Homework 2, due 10/13 Swastik Singh

Intro to Probability, 1st Edition. by Anderson, Seppalainen, and Valko

Problems: 1.15, 1.26, 1.35, 1.40, 1.42 ($AB = A \cap B$), 1.55
Pages 31-36

Problem 1.15: An Urn contains 4 balls, 1 White, 1 Green, 2 Red. Draw 3 w/ replacement. Find the probability we did not see all 3 colors.

a) Use Inclusion-Exclusion

For 1 draw: $\#\Omega = \binom{4}{3}$, $\#W = \#G = \binom{1}{1}$, $\#R = \binom{2}{1}$

$W = \{\text{No white drawn}\}$ | $P(W) = \left(1 - \frac{1}{4}\right)^3 = \left(\frac{3}{4}\right)^3 \rightarrow$ 3 draws

$G = \{\text{No Green drawn}\}$ | $P(G) = \left(1 - \frac{1}{4}\right)^3 = (3/4)^3$

$R = \{\text{No Red drawn}\}$ | $P(R) = \left(1 - \frac{2}{4}\right)^3 = (1/2)^3$

$P(W \cap G) = \left(\frac{1}{2}\right)^3$

$P(G \cap R) = \left(\frac{1}{4}\right)^3 = P(W \cap R)$

$P(G \cap R \cap W) = 0$

$\rightarrow$ none show = impossible

$$\sum_{k=1}^{n} (-1)^{k+1} \cdot \sum_{1 \le i_1 < \dots < i_k \le n} P(A_{i_1} \cap A_{i_2} \cap \dots \cap A_{i_k})$$

$= P(W) + P(G) + P(R) - [P(W \cap G) + P(G \cap R) + P(W \cap R)] + P(G \cap R \cap W)$

$P(G \cup R \cup W) = \left(\frac{3}{4}\right)^3 + \left(\frac{3}{4}\right)^3 + \left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^3 - \left(\frac{1}{4}\right)^3 - \left(\frac{1}{4}\right)^3$

$= \frac{27 + 27 + 8 - 8 - 1 - 1}{64} = \frac{52}{64} = \boxed{\frac{13}{16} = P(G \cup R \cup W)}$

b) Compute Probability by finding compliment.

$\#\Omega = 4 \rightarrow 1$ draw

$4^3 = 64 \rightarrow$ Possibilities in 3 draws

3 balls chosen $\rightarrow 3!$ success conditions [for 1 red ball] $\rightarrow 6$

2 red balls $\rightarrow 6 \times 2$ total success conditions $\Rightarrow 12$

$P([G \cup R \cup W]^c) = \frac{12}{64} = \frac{3}{16}$ $\quad P(G \cup R \cup W) = 1 - \frac{3}{16} = \boxed{\frac{13}{16}}$

multiply for total successes

Problem 1.26: 10 men & 5 women are meeting in a conference room. 4 people are chosen @ random from 15 to form a committee.

a) Probability of 2 men & 2 women

A = {2 men & 2 women} → M{2 men} W{2 women}

Ω = {10 men & 5 women}

$$\frac{\binom{10}{2}\times\binom{5}{2}}{\binom{15}{4}} = \boxed{\frac{30}{91} = P(A)}$$

→ Ways to choose men

→ Ways to choose women

→ Ways to choose ppl

b) Among the 15 is a couple, Bob & Jane. What is the probability that both Bob & Jane are in the committee?

$\#\Omega = \binom{15}{4}$

A = {Bob & Jane picked} = $\begin{array}{cccc} B & J & & \\ \left[\cancel{0}\right. & \cancel{0} & 0 & \left.0\right] \end{array}$ $\binom{15-2}{4-2} = \binom{13}{2} = \#A$

$$\frac{\binom{13}{2}}{\binom{15}{4}} = \boxed{\frac{2}{35} = P(A)}$$

→ Ways ppl can be picked if Bob & Jane are picked

→ Total Sample Space

c) What is the probability that Bob is on the committee & Jane is not?

A = {Bob selected & Jane isn't}

15 → total − Bob [because committee] − Jane [Not committee]

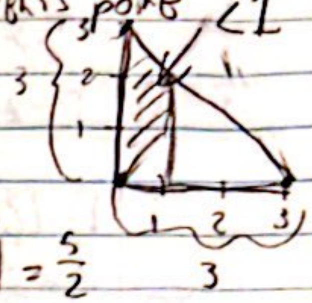
$$\frac{\binom{13}{3}}{\binom{15}{4}} = \boxed{\frac{22}{105} = P(A)}$$

$\binom{13}{3}$ → Ways to select ppl if Bob & not Jane

→ Sample Space

Problem 1.35: Pick a uniformly chosen random point inside the triangle w/ vertices @ (0,0), (3,0), (0,3)

a) What is the probability that the distance of this point to the y axis is less than 1?

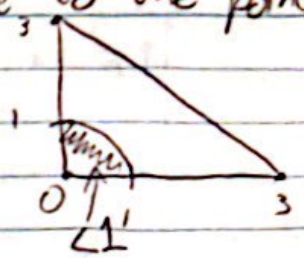



$\Omega = \{\text{All points inside triangle}\} = \frac{bh}{2} = \frac{3 \cdot 3}{2} = \frac{9}{2}$

$A = \{\text{all points in triangle \& less than 1 from y-axis}\} = \frac{3+2}{2} \cdot 1 = \frac{5}{2}$

$\frac{\#A}{\#\Omega} = \frac{5/2}{9/2} = \boxed{\frac{5}{9} = P(A)}$

b) What is the probability that the distance to the point from the origin is greater than 1




$\Omega = \{\text{All points in triangle}\}$

$A = \{\text{All points} > 1 \text{ unit away from origin}\}$

$\#\Omega = \frac{3 \cdot 3}{2} = \frac{9}{2}$

$\#A^c = \frac{\pi r^2}{4} = \frac{\pi \cdot 1^2}{4} = \frac{\pi}{4}$

$P(A^c) = \frac{\pi}{18}$

$\boxed{P(A) = \frac{18 - \pi}{18}}$

Problem 1.40: An urn contains 1 Green ball, 1 Red ball, 1 Yellow ball, & 1 White ball, I draw 4 w/ replacement. What is the probability that there is at least 1 color repeated exactly twice?

$\Omega = \{4 \text{ balls in urn}\} = 4^4$

$A = \{\text{At least 1 color repeated 2 times}\}$

$\hookrightarrow R = \{\text{Red repeated exactly twice}\}$ $P(R) = [\binom{4}{2} \cdot 3 \cdot 3] / 4^4 = P(G) = P(W) = P(Y)$

$\sum_{k=1}^{n} (-1)^{k+1} \cdot \sum_{1 \le i_1 < \dots < i_k} P(A_{i_1} \cap A_{i_2} \cap \dots \cap A_{i_k})$

$P(G \cap W) = \binom{4}{2} = 6/256$

= 2 Green & 2 White
= Same for all others

$P(R \cup W \cup Y \cup G) = P(R) + P(G) + P(Y) + P(W) - (P(R \cap W) + P(R \cap G) + P(R \cap Y) + \dots + P(G \cap W)$ $P(G \cap Y) + P(Y \cap W))$

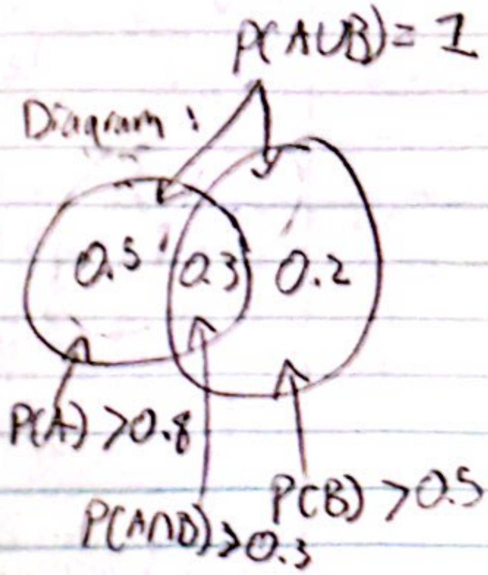
$4 \cdot \frac{54}{256} - 6 \cdot \frac{6}{256} = \boxed{\frac{45}{64} = P(R \cup W \cup Y \cup G)}$

Problem 1.42: Suppose $P(A) > 0.8$ and $P(B) > 0.5$.
Show $P(A \cap B) \geq 0.3$

Inclusion-Exclusion:

$$P(A \cup B) \geq P(A) + P(B) - P(A \cap B)$$
$$1 \geq 0.8 + 0.5 - P(A \cap B)$$
$$1 - (0.8 + 0.5) \geq -P(A \cap B)$$
$$-0.3 \geq -P(A \cap B)$$
$$\boxed{P(A \cap B) \geq 0.3}$$

Diagram:




Problem 1.55: Show that it is not possible to choose a uniform random number from the whole real line.

Problems w/ using whole real line:

Probability Values: Total probabilities must equal 1

P(Full Real line) = 1
- Sum of all probabilities on the whole-number line must be 1
- Whole real line is $\infty$ & as such, we can't assign a probability number to it

$$P(\text{Full Real Line}) = p + p + p + p \ldots = \text{Infinity} \neq 1$$
$$P(\text{Full Real line}) = 0 + 0 + 0 + 0 \ldots = 0 \neq 1$$

Since each value needs to be equally likely & the sum of all probabilities needs to equal 1, we can't assign a value of 0 or anything more than that to find probability on the entire whole real line.