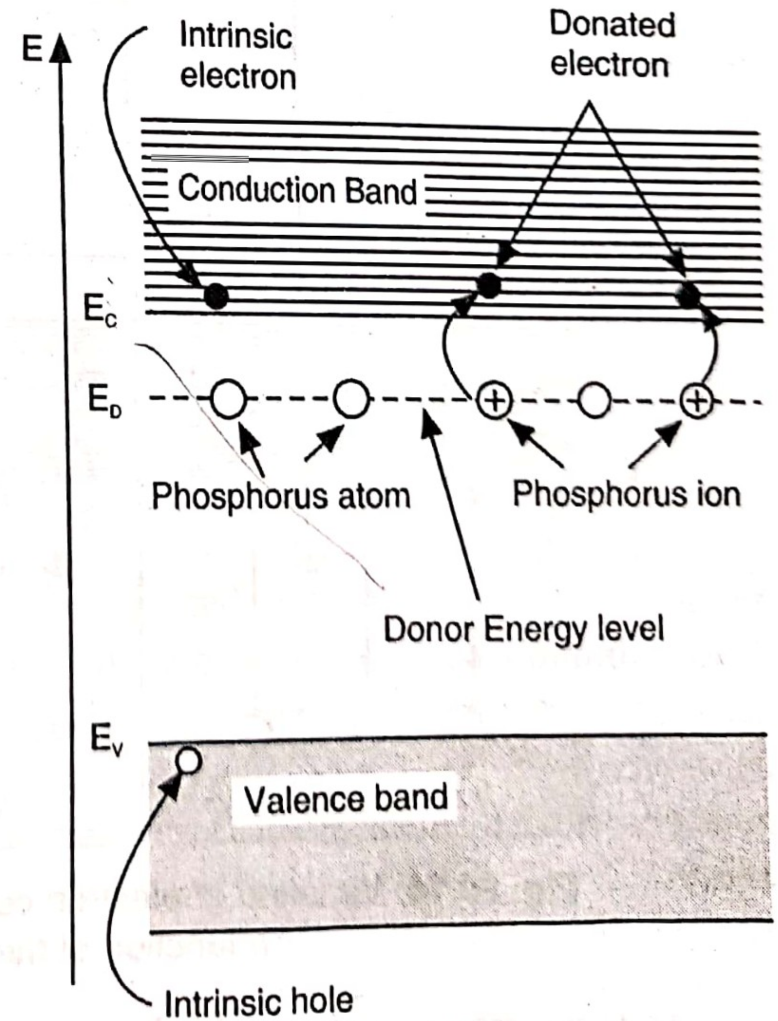
  Example of trivalent dopant: Al, B, Ga, In

n-type semiconductor

Dopant is Pentavalent atom
Know as **DONOR**

Donor Energy Level ***(E_D)*** is very close to conduction band
Need Very low energy (~0.045 ev) to liberate electron

Donor Concentration is N_D



Carrier Concentration in n-type Semiconductors

$$n = Concentration\ of\ electron$$

$$n = N_D^+$$ N_D^+ is the number atom ionized

$$n = N_D - N_D^0$$

$$N_D^+ = \left(N_D - N_D^0\right) = N_D[1 - f(E_D)] = \frac{N_D}{1 + e^{-(E_D - E_F)/kT}}$$

$$n = \frac{N_D}{1 + e^{-(E_D - E_F)/kT}}$$

$$n = N_D e^{(E_D - E_F)/KT}$$

$$n = N_C e^{-(E_C - E_F)/KT}$$

$$N_C e^{-(E_C - E_F)/KT} = N_D e^{(E_D - E_F)/KT}$$

$$N_C e^{-(E_C - E_F)/KT} = N_D e^{(E_D - E_F)/KT}$$

$$\left(\frac{E_D - E_F}{kT}\right) + \left(\frac{E_C - E_F}{kT}\right) = ln\ \frac{N_C}{N_D}$$

$$(E_D + E_C) - 2E_F = (kT)\ ln\ \frac{N_C}{N_D}$$

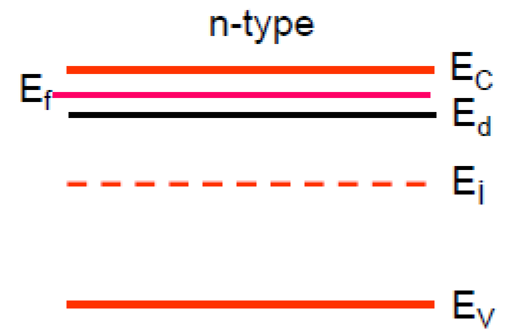
or $$E_F = \frac{E_D + E_C}{2} - \left(\frac{kT}{2}\right) ln\ \frac{N_C}{N_D}$$

or $$E_F = \frac{E_D + E_C}{2} + \left(\frac{kT}{2}\right) ln\ \frac{N_D}{N_C}$$

or $$E_F = \frac{E_D + E_C}{2} + \left(\frac{kT}{2}\right) ln\ \frac{N_D}{2\left(2\pi\ m_e^* kT/h^2\right)^{3/2}}$$

$$E_F = \frac{E_D + E_C}{2}$$

At T= 0K



$$\exp\left[\frac{E_F - E_C}{kT}\right] = \exp\left[\frac{E_D + E_C}{2kT} + \left(\frac{1}{2}\right) ln \frac{N_D}{2\left(2\pi\, m_e^* kT/\mathrm{h}^2\right)^{3/2}} - \frac{E_C}{kT}\right]$$

$$= \exp\left[\frac{E_D - E_C}{2kT} + \left(\frac{1}{2}\right) ln \frac{N_D}{2\left(2\pi\, m_e^* kT/\mathrm{h}^2\right)^{3/2}}\right]$$

$$= \exp\left[\frac{E_D - E_C}{2kT} + ln \sqrt{\frac{N_D}{2\left(2\pi\, m_e^* kT/\mathrm{h}^2\right)^{3/2}}}\right] \qquad \left[\because \frac{1}{2} ln\, x = ln\sqrt{x}\right]$$

$$= \exp\left[\left(\frac{E_D - E_C}{2kT}\right)\right] \cdot \exp\left[ln \sqrt{\frac{N_D}{2\left(2\pi\, m_e^* kT/\mathrm{h}^2\right)^{3/2}}}\right] \qquad \left[\begin{array}{l}\because \exp\ (a+b) = \\ \exp(a) + \exp(b)\end{array}\right]$$

$$= \exp\left[\left(\frac{E_D - E_C}{2kT}\right)\right] \cdot \left[\sqrt{\frac{N_D}{2\left(2\pi\, m_e^* kT/\mathrm{h}^2\right)^{3/2}}}\right] \qquad [\exp\ (ln\, x) = x]$$

$$\therefore \quad n = N_C \exp\left[\frac{E_F - E_C}{kT}\right] = 2\left[\frac{2\pi\, m_e^* kT}{\mathrm{h}^2}\right]^{3/2} \exp\left[\left(\frac{E_D - E_C}{2kT}\right)\right] \cdot \left[\sqrt{\frac{N_D}{2\left(2\pi\, m_e^* kT/\mathrm{h}^2\right)^{3/2}}}\right]$$

or

$$n = (2N_D)^{\frac{1}{2}} \left[\frac{2\pi\, m_e^* kT}{\mathrm{h}^2}\right]^{3/4} \exp\left[\left(\frac{E_D - E_C}{2kT}\right)\right] \qquad (30.6$$

$$n = (2N_D)^{\frac{1}{2}} \left[\frac{2\pi\, m_e^* kT}{h^2} \right]^{3/4} \exp \left[\left(\frac{E_D - E_C}{2kT} \right) \right]$$

Conclusion:

1. The above equation gives the concentration of electron in n-type semiconductor which are majority charge carriers
2. At low temperature concentration of electron depends mostly on N_D donor concentration

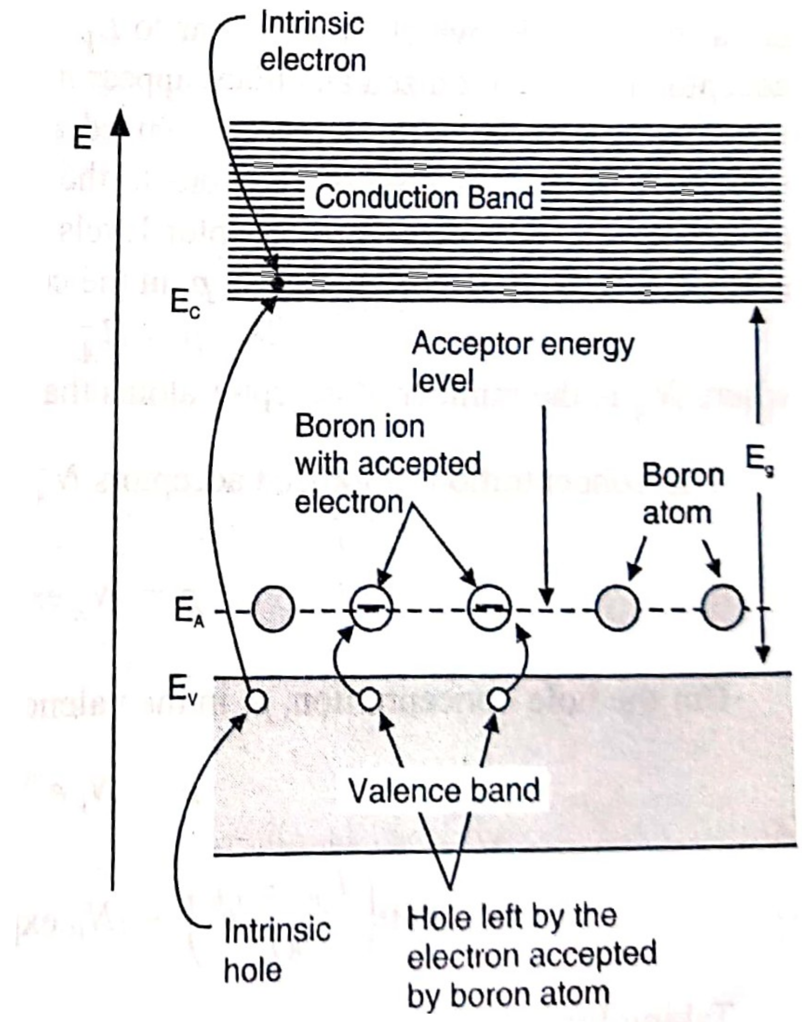
p-type semiconductor

Here dopant is a trivalent atom

Trivalent atoms have vacant energy levels close to valance band. Hence with very little energy electron from valance band can jump to these levels forming hole and negative ion of trivalent atom

As these atoms accept electron they are called ***acceptor*** and energy level is know as ***Acceptor Level*** (E_A)

Concentration of Acceptor atoms denoted by N_A



Hole concentration in p-type semiconductor

'p' represents the hole concentration and it will be equal to the number of negative acceptor ions

$$p = N_A^-$$

Negative ion concentration is equal to probability of atoms in E_A level

$$N_A^- = N_A f(E_A) = N_A \exp\left(\frac{E_F - E_A}{kT}\right)$$

$$p = N_A \exp\left(\frac{E_F - E_A}{kT}\right)$$

But hole concentration in semiconductors is found to be

$$p = N_V e^{-(E_F - E_V)/kT} = N_V \exp\left(\frac{E_V - E_F}{kT}\right)$$

From above two expression for 'p'

$$N_A \exp\left(\frac{E_F - E_A}{kT}\right) = N_V \exp\left(\frac{E_V - E_F}{kT}\right)$$

$$N_A \exp\left(\frac{E_F - E_A}{kT}\right) = N_V \exp\left(\frac{E_V - E_F}{kT}\right)$$

Taking logarithm and rearranging we get

$$\left(\frac{E_F - E_A}{kT}\right) - \left(\frac{E_V - E_F}{kT}\right) = \ln \frac{N_V}{N_A}$$

$$-(E_V + E_A) + 2E_F = (kT) \ln \frac{N_V}{N_A}$$

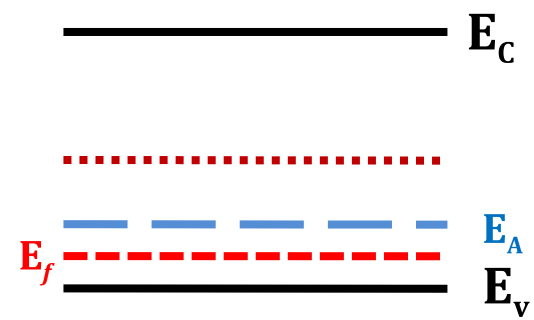
$$E_F = \frac{E_V + E_A}{2} + \left(\frac{kT}{2}\right) \ln \frac{N_V}{N_A}$$

Substituting Value of N_V

$$E_F = \frac{E_V + E_A}{2} - \left(\frac{kT}{2}\right) \ln \frac{N_A}{2\left(2\pi m_h^* kT / h^2\right)^{3/2}}$$

At T=0K

$$E_F = \frac{E_V + E_A}{2}$$



$$\mathrm{Exp}\left[\frac{E_V - E_f}{kT}\right] = \exp\left[\frac{E_V}{kT} - \frac{E_V + E_A}{2kT} + \left(\frac{1}{2}\right)\ln\frac{N_A}{2\left(\frac{2\pi m_h^* kT}{h}\right)^{\frac{3}{2}}}\right]$$

$$= \exp\left[\frac{E_V - E_A}{2kT} + \left(\frac{1}{2}\right) ln\frac{N_A}{2\left(2\pi m_h^* kT/h^2\right)^{3/2}}\right]$$

$$= \exp\left[\frac{E_V - E_A}{2kT} + ln\sqrt{\frac{N_A}{2\left(2\pi m_h^* kT/h^2\right)^{3/2}}}\right] \qquad \left[\because \frac{1}{2} ln\, x = ln\sqrt{x}\right]$$

$$= \exp\left[\left(\frac{E_V - E_A}{2kT}\right)\right]\cdot \exp\left[ln\sqrt{\frac{N_A}{2\left(2\pi\, m_h^* kT/h^2\right)^{3/2}}}\right] \qquad \left[\begin{array}{l}\because \exp\ (a+b) = \\ \exp(a) + \exp(b)\end{array}\right]$$

$$= \exp\left[\left(\frac{E_V - E_A}{2kT}\right)\right]\cdot\left[\sqrt{\frac{N_A}{2\left(2\pi\, m_h^* kT/h^2\right)^{3/2}}}\right] \qquad \left[\because \exp\ (ln\, x) = x\right]$$

$$p = N_V \exp\left[\frac{E_V - E_A}{kT}\right] = 2\left[\frac{2\pi\, m_h^* kT}{h^2}\right]^{3/2} \exp\left[\left(\frac{E_V - E_A}{2kT}\right)\right] \cdot \left[\sqrt{\frac{N_A}{2\left(2\pi\, m_h^* kT/h^2\right)^{3/2}}}\right]$$

$$p = (2N_A)^{1/2}\left[\frac{2\pi m_h^* kT}{h^2}\right]^{\frac{3}{4}} \exp\left[\frac{\mathrm{E_V} - \mathrm{E_A}}{2\mathrm{kT}}\right]$$

Conclusion:

1. The above equation gives the concentration of holes in p-type semiconductor which are majority charge carriers
2. At low temperature concentration of electron depends mostly on N_A acceptor concentration

Temperature Variation of Carrier Concentration in extrinsic semiconductor

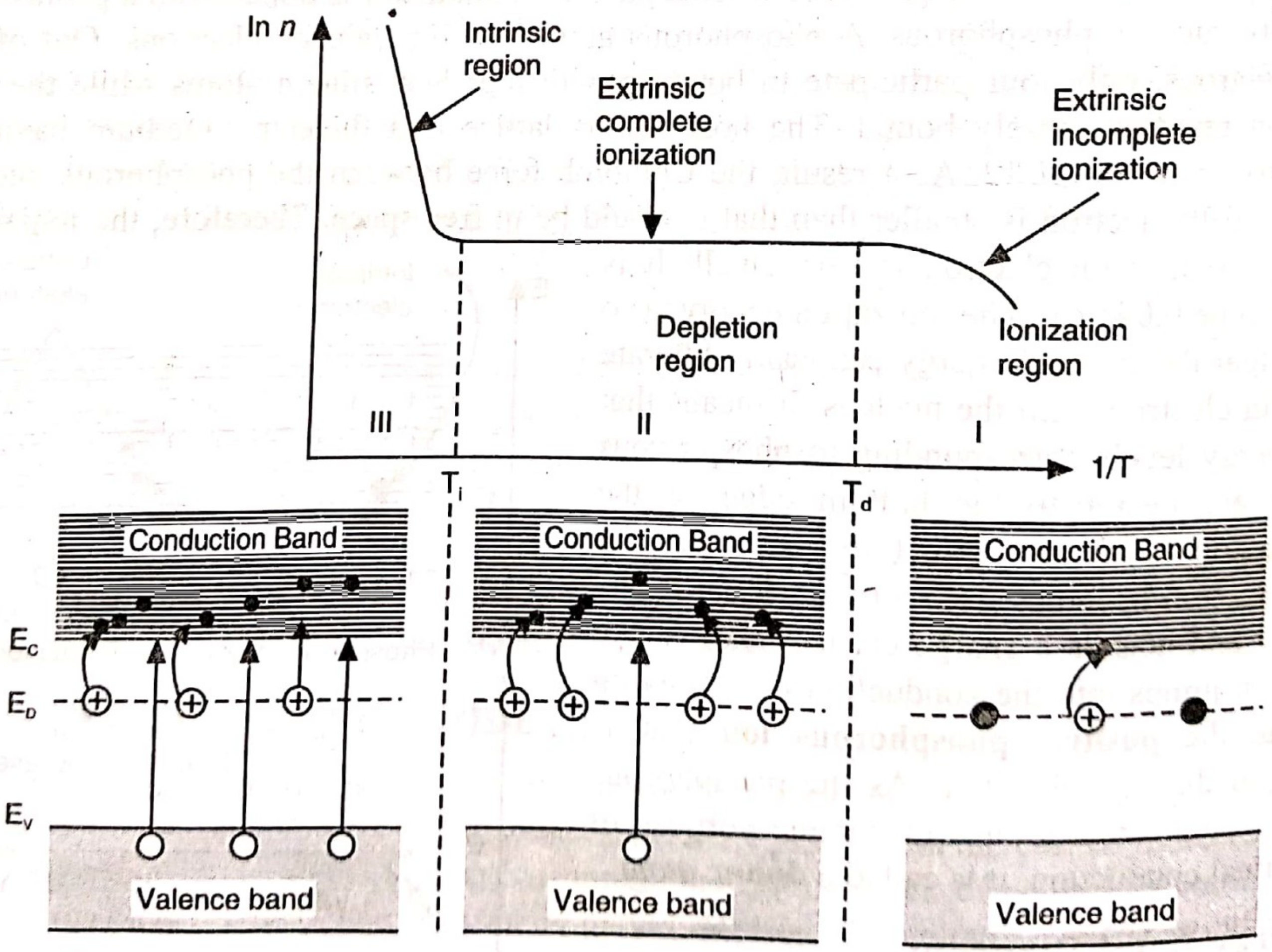
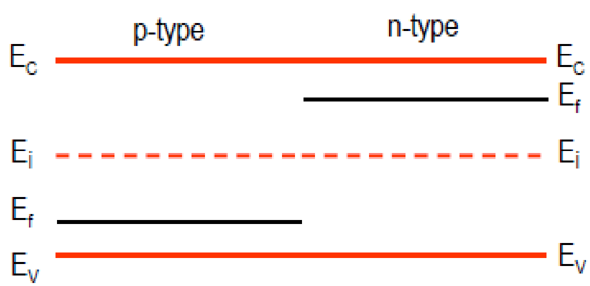
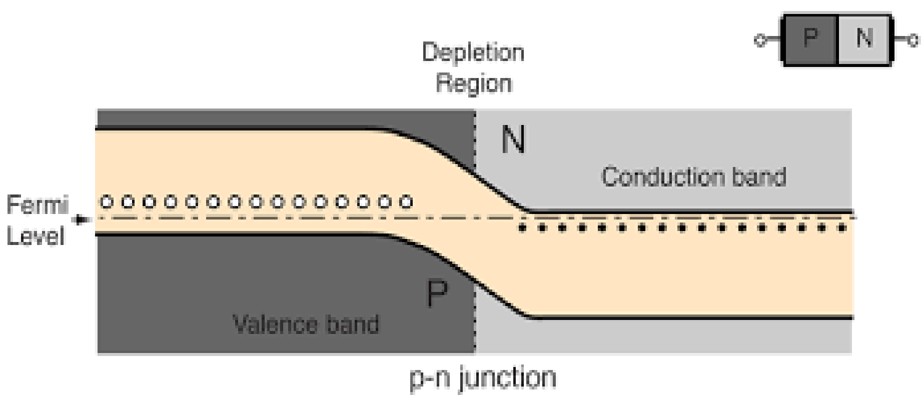


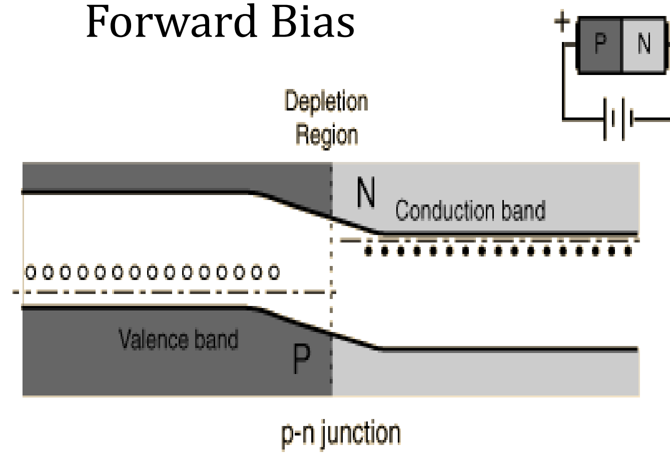
Fig. 30.14. Variation of electron concentration in an n-type semiconductor as a function of the inverse of temperature.

Energy levels in PN Junction





Forward Bias



Reverse Bias