Introduction

What is it that distinguishes the thousands of years of history from what we think of as modern times? The answer goes way beyond the progress of science, technology, capitalism, and democracy.

The distant past was studded with brilliant scientists, mathematicians, inventors, technologists, and political philosophers. Hundreds of years before the birth of Christ, the skies had been mapped, the great library of Alexandria built, and Euclid's geometry taught. Demand for technological innovation in warfare was as insatiable then as it is today. Coal, oil, iron, and copper have been at the service of human beings for millennia, and travel and communication mark the very beginnings of recorded civilization.

The revolutionary idea that defines the boundary between modern times and the past is the mastery of risk: the notion that the future is more than a whim of the gods and that men and women are not passive before nature. Until human beings discovered a way across that boundary, the future was a mirror of the past or the murky domain of oracles and soothsayers who held a monopoly over knowledge of anticipated events.

This book tells the story of a group of thinkers whose remarkable vision revealed how to put the future at the service of the present. By showing the world how to understand risk, measure it, and weigh its consequences, they converted risk-taking into one of the prime catalysts that drives modern Western society. Like Prometheus, they defied the gods and probed the darkness in search of the light that converted the future from an enemy into an opportunity. The transformation in attitudes toward risk management unleashed by their achievements has channeled the human passion for games and wagering into economic growth, improved quality of life, and technological progress.

By defining a rational process of risk-taking, these innovators provided the missing ingredient that has propelled science and enterprise into the world of speed, power, instant communication, and sophisticated finance that marks our own age. Their discoveries about the nature of risk, and the art and science of choice, lie at the core of our modern market economy that nations around the world are hastening to join. Given all its problems and pitfalls, the free economy, with choice at its center, has brought humanity unparalleled access to the good things of life.

The ability to define what may happen in the future and to choose among alternatives lies at the heart of contemporary societies. Risk management guides us over a vast range of decision-making, from allocating wealth to safeguarding public health, from waging war to planning a family, from paying insurance premiums to wearing a seatbelt, from planting corn to marketing cornflakes.

In the old days, the tools of farming, manufacture, business management, and communication were simple. Breakdowns were frequent, but repairs could be made without calling the plumber, the electrician, the computer scientist—or the accountants and the investment advisers. Failure in one area seldom had direct impact on another. Today, the tools we use are complex, and breakdowns can be catastrophic, with far-reaching consequences. We must be constantly aware of the likelihood of malfunctions and errors. Without a command of probability theory and other instruments of risk management, engineers could never have designed the great bridges that span our widest rivers, homes would still be heated by fireplaces or parlor stoves, electric power utilities would not exist, polio would still be maiming children, no airplanes would fly, and space travel would be just a dream.* Without insurance in its many varieties, the death of the breadwinner would reduce young families to starvation or charity, even more people would be denied health care, and only the wealthiest could afford to own a home. If farmers were unable to sell their crops at a price fixed before harvest, they would produce far less food than they do.

*The scientist who developed the Saturn 5 rocket that launched the first Apollo mission to the moon put it this way: "You want a valve that doesn't leak and you try everything possible to develop one. But the real world provides you with a leaky valve. You have to determine how much leaking you can tolerate." (Obituary of Arthur Rudolph, in *The New York Times*, January 3, 1996.)

If we had no liquid capital markets that enable savers to diversify their risks, if investors were limited to owning just one stock (as they were in the early days of capitalism), the great innovative enterprises that define our age—companies like Microsoft, Merck, DuPont, Alcoa, Boeing, and McDonald's—might never have come into being. The capacity to manage risk, and with it the appetite to take risk and make forward-looking choices, are key elements of the energy that drives the economic system forward.

The modern conception of risk is rooted in the Hindu-Arabic numbering system that reached the West seven to eight hundred years ago. But the serious study of risk began during the Renaissance, when people broke loose from the constraints of the past and subjected long-held beliefs to open challenge. This was a time when much of the world was to be discovered and its resources exploited. It was a time of religious turmoil, nascent capitalism, and a vigorous approach to science and the future.

In 1654, a time when the Renaissance was in full flower, the Chevalier de Méré, a French nobleman with a taste for both gambling and mathematics, challenged the famed French mathematician Blaise Pascal to solve a puzzle. The question was how to divide the stakes of an unfinished game of chance between two players when one of them is ahead. The puzzle had confounded mathematicians since it was posed some two hundred years earlier by the monk Luca Paccioli. This was the man who brought double-entry bookkeeping to the attention of the business managers of his day—and tutored Leonardo da Vinci in the multiplication tables. Pascal turned for help to Pierre de Fermat, a lawyer who was also a brilliant mathematician. The outcome of their collaboration was intellectual dynamite. What might appear to have been a seventeenth-century version of the game of Trivial Pursuit led to the discovery of the theory of probability, the mathematical heart of the concept of risk.

Their solution to Paccioli's puzzle meant that people could for the first time make decisions and forecast the future with the help of numbers. In the medieval and ancient worlds, even in preliterate and peasant societies, people managed to make decisions, advance their interests, and carry on trade, but with no real understanding of risk or *the nature of decision-making.* Today, we rely less on superstition and tradition than people did in the past, not because we are more rational, but because our understanding of risk enables us to make decisions in a rational mode.

At the time Pascal and Fermat made their breakthrough into the fascinating world of probability, society was experiencing an extraordinary wave of innovation and exploration. By 1654, the roundness of the earth was an established fact, vast new lands had been discovered, gunpowder was reducing medieval castles to dust, printing with movable type had ceased to be a novelty, artists were skilled in the use of perspective, wealth was pouring into Europe, and the Amsterdam stock exchange was flourishing. Some years earlier, in the 1630s, the famed Dutch tulip bubble had burst as a result of the issuing of options whose essential features were identical to the sophisticated financial instruments in use today.

These developments had profound consequences that put mysticism on the run. By this time Martin Luther had had his say and halos had disappeared from most paintings of the Holy Trinity and the saints. William Harvey had overthrown the medical teachings of the ancients with his discovery of the circulation of blood—and Rembrandt had painted "The Anatomy Lesson," with its cold, white, naked human body. In such an environment, someone would soon have worked out the theory of probability, even if the Chevalier de Méré had never confronted Pascal with his brainteaser.

As the years passed, mathematicians transformed probability theory from a gamblers' toy into a powerful instrument for organizing, interpreting, and applying information. As one ingenious idea was piled on top of another, quantitative techniques of risk management emerged that have helped trigger the tempo of modern times.

By 1725, mathematicians were competing with one another in devising tables of life expectancies, and the English government was financing itself through the sale of life annuities. By the middle of the century, marine insurance had emerged as a flourishing, sophisticated business in London.

In 1703, Gottfried von Leibniz commented to the Swiss scientist and mathematician Jacob Bernoulli that "[N]ature has established patterns originating in the return of events, but only for the most part,"[1]

thereby prompting Bernoulli to invent the Law of Large Numbers and methods of statistical sampling that drive modern activities as varied as opinion polling, wine tasting, stock picking, and the testing of new drugs.* Leibniz's admonition—"but only for the most part"—was more profound than he may have realized, for he provided the key to why there is such a thing as risk in the first place: without that qualification, everything would be predictable, and in a world where every event is identical to a previous event no change would ever occur.

In 1730, Abraham de Moivre suggested the structure of the normal distribution—also known as the bell curve—and discovered the concept of standard deviation. Together, these two concepts make up what is popularly known as the Law of Averages and are essential ingredients of modern techniques for quantifying risk. Eight years later, Daniel Bernoulli, Jacob's nephew and an equally distinguished mathematician and scientist, first defined the systematic process by which most people make choices and reach decisions. Even more important, he propounded the idea that the satisfaction resulting from any small increase in wealth "will be inversely proportionate to the quantity of goods previously possessed." With that innocent-sounding assertion, Bernoulli explained why King Midas was an unhappy man, why people tend to be risk-averse, and why prices must fall if customers are to be persuaded to buy more. Bernoulli's statement stood as the dominant paradigm of rational behavior for the next 250 years and laid the groundwork for modern principles of investment management.

Almost exactly one hundred years after the collaboration between Pascal and Fermat, a dissident English minister named Thomas Bayes made a striking advance in statistics by demonstrating how to make better-informed decisions by mathematically blending new information into old information. Bayes's theorem focuses on the frequent occasions when we have sound intuitive judgments about the probability of some event and want to understand how to alter those judgments as actual events unfold.

All the tools we use today in risk management and in the analysis of decisions and choice, from the strict rationality of game theory to the

*Chapter 7 describes Jacob Bernoulli's achievements in detail. The Law of Large Numbers says in essence that the difference between the observed value of a sample and its true value will diminish as the number of observations in the sample increases.

challenges of chaos theory, stem from the developments that took place between 1654 and 1760, with only two exceptions:

In 1875, Francis Galton, an amateur mathematician who was Charles Darwin's first cousin, discovered regression to the mean, which explains why pride goeth before a fall and why clouds tend to have silver linings. Whenever we make any decision based on the expectation that matters will return to "normal," we are employing the notion of regression to the mean.

In 1952, Nobel Laureate Harry Markowitz, then a young graduate student studying operations research at the University of Chicago, demonstrated mathematically why putting all your eggs in one basket is an unacceptably risky strategy and why diversification is the nearest an investor or business manager can ever come to a free lunch. That revelation touched off the intellectual movement that revolutionized Wall Street, corporate finance, and business decisions around the world; its effects are still being felt today.

The story that I have to tell is marked all the way through by a persistent tension between those who assert that the best decisions are based on quantification and numbers, determined by the patterns of the past, and those who base their decisions on more subjective degrees of belief about the uncertain future. This is a controversy that has never been resolved.

The issue boils down to one's view about the extent to which the past determines the future. We cannot quantify the future, because it is an unknown, but we have learned how to use numbers to scrutinize what happened in the past. But to what degree should we rely on the patterns of the past to tell us what the future will be like? Which matters more when facing a risk, the facts as we see them or our subjective belief in what lies hidden in the void of time? Is risk management a science or an art? Can we even tell for certain precisely where the dividing line between the two approaches lies?

It is one thing to set up a mathematical model that appears to explain everything. But when we face the struggle of daily life, of constant trial and error, the ambiguity of the facts as well as the power of the human heartbeat can obliterate the model in short order. The late Fischer Black, a pioneering theoretician of modern finance who moved from M.I.T. to Wall Street, said, "Markets look a lot less efficient from the banks of the Hudson than from the banks of the Charles."[2]

Over time, the controversy between quantification based on observations of the past and subjective degrees of belief has taken on a deeper significance. The mathematically driven apparatus of modern risk management contains the seeds of a dehumanizing and self-destructive technology. Nobel laureate Kenneth Arrow has warned, "[O]ur knowledge of the way things work, in society or in nature, comes trailing clouds of vagueness. Vast ills have followed a belief in certainty."[3] In the process of breaking free from the past we may have become slaves of a new religion, a creed that is just as implacable, confining, and arbitrary as the old.

Our lives teem with numbers, but we sometimes forget that numbers are only tools. They have no soul; they may indeed become fetishes. Many of our most critical decisions are made by computers, contraptions that devour numbers like voracious monsters and insist on being nourished with ever-greater quantities of digits to crunch, digest, and spew back.

To judge the extent to which today's methods of dealing with risk are either a benefit or a threat, we must know the whole story, from its very beginnings. We must know why people of past times did—or did not—try to tame risk, how they approached the task, what modes of thinking and language emerged from their experience, and how their activities interacted with other events, large and small, to change the course of culture. Such a perspective will bring us to a deeper understanding of where we stand, and where we may be heading.

Along the way, we shall refer often to games of chance, which have applications that extend far beyond the spin of the roulette wheel. Many of the most sophisticated ideas about managing risk and making decisions have developed from the analysis of the most childish of games. One does not have to be a gambler or even an investor to recognize what gambling and investing reveal about risk.

The dice and the roulette wheel, along with the stock market and the bond market, are natural laboratories for the study of risk because they lend themselves so readily to quantification; their language is the language of numbers. They also reveal a great deal about ourselves. When we hold our breath watching the little white ball bounce about on the spinning roulette wheel, and when we call our broker to buy or sell some shares of stock, our heart is beating along with the numbers. So, too, with all important outcomes that depend on chance.

The word "risk" derives from the early Italian *risicare*, which means "to dare." In this sense, risk is a choice rather than a fate. The actions we dare to take, which depend on how free we are to make choices, are what the story of risk is all about. And that story helps define what it means to be a human being.

TO 1200: BEGINNINGS

1

The Winds of the Greeks and the Role of the Dice

Why is the mastery of risk such a uniquely modern concept? Why did humanity wait the many thousands of years leading up to the Renaissance before breaking down the barriers that stood in the way of measuring and controlling risk?

These questions defy easy answers. But we begin with a clue. Since the beginning of recorded history, gambling—the very essence of risk-taking—has been a popular pastime and often an addiction. It was a game of chance that inspired Pascal and Fermat's revolutionary breakthrough into the laws of probability, not some profound question about the nature of capitalism or visions of the future. Yet until that moment, throughout history, people had wagered and played games without using any system of odds that determines winnings and losings today. The act of risk-taking floated free, untrammeled by the theory of risk management.

Human beings have always been infatuated with gambling because it puts us head-to-head against the fates, with no holds barred. We enter this daunting battle because we are convinced that we have a powerful ally: Lady Luck will interpose herself between us and the fates (or the odds) to bring victory to our side. Adam Smith, a masterful student of human nature, defined the motivation: "The overweening conceit which the greater part of men have of their own abilities [and] their absurd presumption in their own good fortune."[1] Although Smith was keenly aware that the human propensity to take risk propelled economic progress, he feared that society would suffer when that propensity ran amuck. So he was careful to balance moral sentiments against the benefits of a free market. A hundred and sixty years later, another great English economist, John Maynard Keynes, agreed: "When the capital development of a country becomes the by-product of the activities of a casino, the job is likely to be ill-done."[2]

Yet the world would be a dull place if people lacked conceit and confidence in their own good fortune. Keynes had to admit that "If human nature felt no temptation to take a chance . . . there might not be much investment merely as a result of cold calculation."[3] Nobody takes a risk in the expectation that it will fail. When the Soviets tried to administer uncertainty out of existence through government fiat and planning, they choked off social and economic progress.

Gambling has held human beings in thrall for millennia. It has been engaged in everywhere, from the dregs of society to the most respectable circles.

Pontius Pilate's soldiers cast lots for Christ's robe as He suffered on the cross. The Roman Emperor Marcus Aurelius was regularly accompanied by his personal croupier. The Earl of Sandwich invented the snack that bears his name so that he could avoid leaving the gaming table in order to eat. George Washington hosted games in his tent during the American Revolution.[4] Gambling is synonymous with the Wild West. And "Luck Be a Lady Tonight" is one of the most memorable numbers in *Guys and Dolls*, a musical about a compulsive gambler and his floating crap game.

The earliest-known form of gambling was a kind of dice game played with what was known as an astragalus, or knuckle-bone.[5] This early ancestor of today's dice was a squarish bone taken from the ankles of sheep or deer, solid and without marrow, and so hard as to be virtually indestructible. Astragali have surfaced in archeological digs in many

parts of the world. Egyptian tomb paintings picture games played with astragali dating from 3500 BC, and Greek vases show young men tossing the bones into a circle. Although Egypt punished compulsive gamblers by forcing them to hone stones for the pyramids, excavations show that the pharaohs were not above using loaded dice in their own games. Craps, an American invention, derives from various dice games brought into Europe via the Crusades. Those games were generally referred to as "hazard," from *al zahr*, the Arabic word for dice.[6]

Card games developed in Asia from ancient forms of fortune-telling, but they did not become popular in Europe until the invention of printing. Cards originally were large and square, with no identifying figures or pips in the corners. Court cards were printed with only one head instead of double-headed, which meant that players often had to identify them from the feet—turning the cards around would reveal a holding of court cards. Square corners made cheating easy for players who could turn down a tiny part of the corner to identify cards in the deck later on. Double-headed court cards and cards with rounded corners came into use only in the nineteenth century.

Like craps, poker is an American variation on an older form—the game is only about 150 years old. David Hayano has described poker as "Secret ploys, monumental deceptions, calculated strategies, and fervent beliefs [with] deep, invisible structures . . . . A game to experience rather than to observe."[7] According to Hayano, about forty million Americans play poker regularly, all confident of their ability to outwit their opponents.

The most addictive forms of gambling seem to be the pure games of chance played at the casinos that are now spreading like wildfire through once staid American communities. An article in *The New York Times* of September 25, 1995, datelined Davenport, Iowa, reports that gambling is the fastest-growing industry in the United States, "a $40 billion business that draws more customers than baseball parks or movie theaters."[8] The *Times* cites a University of Illinois professor who estimates that state governments pay three dollars in costs to social agencies and the criminal justice system for every dollar of revenue they take in from the casinos—a calculus that Adam Smith might have predicted.

Iowa, for example, which did not even have a lottery until 1985, had ten big casinos by 1995, plus a horse track and a dog track with 24-hour slot machines. The article states that "nearly nine out of ten

Iowans say they gamble," with 5.4% of them reporting that they have a gambling problem, up from 1.7% five years earlier. This in a state where a Catholic priest went to jail in the 1970s on charges of running a bingo game. *Al zahr* in its purest form is apparently still with us.

Games of chance must be distinguished from games in which skill makes a difference. The principles at work in roulette, dice, and slot machines are identical, but they explain only part of what is involved in poker, betting on the horses, and backgammon. With one group of games the outcome is determined by fate; with the other group, choice comes into play. The odds—the probability of winning—are all you need to know for betting in a game of chance, but you need far more information to predict who will win and who will lose when the outcome depends on skill as well as luck. There are cardplayers and racetrack bettors who are genuine professionals, but no one makes a successful profession out of shooting craps.

Many observers consider the stock market itself little more than a gambling casino. Is winning in the stock market the result of skill combined with luck, or is it just the result of a lucky gamble? We shall return to this question in Chapter 12.

Losing streaks and winning streaks occur frequently in games of chance, as they do in real life. Gamblers respond to these events in asymmetric fashion: they appeal to the law of averages to bring losing streaks to a speedy end. And they appeal to that same law of averages to suspend itself so that winning streaks will go on and on. The law of averages hears neither appeal. The last sequence of throws of the dice conveys absolutely no information about what the next throw will bring. Cards, coins, dice, and roulette wheels have no memory.

Gamblers may think they are betting on red or seven or four-of-a-kind, but *in reality they are betting on the clock*. The loser wants a short run to look like a long run, so that the odds will prevail. The winner wants a long run to look like a short run, so that the odds will be suspended. Far away from the gaming tables, the managers of insurance companies conduct their affairs in the same fashion. They set their premiums to cover the losses they will sustain in the long run; but when earthquakes

and fires and hurricanes all happen at about the same time, the short run can be very painful. Unlike gamblers, insurance companies carry capital and put aside reserves to tide them over during the inevitable sequences of short runs of bad luck.

Time is the dominant factor in gambling. Risk and time are opposite sides of the same coin, for if there were no tomorrow there would be no risk. Time transforms risk, and the nature of risk is shaped by the time horizon: the future is the playing field.

Time matters most when decisions are irreversible. And yet many irreversible decisions must be made on the basis of incomplete information. Irreversibility dominates decisions ranging all the way from taking the subway instead of a taxi, to building an automobile factory in Brazil, to changing jobs, to declaring war.

If we buy a stock today, we can always sell it tomorrow. But what do we do after the croupier at the roulette table cries, "No more bets!" or after a poker bet is doubled? There is no going back. Should we refrain from acting in the hope that the passage of time will make luck or the probabilities turn in our favor?

Hamlet complained that too much hesitation in the face of uncertain outcomes is bad because "the native hue of resolution is sicklied o'er with the pale cast of thought . . . and enterprises of great pith and moment . . . lose the name of action." Yet once we act, we forfeit the option of waiting until new information comes along. As a result, not-acting has value. The more uncertain the outcome, the greater may be the value of procrastination. Hamlet had it wrong: he who hesitates is halfway home.

To explain the beginning of everything, Greek mythology drew on a giant game of craps to explain what modern scientists call the Big Bang. Three brothers rolled dice for the universe, with Zeus winning the heavens, Poseidon the seas, and Hades, the loser, going to hell as master of the underworld.

Probability theory seems a subject made to order for the Greeks, given their zest for gambling, their skill as mathematicians, their mastery of logic, and their obsession with proof. Yet, though the most civilized of all the ancients, they never ventured into that fascinating world. Their failure to do so is astonishing because the Greeks had the only recorded civilization up to that point untrammeled by a dominating priesthood that claimed a monopoly on the lines of communication with the powers of mystery. Civilization as we know it might have progressed at a much faster pace if the Greeks had anticipated what their intellectual progeny—the men of the Renaissance—were to discover some thousand years later.

Despite the emphasis that the Greeks placed on theory, they had little interest in applying it to any kind of technology that would have changed their views of the manageability of the future. When Archimedes invented the lever, he claimed that he could move the earth if only he could find a place to stand. But apparently he gave no thought to changing it. The daily life of the Greeks, and their standard of living, were much the same as the way that their forebears had subsisted for thousands of years. They hunted, fished, grew crops, bore children, and used architectural techniques that were only variations on themes developed much earlier in the Tigris-Euphrates valley and in Egypt.

Genuflection before the winds was the only form of risk management that caught their attention: their poets and dramatists sing repeatedly of their dependence on the winds, and beloved children were sacrificed to appease the winds. Most important, the Greeks lacked a numbering system that would have enabled them to *calculate* instead of just recording the results of their activities.[9]

I do not mean to suggest that the Greeks gave no thought to the nature of probability. The ancient Greek word εικος (*eikos*), which meant plausible or probable, had the same sense as the modern concept of probability: "to be expected with some degree of certainty." Socrates defines εικος as "likeness to truth."[10]

Socrates' definition reveals a subtle point of great importance. *Likeness to truth is not the same thing as truth.* Truth to the Greeks was only what could be proved by logic and axioms. Their insistence on proof set truth in direct contrast to empirical experimentation. For example, in *Phaedo*, Simmias points out to Socrates that "the proposition that the soul is in harmony has not been demonstrated at all but rests only on probability." Aristotle complains about philosophers who, ". . . while they speak plausibly, . . .do not speak what is true." Elsewhere, Socrates

anticipates Aristotle when he declares that a "mathematician who argues from probabilities in geometry is not worth an ace."[11] For another thousand years, *thinking* about games and *playing* them remained separate activities.

Shmuel Sambursky, a distinguished Israeli historian and philosopher of science, provides the only convincing thesis I could find to explain why the Greeks failed to take the strategic step of developing a quantitative approach to probability.[12] With their sharp distinction between truth and probability, Sambursky contends in a paper written in 1956, the Greeks could not conceive of any kind of solid structure or harmony in the messy nature of day-to-day existence. Although Aristotle suggested that people should make decisions on the basis of "desire and reasoning directed to some end," he offered no guidance to the likelihood of a successful outcome. Greek dramas tell tale after tale of the helplessness of human beings in the grasp of impersonal fates. When the Greeks wanted a prediction of what tomorrow might bring, they turned to the oracles instead of consulting their wisest philosophers.

The Greeks believed that order is to be found only in the skies, where the planets and stars regularly appear in their appointed places with an unmatched regularity. To this harmonious performance, the Greeks paid deep respect, and their mathematicians studied it intensely. But the perfection of the heavens served only to highlight the disarray of life on earth. Moreover, the predictability of the firmament contrasted sharply with the behavior of the fickle, foolish gods who dwelt on high.

The old Talmudic Jewish philosophers may have come a bit closer to quantifying risk. But here, too, we find no indication that they followed up on their reasoning by developing a methodical approach to risk. Sambursky cites a passage in the Talmud, *Kethuboth 9q*, where the philosopher explains that a man may divorce his wife for adultery without any penalty, but not if he claims that the adultery occurred before marriage.[13]

"It is a double doubt," declares the Talmud. If it is established (method unspecified) that the bride came to the marriage bed no longer a virgin, one side of the double doubt is whether the man responsible was the prospective groom himself—whether the event occurred "under him . . . or not under him." As to the second side of the doubt, the argument continues: "And if you say that it was under him, there is doubt whether it was by violence or by her free will." Each side of the double doubt is given a 50–50 chance. With impressive statistical sophistication, the philosophers conclude that there is only one chance in four ($1/2 \times 1/2$) that the woman committed adultery before marriage. Therefore, the husband cannot divorce her on those grounds.

One is tempted to assume that the lapse of time between the invention of the astragalus and the invention of the laws of probability was nothing more than a historical accident. The Greeks and the Talmudic scholars were so maddeningly close to the analysis that Pascal and Fermat would undertake centuries later that only a slight push would have moved them on to the next step.

That the push did not occur was not an accident. Before a society could incorporate the concept of risk into its culture, change would have to come, not in views of the present, but in attitudes about the future.

Up to the time of the Renaissance, people perceived the future as little more than a matter of luck or the result of random variations, and most of their decisions were driven by instinct. When the conditions of life are so closely linked to nature, not much is left to human control. As long as the demands of survival limit people to the basic functions of bearing children, growing crops, hunting, fishing, and providing shelter, they are simply unable to conceive of circumstances in which they might be able to influence the outcomes of their decisions. A penny saved is not a penny earned unless the future is something more than a black hole.

Over the centuries, at least until the Crusades, most people met with few surprises as they ambled along from day to day. Nestled in a stable social structure, they gave little heed to the wars that swept across the land, to the occasions when bad rulers succeeded good ones, and even to the permutations of religions. Weather was the most apparent variable. As the Egyptologist Henri Frankfort has remarked, "The past and the future—far from being a matter of concern—were wholly implicit in the present."[14]

Despite the persistence of this attitude toward the future, civilization made great strides over the centuries. Clearly the absence of modern views about risk was no obstacle. At the same time, the advance of

civilization was not in itself a sufficient condition to motivate curious people to explore the possibilities of scientific forecasting.

As Christianity spread across the western world, the will of a single God emerged as the orienting guide to the future, replacing the miscellany of deities people had worshiped since the beginning of time. This brought a major shift in perception: the future of life on earth remained a mystery, but it was now prescribed by a power whose intentions and standards were clear to all who took the time to learn them.

As contemplation of the future became a matter of moral behavior and faith, the future no longer appeared quite as inscrutable as it had. Nevertheless, it was still not susceptible to any sort of mathematical expectation. The early Christians limited their prophecies to what would happen in the afterlife, no matter how fervidly they beseeched God to influence worldly events in their favor.

Yet the search for a better life on earth persisted. By the year 1000, Christians were sailing great distances, meeting new peoples, and encountering new ideas. Then came the Crusades—a seismic culture shock. Westerners collided with an Arab empire that had been launched at Mohammed's urging and that stretched as far eastward as India. Christians, with faith in the future, met Arabs who had achieved an intellectual sophistication far greater than that of the interlopers who had come to dislodge them from the holy sites.

The Arabs, through their invasion of India, had become familiar with the Hindu numbering system, which enabled them to incorporate eastern intellectual advances into their own scholarship, scientific research, and experimentation. The results were momentous, first for the Arabs and then for the West.*

In the hands of the Arabs, the Hindu numbers would transform mathematics and measurement in astronomy, navigation, and commerce. New methods of calculation gradually replaced the abacus,

*Peter Kinder has pointed out to me a great historical irony in all this. The Vikings and other Norsemen who laid waste to Roman civilization and destroyed the repositories of learning in the ninth century reappear in history as the Normans who brought back to the West the achievements of Arabic learning in the twelfth century.

which for centuries had been the only tool for doing arithmetic everywhere from the Mayans in the western hemisphere, across Europe, to India and the Orient. The word *abacus* derives from the Greek word *abax*, which means sand-tray. Within the trays, columns of pebbles were laid out on the sand.[15] The word *calculate* stems from *calculus*, the Latin word for pebble.

Over the next five hundred years, as the new numbering system took the place of the simple abacus, writing replaced movable counters in making calculations. Written computation fostered abstract thinking, which opened the way to areas of mathematics never conceived of in the past. Now sea voyages could be longer, time-keeping more accurate, architecture more ambitious, and production methods more elaborate. The modern world would be quite different if we still measured and counted with I, V, X, L, C, D, and M—or with the Greek or Hebrew letters that stood for numbers.

But Arabic numbers were not enough to induce Europeans to explore the radical concept of replacing randomness with systematic probability and its implicit suggestion that the future might be predictable and even controllable to some degree. That advance had to await the realization that human beings are not totally helpless in the hands of fate, nor is their worldly destiny always determined by God.

The Renaissance and the Protestant Reformation would set the scene for the mastery of risk. As mysticism yielded to science and logic after 1300 AD, Greek and Roman architectural forms began to replace Gothic forms, church windows were opened to the light, and sculptures showed men and women standing firmly on the ground instead posing as stylized figures with neither muscle nor weight. The ideas that propelled changes in the arts also contributed to the Protestant Reformation and weakened the dominance of the Catholic Church.

The Reformation meant more than just a change in humanity's relationship with God. By eliminating the confessional, it warned people that henceforth they would have to walk on their own two feet and would have to take responsibility for the consequences of their decisions.

But if men and women were not at the mercy of impersonal deities and random chance, they could no longer remain passive in the face of an unknown future. They had no choice but to begin making decisions over a far wider range of circumstances and over far longer periods of

time than ever before. The concepts of thrift and abstinence that characterize the Protestant ethic evidenced the growing importance of the future relative to the present. With this opening up of choices and decisions, people gradually recognized that the future offered opportunity as well as danger, that it was open-ended and full of promise. The 1500s and 1600s were a time of geographical exploration, confrontation with new lands and new societies, and experimentation in art, poetic forms, science, architecture, and mathematics. The new sense of opportunity led to a dramatic acceleration in the growth of trade and commerce, which served as a powerful stimulus to change and exploration. Columbus was not conducting a Caribbean cruise: he was seeking a new trade route to the Indies. The prospect of getting rich is highly motivating, and few people get rich without taking a gamble.

There is more to that blunt statement than meets the eye. Trade is a mutually beneficial process, a transaction in which both parties perceive themselves as wealthier than they were before. What a radical idea! Up to that point, people who got rich had done so largely by exploitation or by plundering another's wealth. Although Europeans continued to plunder across the seas, at home the accumulation of wealth was open to the many rather than the few. The newly rich were now the smart, the adventuresome, the innovators—most of them businessmen—instead of just the hereditary princes and their minions.

Trade is also a risky business. As the growth of trade transformed the principles of gambling into the creation of wealth, the inevitable result was capitalism, the epitome of risk-taking. But capitalism could not have flourished without two new activities that had been unnecessary so long as the future was a matter of chance or of God's will. The first was bookkeeping, a humble activity but one that encouraged the dissemination of the new techniques of numbering and counting. The other was forecasting, a much less humble and far more challenging activity that links risk-taking with direct payoffs.

You do not plan to ship goods across the ocean, or to assemble merchandise for sale, or to borrow money without first trying to determine what the future may hold in store. Ensuring that the materials you order are delivered on time, seeing to it that the items you plan to sell are produced on schedule, and getting your sales facilities in place all must be planned before that moment when the customers show up and lay their money on the counter. The successful business executive is a forecaster first; purchasing, producing, marketing, pricing, and organizing all follow.

The men you will meet in the coming chapters recognized the discoveries of Pascal and Fermat as the beginning of wisdom, not just a solution to an intellectual conundrum involving a game of chance. They were bold enough to tackle the many facets of risk in the face of issues of growing complexity and practical importance and to recognize that these are issues involving the most fundamental philosophical aspects of human existence.

But philosophy must stand aside for the moment, as the story should begin at the beginning. Modern methods of dealing with the unknown start with measurement, with odds and probabilities. The numbers come first. But where did the numbers come from?

2

As Easy as I, II, III

Without numbers, there are no odds and no probabilities; without odds and probabilities, the only way to deal with risk is to appeal to the gods and the fates. Without numbers, risk is wholly a matter of gut.

We live in a world of numbers and calculations, from the clock we squint at when we wake up, to the television channel we switch off at bedtime. As the day proceeds, we count the measures of coffee we put into the coffeemaker, pay the housekeeper, consult yesterday's stock prices, dial a friend's telephone number, check the amount of gas in the car and the speed on the speedometer, press the elevator button in our office building, and open the office door with our number on it. And the day has hardly started!

It is hard for us to imagine a time without numbers. Yet if we were able to spirit a well-educated man from the year 1000 to the present, he probably would not recognize the number zero and would surely flunk third-grade arithmetic; few people from the year 1500 would fare much better.

The story of numbers in the West begins in 1202, when the cathedral of Chartres was nearing completion and King John was finishing his third year on the throne of England. In that year, a book titled *Liber Abaci*, or *Book of the Abacus*, appeared in Italy. The fifteen chapters of the book were entirely handwritten; almost three hundred years would pass before the invention of printing. The author, Leonardo Pisano, was only 27 years old but a very lucky man: his book would receive the endorsement of the Holy Roman Emperor, Frederick II. No author could have done much better than that.[1]

Leonardo Pisano was known for most of his life as Fibonacci, the name by which he is known today. His father's first name was Bonacio, and Fibonacci is a contraction of son-of-Bonacio. Bonacio means "simpleton" and Fibonacci means "blockhead." Bonacio must have been something less than a simpleton, however, for he represented Pisa as consul in a number of different cities, and his son Leonardo was certainly no blockhead.

Fibonacci was inspired to write *Liber Abaci* on a visit to Bugia, a thriving Algerian city where his father was serving as Pisan consul. While Fibonacci was there, an Arab mathematician revealed to him the wonders of the Hindu-Arabic numbering system that Arab mathematicians had introduced to the West during the Crusades to the Holy Land. When Fibonacci saw all the calculations that this system made possible—calculations that could not possibly be managed with Roman letter-numerals—he set about learning everything he could about it. To study with the leading Arab mathematicians living around the Mediterranean, he set off on a trip that took him to Egypt, Syria, Greece, Sicily, and Provence.

The result was a book that is extraordinary by any standard. *Liber Abaci* made people aware of a whole new world in which numbers could be substituted for the Hebrew, Greek, and Roman systems that used letters for counting and calculating. The book rapidly attracted a following among mathematicians, both in Italy and across Europe.

Liber Abaci is far more than a primer for reading and writing with the new numerals. Fibonacci begins with instructions on how to determine from the number of digits in a numeral whether it is a unit, or a multiple of ten, or a multiple of 100, and so on. Later chapters exhibit a higher level of sophistication. There we find calculations using whole numbers and fractions, rules of proportion, extraction of square roots and roots of higher orders, and even solutions for linear and quadratic equations.

Ingenious and original as Fibonacci's exercises were, if the book had dealt only with theory it would probably not have attracted much attention beyond a small circle of mathematical cognoscenti. It commanded

an enthusiastic following, however, because Fibonacci filled it with practical applications. For example, he described and illustrated many innovations that the new numbers made possible in commercial bookkeeping, such as figuring profit margins, money-changing, conversions of weights and measures, and—though usury was still prohibited in many places—he even included calculations of interest payments.

Liber Abaci provided just the kind of stimulation that a man as brilliant and creative as the Emperor Frederick would be sure to enjoy. Though Frederick, who ruled from 1211 to 1250, exhibited cruelty and an obsession with earthly power, he was genuinely interested in science, the arts, and the philosophy of government. In Sicily, he destroyed all the private garrisons and feudal castles, taxed the clergy, and banned them from civil office. He also set up an expert bureaucracy, abolished internal tolls, removed all regulations inhibiting imports, and shut down the state monopolies.

Frederick tolerated no rivals. Unlike his grandfather, Frederick Barbarossa, who was humbled by the Pope at the Battle of Legnano in 1176, this Frederick reveled in his endless battles with the papacy. His intransigence brought him not just one excommunication, but two. On the second occasion, Pope Gregory IX called for Frederick to be deposed, characterizing him as a heretic, rake, and anti-Christ. Frederick responded with a savage attack on papal territory; meanwhile his fleet captured a large delegation of prelates on their way to Rome to join the synod that had been called to remove him from power.

Frederick surrounded himself with the leading intellectuals of his age, inviting many of them to join him in Palermo. He built some of Sicily's most beautiful castles, and in 1224 he founded a university to train public servants—the first European university to enjoy a royal charter.

Frederick was fascinated with *Liber Abaci*. Some time in the 1220s, while on a visit to Pisa, he invited Fibonacci to appear before him. In the course of the interview, Fibonacci solved problems in algebra and cubic equations put to him by one of Frederick's many scientists-in-residence. Fibonacci subsequently wrote a book prompted by this meeting, *Liber Quadratorum*, or *The Book of Squares*, which he dedicated to the Emperor.

Fibonacci is best known for a short passage in *Liber Abaci* that led to something of a mathematical miracle. The passage concerns the problem of how many rabbits will be born in the course of a year from an original pair of rabbits, assuming that every month each pair produces another pair and that rabbits begin to breed when they are two months old. Fibonacci discovered that the original pair of rabbits would have spawned a total of 233 pairs of offspring in the course of a year.

He discovered something else, much more interesting. He had assumed that the original pair would not breed until the second month and then would produce another pair every month. By the fourth month, their first two offspring would begin breeding. After the process got started, the total number of pairs of rabbits at the end of each month would be as follows: 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233. Each successive number is the sum of the two preceding numbers. If the rabbits kept going for a hundred months, the total number pairs would be 354,224,848,179,261,915,075.

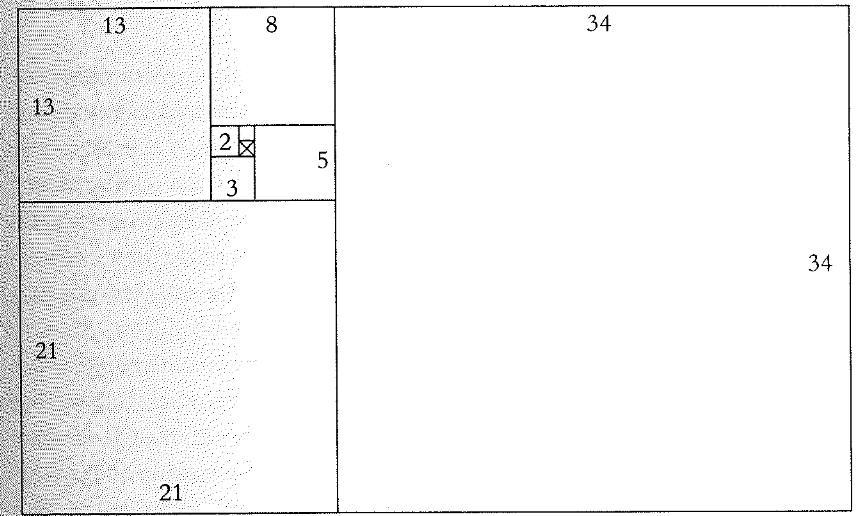
The Fibonacci series is a lot more than a source of amusement. Divide any of the Fibonacci numbers by the next higher number. After 3, the answer is always 0.625. After 89, the answer is always 0.618; after higher numbers, more decimal places can be filled in.* Divide any number by its preceding number. After 2, the answer is always 1.6. After 144, the answer is always 1.618.

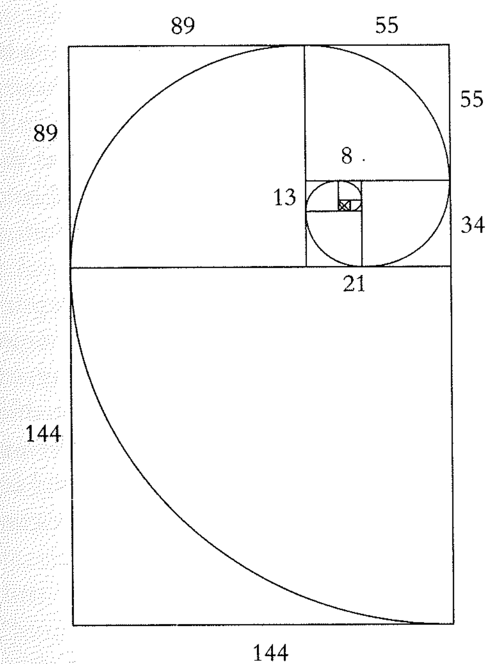
The Greeks knew this proportion and called it "the golden mean." The golden mean defines the proportions of the Parthenon, the shape of playing cards and credit cards, and the proportions of the General Assembly Building at the United Nations in New York. The horizontal member of most Christian crosses separates the vertical member by just about the same ratio: the length above the crosspiece is 61.8% of the length below it. The golden mean also appears throughout nature—in flower patterns, the leaves of an artichoke, and the leaf stubs on a palm tree. It is also the ratio of the length of the human body above the navel to its length below the navel (in normally proportioned people, that is). The length of each successive bone in our fingers, from tip to hand, also bears this ratio.†

In one of its more romantic manifestations, the Fibonacci ratio defines the proportions and shape of a beautiful spiral. The accompanying illustrations demonstrate how the spiral develops from a series of

*One of those odd quirks that numbers can produce reveals that you can derive 0.618 if you take the square root of 5, which is 2.24, subtract 1, and then divide by 2; this result is the algebraic proof of Fibonacci's sequence of numbers.

†In technical terms, the formula for the Fibonacci ratio is as follows: the ratio of the smaller part to the larger part equals the ratio of the larger part to the whole.





Construction of an equiangular spiral using Fibonacci proportions.

Begin with a 1-unit square, attach another 1-unit square, then a 2-unit square then a 2-unit square where it fits, followed by a 3-unit square where it fits and, continuing in the same direction, attach squares of 5, 8, 13, 21, and 34 units and so on.

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squares whose successive relative dimensions are determined by the Fibonacci series. The process begins with two small squares of equal size. It then progresses to an adjacent square twice the size of the first two, then to a square three times the size of the first two, then to five times, and so on. Note that the sequence produces a series of rectangles with the proportions of the golden mean. Then quarter-circle arcs connect the opposite corners of the squares, starting with the smallest squares and proceeding in sequence.

This familiar-looking spiral appears in the shape of certain galaxies, in a ram's horn, in many seashells, and in the coil of the ocean waves that surfers ride. The structure maintains its form without change as it is made larger and larger and regardless of the size of the initial square with which the process is launched: form is independent of growth. The journalist William Hoffer has remarked, "The great golden spiral seems to be nature's way of building quantity without sacrificing quality."[2]

Some people believe that the Fibonacci numbers can be used to make a wide variety of predictions, especially predictions about the stock market; such predictions work just often enough to keep the enthusiasm going. The Fibonacci sequence is so fascinating that there is even an American Fibonacci Association, located at Santa Clara University in California, which has published thousands of pages of research on the subject since 1962.

Fibonacci's *Liber Abaci* was a spectacular first step in making measurement the key factor in the taming of risk. But society was not yet prepared to attach numbers to risk. In Fibonacci's day, most people still thought that risk stemmed from the capriciousness of nature. People would have to learn to recognize man-made risks and acquire the courage to do battle with the fates before they would accept the techniques of taming risk. That acceptance was still at least two hundred years in the future.

We can appreciate the full measure of Fibonacci's achievement only by looking back to the era before he explained how to tell the difference between 10 and 100. Yet even there we shall discover some remarkable innovators.

Primitive people like the Neanderthals knew how to tally, but they had few things that required tallying. They marked the passage of days on a stone or a log and kept track of the number of animals they killed. The sun kept time for them, and five minutes or a half-hour either way hardly mattered.

The first systematic efforts to measure and count were undertaken some ten thousand years before the birth of Christ.[3] It was then that humans settled down to grow food in the valleys washed by such great rivers as the Tigris and the Euphrates, the Nile, the Indus, the Yangtse, the Mississippi, and the Amazon. The rivers soon became highways for trade and travel, eventually leading the more venturesome people to the oceans and seas into which the rivers emptied. To travelers ranging over longer and longer distances, calendar time, navigation, and geography mattered a great deal and these factors required ever more precise computations.

Priests were the first astronomers, and from astronomy came mathematics. When people recognized that nicks on stones and sticks no longer sufficed, they began to group numbers into tens or twenties, which were easy to count on fingers and toes.

Although the Egyptians became experts in astronomy and in predicting the times when the Nile would flood or withdraw, managing or influencing the future probably never entered their minds. Change was not part of their mental processes, which were dominated by habit, seasonality, and respect for the past.

About 450 BC, the Greeks devised an alphabetic numbering system that used the 24 letters of the Greek alphabet and three letters that subsequently became obsolete. Each number from 1 to 9 had its own letter, and the multiples of ten each had a letter. For example, the symbol "pi" comes from the first letter of the Greek word "penta," which represented 5; delta, the first letter of "deca," the word for 10, represented 10; alpha, the first letter of the alphabet, represented 1, and rho represented 100. Thus, 115 was written rho–deca–penta, or $\rho\delta\pi$. The Hebrews, although Bemitic rather than Indo-European, used the same kind of cipher-alphabet system.[4]

Handy as these letter–numbers were in helping people to build stronger structures, travel longer distances, and keep more accurate time, the system had serious limitations. You could use letters only with great difficulty—and almost never in your head—for adding or subtracting or multiplying or dividing. These substitutes for numbers provided nothing more than a means of recording the *results* of calculations performed by other methods, most often on a counting frame or abacus. The abacus—the oldest counting device in history—ruled the world of mathematics until the Hindu-Arabic numbering system arrived on the scene between about 1000 and 1200 AD.

The abacus works by specifying an upper limit for the number of counters in each column; in adding, as the furthest right column fills up, the excess counters move one column to the left, and so on. Our concepts of "borrow one" or "carry over three" date back to the abacus.[5]

Despite the limitations of these early forms of mathematics, they made possible great advances in knowledge, particularly in geometry—the language of shape—and its many applications in astronomy, navigation, and mechanics. Here the most impressive advances were made by the Greeks and by their colleagues in Alexandria. Only the Bible has appeared in more editions and printings than Euclid's most famous book, *Elements*.

Still, the greatest contribution of the Greeks was not in scientific innovation. After all, the temple priests of Egypt and Babylonia had learned a good bit about geometry long before Euclid came along. Even the famous theorem of Pythagoras—the square of the hypotenuse of a right triangle is equal to the sum of the square of the other two sides—was in use in the Tigris-Euphrates valley as early as 2000 BC.

The unique quality of the Greek spirit was the insistence on *proof.* "Why?" mattered more to them than "What?" The Greeks were able to reframe the ultimate questions because theirs was the first civilization in history to be free of the intellectual straitjacket imposed by an all-powerful priesthood. This same set of attitudes led the Greeks to become the world's first tourists and colonizers as they made the Mediterranean basin their private preserve.

More worldly as a consequence, the Greeks refused to accept at face value the rules of thumb that older societies passed on to them. They were not interested in samples; their goal was to find concepts

that would apply everywhere, in every case. For example, mere measurement would confirm that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. But the Greeks asked why that should be so, in all right triangles, great and small, without a single exception to the rule. Proof is what Euclidean geometry is all about. And proof, not calculation, would dominate the theory of mathematics forever after.

This radical break with the analytical methodologies of other civilizations makes us wonder again why it was that the Greeks failed to discover the laws of probability, and calculus, and even simple algebra. Perhaps, despite all they achieved, it was because they had to depend on a clumsy numbering system based on their alphabet. The Romans suffered from the same handicap. As simple a number as 9 required two letters: IX. The Romans could not write 32 as III II, because people would have no way of knowing whether it meant 32, 302, 3020, or some larger combination of 3, 2, and 0. Calculations based on such a system were impossible.

But the discovery of a superior numbering system would not occur until about 500 AD, when the Hindus developed the numbering system we use today. Who contrived this miraculous invention, and what circumstances led to its spread throughout the Indian subcontinent, remain mysteries. The Arabs encountered the new numbers for the first time some ninety years after Mohammed established Islam as a proselytizing religion in 622 and his followers, united into a powerful nation, swept into India and beyond.

The new system of numbering had a galvanizing effect on intellectual activity in lands to the west. Baghdad, already a great center of learning, emerged as a hub of mathematical research and activity, and the Caliph retained Jewish scholars to translate the works of such pioneers of mathematics as Ptolemy and Euclid. The major works of mathematics were soon circulating throughout the Arab empire and by the ninth and tenth centuries were in use as far west as Spain.

Actually, one westerner had suggested a numbering system at least two centuries earlier than the Hindus. About 250 AD, an Alexandrian mathematician named Diophantus wrote a treatise setting forth the advantages of a system of true numbers to replace letters substituting for numbers.[6]

Not much is known about Diophantus, but the little we do know is amusing. According to Herbert Warren Turnbull, a historian of mathematics, a Greek epigram about Diophantus states that "his boyhood lasted 1/6th of his life; his beard grew after 1/12th more; he married after 1/7th more, and his son was born five years later; the son lived to half his father's age, and the father died four years after his son." How old was Diophantus when he died?[7] Algebra enthusiasts will find the answer at the end of this chapter.

Diophantus carried the idea of symbolic algebra—the use of symbols to stand for numbers—a long way, but he could not quite make it all the way. He comments on "the impossible solution of the absurd equation $4 = 4x + 20$."[8] Impossible? Absurd? The equation requires x to be a negative number: -4. Without the concept of zero, which Diophantus lacked, a negative number is a logical impossibility.

Diophantus's remarkable innovations seem to have been ignored. Almost a millennium and a half passed before anyone took note of his work. At last his achievements received their due: his treatise played a central role in the flowering of algebra in the seventeenth century. The algebraic equations we are all familiar with today—equations like $a + bx = c$—are known as Diophantine equations.

The centerpiece of the Hindu-Arabic system was the invention of zero—*sunya* as the Indians called it, and *cifr* as it became in Arabic.[9] The term has come down to us as "cipher," which means empty and refers to the empty column in the abacus or counting frame.*

The concept of zero was difficult to grasp for people who had used counting only to keep track of the number of animals killed or the number of days passed or the number of units traveled. Zero had nothing to do with what counting was for in that sense. As the twentieth-century English philosopher Alfred North Whitehead put it,

*The Arabic term survives even in Russian, where it appears as *tsifra*, which is the word for number.

> The point about zero is that we do not need to use it in the operations of daily life. No one goes out to buy zero fish. It is in a way the most civilized of all the cardinals, and its use is only forced on us by the needs of cultivated modes of thought.[10]

Whitehead's phrase "cultivated modes of thought" suggests that the concept of zero unleashed something more profound than just an enhanced method of counting and calculating. As Diophantus had sensed, a proper numbering system would enable mathematics to develop into a science of the abstract as well as a technique for measurement. Zero blew out the limits to ideas and to progress.

Zero revolutionized the old numbering system in two ways. First, it meant that people could use only ten digits, from zero to nine, to perform every conceivable calculation and to write any conceivable number. Second, it meant that a sequence of numbers like 1, 10, 100 would indicate that the next number in the sequence would be 1000. Zero makes the whole structure of the numbering system immediately visible and clear. Try that with the Roman numerals I, X, and C, or with V, L, and D—what is the next number in those sequences?

The earliest known work in Arabic arithmetic was written by al-Khowârizmî, a mathematician who lived around 825, some four hundred years before Fibonacci.[11] Although few beneficiaries of his work are likely to have heard of him, most of us know of him indirectly. Try saying "al-Khowârizmî" fast. That's where we get the word "algorithm," which means rules for computing.[12] It was al-Khowârizmî who was the first mathematician to establish rules for adding, subtracting, multiplying, and dividing with the new Hindu numerals. In another treatise, *Hisâb al-jabr w'almuqâbalah,* or "Science of transposition and cancellation," he specifies the process for manipulating algebraic equations. The word *al-jabr* thus gives us our word *algebra,* the science of equations.[13]

One of the most important, surely the most famous, early mathematician was Omar Khayyam, who lived from about 1050 to about 1130 and was the author of the collection of poems known as the *Rubaiyat.*[14] His haunting sequence of 75 four-line poems (the word

Rubaiyat defines the poetic form) was translated in Victorian times by the English poet Edward Fitzgerald. This slim volume has more to do with the delights of drinking wine and taking advantage of the transitory nature of life than with science or mathematics. Indeed, in number XXVII, Omar Khayyam writes:

> Myself when young did eagerly frequent
> Doctor and Saint, and heard great Argument
> About it and about; but evermore
> Came out by the same Door as in I went.

According to Fitzgerald, Omar Khayyam was educated along with two friends, both as bright as he: Nizam al Mulk and Hasan al Sabbah. One day Hasan proposed that, since at least one of the three would attain wealth and power, they should vow that "to whomsoever this fortune falls, he shall share it equally with the rest, and preserve no pre-eminence for himself." They all took the oath, and in time Nizam became vizier to the sultan. His two friends sought him out and claimed their due, which he granted as promised.

Hasan demanded and received a place in the government, but, dissatisfied with his advancement, left to become head of a sect of fanatics who spread terror throughout the Mohammedan world. Many years later, Hasan would end up assassinating his old friend Nizam.

Omar Khayyam asked for neither title nor office. "The greatest boon you can confer on me," he said to Nizam, "is to let me live in a corner under the shadow of your fortune, to spread wide the advantages of science and pray for your long life and prosperity." Although the sultan loved Omar Khayyam and showered favors on him, "Omar's epicurean audacity of thought and speech caused him to be regarded askance in his own time and country."

Omar Khayyam used the new numbering system to develop a language of calculation that went beyond the efforts of al-Khowârizmî and served as a basis for the more complicated language of algebra. In addition, Omar Khayyam used technical mathematical observations to reform the calendar and to devise a triangular rearrangement of numbers that facilitated the figuring of squares, cubes, and higher powers of mathematics; this triangle formed the basis of concepts developed by the seventeenth-century French mathematician Blaise Pascal, one of the fathers of the theory of choice, chance, and probability.

The impressive achievements of the Arabs suggest once again that an idea can go so far and still stop short of a logical conclusion. Why, given their advanced mathematical ideas, did the Arabs not proceed to probability theory and risk management? The answer, I believe, has to do with their view of life. Who determines our future: the fates, the gods, or ourselves? The idea of risk management emerges only when people believe that they are to some degree free agents. Like the Greeks and the early Christians, the fatalistic Muslims were not yet ready to take the leap.

By the year 1000, the new numbering system was being popularized by Moorish universities in Spain and elsewhere and by the Saracens in Sicily. A Sicilian coin, issued by the Normans and dated "1134 Annoy Domini," is the first known example of the system in actual use. Still, the new numbers were not widely used until the thirteenth century.

Despite Emperor Frederick's patronage of Fibonacci's book and the book's widespread distribution across Europe, introduction of the Hindu-Arabic numbering system provoked intense and bitter resistance up to the early 1500s. Here, for once, we can explain the delay. Two factors were at work.

Part of the resistance stemmed from the inertial forces that oppose any change in matters hallowed by centuries of use. Learning radically new methods never finds an easy welcome.

The second factor was based on more solid ground: it was easier to commit fraud with the new numbers than with the old. Turning a 0 into a 6 or a 9 was temptingly easy, and a 1 could be readily converted into a 4, 6, 7, or 9 (one reason Europeans write 7 as 7̶). Although the new numbers had gained their first foothold in Italy, where education levels were high, Florence issued an edict in 1229 that forbade bankers from using the "infidel" symbols. As a result, many people who wanted to learn the new system had to disguise themselves as Moslems in order to do so.[15]

The invention of printing with movable type in the middle of the fifteenth century was the catalyst that finally overcame opposition to the full use of the new numbers. Now the fraudulent alterations were no longer possible. Now the ridiculous complications of using Roman numerals became clear to everyone. The breakthrough gave a great lift to commercial transactions. Now al-Khowârizmî's multiplication tables became something that all school children have had to learn forever after. Finally, with the first inklings of the laws of probability, gambling took on a whole new dimension.

The algebraic solution to the epigram about Diophantus is as follows. If x was his age when he died, then:

$$x = \frac{x}{6} + \frac{x}{12} + \frac{x}{7} + 5 + \frac{x}{2} + 4.$$

Diophantus lived to be 84 years old.

1200–1700: A THOUSAND OUTSTANDING FACTS