20EE104 - Basic Electrical Engineering

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Contents

UNIT-I

1.1 DC Circuits

1.1.1 Charge (Q)

A **coulomb** (C) of charge is defined as the total charge associated with $6.242X10^{18}$ electrons.

1.1.2 Current (I)

Current is the rate of flow of electric charge in a circuit. Its unit of measurement is ampere (A)

If 1C of charge ($6.242X10^{18}$ electrons) drift at uniform velocity through the imaginary circular cross section of a conductor as shown in figure 1.1 in 1 second, the flow of charge, or current, is said to be 1 ampere (A). The current (I) in amperes can be

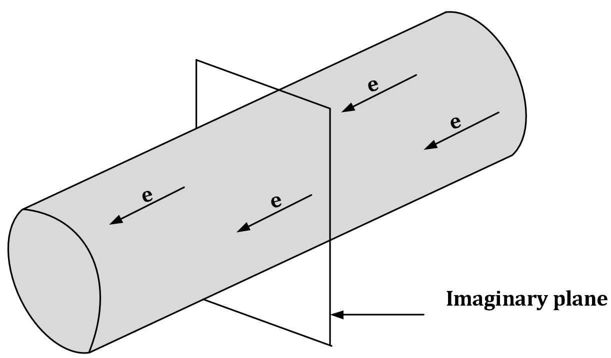


Figure 1.1: Defining the unit of measurement for current

calculated as:

$$I = \frac{Q}{t} \tag{1.1}$$

Where Q =charge in coulomb, t = time in sec

1.1.3 Electric Potential (V)

The electrical potential at any point in a charged conductor is defined as the energy exchanged to bring a unit charge from point of reference (generally infinity) to that point. Its unit of measurement is volts.

The potential difference (p.d) between any two points of a charged conductor is the energy exchanged in moving a unit charge between two points. A potential difference of 1 volt (V) exists between two points if 1 joule (J) of energy is exchanged in moving 1 coulomb (C) of charge between the two points.

Pictorially, if one joule of energy (1J) is required to move one coulomb (1C) of charge of figure 1.2 from position x to y, the potential difference or voltage between the two points is one volt (1V). In general, the potential difference (V) between two

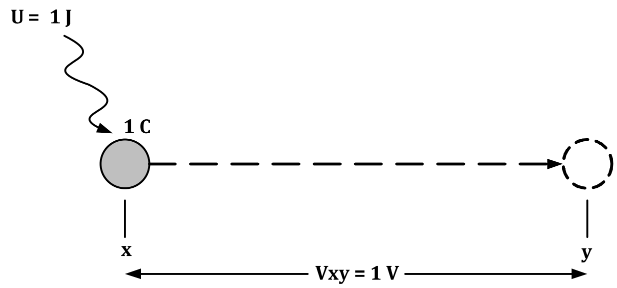


Figure 1.2: Defining the unit of measurement for voltage

points is determined by

$$V = \frac{U}{Q} \tag{1.2}$$

Where U = energy exchanged in joules, Q = charge in coulomb

1.1.4 Resistance (R)

The flow of charge through any material encounters an opposition. This opposition, due to the collisions between electrons and between electrons and other atoms in the material, *which converts electrical energy into another form of energy such as heat, is called the **resistance** of the material.* Unit of measurement of resistance is ohm, for which the symbol is Ω, the capital Greek letter omega. The circuit symbol for resistance appears in figure 1.3. The resistance of any material with a uniform

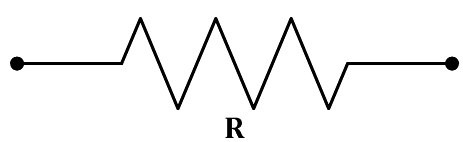


Figure 1.3: Resistance symbol and notation

cross-sectional area is determined by following factors:

1. Material
2. Length
3. Cross-sectional area

4. Temperature

At a fixed temperature of 20^oC (room temperature), the resistance is related to the other three factors by

$$R\alpha\frac{l}{A} \tag{1.3}$$

$$R = \frac{\rho l}{A} \tag{1.4}$$

Where ρ (Greek letter rho) is a characteristic of the material called the resistivity, l is the length of the sample, and A is the cross-sectional area of the sample.

1.1.5 Capacitance (C)

A capacitor is a device which can store electric charge for short periods of time. Like resistors, capacitors can be connected in series and in parallel.
The property of a capacitor to store an electric charge when its plates are at different potentials is referred to as its capacitance.
The unit of capacitance is termed the farad (abbreviation F) which may be defined as the capacitance of a capacitor between the plates of which there appears a potential difference of 1 volt when it is charged by 1 coulomb of electricity.
The circuit symbol for capacitance appears in figure 1.4. The capacitance of the ca-

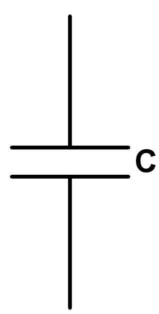


Figure 1.4: Capacitance symbol and notation

pacitor is given by

$$Capacitance = \frac{Charge}{Applied p.d.} \tag{1.5}$$

$$C = \frac{Q}{V} \tag{1.6}$$

In practice, the farad is found to be inconveniently large and the capacitance is usually expressed in *microfarads*(μF) or or in *picofarads* (pF) where
$1\mu F = 10^{-6}F$ and $1pF = 10^{-12}$

1.1.6 Electromotive force (E)

An electromotive force is that which tends to produce an electric current in a circuit, and the unit of measurement of e.m.f. is the volt.

1.1.7 Ohms law

Ohms Law states that the current I flowing in a conductor (circuit) is directly proportional to the applied voltage V provided the temperature remains constant.

$$I \alpha V \tag{1.7}$$

$$I = \frac{V}{R} \tag{1.8}$$

where $\frac{1}{R}$ is constant of proportionality

1.1.8 Kirchhoff's Laws

Kirchhoff's Current Law

At any instant, the algebraic sum of the currents at a junction in a network is zero. In other words, **the algebraic sum of the currents entering and leaving an area, system, or junction is zero.** The Currents flowing towards the junction have been considered positive and those flowing away from the junction are negative.

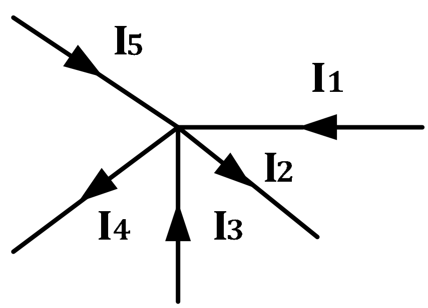


Figure 1.5: Kirchhoff's Current Law

From the figure 1.5,

$$I_1 - I_2 + I_3 - I_4 + I_5 = 0 \tag{1.9}$$

Kirchhoff's Voltage Law

At any instant in a closed loop, the algebraic sum of the e.m.f.s acting round the loop is equal to the algebraic sum of the potential drops round the loop. In other words, **the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.**

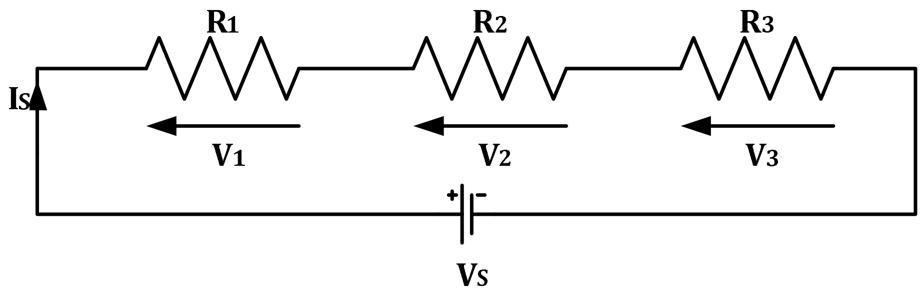


Figure 1.6: Kirchhoff's Voltage Law

From the figure 1.6,

$$V_s - V_1 - V_2 - V_3 = 0$$
$$V_s = V_1 + V_2 + V_3 \tag{1.10}$$

Note:
The polarity of a voltage source is unaffected by the direction of assigned loop currents.
Case a. If assumed currents direction is from ve terminal to +ve terminal in a voltage source, then it is taken as voltage rise as ssown in figure 1.7a
Case b. If assumed currents direction is from +ve terminal to -ve terminal in a voltage source, then it is taken as voltage drop as shown in figure 1.7b. Voltage always drops in a resistor, the polarity of the voltage drop across the resistor depends upon the direction of current. Therefore, polarity of the current entering terminal is marked as +ve and the polarity of the current leaving terminal is marked as ve.
Case c. Assumed current is entering the upper terminal of the resistor and leaving the bottom terminal, therefore, voltage drop polarity is shown in figure 1.7c.
Case d. Assumed current is entering the lower terminal of the resistor and leaving the upper terminal, therefore, voltage drop polarity is shown in figure 1.7d.

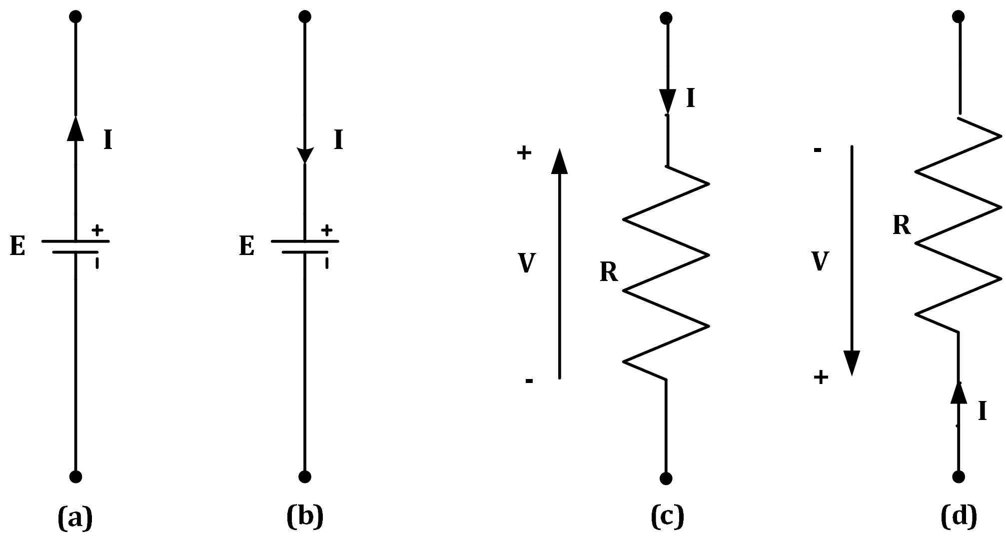


Figure 1.7: Polarity of voltage source and polarity v.d. across resistor

1.2 Numerical Problems

1. The charge flowing through the imaginary surface of figure 1.1 is 0.16 C every 64 ms. Determine the current in amperes.
   Solution:
   $$I = \frac{Q}{t} = \frac{0.16}{64X10^{-3}} = 2.5A$$
2. Find the potential difference between two points in an electrical system if 60J of energy is expended by a charge of 20C between these two points.
   Solution:
   $$V = \frac{U}{Q} = \frac{60}{20} = 3V$$
3. A capacitor having a capacitance of 80 μ F is connected across a 500 V d.c. supply. Calculate the charge.
   Solution:
   $$C = \frac{Q}{V} \quad \Rightarrow Q = CV = (80X10^{-6}) \times 500 = 0.04C = 40mC$$
4. Determine the current resulting from the application of a 9-V battery across a network with a resistance of 2.2Ω.
   Solution:
   $$I = \frac{V}{R} = \frac{9}{2.2} = 4.09A$$
5. Calculate the resistance of a $60W$ bulb if a current of $500mA$ results from an applied voltage of $120V$.

Solution:

$$I = \frac{V}{R} => R = \frac{V}{I} = \frac{120}{500X10^{-3}} = 240\Omega$$

6. For the network shown in figure 1.8 , determine the supply current and the source e.m.f.

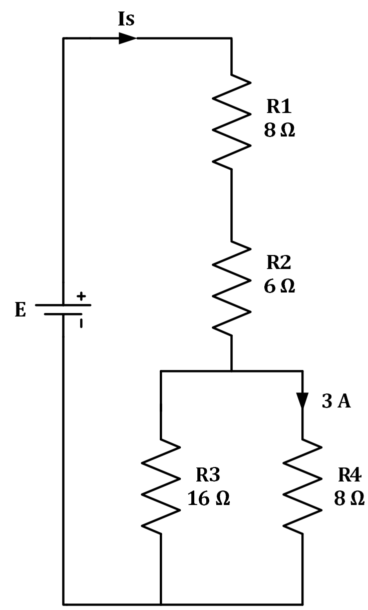


Figure 1.8: Circuit diagram for example **Ex 1**

Solution: Consider the circuit in the figure 1.9,

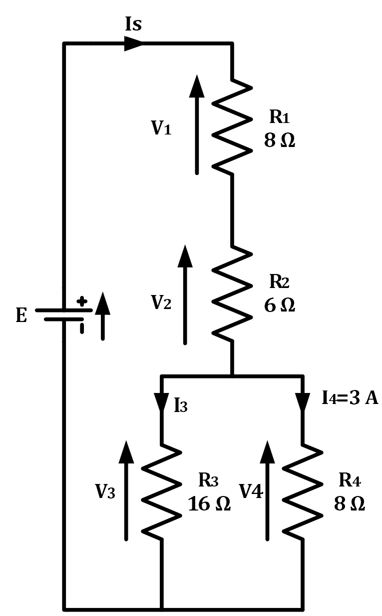


Figure 1.9

Given

$$I_4 = 3A$$
$$\therefore V_4 = R_4 I_4 = 8X3 = 24V$$

Since R_3 and R_4 are in parallel,

$$V_3 = V_4 = 24V$$
$$\therefore V_3 = R_3 I_3 \implies I_3 = \frac{V_3}{I_3} = \frac{24}{16} = 1.5A$$

By KCL,

$$I_s = I_3 + I_4 = 1.5 + 3 = 4.5A$$

Also,

$$V_1 = R_1 I_s = 8X4.5 = 36V \text{ and}$$
$$V_2 = R_2 I_s = 6X4.5 = 27V$$

By KVL,

$$E = V_1 + V_2 + V_3 = 36 + 27 + 24 = 87V$$

1.3 Series and Parallel Circuits

1.3.1 Series Circuits

Consider three resistors R_1, R_2 and R_3 connected in series as shown in figure 1.10. From Ohm's Law, $V_1 = I_s R_1, V_2 = I_s R_2$ and $V_3 = I_s R_3$, the current I_s being the same

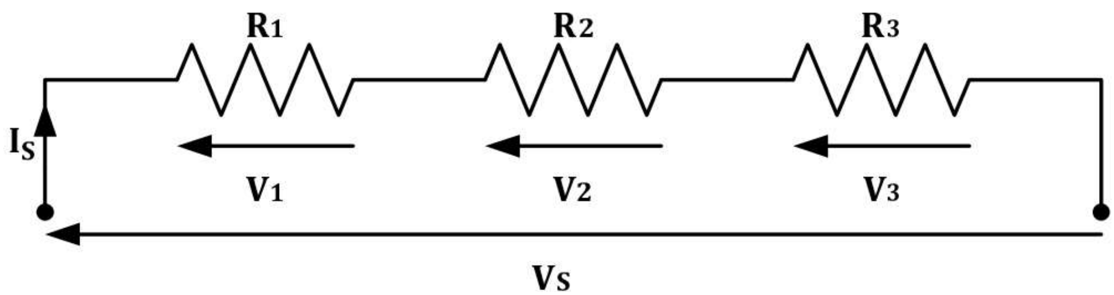


Figure 1.10: Resistors in series

in each resistor. Using KVL, the loop equation can be written as

$$\begin{aligned} V_s &= V_1 + V_2 + V_2 \\ &= I_s R_1 + I_s R_2 + I_s R_3 \\ &= I_s \left(R_1 + R_2 + R_3\right) \end{aligned} \tag{1.11}$$

Now consider a resistance R_{eq}, when connected across a supply of V_s volts draws a current of I_sA as shown in figure 1.11.

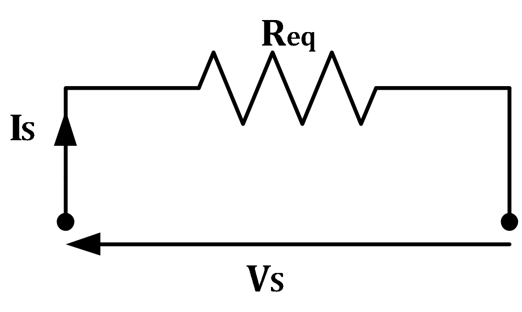


Figure 1.11: Equivalent circuit of series connected resistors

The voltage equation can be written as:

$$V_s = I_s R_{eq} \tag{1.12}$$

From (1.11) and (1.12), equivalent resistance R_{eq} can be written as:

$$R_{eq} = R_1 + R_2 + R_3 \tag{1.13}$$

1.3.2 Potential divider

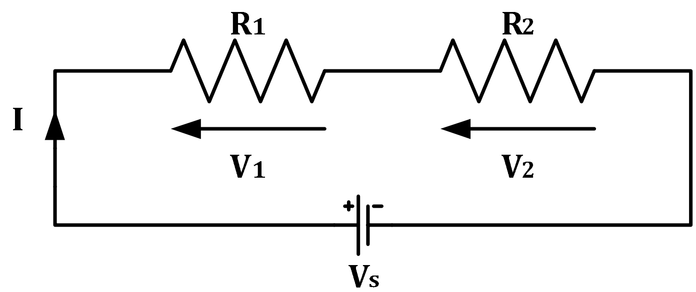


Figure 1.12: Potential divider circuit

Consider two resistors R_1 and R_2 connected in series as shown in figure 1.12. The total resistance of the circuit is given by

$$R_{eq} = R_1 + R_2 \tag{1.14}$$

and therefore the current in the circuit is

$$I_s = \frac{V_s}{R_{eq}} \tag{1.15}$$

The voltage drop across resistor R_1 is given by

$$\begin{aligned} V_1 &= R_1 I_s \\ &= R_1 \frac{V_s}{R_{eq}} \end{aligned}$$

$$V_1 = R_1 \frac{V_s}{R_1 + R_2} \tag{1.16}$$

Similarly, Voltage drop across resistor R_2 is given by

$$V_2 = R_2 \frac{V_s}{R_1 + R_2} \tag{1.17}$$

Therefore, the voltage distribution for the circuit shown in figure 1.12 is given by:

$$\begin{aligned} V_1 &= \left(\frac{R_1}{R_1 + R_2}\right) V_s \\ V_2 &= \left(\frac{R_2}{R_1 + R_2}\right) V_s \end{aligned}$$

1.3.3 Parallel Circuit

Resistors connected in parallel is shown in figure 1.13.

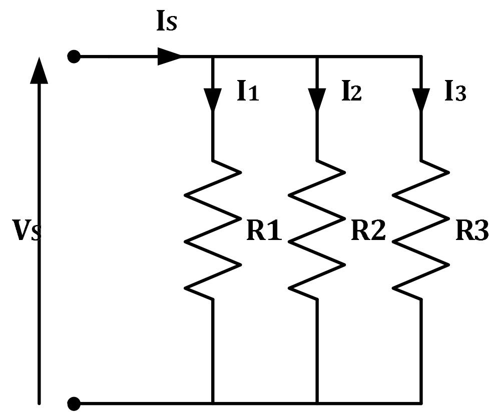


Figure 1.13: Resistors connected in parallel

Using KCL, source current I_s can be written as:

$$I_s = I_1 + I_2 + I_3 \tag{1.18}$$

Using Ohm's law, I_1, I_2 and I_3 can be expressed as:

$$I_1 = \frac{V_s}{R_1}, \quad I_2 = \frac{V_s}{R_2} \quad \text{and} \quad I_3 = \frac{V_s}{R_3} \tag{1.19}$$

Substituting (1.19) in (1.18) leads to:

$$I_s = \frac{V_s}{R_1} + \frac{V_s}{R_2} + \frac{V_s}{R_3} \tag{1.20}$$

$$= V_s \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \tag{1.21}$$

Now consider a resistance R_{eq}, when connected across a supply of V_s volts draws a current of I_sA as shown in figure 1.14. The source current I_s can be written as :

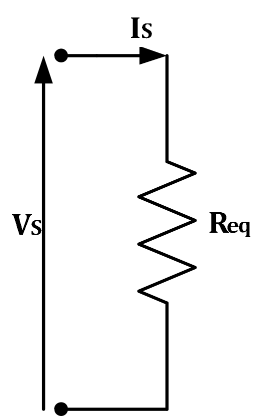


Figure 1.14: Equivalent circuit of resistors connected in parallel

$$I_s = \frac{V_s}{R_{eq}} \tag{1.22}$$

Comparing (1.20) and (1.22), equivalent resistance R_{eq} in parallel circuit can be written as :

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{1.23}$$

1.3.4 Current division

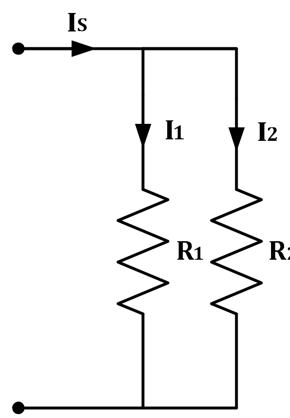


Figure 1.15: Current division in parallel circuit

For the circuit shown in figure 1.15, the equivalent resistance, $R_{eq}$is given by:

$$R_{eq} = \left(\frac{R_1 R_2}{R_1 + R_2}\right) \tag{1.24}$$

Equivalent circuit of resistors connected in parallel is as shown in 1.16 Using Ohm's

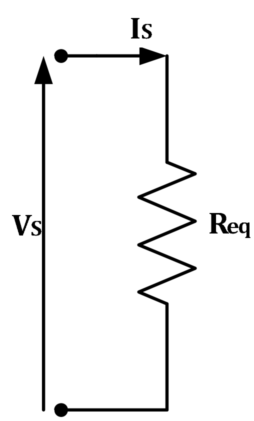


Figure 1.16: Equivalent circuit of resistors connected in parallel

law, V_s can be written as :

$$V_s = R_{eq} I_s \tag{1.25}$$

Using Ohm's law, Branch current I_1 can be expressed as:

$$I_1 = \frac{V_s}{R_1} \tag{1.26}$$

Substitute (1.25) in (1.26), we get

$$\begin{aligned} I_1 &= \frac{1}{R_1} R_{eq} I_s \\ &= \frac{1}{R_1}\left(\frac{R_1 R_2}{R_1 + R_2}\right) I_s \end{aligned} \tag{1.27}$$

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_s \tag{1.28}$$

Similarly, branch current I_2 is given by,

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_s \tag{1.29}$$

Branch currents I_1 and I_2 can be expressed as:

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_s$$
$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_s$$

1.4 Numerical Problems

1. For the circuit shown in figure 1.17, determine

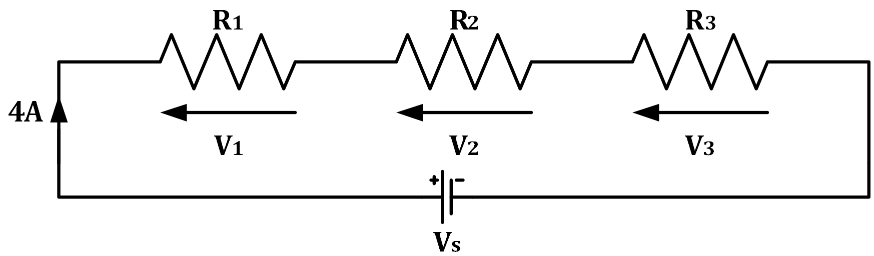


Figure 1.17: Circuit for the exercise 1

(a) Battery voltage V

(b) Total resistance of the circuit and

(c) Resistance of resistors R_1, R_2 and R_3 given that the potential difference across R_1, R_2 and R_3 is 5V, 2V and 6V respectively.

Solution:

From the figure 1.18

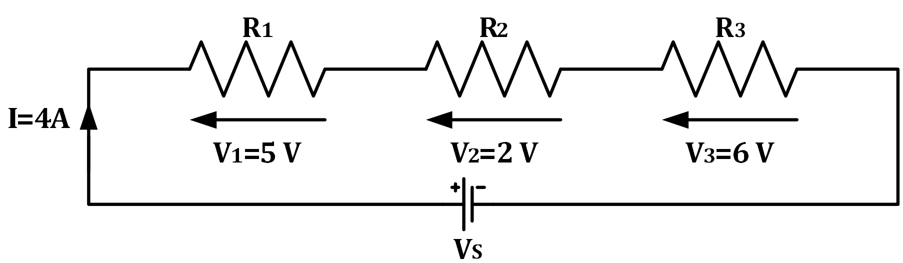


Figure 1.18: Circuit for the exercise 1

(a) Battery Voltage

$$V_s = V_1 + V_2 + V_3 = 5 + 2 + 6 = 13V$$

(b) Total Circuit Resistance =

$$\frac{V_s}{I} = \frac{13}{4} = 3.25\Omega$$

(c) Resistance

$$R_1 = \frac{V_1}{I} = \frac{5}{4} = 1.25\Omega$$
$$R_2 = \frac{V_2}{I} = \frac{2}{4} = 0.5\Omega$$
$$R_3 = \frac{V_3}{I} = \frac{6}{4} = 1.5\Omega$$

2. For the network shown in figure 1.19, calculate the effective resistance and compute the source current.

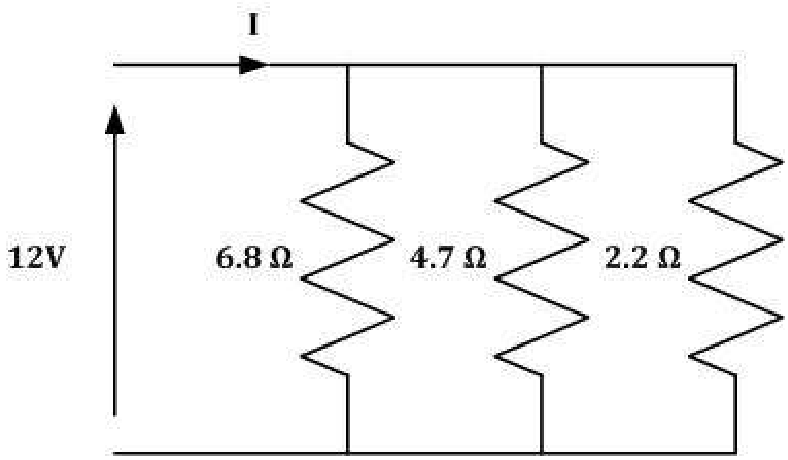


Figure 1.19: Circuit for the exercise 2

Solution:

Equivalent resistance in parallel circuit can be written as:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{1.30}$$
$$= \frac{1}{6.8} + \frac{1}{4.7} + \frac{1}{2.2} \tag{1.31}$$
$$= 0.815 \tag{1.32}$$

Therefore, $R_{eq} = \frac{1}{0.815} = 1.23\Omega$

Source current $I = \frac{V}{R_{eq}} = \frac{12}{1.23} = 9.76A$

3. A battery has an e.m.f. of 12V is connected across terminals AB of the circuit shown in figure 1.20, Find

(a) Equivalent resistance across terminals AB

(b) Current flowing in each resistance and

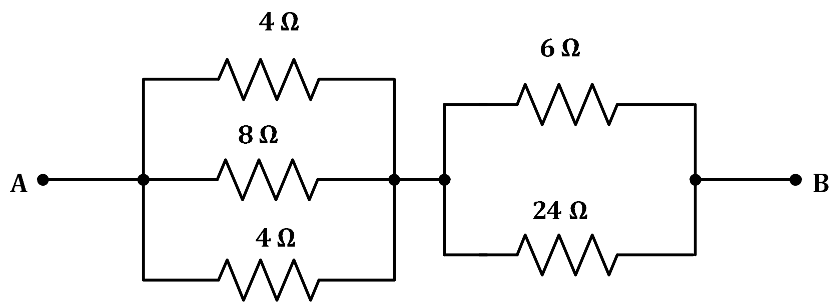


Figure 1.20: Circuit for the exercise 3

(c) Total power dissipated by the circuit

Solution

(a) Equivalent resistance across terminals AB

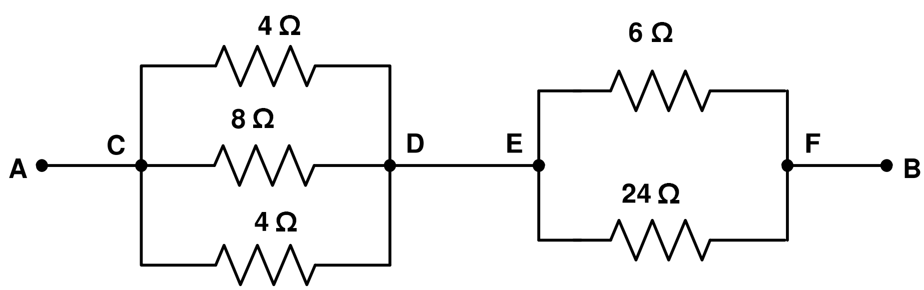


Figure 1.21: Circuit for the exercise 3 after naming every node

From the figure 1.21, equivalent resistance between the nodes C and D is:

$$\frac{1}{R_{CD}} = \frac{1}{4} + \frac{1}{8} + \frac{1}{4} = \frac{5}{8}$$
$$=> R_{CD} = \frac{8}{5} = 1.6\Omega$$

Resistance between the nodes E and F is computed as:

$$R_{EF} = \frac{6 * 24}{6 + 24} = 4.8\Omega$$

Using R_{CD} and R_{EF}, the equivalent circuit will be as in figure 1.22 Resis-

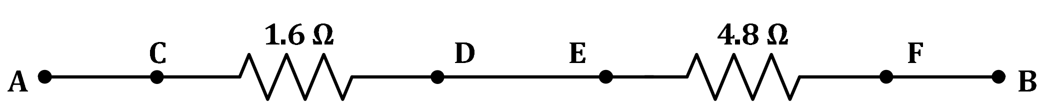


Figure 1.22: Equivalent circuit using R_{CD} and R_{EF}

tance between the terminals A and B is calculated as:

$$R_{AB} = R_{CD} + R_{EF} = 1.6 + 4.8 = 6.4\Omega$$

Equivalent circuit with resistance R_{AB} is shown in figure 1.23

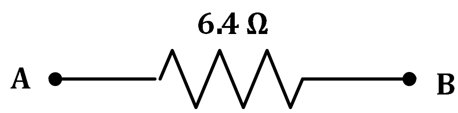


Figure 1.23: Equivalent circuit with R_{AB}

(b) Total circuit current can be computed from the figure 1.24 as:

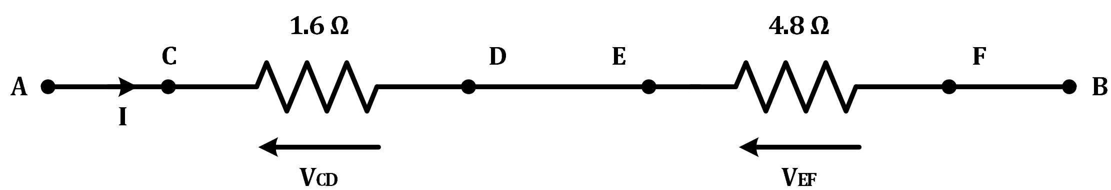


Figure 1.24: Equivalent circuit to compute circuit current

$$I = \frac{V}{R_{AB}} = \frac{12}{6.4} = 1.875A$$

Voltage between C and D, $V_{CD} = IR_{CD} = 1.875 * 1.6 = 3V$
Voltage between E and F, $V_{EF} = IR_{EF} = 1.875 * 4.8 = 9V$
Using the figure 1.25, current through each resistor can be computes as:

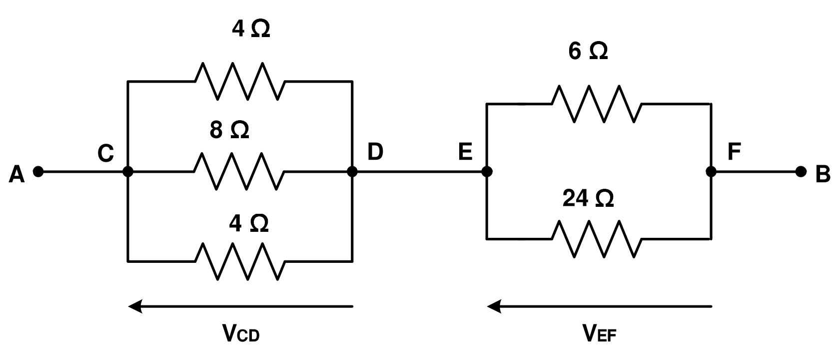


Figure 1.25: Reference circuit to compute current through each resistor

$$\text{Current through } 4\Omega \text{ resistors} = \frac{3}{4} = 0.75A$$
$$\text{Current through } 8\Omega \text{ resistors} = \frac{3}{8} = 0.375A$$

Current through 6Ω resistors $= \dfrac{9}{6} = 1.5A$

Current through 24Ω resistors $= \dfrac{9}{24} = 0.375A$

(c) Power dissipation in the circuit $= I^2R_{AB} = 1.875^2 * 6.4 = 22.5W$

1.5 Mesh and Nodal Analysis

1.5.1 Mesh analysis

The term mesh is derived from the similarities in appearance between the closed loops of a network and a wire mesh fence. Mesh analysis relies on Kirchhoff's laws, the technique proceeds as follows:

1. Circulating currents are allocated to closed loops or meshes in the circuit rather than to branches.

2. An equation for each loop of the circuit is then obtained by equating the algebraic sum of the e.m.f.s round that loop to the algebraic sum of the potential differences (in the direction of the loop, mesh or circulating current), as required by Kirchhoff's voltage law.

3. Branch currents are found thereafter by taking the algebraic sum of the loop currents common to individual branches.

Note: If a resistor has two or more assumed currents through it, the total current through the resistor is the assumed current of the loop in which Kirchhoff's voltage law is being applied, plus the assumed currents of the other loops passing through in the same direction, minus the assumed currents through in the opposite direction.

Example: Apply the mesh analysis method to the network of figure 1.26, find Power absorbed by 4Ω resistor and power supplied by $2V$ voltage source.

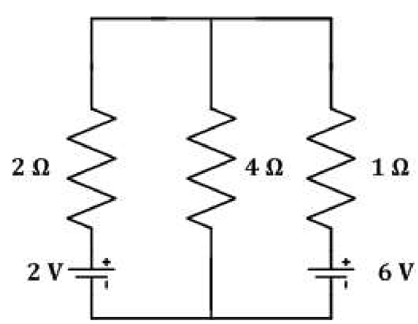


Figure 1.26: Circuit diagram for example

Solution:

Step 1: Two loop currents ($I1$ and $I2$) are assigned in the clockwise direction in the windows of the network shown in figure 1.27. A third loop ($I3$) could have been included around the entire network, but the information carried by this loop is already included in the other two.

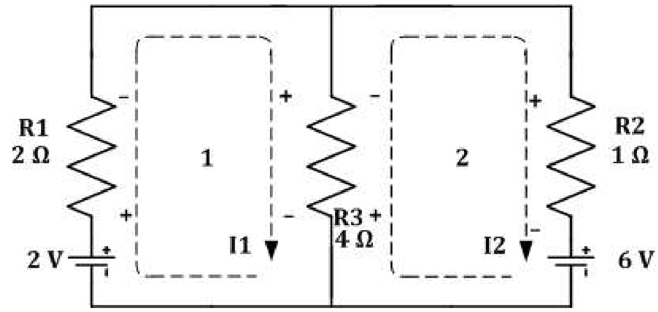


Figure 1.27: loop current directions

Step 2: Polarities are drawn within each window to agree with assumed current directions. Note that for this case, the polarities across the 4Ω resistor are the opposite for each loop current.

Step 3: Kirchhoff's voltage law is applied around each loop in the assumed loop currents direction. The voltage across each resistor is determined by $V = IR$, and for a resistor with more than one current through it, the current is the loop current of the loop being examined plus or minus the other loop currents as determined by their directions.

Loop-1

$$2 - 2(I_1) - 4(I_1 - I_2) = 0$$

Loop-2

$$-4(I_2 - I_1) - 1(I_2) - 6 = 0$$

Step 4: The equations are then rewritten as follows:

$$\text{Loop-1: } 2 - 2(I_1) - 4(-I_1) + 4(I_2) = 0 \implies 2 - 6I_1 + 4I_2 = 0$$
$$\text{Loop-2: } -4(I_2) + 4(I_1) - 1(I_2) - 6 = 0 \implies -5I_2 + 4I_1 - 6 = 0$$

Solving above two equations, results in,

$$I1 = -1A \text{ and}$$
$$I2 = -2A$$

The minus signs indicate that the currents have a direction opposite to that indicated by the assumed loop current. The actual current through the $2V$ source and 2Ω resistor is therefore $1A$ in the other direction, and the current through the $6V$ source and 1Ω resistor is $2A$ in the opposite direction indicated on the circuit.

The current through the 4Ω resistor is determined by the following equation from the original network:

$$I_{4\Omega} = I_1 - I_2 = -1 - (-2) = -1 + 2 = 1A(\downarrow)$$

Power absorbed by $4\Omega = I^2 X 4 = 4W$
Power supplied by 2V battery $= -1X2 = 2W$ (It is absorbing the power, i.e. it is acting as a load $6V$ source)

1.5.2 Nodal Analysis

This technique of circuit solution, also known as the Node Voltage method, is based on the application of Kirchhoff's first (current) law at each junction (node) of the circuit, to find the node voltages. A node is defined as a junction of two or more branches. If we now define one node of any network as a reference (that is, a point of zero potential or ground), the remaining nodes of the network will all have a fixed potential relative to this reference. For a network of N nodes, there exists $(N - 1)$ nodes with a fixed potential relative to the assigned reference node. Equations relating these nodal voltages can be written by applying Kirchhoff's current law at each of the $(N - 1)$ nodes. To obtain the complete solution of a network, these nodal voltages are then evaluated in the same manner in which loop currents were found in loop analysis. The nodal analysis method generally proceeds as follows:

1. Choose a reference node to which all node voltages can be referred. Label all the other nodes with (unknown) values of voltage, V_1, V_2, etc.

2. Assign currents in each connection to each node, except the reference node, in terms of the node voltages, V_1, V_2, etc.

3. Apply Kirchhoff's current law at each node, obtaining as many equations as there are unknown node voltages.

4. Solve the resulting equations to find the node voltages.

Note: For each node, dont be influenced by the direction that an unknown current for another node may have had. Each node is to be treated as a separate entity, independent of the application of Kirchhoff's current law to the other nodes.

1.5.3 Energy (U)

Energy is defined as the amount of work a physical system is capable of performing, that is, the capacity of the system to do work. Its unit of measurement is joules(J). The electrical energy is measured in watt-hour (Wh).
$1J = 1Ws => 1Wh = 3600J$

1.5.4 Power (P)

Power is an indication of how much work (the conversion of energy from one form to another) can be done in a specified amount of time, that is, a rate of doing work. The electrical unit of measurement for power is the watt(W), defined by 1watt(W)=1joule/second (J/s).

Power is determined by

$$P = \frac{U}{t} \tag{1.33}$$

The power delivered to, or absorbed by, an electrical device or system can be found in terms of the current and voltage by substituting (1.2) into (1.33):

$$P = \frac{U}{t} = \frac{QV}{t} = VI \tag{1.34}$$

$$\text{where } I = \frac{Q}{t}$$

Other forms can be written as:

$$P = VI = V\left(\frac{V}{R}\right) = \frac{V^2}{R} \tag{1.35}$$

$$P = VI = (IR)I = I^2R \tag{1.36}$$

Ex 5. Find the power delivered to the dc motor of figure 1.28

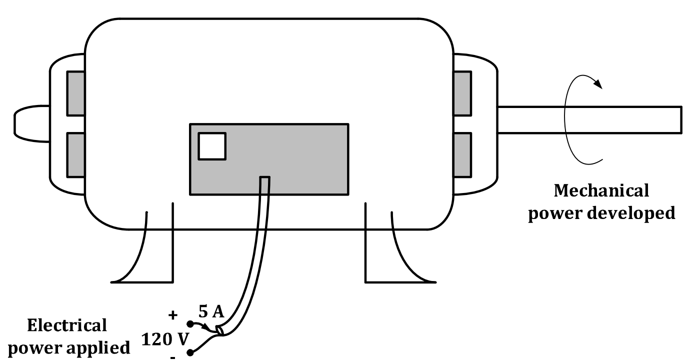


Figure 1.28: Exercise 5

Solution:

$$P = VI = 120 \times 5 = 600W$$

Ex 6. How much energy (in kilowatthours) is required to light a 60W bulb continuously for 1 year (365 days)?

Solution:

$$P = \frac{U}{t} => U = P \times t = 60 \times 365 \times 24 = 525,600Wh = 525.6kWh$$

1.6 Numerical Problems

1. Calculate the current in each branch of the network shown in figure 1.29

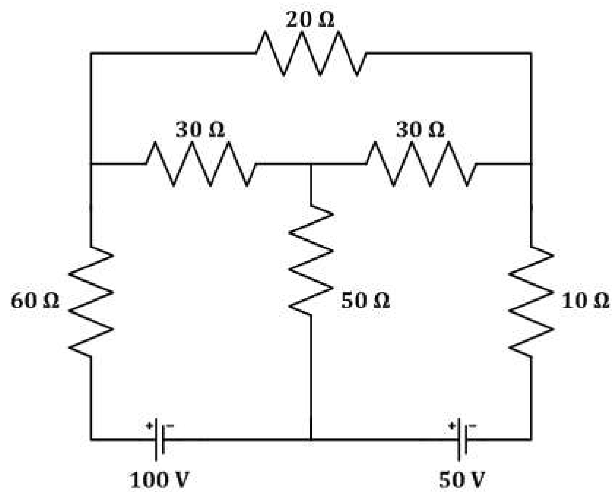


Figure 1.29: Circuit diagram for example

Solution: Assume the direction of currents as in figure 1.30

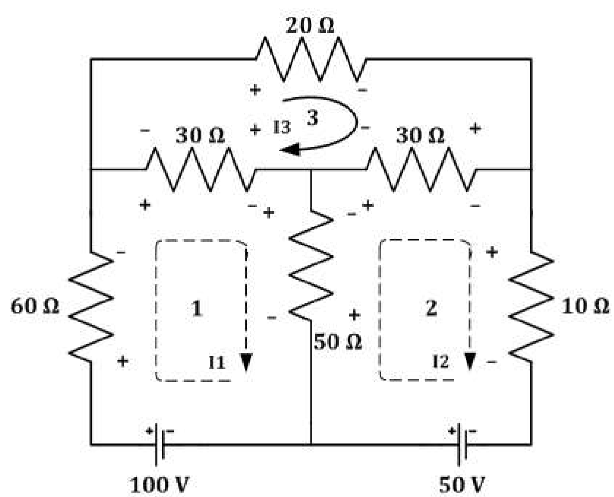


Figure 1.30: Circuit diagram for example

Loop-1:

$$100 = I_1 \times (60 + 30 + 50) - I_2 \times 50 - I_3 \times 30$$

$$\implies 100 = 140I_1 - 50I_2 - 30I_3$$

Loop-2:

$$
\begin{aligned}
50 &= I_2 \times (50 + 30 + 10) - I_1 \times 50 - I_3 \times 30 \\
\implies 50 &= -50I_1 + 90I_2 - 30I_3
\end{aligned}
$$

Loop-3:

$$
\begin{aligned}
0 &= I_3 \times (30 + 30 + 20) - I_1 \times 30 - I_2 \times (30) \\
\implies 0 &= -30I_1 - 30I_2 + 80I_3
\end{aligned}
$$

Solving Loop equations leads to

$$
\begin{aligned}
I_1 &= 1.7123A \\
I_2 &= 1.9667A \\
I_3 &= 1.3796A
\end{aligned}
$$

Current in $60\Omega = I_1 = 1.7123A(\uparrow)$ in direction of I_1
Current in $30\Omega = I_1 - I_3 = 0.3327A(\rightarrow)$ in direction of I_1
Current in $50\Omega = I_1 - I_2 = -0.2544A(\downarrow)$ in direction of I_1
Current in $40\Omega = I_3 - I_2 = -0.5871A(\leftarrow)$ in direction of I_3
Current in $10\Omega = I_2 = 1.9667A(\downarrow)$ in direction of I_2
Current in $20\Omega = I_3 = 1.379A(\rightarrow)$ in direction of I_3

2. Using Nodal analysis, calculate the voltages V_1 and V_2 in the circuit of figure 1.31.

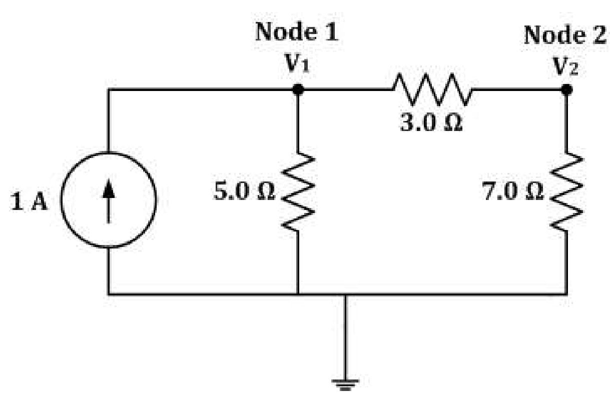


Figure 1.31: Circuit for the exercise 2

Solution: Refer to the four steps previously indicated:

(a) Reference node chosen is shown in figure 1.31. Voltages V_1 and V_2 assigned to the other two nodes.

(b) Assign currents in each connection to each node figure 1.32.

(c) Apply Kirchhoff's current law to sum the currents at each node.

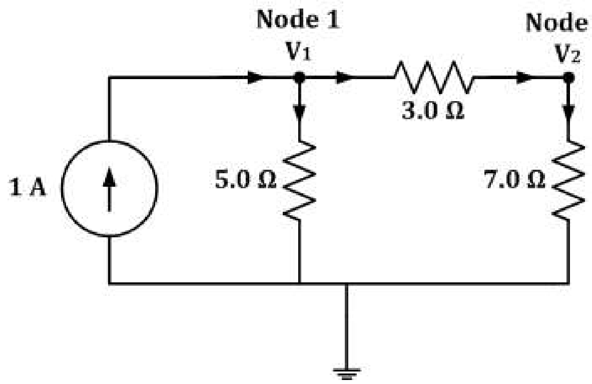


Figure 1.32: Circuit for the exercise 2

At node 1:

$$\frac{V_1}{5} + \left(\frac{V_1 - V_2}{3}\right) = 1$$
$$\Rightarrow V_1\left(\frac{1}{5} + \frac{1}{3}\right) - \frac{V_2}{3} = 1$$

Which simplifies to,

$$8V_1 - 5V_2 = 15 \tag{1.37}$$

At node 2:

$$\left(\frac{V_1 - V_2}{3}\right) = \frac{V_2}{7}$$
$$\Rightarrow \frac{V_1}{3} - V_2\left(\frac{1}{3} + \frac{1}{7}\right) = 0$$

Which simplifies to,

$$7V_1 - 10V_2 = 0 \tag{1.38}$$

(d) Solving (1.37) and (1.38), we get $V_1 = 3.3333V$ and $V_2 = 2.3333V$

3. Using the Nodal analysis method calculate the voltages V_1 and V_2 in figure 1.33. and hence calculate the currents in the 8Ω resistor.

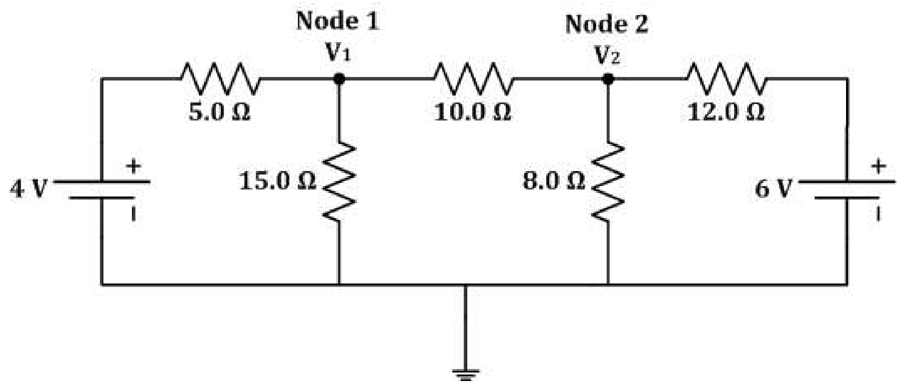


Figure 1.33: Circuit for the exercise 3

Solution:

(a) Reference node chosen is shown in figure 1.34. Voltages V_1 and V_2 assigned to the other two nodes.

(b) Assign currents in each connection to each node (figure 1.34).

(c) Apply Kirchhoff's current law to sum the currents at each node.

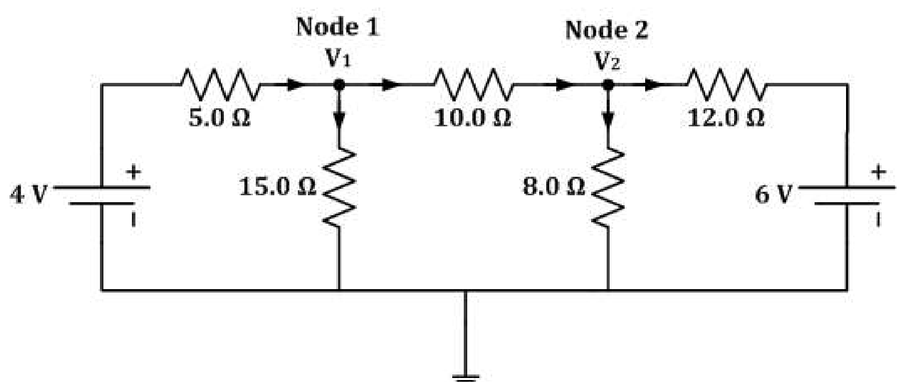


Figure 1.34: Circuit for the exercise 3

At node 1:

$$\frac{4-V_1}{5} = \frac{V_1-V_2}{10} + \frac{V_1}{15}$$
$$24-6V_1 = 3V_1-3V_2+2V_1$$

Taking L.C.M. and simplifying, results in

$$24-6V_1 = 3V_1-3V_2+2V_1$$

Which simplifies to,

$$11V_1 - 3V_2 = 24 \tag{1.39}$$

At node 2:

$$\frac{V_1 - V_2}{10} = \frac{V_2 - 6}{12} + \frac{V_2}{8}$$

Taking L.C.M. and simplifying, results in

$$12V_1 - 12V_2 = 10V_2 - 60 + 15V_2$$

Which simplifies to,

$$12V_1 - 37V_2 = -60 \tag{1.40}$$

(d) Solving 1.39 and 1.40, we get $V_1 = 2.88V$ and $V_2 = 2.55V$. Hence current in 8Ω resistor is, $I_{8\Omega} = \dfrac{V_2}{8} = 0.32A$. (From node 2 to reference node).

1.7 AC fundamentals

In previous chapter we have considered the circuits in which the current has remained constant. However, there remains another type of system the alternating system in which the magnitudes of the voltage and of the current vary in a repetitive manner. Almost every electrical supply to houses and to industry uses alternating current.It flows first in one direction and then in the other. Alternating current can be abbreviated to a.c, hence a system with such an alternating current is known as an a.c system.

1.7.1 Generation of an alternating e.m.f.

Figure 1.35 shows a loop AB carried by a spindle DD rotated at a constant speed in an anticlockwise direction in a uniform magnetic field due to poles NS. The ends of the loop are brought out to two slip-rings C1 and C2, attached to but insulated from DD. Bearing on these rings are carbon brushes E1 and E2, which are connected to an external resistor R.

When the plane of the loop is horizontal, as shown in Fig. 1.36(a), the two sides A

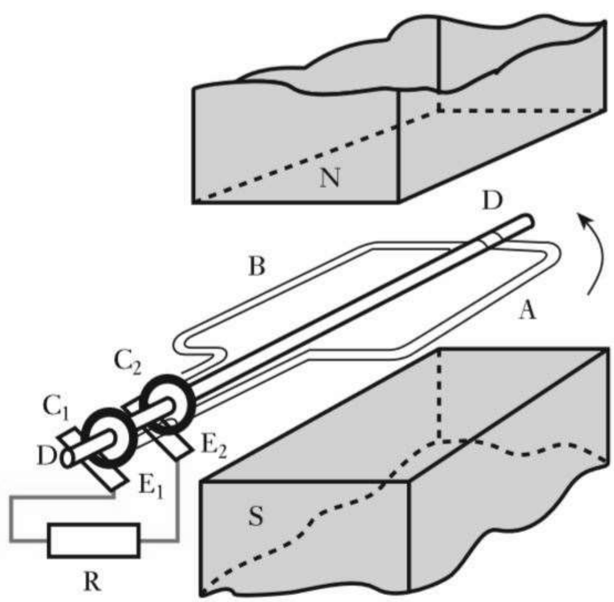


Figure 1.35: Generation of alternating e.m.f

and B are moving parallel to the direction of the magnetic flux; it follows that no flux is being cut and no e.m.f. is being generated in the loop. Subsequent diagrams in Fig. 1.36 show the effects which occur as the coil is rotated. In Fig. 1.36(b), the coil sides are cutting the flux and therefore an e.m.f. is induced in the coil sides. Since the coil sides are moving in opposite directions, the e.m.f.s act in opposite directions,

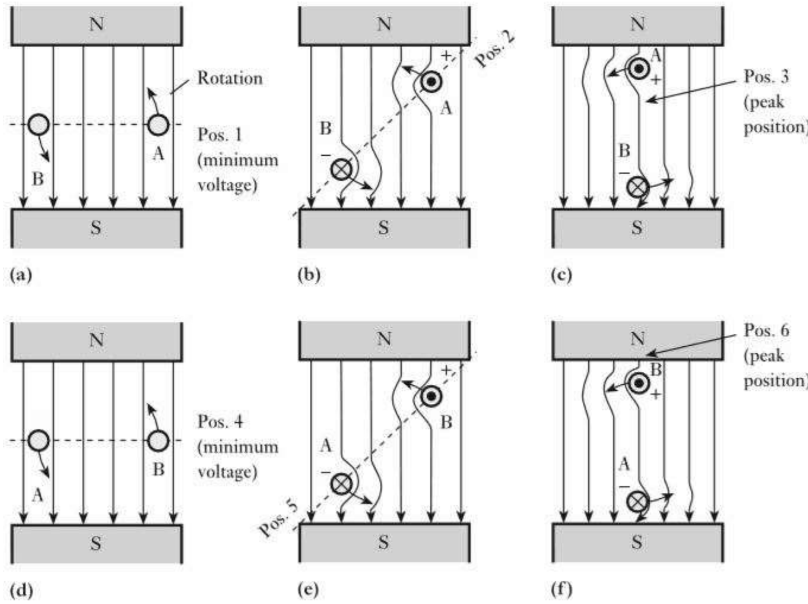


Figure 1.36: Generation of alternating e.m.f

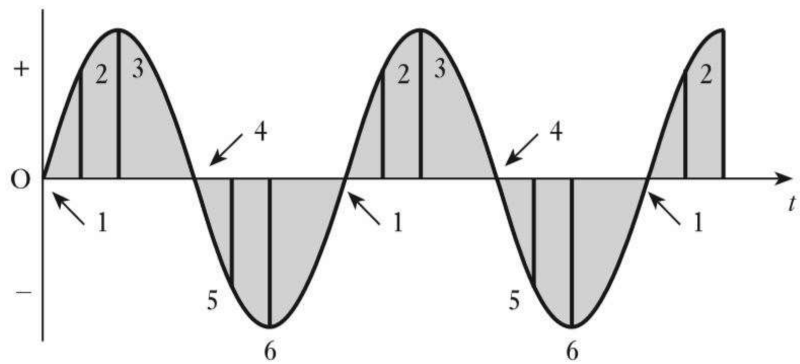


Figure 1.37: Generation of alternating e.m.f

as shown by the dot and cross notation. However, they do act in the same direction around the coil so that the e.m.f. which appears at the brushes is twice that which is induced in a coil side. Once the coil reaches the position shown in Fig. 1.36(c), the rate of cutting reaches a maximum. Thereafter the e.m.f. falls to zero by the time the coil has rotated to the position shown in Fig. 1.36(d).

The induced e.m.f. in the position shown in Fig. 1.36(e) is of particular interest. At first sight, it appears that the diagram is the same as that of Fig. 1.36(b), but in fact it is side A which bears the cross while side B has the dot. This means that the e.m.f. is of the same magnitude but of the opposite polarity. This observation also applies to Fig. 1.36(f). It follows that the variation of induced e.m.f. during the second half of the cycle of rotation is the same in magnitude as during the first half but the polarity of the e.m.f. has reversed.

It is significant that we have concentrated on one cycle of events arising from the single rotation of the coil AB shown in Fig. 1.35. However, alternating e.m.f.s and alternating voltages continue to repeat the cycle, the effect at each of the situations shown in Fig. 1.36 recurs in each subsequent cycle as shown in Fig 1.37.

1.7.2 Frequency of generated voltage

The waveform of the e.m.f. generated in an a.c. generator undergoes one complete cycle of variation when the conductors move past a N and a S pole; and the shape of the wave over the negative half is exactly the same as that over the positive half.
The generator shown in Fig. 1.35 has two poles which can also be described as having one pair of poles. Machines can have two or more pairs of poles. For example, if there were N poles placed top and bottom and S poles to either side then the machine would have two pairs of poles.
If an a.c. generator has P number of poles and if its speed is N revolutions per minutes, then

$$Frequency = f \tag{1.41}$$

$$= \text{No. of cycles per seconds} \tag{1.42}$$

$$= \text{No. of cycles per revolution X number revolutions per minute} \tag{1.43}$$

$$= \frac{P}{2} \times \frac{N}{60} \tag{1.44}$$

$$f = \frac{PN}{120}\ Hz \tag{1.45}$$

1.7.3 Basic concepts of an alternating quantity

Waveform:

The variation of a quantity such as voltage or current shown on a graph to a base of time or rotation is known as a waveform.

Cycle:

The complete set of positive and negative values of an alternating quantity is known as a cycle.

Time Period (T):

The time taken to complete one cycle is termed as time period.

Frequency (f):

The number of cycles that occur in one second is termed as the frequency of that quantity. It is measured in hertz (Hz). It follows that frequency f is related to the period T by the relation f=$1/T$

Instantaneous Value:

The value of an alternating quantity at any instant with reference to zero. Instantaneous values are denoted by lower case symbols such as e, v & i etc.

Peak Amplitude:

The maximum value obtained by an alternating quantity is termed as the amplitude. The amplitude is Em or V_m or I_m. The amplitude is generally described as the maximum value hence the maximum voltage has the symbol V_m.

Phase:

Phase of a particular value of an alternating quantity is the fractional part of time period or cycle through which the quantity has advanced from the selected zero position of reference. When two alternating quantities of the same frequency have different zero points, they are said to have phase difference.

Peak-to-peak value:

The maximum variation between the maximum positive instantaneous value and the maximum negative instantaneous value is the peak-to-peak value. For a sinusoidal waveform, this is twice the amplitude or peak value. The peak-to-peak value is E_{pp} or V_{pp} or I_{pp}.

1.8 Concept of Average and RMS values

1.8.1 Average Value (I_{av}):

This is defined on the basis of amount of charge transferred, which is given by q = I × t. The average value of an alternating current is equal to that (D.C) steady current, which transfers the same amount of charge as transferred by the alternating current across the same circuit and in the same time.
Or, alternatively, average value of current is

$$\frac{\text{Area enclosed over half-cycle}}{\text{Length of base over half-cycle}} \tag{1.46}$$

In the case of sinusoidal wave the average value is obtained by adding or integrating the instantaneous values of the quantity over one alternating or 1/2 cycles only.
Consider a sinusoidal varying alternating current i= I_m sinθ as shown in Fig 1.38 below:

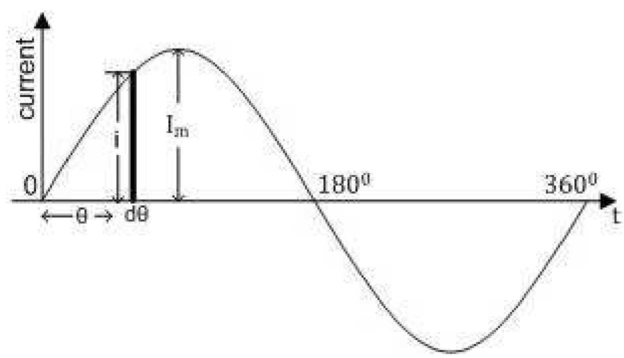


Figure 1.38: Average Value

For a very small interval dθ radians, the area of the shaded strip is i · dθ ampere radians. The use of the unit 'ampere radian 'avoids converting the scale on the horizontal axis from radians to seconds, therefore, total area enclosed by the current wave over half-cycle is

$$\int_0^{\pi} id\theta = \int_0^{\pi} I_m sin\theta = [-cos\theta]_0^{\pi} = 2I_m \text{ ampere radians}$$

From Eq. 1.46 , average value of current over a half cycle is $\frac{2I_m}{\pi} = 0.637I_m$
Thus the average value of an alternating quantity is 0.637 times its maximum value.
Note: The average value over full cycle of sinusoidal wave is zero.

1.8.2 Root Mean Square (RMS) value or Effective value: (I):

In a.c., the average value is of comparatively little importance. This is due to the fact that it is the power produced by the electric current that usually matters. Thus, if a sinusoidal varying alternating current i= I_m sinθ is passed through a resistor having resistance R ohms, the heating effect of i is i^2R.
Thus the RMS value of an alternating current is defined as that steady(D.C) current which when flowing through a given resistor for a given time produces the same amount of heat as produced by the alternating current when flowing through the same resistor for the same period of time.
The average heating effect can be expressed as follows:
Average heating effect over half-cycle is

$$\frac{\text{Area enclosed by } i^2R \text{ over half-cycle}}{\text{Length of base}} \tag{1.47}$$

This is a more convenient expression to use when deriving the r.m.s. value of a sinusoidal current. If the current is passed through a resistor having resistance R ohms, instantaneous heating effect = i^2 R watts. The variation of i^2R during a complete cycle is shown in Fig.1.40 below During interval dθ radians, heat generated

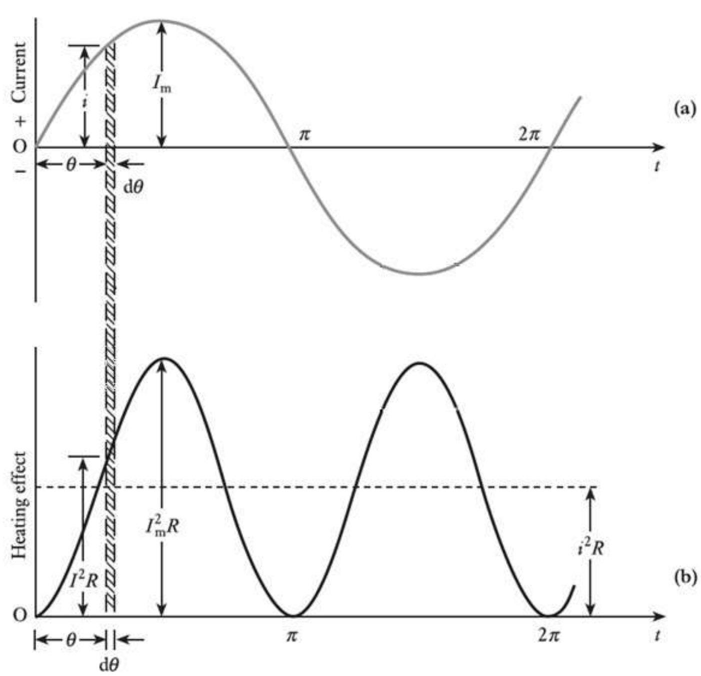


Figure 1.39: RMS Value

is i^2R $\cdot$ dθ watt radians and is represented by the area of the shaded strip. Hence heat generated during the first half-cycle is area enclosed by the i^2R curve and is

$$\begin{aligned}
\int_0^{\pi} i^2 R d\theta &= I_m^2 R \int_0^{\pi} sin^2\theta d\theta \\
&= I_m^2 R \int_0^{\pi} (\frac{1-cos2\theta}{2}) \\
&= I_m^2 R \left[\frac{\theta}{2} - \frac{sin2\theta}{4}\right]_0^{\pi} \\
&= \pi R \frac{I_m^2}{2}
\end{aligned}$$

From Eq. 1.47 average heating effect is

$$\frac{\pi R \frac{I_m^2}{2}}{\pi} = R\frac{I_m^2}{2}$$

If I is the value of direct current through the same resistance to produce the same heating effect

$$\begin{aligned}
I^2 R &= R\frac{I_m^2}{2} \\
I &= \frac{I_m}{\sqrt{2}} = 0.707 I_m
\end{aligned}$$

1.8.3 Form Factor

The ratio of RMS value to average value of an alternating quantity is called form factor. For sine wave current,

$$K_f = \frac{I}{I_{av}} = \frac{I_m/\sqrt{2}}{2I_m/\pi} = 1.11$$

1.8.4 Peak Factor

The ratio of maximum value to the RMS value of an alternating quantity is called peak factor For sine wave current,

$$K_p = \frac{I_m}{I_{rms}} = \frac{I_m}{I_m/\sqrt{2}} = 1.414$$

1.8.5 Power

True Power and Reactive Power:

Power is consumed only in resistance and no power is consumed in pure inductor or pure capacitor. The power received from L and C in one quarter cycle is returned to the source in the next quarter cycle. This circulating power is called reactive power and does no useful work in the circuit. The power which is actually consumed in the circuit is called the true power or active power. Therefore current in-phase with voltage produces true power whereas the current 90° out of phase with voltage contributes to reactive power.
True Power= Voltage × current in phase with voltage
Reactive Power= Voltage × current 90° out of phase with voltage

The current can be resolved into two components:

1. I cosϕ -in phase with voltage
2. I sinϕ -90° out of phase with voltage

∴ True Power,P = V I Cosϕ Watts or KW
Reactive Power,Q= VI Sinϕ VAR(Volt Amp Reactive) or KVAR
Apparent power, S= VI VA(Volt-Amps) or KVA
From Fig. 1.40,

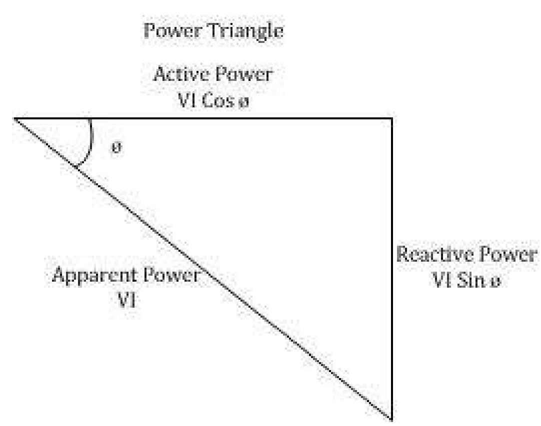


Figure 1.40: Power Triangle

$$S^2 = P^2 + Q^2$$
$$S = \sqrt{P^2 + Q^2}$$

1.8.6 Power Factor of a circuit

The p.f of a circuit can be defined in the following three ways:-

The power factor of a circuit is the cosine of the angle between the voltage and the current. P.f = cos ϕ .

The power factor of a circuit is the ratio of the resistance to the impedance of the circuit. P.f =R / Z.

The power factor of a circuit is the ratio of the real power to the apparent power. P.f = P / EI. The maximum value p.f is unity.

1.9 Phasor representation of an alternating quantity

Consider an alternating voltage as shown in the Fig. 1.41. Let OA represent the instantaneous value of this quantity rotating in the anti-clock wise direction about the point O at a uniform angular velocity ω. Fig.1.41 shows OA when it has rotated through an angle from the position occupied when the voltage was passing through its zero value.

If AB and AC are drawn perpendicular to the horizontal and vertical axes respectively:

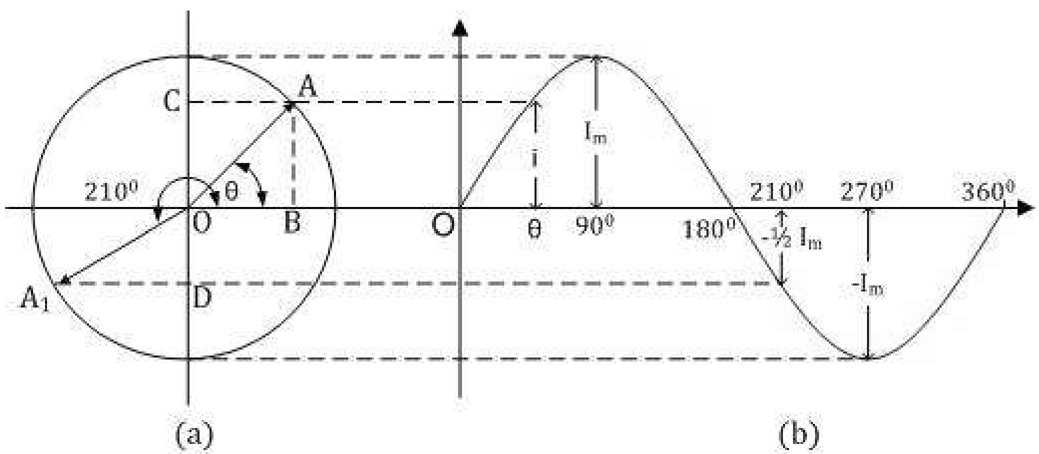


Figure 1.41: Representation of alternating voltage

$$\begin{aligned} OC = AB &= OAsin\theta \\ &= V_m sin\theta \\ &= V, \text{ namely the value of the voltage at that instant} \end{aligned}$$

Hence the projection of OA on the vertical axis represents to scale the instantaneous value of the voltage. Thus when θ=90°, the projection is OA itself; when θ=180°, the projection is zero and corresponds to the voltage passing through zero from a positive to a negative value; when θ=210°, the phasor is in position OA_1, and the projection = OD= (1/2) OA_1 = - 1/2V_m; And when θ=360°, the projection is again zero and corresponds to the voltage passing through zero from a negative to a positive value. It follows that OA rotates through one revolution or 2π radians in one cycle of the voltage wave.

If f is the frequency in hertz, then OA rotates through f revolutions of 2πf radians in 1s. Hence the angular velocity of OA is 2πf radians per second and is denoted by the symbol ω (omega), i.e.

$\omega = 2\pi$f radians per second

If the time taken by OA in Fig.1.41 to rotate through an angle θ radians is t seconds, then

$$\begin{aligned}\theta &= \text{angular velocity} \times \text{time}\\ &= \omega \times \text{t} = 2\pi ft \text{ radians}\end{aligned}$$

We can therefore express the instantaneous value of the voltage thus:

v = V_m sin θ = V_m sin ω t

Therefore v= V_m sin 2πf t.

1.10 Numerical Problems

1. An alternating voltage has the equation v =141.4 sin377t Volts, what are the values of (a) r.m.s voltage (b) frequency (c) the instantaneous voltage when t=3ms.

   Solution:

   Given v =141.4 sin377t

   Comparing this equation with v = V_m sinω t

   ω= angular velocity =2πf radians / second

   a) r.m.s voltage = V = 0.707 $\times$ V_m

   V_m= 141.4V

   V= 0.707$\times$141.4 = 99.96V.

   Angular velocity =ω = 2πf , since ω= 377.

   b) Frequency f=$\omega/2\pi$ =377/2π= 60Hz.

   Given t=3ms.

   c)Instantaneous voltage v= $V_m$sinωt

   v=141.4 sin (377$\times$3$\times$10-3$\times$ 180/π) = 127.94 V (Keep the calculator in degree mode).

2. An alternating current of sinusoidal waveform has an r.m.s. value of 10.0 A. What are the peak values of this current over one cycle?

   Solution:

   I_m= I / 0.707 =10 / 0.707 = 14.14 A

   The peak values therefore are 14.14 A and -14.14 A.

1.11 Analysis of series and parallel circuits

1.11.1 Analysis of a purely resistive circuit

Consider a circuit having resistance R ohms connected across the terminals of an a.c. supply, as in Fig.1.42a. In the circuit shown in figure, R is a pure resistance to which an alternating voltage v = V_m sinωt is applied, due to which an alternating current i flows through it. By Ohms law,

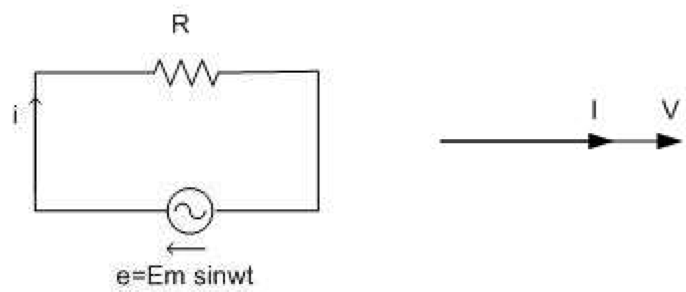


Figure 1.42: a) Pure resistive circuit b) Phasor Diagram

$$i = \frac{v}{R} = \frac{V_m sin\omega t}{R} = I_m sin\omega t \text{ where, } I_m = V_m/R$$

By observing the equations for voltage, v=$V_m$sinωt and i=I_m sinωt, it can be concluded that the current is in phase with the voltage. Vectorially the r.m.s values of voltage and current are represented as shown in Fig.1.42b.

The instantaneous power consumed by the resistance is given by,

$$p = v \times i = V_m sin\omega t I_m sin\omega t = V_m I_m sin^2\omega t$$
$$= V_m I_m \frac{(1 - cos2\omega t)}{2} = \frac{1}{2} V_m I_m - \frac{1}{2} V_m I_m cos2\omega t$$

This equation consists of two parts. The second part $\frac{1}{2}V_m I_m cos2\omega t$ is a periodically varying quantity whose frequency is two times the frequency of the applied voltage and its average value over a period of time is zero. Power is a scalar quantity and hence only its average value has to be taken into account. Hence the power consumed by the resistance is only due to the first part 1/2 $V_m I_m$. $\therefore$ Average Power,

$$P = \frac{1}{2} V_m I_m = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI$$

The waveforms of v, i and P are as shown in the Fig.1.43 below.

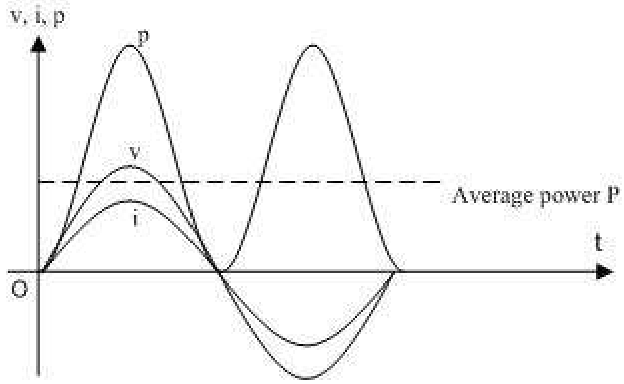


Figure 1.43: a) Waveforms of voltage, current and average power in pure resistive circuit

1.11.2 Analysis of a purely inductive circuit

Consider a circuit with a coil of pure inductance L henry connected across the terminals of an a.c. supply, as in Fig.1.44a. Due to the applied alternating voltage

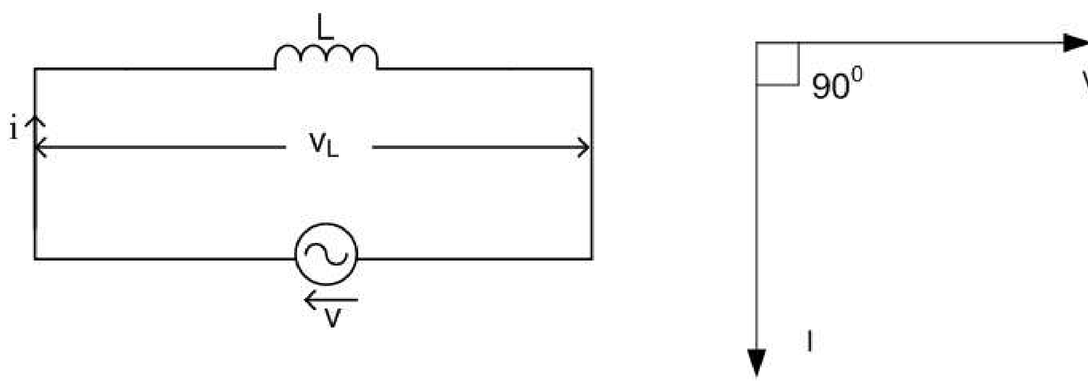


Figure 1.44: a) Pure inductive circuit b) Phasor Diagram

v=$V_m$sinωt is applied, an alternating current i flows through it. This alternating current produces an alternating flux, which links the coil and hence an e.m.f e is induced in it, which opposes the applied voltage and is given by,

$$e = -L\frac{di}{dt} = -v$$

$$\therefore v = L\frac{di}{dt}$$
$$di = \frac{v}{L}dt = \frac{1}{L}V_m sin\omega t \cdot dt$$

Integrating the above equation, we get

$$\begin{aligned} i &= \frac{V_m}{L}\int sin\omega t \cdot dt \\ &= \frac{V_m}{\omega L}(-cos\omega t) \\ &= \frac{V_m}{\omega L}sin(\omega t - \frac{\pi}{2}) \\ &= I_m sin(\omega t - \frac{\pi}{2}) \end{aligned}$$

Where $I_m = V_m/\omega L = V_m/X_L$
$X_L = \omega L = 2\pi$ L = Inductive reactance in Ohms
By observing the equations for voltage and current, we find that the current lags the voltage by an angle $\pi/2$. Vectorial representation of the r.m.s values of voltage and current is as shown in figure 1.44b.
The instantaneous power is given by,

$$\begin{aligned} p = v \cdot i &= V_m sin\omega t \cdot Imsin(\omega t - \frac{pi}{2}) \\ &= V_m I_m sin\omega t(-cos\omega t) = -\frac{1}{2}V_m I_m sin2\omega t \end{aligned}$$

The equation for p consists of a quantity which is periodically varying and having a frequency two times the frequency of the applied voltage and whose average value is zero. Hence, the power consumed by a pure inductance is zero, because the power is a scalar quantity and only its average value has to be considered. The power given to the pure inductance is stored in the form of an electromagnetic field. The waveforms of instantaneous voltage, current & power are as shown in Fig.1.45.

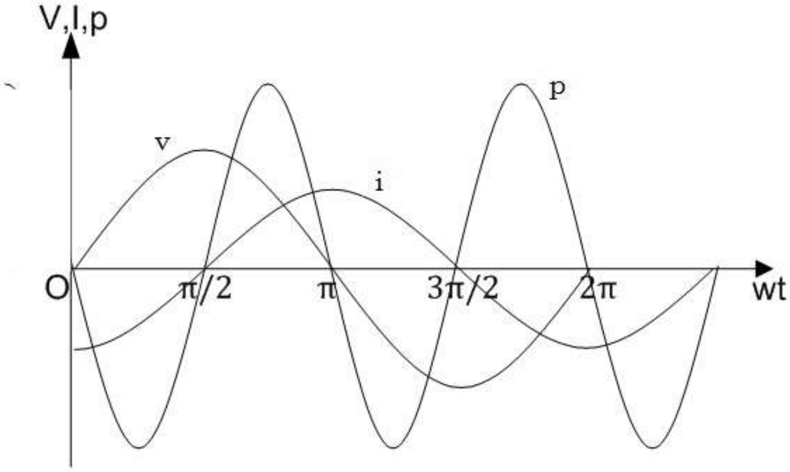


Figure 1.45: Waveforms of v,i and p for Pure inductive circuit

1.11.3 Analysis of a purely capacitive circuit:

Consider a capacitor of pure capacitance C, across which an alternating voltage v=$V_m sin\omega t$is applied, due to which an alternating current i flows, charging the plates of the capacitor with a charge of q coulombs as shown in figure 1.46a.

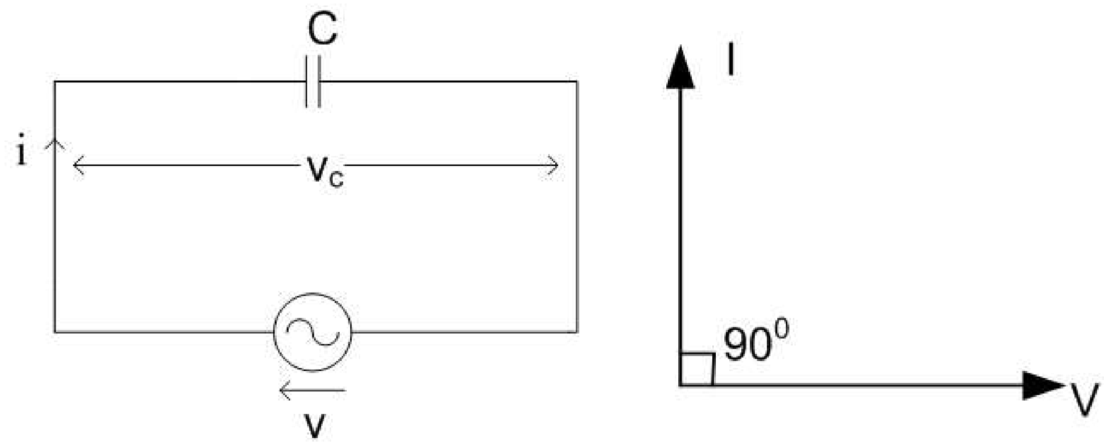


Figure 1.46: a) Pure capacitive circuit b) Phasor Diagram

$$\begin{aligned} i &= \frac{dq}{dt} = \frac{d(Cv)}{dt} = C\frac{dv}{dt} = C\frac{d}{dt}(V_m sin\omega t) \\ &= \omega C V_m cos\omega t = \frac{V_m}{1/\omega C} sin(\omega t + \pi/2) \\ &= \frac{V_m}{X_C} sin(\omega t + \pi/2) \\ &= I_m sin(\omega t + \pi/2) \end{aligned}$$

where, $I_m = \frac{V_m}{X_C}$ and $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$ =Capacitive reactance in ohms.
By observing the equations for voltage and current, we find that the current leads the voltage by an angle $\pi/2$. Vectorial representation of the r.m.s values of voltage and current is as shown in Figure 1.46b.
The instantaneous power is given by,

$$p = v \cdot i = V_m sin\omega t \cdot Imsin(\omega t + \frac{pi}{2})$$
$$= V_m I_m sin\omega t cos\omega t = \frac{1}{2} V_m I_m sin2\omega t$$

The equation for p consists of a quantity which is periodically varying and having a frequency two times the frequency of the applied voltage and whose average value is zero. Hence, the power consumed by a pure capacitance is zero, because the power is a scalar quantity and only its average value has to be considered. The power given to the pure capacitance is stored in the form of an electrostatic field. The waveforms of v, i and p are as shown in Fig.1.47.

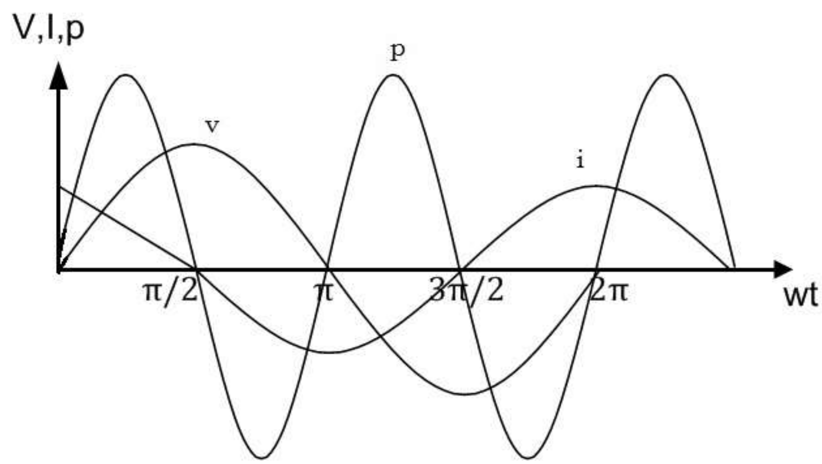


Figure 1.47: Waveforms of v,i and p for Pure capacitive circuit

1.12 Numerical Problems

1. An AC Circuit consist of a pure resistance of 10ohms and is connected across an AC supply of 230 V, 50Hz. Calculate current, power consumed, equation for voltage and current.
   Solution:
   i = v/r = 230/10 = 23A

p = vi = 230 $\times$ 23 = 5290w
V_m = $\sqrt{2}$ V = 325.27V
I_m = $\sqrt{2}$ i = 32.52 A
ω = 2πf = 314Hz
v = $V_m$sinωt = 325.27 sin314t V
i = $I_m$sinωt = 32.52 sin314t A

2. A pure inductive coil allows a current of 10A to flow from 230V, 50Hz supply. Calculate inductive reactance, inductance of the coil, power absorbed, equations for voltage and current.
   Solution:
   Inductive reactance, X_L= v/i= 230/10 = 23 Ohms
   Coil inductance, L = X_L / 2πf = 23 / 2π*50 = 0.073H
   Power absorbed = 0 ($\because$ Power absorbed in pure inductive circuit is zero)
   Peak value of the voltage, V_m=$\sqrt{2}$V=$\sqrt{2}$*230=325.27V
   Peak value of the current, I_m=$\sqrt{2}$ i = $\sqrt{2}$*10 =14.14 A
   ω= 2πf = 314Hz
   Equation of the voltage : v = $V_m$sinωt = 325.27 sin314t V
   Equation of the current : i = $I_m\sin(\omega t - \frac{\pi}{2})$ = 14.14 sin(314 t-$\frac{\pi}{2}$) A

3. A 318uF capacitor is connected across a 230V, 50Hz supply. Calculate capacitive reactance, rms current, equations for the voltage and current.
   Solution:
   Capacitive reactance, X_C= 1 / 2πf C =1 / 2π*50*318u= 10 Ohms
   RMS current, i = V / X_C= 230 / 10 = 23A
   Peak value of the voltage, $V_m = \sqrt{2}V = \sqrt{2}$*230=325.27V
   Peak value of the current, $I_m = \sqrt{2}I = \sqrt{2}$*23 =32.53 A
   ω = 2πf = 314Hz
   Equation of the voltage: v = $V_m sin\omega t$= 325.27 sin314t V
   Equation of the current: i = $I_m sin(\omega t + \pi/2)$= 32.53 sin(314t+$\pi$/2) A

1.13 Analysis of a R-L, R-C and R-L-C Series circuits

1.13.1 Analysis of a R-L Series circuit

Consider an R-L series circuit as shown in Fig.1.48a below to which an alternating voltage of r.m.s value V is applied, due to which an r.m.s value of current I flows through the circuit. The vector diagram taking I as reference vector is as shown in Fig.1.48 b. The vector diagram consists of three voltages, V = I R which is in phase

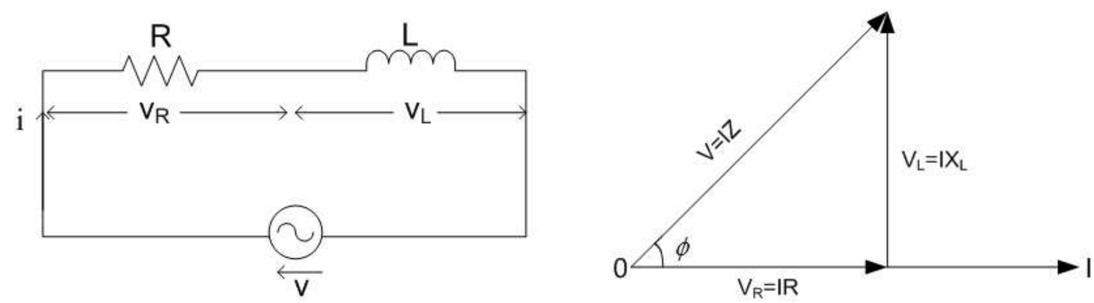


Figure 1.48: a) RL series circuit b) Phasor Diagram

with the current, V =I X which leads the current by 90°. The vector sum of these two voltages is applied voltage V=I Z. here, Z is the impedance of the circuit in ohms.
The impedance of an a.c circuit may be defined as the opposition offered for the flow of alternating current in the circuit. It is the combination of resistance and reactance of the circuit. The impedance triangle for an R-L series circuit is shown in fig 1.49. From the vector diagram, we observe that the current lags the voltage by an angle ϕ.

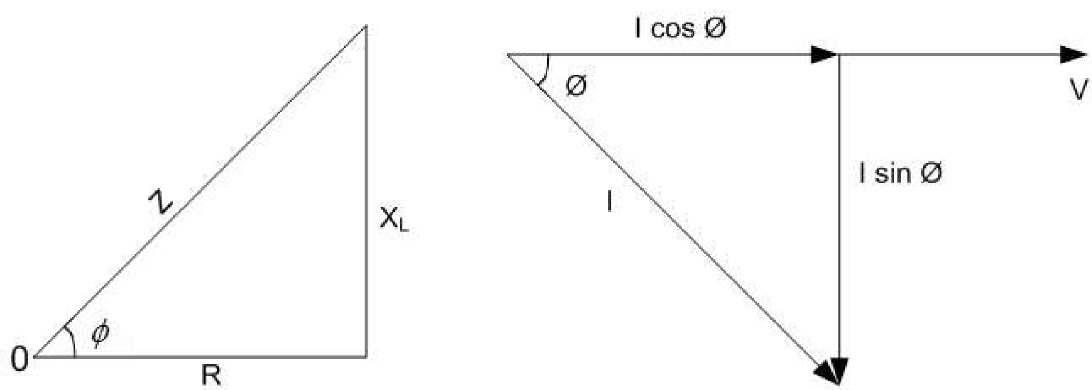


Figure 1.49: a) Impedance triangle

If v = V_m sinω t ,

Then, i= $I_m sin(\omega t - \phi)$
The instantaneous power is given by,

$$\begin{aligned} p = v \cdot i &= V_m sin\omega t \cdot Imsin(\omega t - \phi) \\ &= V_m I_m \frac{1}{2}[cos\phi - cos(2\omega t - \phi)] \\ &= \frac{1}{2} V_m I_m cos\phi - \frac{1}{2} V_m I_m cos(2\omega t - \phi) \end{aligned}$$

The second term in the equation is a periodically varying quantity, whose frequency is two times the frequency of the applied voltage and its average value is zero. As power is always an average value, only the first term represents the power consumed.

$$\begin{aligned} \therefore P &= \frac{1}{2} V_m I_m cos\phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} cos\phi \\ &= VIcos\phi \end{aligned}$$

Where, cosϕ is known as the power factor of the circuit. The waveforms of v, i and p are shown in Fig.1.50. The areas of the +ve power lobes is more than the areas of

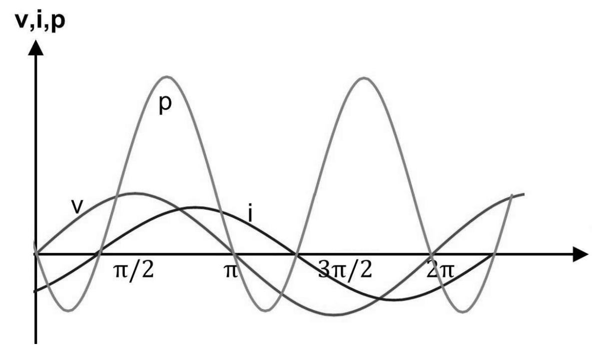
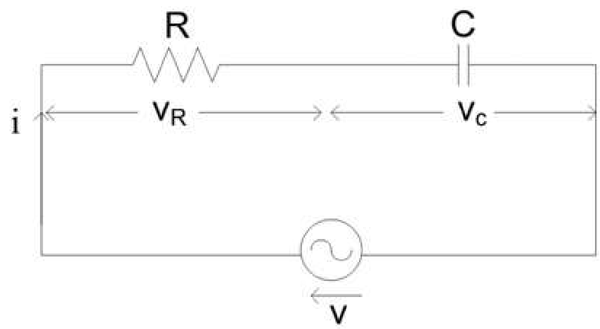


Figure 1.50: Waveforms of v,i and p for RL series circuit

the ve power lobes, indicating that the power received by the circuit is more than the power returned by the circuit. Hence, the circuit consumes a net power given by,P =V I cos ϕ.

1.13.2 Analysis of a R-C Series circuit

Consider an R-C series circuit as shown in Fig.1.51a. to which an alternating voltage of r.m.s value V is applied, due to which an r.m.s value of current I flows through the circuit. I is taken as the reference vector. The diagram consists of three voltages, V_R = I R which is in phase with the current, V_C =I X_C which lags the current by 90°. The vector sum of these two voltages is applied voltage V=IZ. Here, Z is the



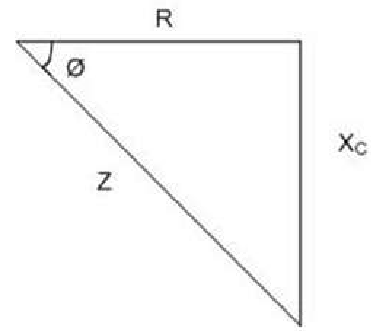


Figure 1.51: a) RC series circuit b) Phasor Diagram

impedance of the circuit in ohms.
V = $V_R + V_C$ (phasor sum)

$$V = \sqrt{V_R^2 + V_C^2} = \sqrt{I^2R^2 + I^2X_C^2}$$
$$= I\sqrt{R^2 + X_C^2} = IZ$$

where, Z= $\sqrt{R^2 + X_C^2}$ and $\phi = tan^{-1}\frac{X_C}{R} = cos^{-1}\frac{R}{Z}$
The current leads the voltage by an angle ϕ.
If v = $V_m sin\omega t$, Then, i= $I_m sin(\omega t + \phi)$
The current in the circuit is given by,I=V/Z
The impedance triangle for an R-C series circuit is shown in Fig.1.51b.
The instantaneous power is given by,

$$p = v \cdot i = V_m sin\omega t \cdot Imsin(\omega t + \phi)$$
$$= V_m I_m \frac{1}{2}[cos(-\phi) - cos(2\omega t + \phi)]$$
$$= \frac{1}{2}V_m I_m cos\phi - \frac{1}{2}V_m I_m cos(2\omega t + \phi)$$

The second term in the equation is a periodically varying quantity, whose frequency is two times the frequency of the applied voltage and its average value is zero. As power is always an average value, only the first term represents the power consumed.

$$\therefore P = \frac{1}{2}V_m I_m cos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}cos\phi$$
$$= VIcos\phi$$

The waveforms of v, i and p are shown in Fig.1.52.

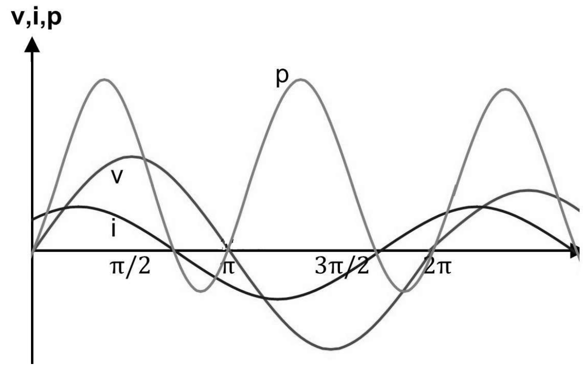


Figure 1.52: Waveforms of v,i and p for RC series circuit

1.13.3 Analysis of a R-L-C Series circuit

Consider an R-L-C series circuit as shown in Fig.1.53. to which an alternating voltage of r.m.s value V is applied, due to which an r.m.s value of current I flows through the circuit. // Three cases of the circuit can be discussed here.

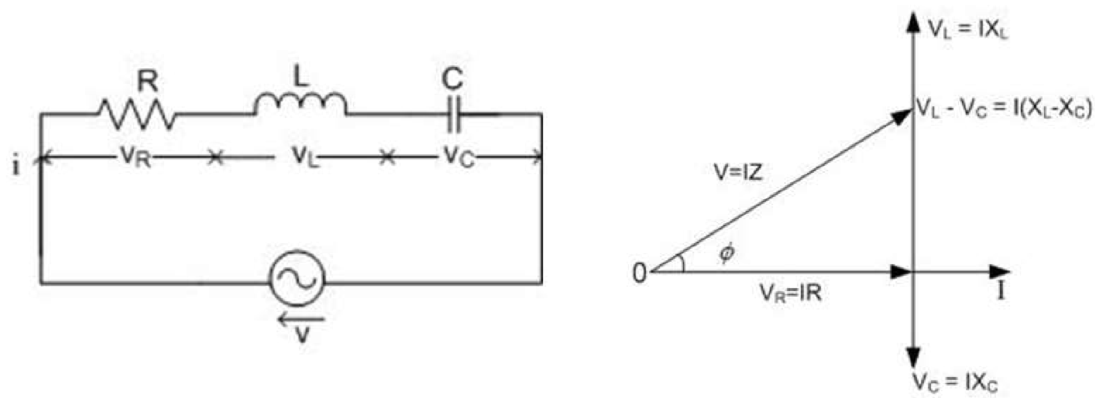


Figure 1.53: a) RLC series circuit b) Phasor Diagram for $X_L > X_C$

Case 1; When $X_L > X_C$
When the inductive reactance is more than the capacitive reactance, the vector diagram of the circuit as shown in Fig.1.53a. From the vector diagram, we observe that the current lags the voltage by an angle ϕ. The impedance triangle is shown in Fig.1.53b

$$I = \frac{V}{Z}$$
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

The circuit behaves similar to an R-L circuit.
If v = $V_m sin\omega t$,
Then, i= $I_m sin(\omega t - \phi)$.

Hence it can be proved that the power consumed is given by P=VI cos ϕ.

Case 2; When $X_L < X_C$

When the inductive reactance is less than the capacitive reactance, the vector diagram of the circuit as shown in Fig.1.54a.From the vector diagram, we observe that the current leads the voltage by an angle ϕ .

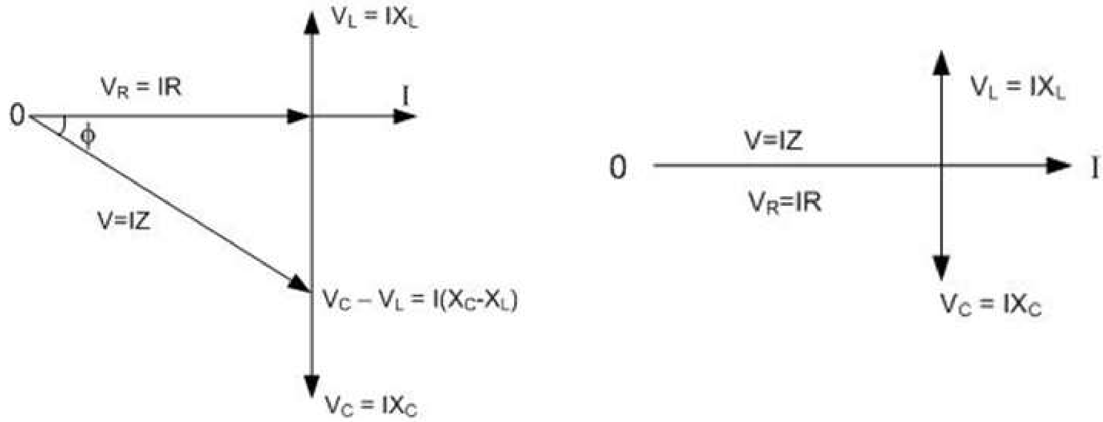


Figure 1.54: Phasor Diagram for a) $X_L < X_C$ b)$X_L = X_C$

$$I = \frac{V}{Z}$$
$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

The circuit is similar to an R-C circuit.

If v = $V_m sin\omega t$,

Then, i= $I_m sin(\omega t + \phi)$.

Hence it can be proved that the power consumed is given by P=VI cos ϕ.

Case 3; When $X_L = X_C$

When the inductive reactance is equal to the capacitive reactance, the vector diagram of the circuit as shown in Fig. 1.54b. V and V cancel out each other. The current is in phase with the voltage and circuit behaves as a pure resistance circuit. Hence Z = R.

If v = $V_m sin\omega t$,

Then, i= $I_m sin\omega t$.

Hence it can be proved that the power consumed is given by P=VI, which is proved in the case of a pure resistance circuit.

1.14 Numerical Problems

1. A resistance of 7 is connected in series with a pure inductance of 31.4mH and the circuit is connected to a 100V, 50Hz sinusoidal supply. Calculate the a) circuit current b) Phase angle
   Solution:
   Given data: R=7 L= 31.4mH V=100V, f=50Hz
   a) Current I = V/Z
   $X_L = 2\pi f L = 2\pi \times 50 \times 31.4 \times 10^{-3} = 9.86$ Ohm
   Impedance Z=$\sqrt{R^2 + X_L^2}$
   Substituting for XL, Z= $\sqrt{7^2 + 9.86^2}$= 12.09 Ohm
   $\therefore$ Current I = 100/12.09=8.27A
   b) Phase angle $\phi = tan^{-1}\frac{X_L}{R} = tan^{-1}\frac{9.86}{7} = 55° lag$ ($\because$ inductive circuit)

2. A capacitor of 8.0 μ F takes a current of 1.0 A when the alternating voltage applied across it is 230 V. Calculate: (a) the frequency of the applied voltage; (b) the resistance to be connected in series with the capacitor to reduce the current in the circuit to 0.5 A at the same frequency; (c) the phase angle of the resulting circuit.
   Solution:
   (a)$X_C = \frac{V}{I} = \frac{230}{1} = 230\Omega$
   $X_C = \frac{1}{2\pi f C}$
   $\therefore f = \frac{1}{2\pi C X_C} = \frac{1}{2\pi \times 8 \times 10^{-3} \times 230} = 86.5$ Hz
   (b) When a resistance is connected in series with the capacitor,
   Z=$\frac{V}{I} = \frac{230}{0.5}$= 460 Ω
   Z=$\sqrt{R^2 + X_C^2} = \sqrt{R^2 + 230^2}$=460$\Omega$
   hence R=398Ω
   (c) $\phi = cos^{-1}\frac{R}{Z} = cos^{-1}\frac{398}{460} = 30°$ lead

3. A 230V, 50Hz AC supply is applied to a coil of 0.06 H inductance and 2.5 Ohms resistance, the connection in series with a 6.8micro farad capacitance. Calculate the impedance, current, phase angle between voltage and current, power factor and the power consumed.
   Solution: $X_L = 2\pi$ f L = 18.85 Ohms
   $X_C = 1/2\pi$ f C = 468 Ohms

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$
$$= 449.2 Ohms$$

i = V/Z = 230/449.2 = 0.512A

To find the phase angle of the resulting circuit:
tan $\phi = (X_C - X_L)$/R = 449.15/2.5 = 179.66
Therefore ϕ = 89.70° lead
Cos ϕ= R/Z = 2.5 / 449.2 = 0.00557 lead
P = VI Cos ϕ = 230 X 0.512 X 0.00557 = 0.656w

4. A circuit having resistance of 12Ω, an inductance of 0.15 Henry and a capacitance of 100μF in series is connected across a 100V, 50Hz supply. Calculate a) the impedance b) the current c) the voltage across R,L and C d) the phase difference between the current and the supply voltage.
Solution:
Data given: R=12Ω, L=0.15H, C=100μF, V=100V, f=50Hz
$X_L = 2\pi$ f L = 47.12 Ω
$X_C = 1/2\pi$ f C = 468 Ohms

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$
$$= 19.42 Ohms$$

Current I=V/Z= 100 / 19.42 = 5.149 A
Voltage across resistance V_R= IR= 5.149 × 12= 61.8 V
Voltage across inductance V_L= IX_L= 5.149 × 47.12= 242.66 V
Voltage across capacitance V_C= IX_L= 5.149 × 31.83= 164 V
Phase difference ϕ=cos^{-1} (R/Z) = cos^{-1} (12/19.42) =51.83°

1.15 Analysis of Parallel circuits

Admittance(Y):

Admittance is the reciprocal of impedance. i.e Y=$\frac{1}{Z}$
Its unit is mho (℧). In S.I units, the unit of admittance is Siemens.
If Z_1 , Z_2 ,Z_3 are the three impedances connected in parallel , then the total impedance is given by ,

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$
$$Y = Y_1 + Y_2 + Y_3$$

Hence admittances connected in parallel can be directly added .Hence the method of admittance is most suitable for the solution of parallel circuits.
Consider a parallel circuit comprising of three impedances Z_1 , Z_2 ,Z_3 as shown in Fig.1.55
As per KCL, $I = I_1 + I_2 + I_3$

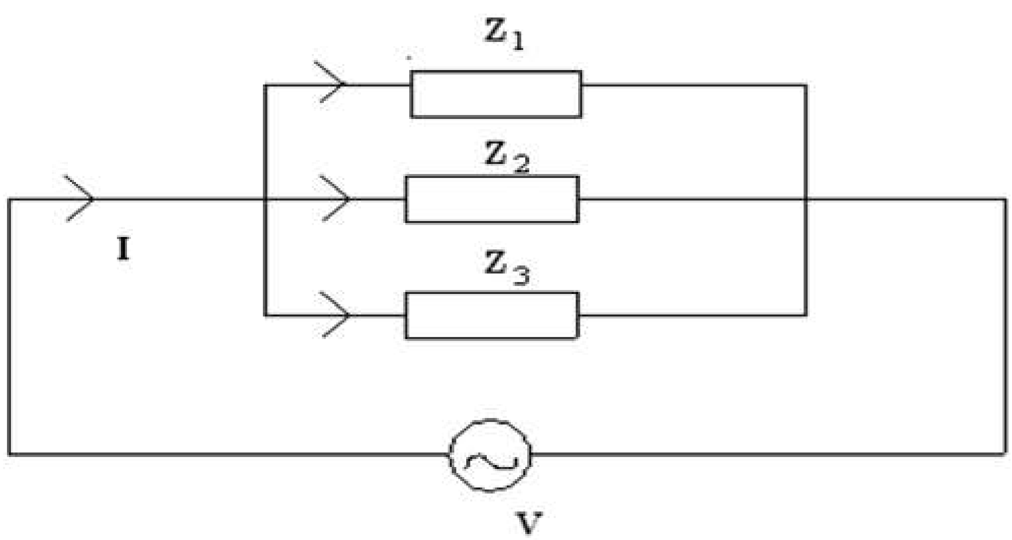


Figure 1.55: Parallel circuit

where, $I_1 = \frac{V}{Z_1}$, $I_2 = \frac{V}{Z_2}$ and $I_3 = \frac{V}{Z_3}$

$$\begin{aligned}
\therefore I &= \frac{V}{Z_1} + \frac{V}{Z_2} + \frac{V}{Z_3} \\
&= V(\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}) \\
&= V(Y_1 + Y_2 + Y_3) \\
I &= VY_{eq}
\end{aligned}$$

Where,$Y_{eq} = Y_1 + Y_2 + Y_3$
Thus if the branch admittance and total admittance are known, then branch currents $I_1 = VY_1$, $I_2 = VY_2$, $I_3 = VY_3$ and total current $I = VY_{eq}$

1.16 Numerical Problems

1. Two circuits, the impedance of which are given by Z1=10+j15Ω and Z2=6-j8Ω are connected in parallel. If the total current supplied is 15A, what is the power taken by the each branch? Find also the power factor of the individual circuits and of combination. Draw the vector diagram.
   Solution: Data given: I=15 A= 15 ∠0°, Z1=10+j15Ω, Z2=6-j8Ω
   Equivalent Impedance,

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(10 + j15)(6 - j8)}{(10 + j15 + 6 - j8)} = 9.67 - j3.6 = 10.3\angle - 20.4°\Omega$$

   Applied voltage, V=IZ= 15*$10.3\angle - 20.4° = 154.5\angle - 20.4°$ V
   Branch current,$I_1 = V/Z_1 = 154.5\angle - 20.4°/(10 + j15) = 8.58\angle - 77.4°$ A
   $I_2 = V/Z_2 = 154.5\angle - 20.4°/(6 - j8) = 15.45\angle 32.7°$ A
   A phasor diagrams is drawn as shown in fig. **??** Therefore power taken by first

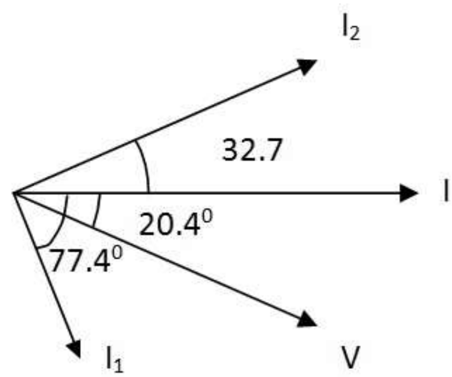


Figure 1.56: Phasor diagram

   branch
   $P_1 = I_1^2 R_1 = 8.58^2 \times 10$ =736 W
   Power taken by second branch
   $P_2 = I_2^2 R_2 = 15.45^2 \times 6$ =1432 W
   From the phasor diagram, I_1 lags behind V by 57° and I_2 leads it by 53.1°
   ∴ p.f of first branch = cos 57° 0.544 (lagging)
   p.f of second branch = cos 53.1° 0.6 (leading)
   Phase angle difference behind V and I is 20.4°. Hence power factor of the circuit

$= \cos 20.4° = 0.937$ (leading)

1.17 Three-phase AC Circuits

Electricity supply systems have to deliver power to many types of load. The greater the power supplied, for a given voltage, the greater the current. Three-phase systems are well suited to electricity supply applications because of their ability to transmit high powers efficiently and to provide powerful motor drives.

1.17.1 Necessity and Advantages of three phase system over single phase system

Three-phase systems have the following advantages over single-phase systems:

1. Three phase induction motors are self starting whereas, single phase induction motors are not self starting unless it was fitted with an auxiliary winding.

2. Three phase induction motor has better efficiency and power factor than the corresponding single-phase machine.

3. Three phase motors produce uniform torque whereas; the torque produced by single phase motors is pulsating.

4. For the same capacity, a three phase apparatus costs less than a single phase apparatus.

5. A three phase apparatus is more efficient than a single phase apparatus.

6. For the same capacity, a three phase apparatus is smaller in size compared to single phase apparatus.

7. The single phase generators when connected in parallel give rise to harmonics, whereas the three phase generators can be conveniently connected in parallel without giving rise to harmonics.

8. In three phase star system, two different voltages can be obtained, one between lines and other between the line and phase, whereas only one voltage can be obtained in a single phase system.

1.17.2 Delta connection of three-phase windings

The arrangement of three-phase windings with six line conductors shown in figure 1.57.a necessitates six line conductors and is therefore cumbersome and expensive, so let us consider how it may be simplified.

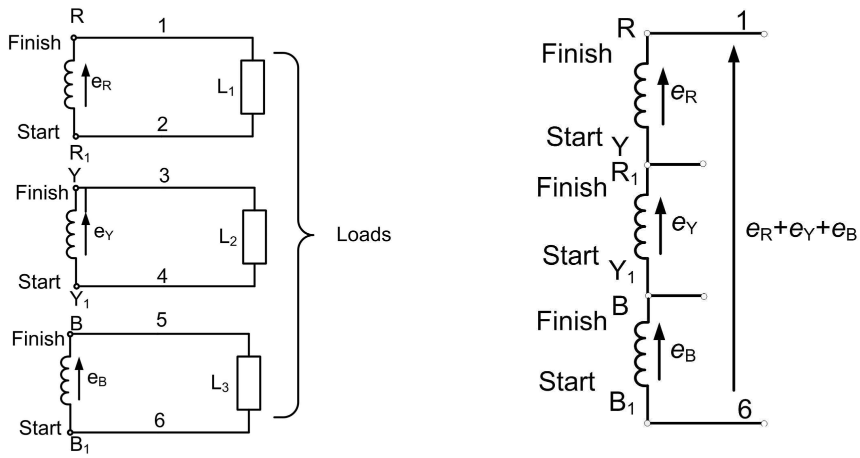


Figure 1.57: a. Three-phase windings with six line conductors b. Resultant e.m.f. if the three windings are connected in series

Let us join R_1 and Y together as in figure 1.57.b, thereby enabling conductors 2 and 3 of figure 1.57.a to be replaced by a single conductor. Similarly, let us join Y_1 and B together so that conductors 4 and 5 may be replaced by another single conductor. If we join B_1 to R, there will be three e.m.f.s chasing each other around the loop and these would produce a circulating current in that loop. However, we can show that the resultant e.m.f. between these two points is zero and that there is therefore no circulating current when these points are connected together. Instantaneous value of

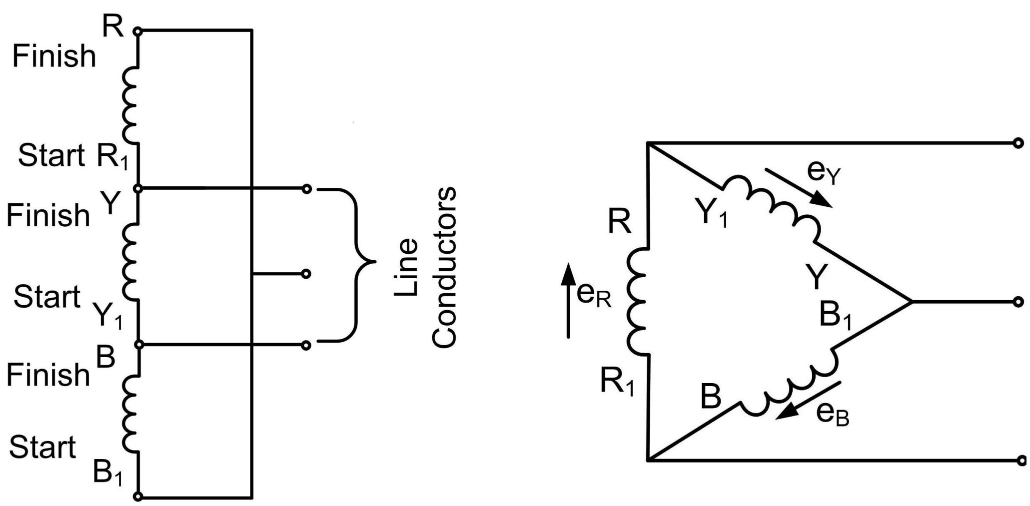


Figure 1.58: a. Delta connection of three-phase winding b. Conventional representation of a delta or mesh-connected winding

total e.m.f. acting from B_1 to R is

$$\begin{aligned} e_R &+ e_Y + e_B \\ &= E_m \sin\theta + \sin(\theta - 120^\circ) + \sin(\theta - 240^\circ) \end{aligned}$$

$$
\begin{aligned}
&= E_m(\sin\theta + \sin\theta\cos 120^\circ - \cos\theta\sin 120^\circ + \sin\theta\cos 240^\circ - \cos\theta\sin 240^\circ) \\
&= Em(\sin\theta - 0.5\sin\theta - 0.866\cos\theta - 0.5\sin\theta + 0.866\cos\theta) \\
&= 0
\end{aligned}
$$

Since this condition holds for every instant, it follows that R and B_1 can be joined together, as shown in figure 1.58a, without any circulating current being set up around the circuit. The circuit derived in figure 1.58a is usually drawn as in figure 1.58b and the arrangement is referred to as delta connection, also known as a mesh connection.

1.17.3 Star connection of three-phase windings

If we join together the three 'starts', R_1, Y_1 and B_1 at N, as in figure 1.59a., so that the three conductors 2, 4 and 6 of figure 1.57 can be replaced by the single conductor NM of figure 1.59a. If i_R, i_Y and i_B are the instantaneous values of the currents in the three phases, the instantaneous value of the current in the common wire MN is $(i_R + i_Y + i_B)$, having its positive direction from M to N.

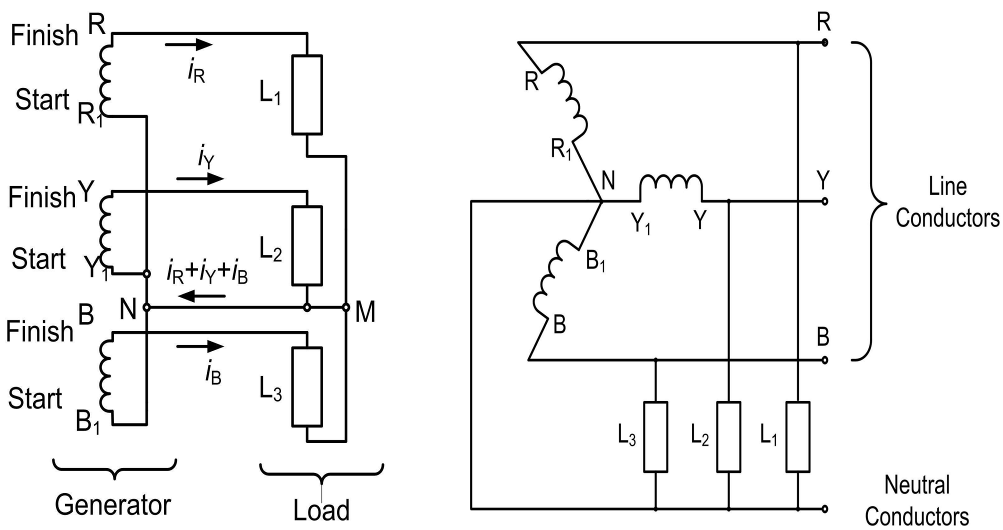


Figure 1.59: a. Star connection of three-phase winding b. Four-wire star-connected system

This arrangement is referred to as a four-wire star-connected system and is more conveniently represented as in figure 1.59b, and junction N is referred to as the star or neutral point. Three-phase motors are connected to the line conductors R, Y and B, whereas lamps, heaters, etc. are usually connected between the line and neutral conductors, as indicated by L_1, L_2 and L_3, the total load being distributed as equally as possible between the three lines. If these three loads are exactly alike, the phase currents have the same peak value, I_m, and differ in phase by 120°. Hence if the instantaneous value of the current in load L_1 is represented by

$$i_1 = I_m \sin\theta$$

instantaneous current in L_2 is

$$i_2 = I_m \sin(\theta - 120°)$$

and instantaneous current in L_3 is

$$i_3 = I_m \sin(\theta - 240°)$$

Hence instantaneous value of the resultant current in neutral conductor MN (figure 1.59a) is

$$\begin{aligned} i_1 + i_2 + i_3 &= I_m \sin\theta + \sin(\theta - 120°) + \sin(\theta - 240°) \\ &= I_m \times 0 = 0 \end{aligned}$$

Thus with a balanced load the resultant current in the neutral conductor is zero at every instant; hence this conductor can be dispensed with, thereby giving us the three-wire star-connected.

1.17.4 Relationship between line and phase quantities of Star-connected system

Let us again assume the e.m.f. in each phase to be positive when acting from the neutral point outwards, so that the r.m.s. values of the e.m.f.s generated in the three phases can be represented by V_R , V_Y and V_B in figure 1.60a, 1.60b and figure 1.60c.

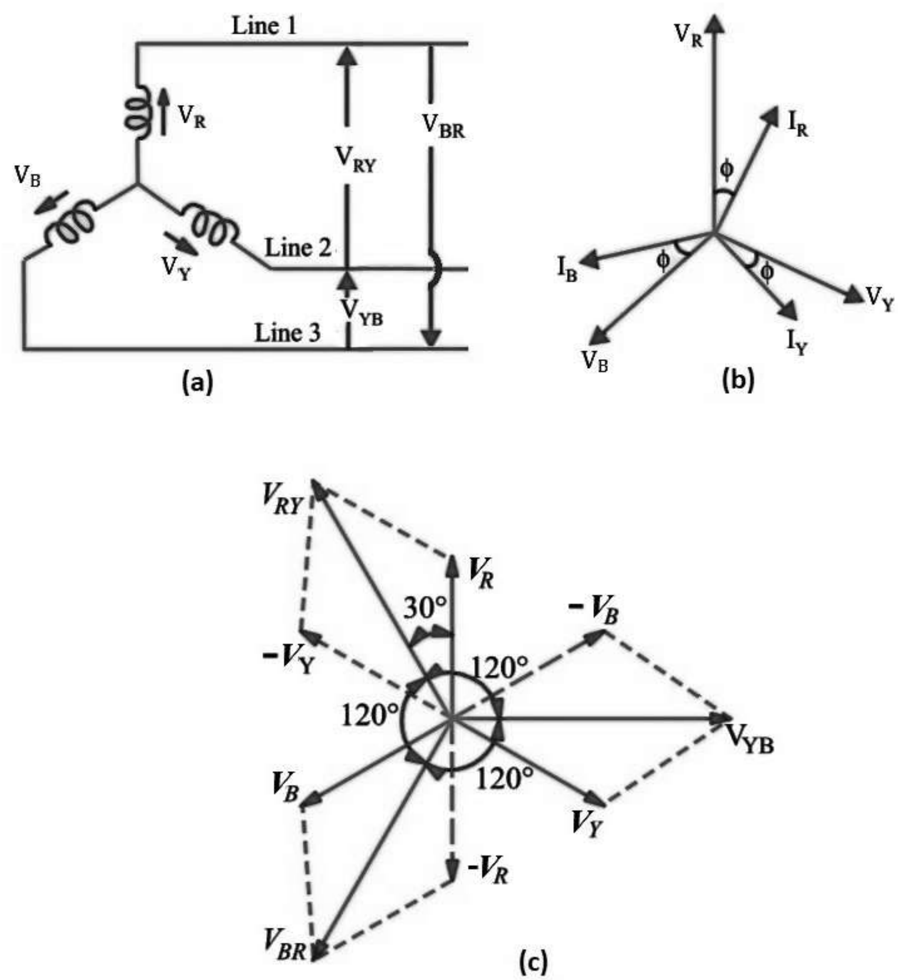


Figure 1.60: a. Star-connected generator b.Phase voltages and phase currents c. Phasor diagram for Fig. 1.60a.

When the relationships between line and phase quantities are being derived, it is essential to relate the phasor diagram to a circuit diagram and to indicate on each phase the direction in which the voltage or current is assumed to be positive. The value of the e.m.f. acting from Y via N to R is the phasor difference of V_R and V_Y. Hence $-V_Y$ is drawn equal and opposite to V_Y and added to V_R , giving V_{RY} as the e.m.f. acting from Y to R via N. Here V_{BR} is obtained by subtracting V_R from V_B , and V_{YB} is obtained by subtracting V_B from V_Y , as shown in figure 1.60c. From the symmetry of this diagram it is evident that the line voltages are equal and are spaced 120° apart. Further, since the sides of all the parallelograms are of equal length, the diagonals bisect one another at right angles. Also, they bisect the angles of their respective parallelograms; and, since the angle between V_R and $-V_Y$ is 60°,

$$\therefore V_{RY} = 2V_R \cos 30^\circ = \sqrt{3}\ V_R$$

i.e. Line voltage = $\sqrt{3}$ × star (or phase) voltage

From figure 1.60a it is obvious that in a star-connected system the current in a line conductor is the same as that in the phase to which that line conductor is connected. Hence, in general, if

$$V_L = \text{p.d. between any two line conductors} = \text{line voltage}$$

and

$$\begin{aligned} V_P &= \text{p.d. between a line conductor and the neutral point} \\ &= \text{star voltage (or voltage to neutral)} \end{aligned}$$

and if I_L and I_P are line and phase currents respectively, then for a star-connected system,

$$V_L = \sqrt{3}V_P \text{ and } I_L = I_P$$

The voltage given for a three-phase system is always the line voltage unless it is stated otherwise.

1.17.5 Delta-connected system

Let I_1 , I_2 and I_3 be the r.m.s. values of the phase currents having their positive directions as indicated by the arrows in Fig. 1.61a. Since the load is assumed to be balanced, these currents are equal in magnitude and differ in phase by 120°, as shown in figure 1.61b.

From figure 1.61a it will be seen that I_1 , when positive, flows away from line conductor R, whereas I_3 , when positive, flows towards it. Consequently, I_R is obtained by subtracting I_3 from I_1 , as in figure 1.61b. Similarly, I_Y is the phasor difference of I_2 and I_1 , and I_B is the phasor difference of I_3 and I_2 . From figure 1.61a it is evident that the line currents are equal in magnitude and differ in phase by 120°.

Also $I_R = 2I_1 \cos 30^\circ = \sqrt{3}I_1$

Hence for a delta-connected system with a balanced load

Line current = $\sqrt{3}$ × phase current i.e. $I_L = \sqrt{3}I_P$

From figure 1.61a. it can be seen that, in a delta-connected system, the line and the phase voltages are the same, i.e. $V_L = V_P$

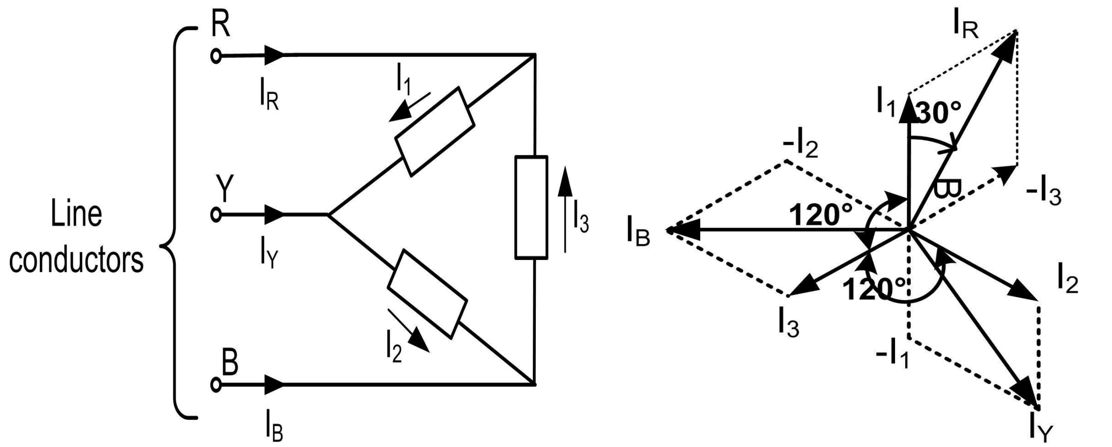


Figure 1.61: a. Delta-connected system with balanced load b. Phasor diagram for Fig.1.61a.

1.17.6 Expression for three-phase power

If I_P is the r.m.s. value of the current in each phase and V_P the r.m.s. value of the p.d. across each phase,

$$\begin{aligned} \text{Active power per phase} &= I_P \times V_P \times \text{power factor} \\ \text{Total active power} &= 3I_PV_P \times \text{power factor} \end{aligned}$$

$$\therefore P = 3I_PV_P \cos\phi \tag{1.48}$$

If I_L and V_L are the r.m.s. values of the line current and voltage respectively, then for a star-connected system,

$$V_P = \frac{V_L}{\sqrt{3}} \text{ and } I_P = I_L$$

Substituting for I_P and V_P in equation 1.48, we have

$$\text{Total active power in watts} = \sqrt{3}\ I_L\ V_L \times \text{power factor}$$

For a delta-connected system

$$V_P = V_L\text{and } I_P = \frac{I_L}{\sqrt{3}}$$

Again, substituting for I_P and V_P in equation 1.48, we have

$$\text{Total active power in watts} = \sqrt{3}\ I_L\ V_L \times \text{power factor}$$

Hence it follows that, for any balanced load,

$$\begin{aligned} \text{Active power in watts} &= \sqrt{3} \times \text{line current } \times \text{line voltage } \times \text{power factor} \\ &= \sqrt{3} I_L V_L \times \text{power factor} \end{aligned}$$

$$P = \sqrt{3} V_L I_L cos\phi \tag{1.49}$$

1.18 Numerical Problems

1. A three-phase motor operating off a 400 V system is developing 20 kW at an efficiency of 0.87 p.u. and a power factor of 0.82. Calculate: (a) the line current; (b) the phase current if the windings are delta-connected.

   Solution : (a) Since

$$\begin{aligned}\text{Efficiency} &= \frac{\text{output power in watts}}{\text{input power in watts}}\\ \eta &= \frac{\text{output power in watts}}{1.73 I_L V_L \times \text{ p.f.}}\end{aligned}$$

$$0.87 = 20 \times 1000 \ / \ \sqrt{3} \times \text{I L} \times 400 \times 0\ .\ 82$$
$$\text{Line current} = I_L = 40.0 \text{ A}$$

   (b) For a delta-connected winding
   Phase current = line current $/\sqrt{3}$= $40.0/\sqrt{3}$ = 23.1 A

2. Three similar inductors, each of resistance 10 Ω and inductance 0.0159 H, are star-connected to a three-phase, 400 V, 50 Hz sinusoidal supply. Calculate: (i) the value of the line current; (ii) the power factor; (iii) the active power input to the circuit.

   i) X_L =2 π fL = 2 π × 50 × 0.0159 = 5.0 Ω
   Z= $\sqrt{R^2 + X_L^2} = \sqrt{10^2 + 5^2}$ = 11.18 Ω
   In star-connected system: $V_L = \sqrt{3}V_P$
   $\therefore V_P = V_L \ / \ \sqrt{3} = 400 \ /\sqrt{3}$ = 230.94 Volts
   Phase current $I_P = V_P$/Z = 230.94/ 11.18 = 20.66 A

   Line current = Phase current= 20.66 A

   ii) Power factor, cosϕ = R/Z =10 /11.18 = 0.8944 lagging ($\because$ Inductive circuit)

   iii) Active Power, P= $\sqrt{3}\ V_L\ I_L \cos\phi = \sqrt{3}$ 400 × 11.18 × 0.8944 = 6927.78 W

1.19 Measurement of three-phase power using two wattmeter method

Consider a balanced star-connected three-phase, three-wire system as shown in figure 1.62. Suppose L in figure 1.62 to represent three similar loads connected in star, and suppose V_{RN} , V_{YN} and V_{BN} to be the r.m.s. values of the phase voltages and I_R , I_Y and I_B to be the r.m.s. values of the currents. Since these voltages and currents are assumed sinusoidal, they can be represented by phasors, as in figure 1.63, the currents being assumed to lag the corresponding phase voltages by an angle ϕ. Two wattmeters are connected with each of its current coil in one line and the voltage coil between the lines. Current through current coil of W_1 is I_R . Potential difference

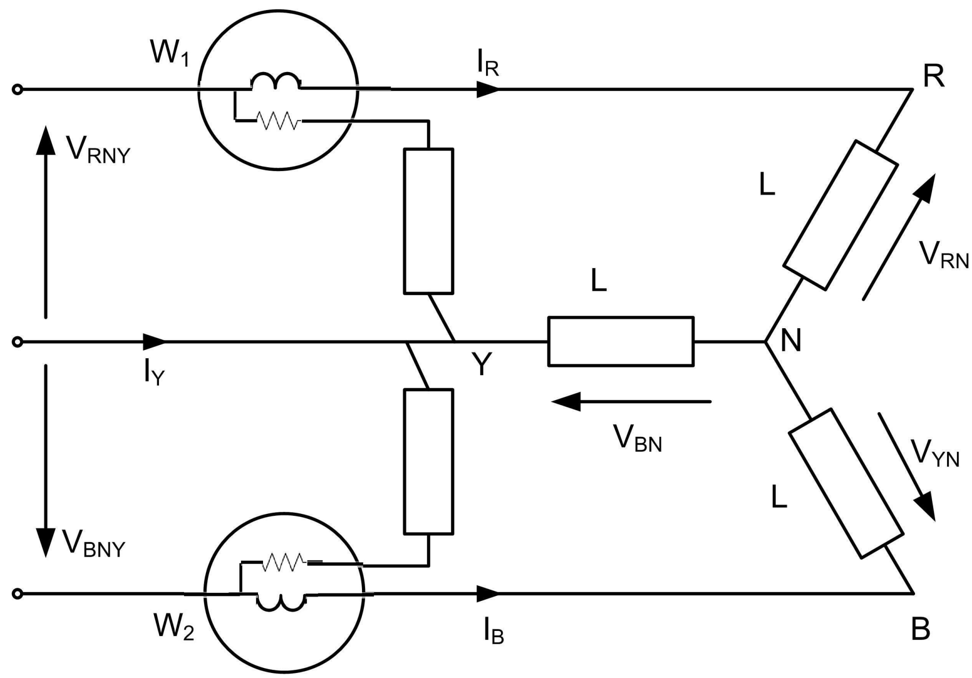


Figure 1.62: Measurement of active power and power factor by two wattmeters

across voltage circuit of W_1 is

Phasor difference of V_{RN} and $V_{YN} = V_{RNY}$

Phase difference between I_R and $V_{RNY} = 30° + \phi$.

Therefore reading on W_1 is

$$P_1 = I_R V_{RNY} \cos(30° + \phi)$$

Current through current coil of $W_2 = I_B$. Potential difference across voltage circuit of W_2 is

Phasor difference of V_{BN} and $V_{YN} = V_{BNY}$

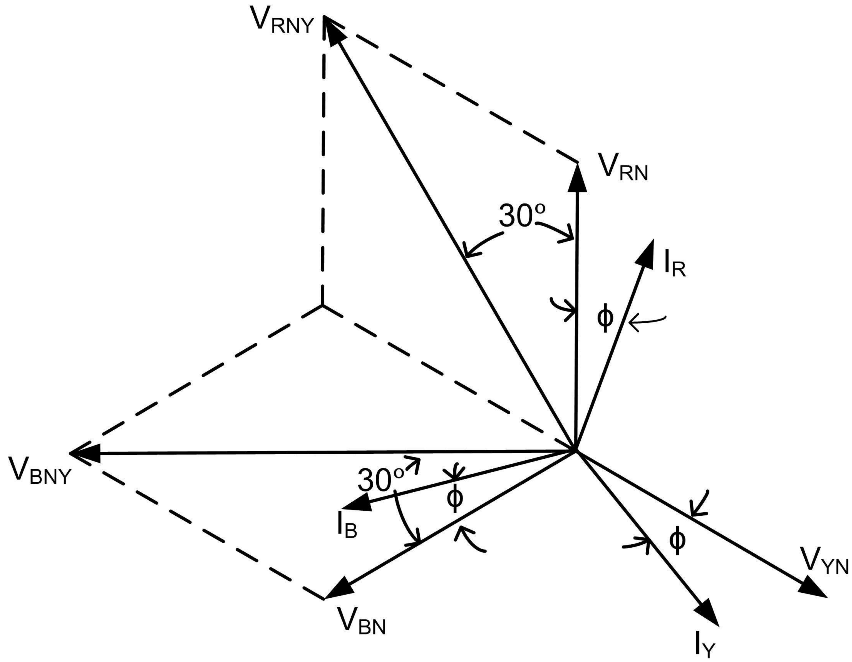


Figure 1.63: Phasor diagram for Fig. 1.62

Phase difference between I_B and $V_{BNY} = 30° - \phi$.
Therefore reading on W_2 is

$$P_2 = I_B V_{BNY} cos(30° - \phi)$$

Since the load is balanced,

$$I_R = I_Y = I_B = (say) I_L, \text{ numerically}$$

and

$$V_{RNY} = V_{BNY} = (say) V_L, \text{ numerically}$$

Hence

$$P_1 = I_L V_L \cos(30° + \phi) \tag{1.50}$$

and

$$P_2 = I_L V_L \cos(30° - \phi) \tag{1.51}$$

$$P_1 + P_2 \quad = I_L V_L \text{cos}(30° + \phi) + \cos(30° - \phi)$$
$$P_1 + P_2 \quad = I_L V_L (\cos 30°. \cos\phi - \sin 30°. \sin\phi$$
$$+ \cos 30°. \cos\phi + \sin 30°. \sin\phi)$$

$$P_1 + P_2 = \sqrt{3} I_L V_L \cos\phi \tag{1.52}$$

This expression is for the total active power in a balanced three-phase system. This proves that the sum of the two wattmeter readings gives the total active power, by assuming a balanced load and sinusoidal voltages and currents.

When the power factor of the load is 0.5 lagging, ϕ is $60°$ and from equation 1.50, the reading on $W_1 = I_L V_L \cos 90° = 0$. When the power factor is less than 0.5 lagging, ϕ is greater than $60°$ and $(30° + \phi\ )$ is therefore greater than $90°$. Hence the reading on W_1 is negative. To measure this active power it is necessary to reverse the connections to either the current or the voltage coil, but the reading thus obtained must be taken as negative when the total active power and the ratio of the wattmeter readings are being calculated. From equations 1.50, 1.51 and 1.52

$$P_2 - P_1 = V_L I_L \sin\phi \tag{1.53}$$

multiplying by $\sqrt{3}$ on both side

$$\sqrt{3}(P_2 - P_1) = \sqrt{3} V_L I_L \sin\phi = Q \tag{1.54}$$

Eq1.54 can be used to find the average reactive power consumed by the load. Dividing Eq1.53 by Eq1.52, we get

$$\frac{(P_2 - P_1)}{(P_2 + P_1)} = \frac{V_L I_L \sin\phi}{\sqrt{3} V_L I_L \cos\phi}$$

$$\implies \frac{(P_2 - P_1)}{(P_2 + P_1)} = \frac{\tan\phi}{\sqrt{3}}$$

$$\implies \phi = \tan^{-1}\left\{\frac{\sqrt{3}(P_2 - P_1)}{P_1 + P_2}\right\} \tag{1.55}$$

$$\implies \cos\phi = \cos\left[\tan^{-1}\left\{\frac{\sqrt{3}(P_2 - P_1)}{P_1 + P_2}\right\}\right] \tag{1.56}$$

Hence, ϕ and $\cos\phi$ can be determined using Eq1.55 and Eq1.55 respectively.

1.20 Numerical Problems

1. The input power to a three-phase motor was measured by the two- wattmeter method. The readings were 5.2 kW and -1.7 kW, and the line voltage was 400 V. Calculate: (a) the total active power (b) the power factor (c) the line current.

   Solution: (a) Total power = 5.2k +(- 1.7k) = 3.5 kW.

   (b) From equation 1.55

$$\implies \phi = \tan^{-1}\left\{\frac{\sqrt{3}(P_2 - P_1)}{P_1 + P_2}\right\}$$

$$\phi = \tan^{-1}\sqrt{3}\frac{(5.2 - (-1.7))}{(5.2 + (-1.7))}$$

$$\phi = 73.67°$$

   and power factor = $\cos\phi = 0.281$

   From the data it is impossible to state whether the power factor is lagging or leading.

   (c) From equation 1.49,
   $P_{3\phi} = \sqrt{3}V_L I_L \cos\phi$
   $3500 = \sqrt{3} \times I_L \times 400\times\ 0.281$
   $\implies I_L = 18.0$ A.

2. In a three-phase circuit two wattmeters used to measure power indicate 1200 W and 600 W respectively. Find the power factor of the circuit:
   i. When both wattmeter readings are positive.
   ii. When the latter is obtained by reversing the current coil connections.
   Solution:
   i. When both wattmeter readings are positive: W˙1=600 W, W˙2 = 1200 W.

$$\implies \phi = \tan^{-1}\left\{\frac{\sqrt{3}(P_2 - P_1)}{P_1 + P_2}\right\}$$

$$\implies \phi = \tan^{-1}\left\{\frac{\sqrt{3}(1200 - 600)}{600 + 1200}\right\}$$

$$\phi = 30°$$

   and power factor = $\cos\phi = 0.866$
   ii. When one of the wattmeter's current coil connection is reversed : W˙1=-600

W, W˙2 = 1200 W.

$$\implies \phi = \tan^{-1}\left\{\frac{\sqrt{3}(P_2 - P_1)}{P_1 + P_2}\right\}$$

$$\implies \phi = \tan^{-1}\left\{\frac{\sqrt{3}(1200 - (-600))}{-600 + 1200}\right\}$$

$$\phi = 79.1^\circ$$

and power factor = $\cos\phi = 0.1899$

UNIT-II

2.1 Review of Electromagnetism

The source of magnetic flux is either a permanent magnet or a current carrying coil. The lines of the magnetic flux always form a closed path. The closed path followed by the lines of magnetic flux is called a magnetic circuit. Thus, a magnetic circuit provides a closed path for the magnetic flux and is similar to an electric circuit which provides a closed path for the flow of electric current. In this chapter, definitions about the various magnetic quantities and simple analysis of magnetic circuits are provided.

2.1.1 Definition of Magnetic Quantities

Magneto Motive Force (MMF)

MMF is the source of producing flux in a magnetic circuit. For a current I flowing through a coil of N turns, the magnetic flux is obtained as a product of I and N and its unit is Amp-turns.

$$MMF = IN\text{AT} \tag{2.1}$$

Magnetising Force (H)

The Magnetising Force, otherwise called as Magnetic Field Intensity, is defined as the magneto motive force per unit length of the magnetic flux path.
Magnetising Force is a measure of the ability of a magnetised body to produce magnetic induction in other magnetic substances. H is the variable symbol to denote magnetising force and Ampere-Turns/metre (AT/m) is the unit.

$$H = \frac{MMF}{l} = \frac{IN}{l}\text{AT/m.} \tag{2.2}$$

Magnetic Flux (ϕ)

The magnetic lines of force or the amount of lines of a magnetic field provided by a magnet is called the magnetic flux. It is represented by the variable symbol ϕ and its unit is webers (Wb).
1 Wb = 10^8 Magnetic lines of force= 10^8 maxwells.

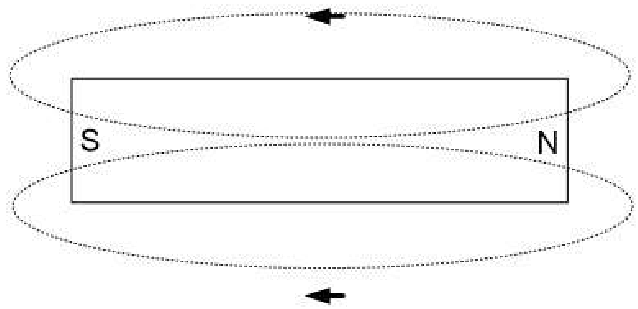


Figure 2.1

Magnetic Flux Density (B)

Magnetic flux density or Magnetic Induction is defined as the magnetic flux per unit area at right angles to the direction of the flux. B is the variable symbol used to denote the magnetic flux density and its unit is weber per square metre (Wb/m^2) or tesla (T).

$$B = \frac{\phi}{A}\text{T}. \tag{2.3}$$

Permeability

This is the ability of the medium to set up a magnetic flux density (B) by the magnetising force (H).

Permeability of free space The flux density established in a vacuum changes linearly with respect to the magnetising force and the proportionality constant is called the permeability of free space. It is denoted by μ_o and has the unit of Henry per metre (H/m).

$$\mu_o = 4\pi \times 10^{-7}\text{H/m}. \tag{2.4}$$

Relative permeability In ferromagnetic materials, like steel, by virtue of their inherent property, a given magnetising force sets up much more magnetic flux density compared with that in a vacuum. The ratio of flux density produced in a medium or material to the flux density produced in a vacuum by the same magnetising force is called as the relative permeability, denoted by μ_r.

$$\mu_r = \frac{Fluxdensityinthemedium}{Fluxdensityinthevacuum}$$

$$\mu_r = \frac{B}{\mu_o H}$$

$$\Rightarrow B = \mu_o \mu_r \text{H}. \tag{2.5}$$

For many magnetic materials, the value of μ_r itself changes with different values of the magnetising force. The value of relative permeability in different media are
μ_r = 1000-10000 for magnetic materials and
μ_r = 1 for non-magnetic materials.
Absolute Permeability The product of relative permeability and permeability of free space is called the absolute permeability and denoted by μ.

$$\mu = \mu_o \mu_r \tag{2.6}$$

$$and B = \mu H \tag{2.7}$$

Reluctance and Permeance

Reluctance The opposition offered by a magnetic circuit to the establishment of a magnetic flux is called as reluctance of the magnetic circuit. R or S is the variable symbol used to denote the reluctance of the magnetic circuit and its unit is Ampere-Turn/Weber (AT/Wb).
The reluctance in a magnetic circuit is directly proportional to the length of the field path,l, and inversely proportional to the area of a cross-section, A, of the magnetic field path.

$$\Re \propto \frac{l}{A}$$

$$\Re = \frac{l}{\mu A} \text{AT/Wb}. \tag{2.8}$$

Further from equations 2.2,2.3 and 2.6

$$\begin{aligned} MMF &= Hl \\ &= \frac{B}{\mu} l \\ &= B \times \frac{A}{A} \frac{l}{\mu} \\ &= BA \frac{l}{\mu A} \\ MMF &= \phi \Re \end{aligned}$$

$$Thus \Re = \frac{MMF}{\phi} \tag{2.9}$$

Permeance The reciprocal of reluctance is called permeance. Therefore, the permeance of the magnetic circuit is the readiness with which a magnetic flux is developed. P is the variable symbol and its unit is either webers/Ampere-Turn (Wb/AT).

$$P = \frac{1}{\Re}\text{Wb/AT} \tag{2.10}$$

2.1.2 Exercise

1. The flux produced in the air-gap between two electromagnetic pole faces is 6×10^{-2} Wb. Length of air gap is 1.4 cm and cross-sectional area of the gap is $0.3m^2$. Find

   (a) the flux density,

   (b) magnetic field intensity,

   (c) reluctance,

   (d) permeance and

   (e) mmf dropped.

   Solution:
   Given: $\phi = 6 \times 10^{-2}$; $l_g = 1.4 \times 10^{-2}$; $A = 0.3m^2$; $\mu_r = 1$(air-gap).

   (a) Flux density

   $$\begin{aligned} B =& \frac{\phi}{A} \\ =& \frac{6 \times 10^{-2}}{0.3} \\ B =& 0.2\text{T}. \end{aligned}$$

   (b) Magnetic field intensity

   $$\begin{aligned} H =& \frac{B}{\mu_o} (\because \mu_r = 1 \text{for air}) \\ =& \frac{0.2}{4\pi \times 10^{-7}} \\ H =& 159.155 \times 10^3 \text{AT/m}. \end{aligned}$$

(c) Reluctance

$$\begin{aligned}\Re =&\frac{l_g}{\mu_o A}\\ =&\frac{1.4\times 1^{-2}}{(4\pi\times 10^{-7})\times 0.3}\\ \Re =&37.136\times 10^{3}\text{AT/Wb.}\end{aligned}$$

(d) Permeance

$$\begin{aligned}P =&\frac{1}{\Re}\\ =&\frac{1}{37.136\times 10^{3}}\\ \Re =&26.928\times 10^{-6}\text{Wb/AT.}\end{aligned}$$

(e) MMF for air-gap

$$\begin{aligned}MMF =&\phi\Re\\ =&\left(6\times 10^{-2}\right)\times\left(37.136\times 10^{3}\right)\\ MMF =&2228.16\text{AT.}\end{aligned}$$

2. A magnetic core, in the form of a closed ring, has a mean length of 30 cm and a cross-sectional area of 1.2 cm^2. The relative permeability of iron is 2500.

   (a) What current will be required to pass on through a coil of 2000 turns uniformly wound round the ring to create a flux of 0.5 mWb in the iron?

3. An iron rod of 1.8 cm diameter is bent to form a ring of mean diameter 25 cm and wound with 250 turns of wire. A gap of 1 mm exists in between the end faces. Calculate the current required to produce a flux of 0.6 mWb. Take relative permeability of iron as 1200.

.

2.1.3 Permanent Magnet

Any body which is capable of attracting iron pieces is called a magnet. If the magnet possess this capability on its own, without any external aid, the magnet is called a permanent magnet.

Magnetic Materials

Permanent magnets ore available as natural ore as well as fabricated type. Ferrous oxide (Fe_3O_4) possesses the magnetic property as an ore itself.
On the other hand, permanent magnets can also be fabricated to the required size and shape and they are initially magnetised by electrical power.
Once these materials are magnetised, they retain the magnetic property for a long period. The following are examples of fabricated type permanent magnets: (i) Ferrites (ii) Alnico (iii) Neomax (iv) Samarium Cobalt etc.

Magnetic Field and its Behaviour

Around a magnet, infinite lines of magnetic flux are present as shown in Fig.2.2. Although these lines of flux do not physically exist, their effect is felt. The following are the important characteristics of lines of magnetic flux.

- The direction of a line of magnetic flux at any point in a nonmagnetic medium, such as air, is that of a North seeking pole of a compass needle placed at that point i.e from N-North pole to S- South pole.
- Each line of magnetic flux forms a closed loop.
- Lines of magnetic flux do not intersect each other.
- Lines of magnetic flux which are parallel and in the same direction repel one another

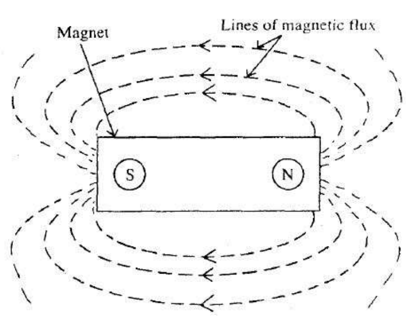


Figure 2.2: Magnetic flux around a bar magnet

The advantages of permanent magnets are:

- Smaller in size.
- No extra source is required.

The disadvantage of the permanent magnet is that once a magnet is fabricated to a particular size, the magnitude and direction of the magnetic flux is fixed. Hence, control over the flux produced is lost.

2.1.4 Electromagnet

For getting the desired performance from electrical machines, the magnetic flux present in the machine should be controllable. The electromagnet provides this facility.

Magnetic Effect of Electric Current

Whenever a conductor carries a current, a magnetic field is set up as an integral part of the electrical phenomenon of current flow. The magnetic flux lines are considered to be circular in shape and are present in a plane perpendicular to the flow of the current. The magnitude and direction of the magnetic field depends on the magnitude and direction of the current flow. In Fig. 5.2 the solid circle denotes the cross-sectional view of a round conductor. When a current is passed through this conductor, a circular shaped magnetic field, shown by dotted lines, is set up in a plane perpendicular to the current flow.
A circular field can have any one of the two circular directions, namely, clockwise or anticlockwise.
In Fig.2.3(a) the conductor carries a current in an inward direction which is shown by a cross(x) mark inside the conductor. Corresponding to this direction of current flow, the magnetic field takes a clockwise direction.
The direction of the magnetic field can be changed by changing the direction of the

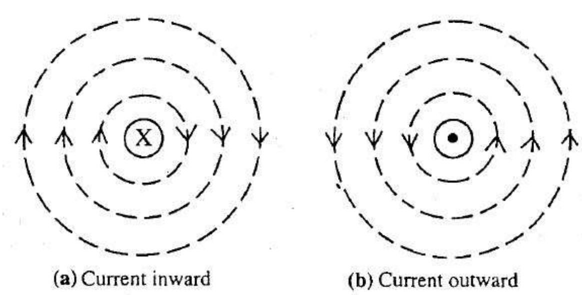


Figure 2.3: Magnetic field around current carrying conductor

current flow through the conductor. For an outward flow of current direction which is shown by a dot(-) mark inside the conductor in Fig.2.3(b), the magnetic field takes an anticlockwise direction.
To remember the direction of magnetic field in accordance with the direction of current, any one of the following rules can be used.

Right Hand Grip Rule

If the current carrying conductor is gripped by the right hand, of course with sufficient insulation, in such a way that the thumb points towards the direction of the current flow as shown in Fig.2.4, then the direction of the magnetic field is given by the direction in which the other four fingers surround the conductor for gripping it.

In Fig.2.4, the current flow is from right to left. The conductor is gripped by the right hand so that the thumb points towards left. The lines of magnetic field are directed clockwise when viewed from the right end of the conductor as given by the direction of the four fingers.

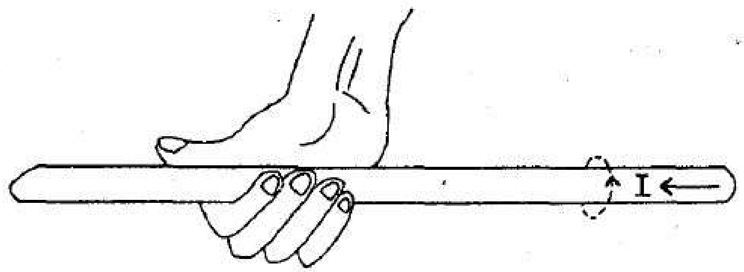


Figure 2.4: Right hand grip rule

Right Handed Cork Screw Rule

In the conductor shown in Fig. 2.5(b) the current direction is shown from left to right. As seen from the left edge of the conductor, the current direction is inward and it sets up a magnetic field in the clockwise direction.
The direction of the magnetic field set up by a current carrying conductor is in the same direction as that of the rotation of a right handed cork screw, so that the screw moves in the same direction as that of the current flow2.5(a).

Now let us apply the foresaid principles to a current carrying solenoid shown in Fig.2.6.

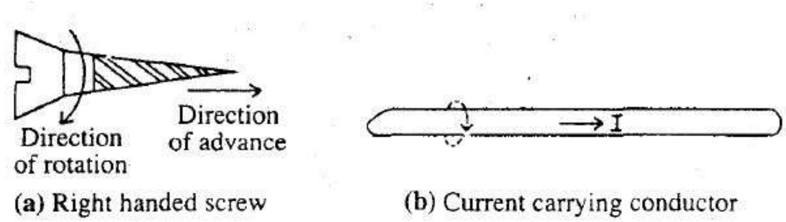


Figure 2.5: Right handed cork screw rule

A solenoid is a piece of iron wound with a coil. The connection of a battery B to the solenoid in the direction shown in Fig.2.6 causes a current I to flow through the solenoid. The direction of the current in the conductors looked at the top is outward. These current carrying conductors set up lines of magnetic field in the anticlockwise direction. The conductors looked at the bottom carry the current inward and hence they set up lines of magnetic field in clockwise direction. In the resultant, the lines of the magnetic field set up by the solenoid now looks very similar to those available around the permanent magnet shown in Fig.2.2. Therefore, by this arrangement we are able to produce a two- pole magnetic field-North pole on the right hand side and South pole on the left hand side. Now to the same solenoid, if the direction of the current is reversed by connecting the battery in the opposite direction, the North pole will appear on the left hand side and the South pole on the right hand side.

Thus, in the electromagnet, apart from establishing a magnetic field using electrical

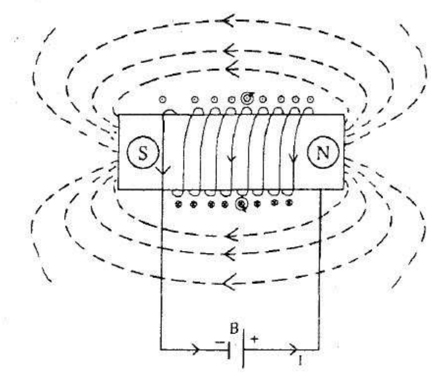


Figure 2.6: Electromagnet

power, we are in a position to control the magnitude and direction of the magnetic field.

2.2 Electromagnetic Induction

2.2.1 Faradays Law I Law:

Whenever the magnetic flux linking with a conductor or coil changes, an emf is induced in it.

2.2.2 Faradays Law II Law:

The magnitude of the emf induced in a conductor or coil is directly proportional to the rate of change of flux linkages.
i.e.,

$$\mathbf{e} \propto \frac{d\phi}{dt}$$

Illustration: Consider a coil connected to a galvanometer and a permanent magnet NS as shown in fig.2.7. When the magnet is brought towards the coil the galvanometer deflects in one direction (due to increase in flux linking the coil) and while the magnet is withdrawn from the coil, the galvanometer deflects in opposite direction (due to reduction of flux linking the coil). The magnitude of induced e.m.f. is proportional to the rate at which the conductor cuts the magnetic flux. Suppose the coil has N

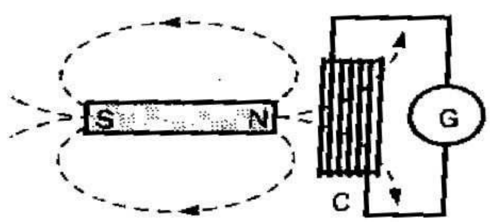


Figure 2.7

number of turns and flux through it changes from a initial value of ϕ_1 Wb to the final value of ϕ_2 Wb in time dt seconds. Then initial flux linkages= N*ϕ_1
And final flux linkages = N*ϕ_2
So induced emf

$$e \propto \frac{(N * \phi_2 - N * \phi_1)}{dt} \text{Wb/sec or Volts}$$
$$\text{or} e \propto N \frac{d\phi}{dt}$$

2.2.3 Lenzs Law:

The direction of an induced e.m.f. is always such that it tends to set up a current opposing the motion or the change of flux responsible for inducing that e.m.f.
The direction of the induced emf or induced current is such that it opposes the change that is producing it. If the current is induced due to motion of the magnet, then the induced current in the coil sets itself to stop the motion of the magnet. If the current is induced due to change in current in the primary coil, then induced current is such that it tends to stop the change.
Illustration: Consider Fig 2.8. When switch is closed, the magnetic flux developed in the ring by the applied current in coil A is in clockwise direction (by applying screw or thumb rule). Consequently the induced current in coil C should try to produce a flux in an anticlockwise direction in the ring. The anticlockwise flux in the ring would require the current in coil C to be passing from X to Y. Hence, this must also be the direction of the e.m.f. induced in C.
Let N be the number of turns in the coil and the flux linking the coil changes from

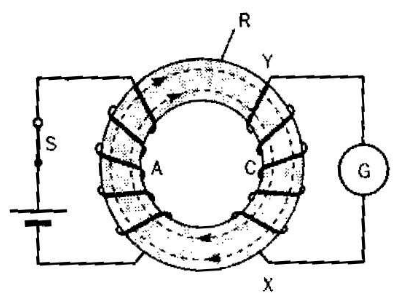


Figure 2.8

an initial value of ϕ_1 to ϕ_2 webers in dt seconds.
Hence, the induced e.m.f.

$$e = -N\frac{(\phi_2 - \phi_1)}{dt}\text{volts}$$

Where, Nϕ_1 Wb turns is called flux linkages. The above Equation can be expressed in differential form by writing

$$e = -N\frac{d\phi}{dt}\text{volts.}$$

The last expression is associated with negative sign to imply that the induced e.m.f. sets up a current which tends to develop a flux to oppose the very cause of its production.

2.2.4 Electromagnetically induced EMF

Types of induced E.M.F.: Induced e.m.f. can be classified into two types-i) Statically induced e.m.f. and ii) Dynamically induced e.m.f.

Statically induced e.m.f.:

When the conductor (coil) is stationary and the magnetic field linking the conductor is changing (moving), the e.m.f. induced in the conductor (coil) is called statically induced e.m.f.
Statically induced e.m.f. is classified into two
a) Self induced e.m.f. b) Mutually induced e.m.f.

Self induced e.m.f.:

This is the e.m.f. induced across the terminals of a coil, by the virtue of the change of flux linking with the coil, due to the change of the current flowing through the same coil. This self induced e.m.f. persists as long as the flux linking with the coil is changing due to the change of current through the same coil.
When a coil is carrying current, a magnetic field is established through the coil. If the current in the coil changes then the flux linking with the coil also changes. Hence an emf is induced in the coil. This is known as self induced emf. The direction of this emf is such as to oppose the cause producing it i.e., it opposes the change of current.

Self Inductance (inductance)(L): Any circuit, in which a change of current is accompanied by a change of flux, and therefore an induced e.m.f. is said to be inductive or to possess self inductance or merely inductance. This is quantified by coefficient of self inductance (L) expressed in the unit of Henry. Self inductance of a coil is defined as the property of the coil due to which it opposes the change in the amount of current flowing through it.
Consider a coil of N turns which is connected to an alternating voltage V due to which an alternating current I flows through the coil. This alternating current produces a alternating flux ϕ, which links the coil and induces an emf in the coil.

$$
\begin{aligned}
E &= -N * \frac{d\phi}{dt} \\
&= -N * \frac{d\phi}{di} * \frac{di}{dt} \\
&= -L * \frac{di}{dt}
\end{aligned}
$$

where

$$
\begin{aligned}
L &= N * \frac{d\phi}{di} \\
L &= N \frac{\phi}{I} \text{H}
\end{aligned}
$$

Here L is the self inductance of the coil.
Self inductance is defined as the ratio of Weber turns per ampere in the coil,that is,

$$L = \frac{N\phi}{I} \text{H}$$

Substituting $\phi = N * I/\Re$
Where, $\Re = \dfrac{l}{\mu_0 \mu_r A}$ is Reluctance of the magnetic circuit on which the coil is wound.
l =length of magnetic circuit, m
μ_0 =permeability of free space in H/m
μ_r =relative permeability of the medium (a constant)
A = area of cross section of magnetic circuit in m^2
We get,
$L = \dfrac{\mu_0 \mu_r A N^2}{l}$ Henry.
Also, $e = -L\dfrac{dI}{dt}$ volts.
Hence the self inductance of the coil is said to be one Henry, if an e.m.f. of one volt is induced across the terminals of a coil with a rate of change of current of one ampere/second.

Mutually induced e.m.f.:

Consider two neighbouring coils A and C, as shown in Figure 2.9, which are magnetically coupled with each other. Whenever the current in a coil A changes, the flux developed due to this current also changes. The changing flux so developed if links the neighboring coil C, an e.m.f. induces across the coil C, which is called mutually induced e.m.f.

The mutually induced e.m.f. persists as long as the current in the coil A is changing and is proportional to the number of turns in coil C and the rate at which the flux linking the coil C is changing. The direction of the mutually induced e.m.f. also follows Lenzs law i.e., it tries to oppose the very cause of its production. **The coef-**

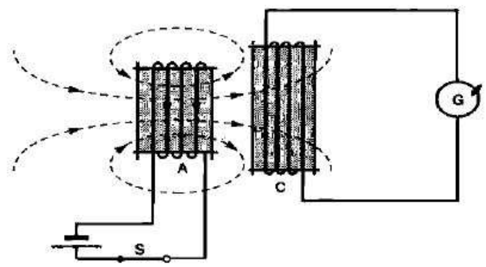


Figure 2.9

ficient of mutual inductance(M): The coefficient of mutual inductance between two coils is defined as the Weber turns in one coil due to one ampere of current in other coil.

That is,

$$M = \frac{N_2 \phi_1}{I_1} \text{Henry} \tag{2.11}$$

Multiplying and dividing RHS of Eqn 2.11 by N_1

$$M = \frac{N_1 N_2 \phi_1}{N_1 I_1}$$
$$= \frac{N_1 N_2}{\Re}$$

where $\Re$ is reluctance of the magnetic circuit on which the two coils are wound.

Or

$$M = \frac{\mu_0 \mu_r A N_1 N_2}{l} \text{Henry} \tag{2.12}$$

Rearranging Eqn 2.11 and differentiating both sides,

$$\begin{aligned} M\frac{dI_1}{dt} &= N_2\frac{d\phi_1}{dt} \\ \text{that is, } e_m &= -N_2\frac{d\phi_1}{dt} \\ e_m &= -M\frac{dI_1}{dt}\text{Volt} \end{aligned}$$

Hence, Mutual inductance between two coils is of one Henry when the e.m.f induced in the second coil is one volt due to a rate of change of current of one ampere per second in the first coil.

2.3 Coefficient of coupling

Consider two neighboring coils A and C having turns N_1 andN_2 respectively, which are magnetically coupled. We can express the coefficient of self inductance of coil A as $L_1 = \frac{\mu_0 \mu_r A {N_1}^2}{l}$ H and the coefficient of self inductance of coil C as $L_2 = \frac{\mu_0 \mu_r A {N_2}^2}{l}$ H.
When a current I_1 ampere flows in coil A, the flux produced is $\phi_1 = \frac{N_1 I_1 \mu_0 \mu_r A}{l}$ wb.

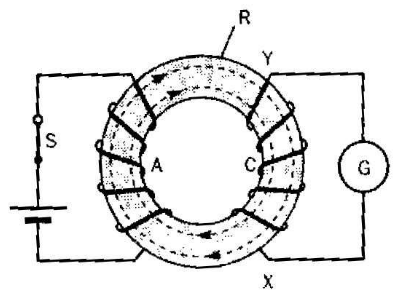


Figure 2.10

If a fraction k_1 of ϕ_1 that is,$k_1\phi_1$ links with coil C, then $M = \frac{N_2 k_1 \phi_1}{I_1}$ H, where $k_1 \leq 1$. Hence,

$$M = \frac{N_2 k_1 N_1 \mu_0 \mu_r A}{l} \text{H}. \tag{2.13}$$

Similarly, if a current I_2 ampere flows in coil C, the flux produced is $\phi_2 = \frac{N_2 I_2 \mu_0 \mu_r A}{l}$ wb.
If a fraction k_2 of ϕ_2 that is,$k_2\phi_2$ links with coil C, where $k_2 \leq 1$ then, the mutual inductance can be expressed also as, Hence,

$$M = \frac{N_1 k_2 N_2 \mu_0 \mu_r A}{l} \text{H}. \tag{2.14}$$

Multiplying eqns 2.13 and 2.14 and taking $k = k_1 = k_2$, the expression for M becomes

$$M^2 = k^2 L_1 L_2 H.$$

The coefficient of coupling is expressed as,

$$k = \frac{M}{\sqrt{L_1 L_2}}. \tag{2.15}$$

2.4 Numerical Problems

1. 1. An air cored solenoid has a length of 50 cm and diameter of 2 cm. Calculate the inductance, if it has 1000 turns.

   Solution

   Area of the core $A = \pi \frac{d^2}{4} = \pi \frac{(2x10^{-2})^2}{4} = 3.1416 * 10 - 4m^2$

   Length of the core l=0.5 m

   Inductance $L = \frac{N^2 \mu_0 \mu_r A}{l}$

   $= \frac{1000 * 1000 * 4 * \pi x 10^{-7} * 3.1416 * 10^{-4} * 1}{0.5} = 0.79$ mH

2. Two coils of 30 and 520 turns are wound side by side on a closed iron circuit of area 100 sqcm and mean length 150 cm. i) estimate the M if the relative permeability of iron is 2000. ii) a current in first grows steadily from 0 to 10 A in 0.01 . Find the e.m.f. induced in the other coil.

   Solution

   Given: N˙1=50, N˙2=520, A=100*10ˆ-4 mˆ2, l=150x10ˆ-2, μ_r=2000.

   i. Mutual Inductance $M = \frac{N_1 N_2 \mu_0 \mu_r A}{l}$

   $M = \frac{30 * 520 * 4 * \pi x 10^{-7} * 2000 * 100 * 10^{-4} * 1}{150x10^{-2}} = 0.2614$ H

   ii. $e_2 = M \frac{dI_2}{dt} = 0.2614 * \frac{10 - 0}{0.01}$

   $e_2 = 26.14$ V

3. Two coils A and B have self inductances of 120 μ H and 200 μ H respectively. When a current of 3 Amp through coil A is reversed, it is found that the flux linkages across coil B are 600 μ wb-turns. Calculate i) Mutual inductance between coils, ii) the average e.m.f. induced in coil B if the flux is reversed in 0.1s iii) coefficient of coupling.

   Solution

   i. Mutual inductance =Change of flux linkage in second coil due to change of current in first coil

   $M = \frac{N_B * \phi_{linkingB}}{I_B}$

   $= \frac{600x10^{-6}}{6} = 100\mu$ H

   ii. Average emf induced in coil B=$e_B = M \frac{dI_A}{dt}$

   $e_B = 100x10^{-6} \frac{-3 - 3}{0.1} = -6m$ V

$|e_2|= 6m\text{V}$

iii. Coefficient of coupling $k = \dfrac{M}{\sqrt{L_A L_B}} = \dfrac{100x10^{-6}}{\sqrt{120x10^{-6}x200x10^{-6}}}$=0.645

2.5 Transformers

One of the most important and widely used electrical machines is the transformer. It receives power at one voltage and delivers it at another. This conversion aids the efficient long-distance transmission of electrical power from generating stations. Since power lines incur significant I^2R losses, it is important to minimize these losses by the use of high voltages. The same power can be delivered by high-voltage circuits at a fraction of the current required for low-voltage circuits.

2.5.1 Construction and Principle of operation of single phase transformer

Figure 2.11 shows the general arrangement of a transformer. A steel core C consists of laminated sheets, about 0.35 - 0.7 mm thick, insulated from one another. The purpose of laminating the core is to reduce the eddy-current loss. The vertical portions of the core are referred to as limbs and the top and bottom portions are the yokes. Coils P and S are wound on the limbs. Coil P is connected to the supply and is therefore termed the primary; coil S is connected to the load and is termed the secondary.

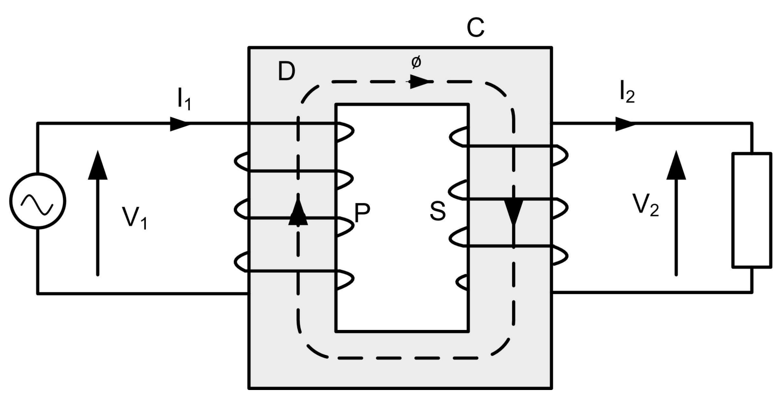


Figure 2.11: A transformer

An alternating voltage applied to P circulates an alternating current through P and this current produces an alternating flux in the steel core, the mean path of this flux being represented by the dotted line D. If the whole of the flux produced by P passes through S, the e.m.f. induced in each turn is the same for P and S. Hence, if N_1 and N_2 are the number of turns on P and S respectively,

$$\frac{\text{Total e.m.f. induced in S}}{\text{Total e.m.f. induced in P}} = \frac{N_2 \times \text{e.m.f. per turn}}{N_1 \times \text{e.m.f. per turn}} = \frac{N_2}{N_1}$$

When the secondary is on open circuit, its terminal voltage is the same as the induced e.m.f. The primary current is then very small, so that the applied voltage V 1 is practically equal and opposite to the e.m.f. induced in P. Hence:

$$\frac{V_2}{V_1} \simeq \frac{N_2}{N_1}$$

Since the full-load efficiency of a transformer is nearly 100 per cent,

$$I_1V_1 \times \text{primary power factor} \simeq I_2V_2 \times \text{secondary power factor}$$

But the primary and secondary power factors at full load are nearly equal,

$$\therefore \frac{I_1}{I_2} \simeq \frac{V_2}{V_1}$$

When the secondary is on open circuit, the primary current is such that the primary ampere-turns are just sufficient to produce the flux necessary to induce an e.m.f. that is practically equal and opposite to the applied voltage. This magnetizing current is usually about 3-5 per cent of the full-load primary current. full-load primary ampere-turns are approximately equal to full-load secondary ampere-turns, i.e.

$$I_1N_1 \simeq I_2N_2$$

so that

$$\frac{I_1}{I_2} \simeq \frac{N_2}{N_1} \simeq \frac{V_2}{V_1}$$

It will be seen that the magnetic flux forms the connecting link between the primary and secondary circuits and that any variation of the secondary current is accompanied by a small variation of the flux and therefore of the e.m.f. induced in the primary, thereby enabling the primary current to vary approximately proportionally to the secondary current.

2.5.2 Classification of Transformers based on construction

The types of transformers differ in the manner in which the primary and secondary coils are provided around the laminated steel core. According to the design, transformers can be classified into two:

Core- Type Transformer

If the windings are wound around the core in such a way that they surround the core ring on its outer edges, then the construction is known as the closed core type construction of the transformer core. In this type, half of the winding is wrapped around each limb of the core, and is enclosed such that no magnetic flux losses can occur and the flux leakages can be minimized.

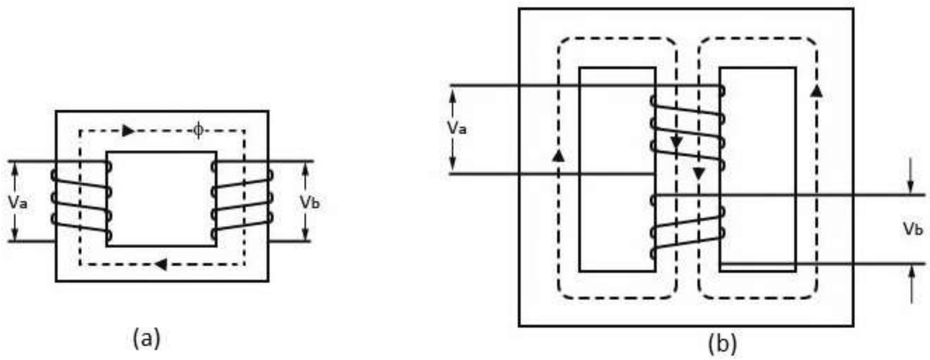


Figure 2.12: Types of Transformers:(a) Core-type (b) Shell-type

Shell- Type Transformer

In shell type construction of the core, the windings pass through the inside of the core ring such that the core forms a shell outside the windings. This arrangement also prevents the flux leakages since both the windings are wrapped around the same center limb.

2.5.3 EMF equation

Suppose the maximum value of the flux to be ϕ_m webers and the frequency to be f hertz. From figure 2.13 it is seen that the flux has to change from $+\phi_m$ to $-\phi_m$ in half a cycle, namely in 1/2f seconds.

$$\begin{aligned}\therefore \text{Average rate of change of flux} &= 2\phi_m \div (1/2f) \\ &= 4f\phi_m \text{ webers per second}\end{aligned}$$

and average e.m.f. induced per turn is 4f ϕ_m volts

But for a sinusoidal wave the r.m.s. or effective value is 1.11 times the average value,

$\therefore$ RMS value of e.m.f. induced per turn = 1.11 $\times$ 4f ϕ_m

Hence, r.m.s. value of e.m.f. induced in primary is

$$E_1 = 4.44 N_1 f \phi_m \text{ volts} \tag{2.16}$$

and r.m.s. value of e.m.f. induced in secondary is

$$E_2 = 4.44 N_2 f \phi_m \text{ volts} \tag{2.17}$$

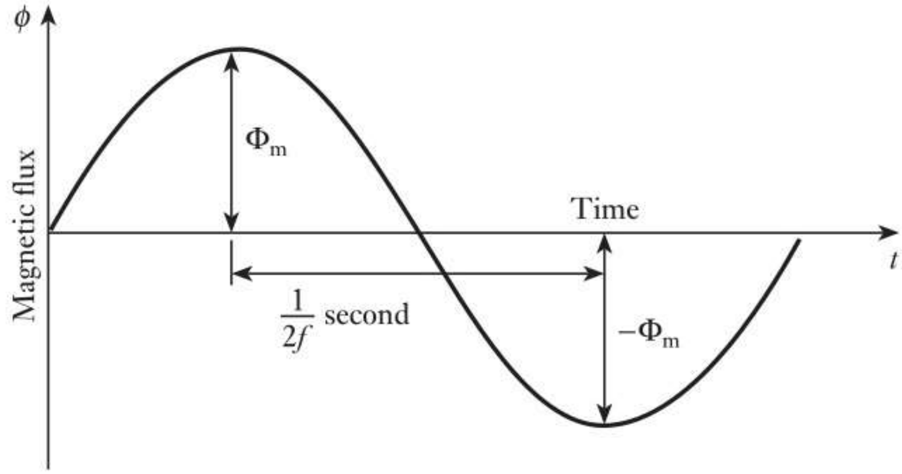


Figure 2.13: Waveform of flux variation

2.5.4 Losses in a transformer

The flux linking coils can be greatly improved by the introduction of a ferromagnetic core. When the core is energized from an a.c. source, the magnetizing force rises and falls in accordance with the magnetizing current which is basically sinusoidal. This variation does not cause B and H to vary according to the magnetic characteristic, but rather as shown in figure 2.14. This loop is called the hysteresis loop.

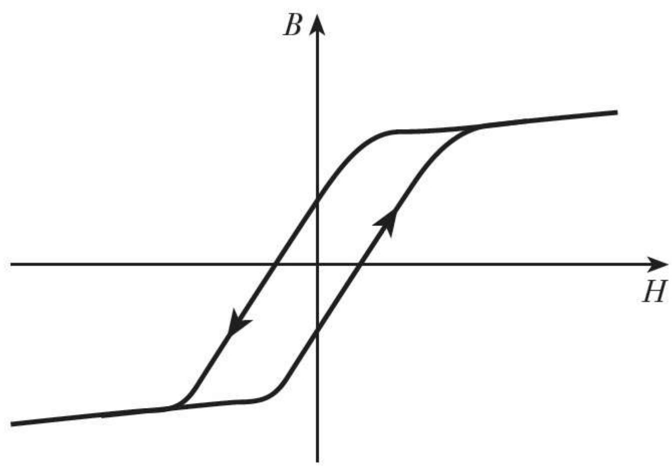


Figure 2.14: Hysteresis loop

The larger the hysteresis loop the greater the energy required to create the magnetic field and this energy has to be supplied during each cycle of magnetization. This requirement of supplying energy to magnetize the core is known as the hysteresis loss.

The varying flux in the core induces e.m.f.s and hence currents in the core material. These give rise to I^2R losses. These losses are called eddy-current losses. The sum of the hysteresis loss and the eddy-current losses is known as the core loss.

The losses which occur in a transformer on load can be divided into two groups:
1. I^2R losses in primary and secondary windings, namely $I_1^2R_1 + I_2^2R_2$.
2. Core losses due to hysteresis and eddy currents.

Since the maximum value of the flux in a normal transformer does not vary by more than about 2 per cent between no load and full load, it is usual to assume the core loss constant at all loads. Hence, if P˙c = total core loss, total losses in transformer are

$$P_c + I_1^2R_1 + I_2^2R_2$$

2.5.5 Efficiency of a transformer

The efficiency of a transformer at a particular load and power factor is defined as the ratio of power output to power input.

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power + losses}}$$

$$\text{Efficiency} = \frac{I_2V_2 \times p.f.}{I_2V_2 \times p.f. + P_c + I_1^2R_1 + I_2^2R_2}$$

In general:

$$\text{Efficiency} = \frac{\text{output power}}{\text{input power}} = \frac{\text{input power - losses}}{\text{intput power}}$$

$$\eta = 1 - \frac{losses}{\text{intput power}}$$

2.5.6 Condition for maximum efficiency of a transformer

If R_{2e} is the equivalent resistance of the primary and secondary windings referred to the secondary circuit,

$$\begin{aligned} R_{2e} &= R_1\left(\frac{N_2}{N_1}\right)^2 + R_2 \\ &= \text{a constant for a given transformer} \end{aligned}$$

Hence for any load current I_2: Total I^2R loss= I^2R_{2e}

$$\text{and Efficiency} = \frac{I_2V_2 \times p.f.}{I_2V_2 \times p.f. + P_c + I_2^2R_{2e}}$$

$$\text{Efficiency} = \frac{V_2 \times p.f.}{V_2 \times p.f. + (P_c/I_2) + R_{2e}}$$

For a normal transformer, V_2 is approximately constant, hence for a load of given power factor the efficiency is a maximum when the denominator of above equation is a minimum, i.e. when

$$\frac{d}{dI_2}\left(V_2 \times p.f. + (P_c/I_2) + R_{2e}\right) = 0$$

$$\therefore -(P_c/I_2^2) + R_{2e} = 0$$

or $I_2^2 R_{2e} = P_c$

Hence the efficiency is a maximum when the variable I^2R loss is equal to the constant core loss.

2.5.7 Voltage regulation of a transformer

The voltage regulation of a transformer is defined as the variation of the secondary voltage between no load and full load, expressed as either a per-unit or a percentage of the no-load voltage, the primary voltage being assumed constant, i.e.

$$\text{Voltage Regulation} = \frac{\text{no-load voltage - full-load voltage}}{\text{no-load voltage}}$$

2.6 Numerical Problems

1. A 250 kVA, 11 000 V/400 V, 50 Hz single-phase transformer has 80 turns on the secondary. Calculate:
   (a) the approximate values of the primary and secondary currents;
   (b) the approximate number of primary turns;
   (c) the maximum value of the flux.
   Solution
   (a) Full-load primary current

$$\simeq 250 \times 1000/11000 = 22.7A$$

   and full-load secondary current

$$\simeq 250 \times 1000/400 = 625A$$

   (b) No. of primary turns

$$\simeq 80 \times 11000/400 = 2200$$

   (c) From expression 2.17

$$\begin{aligned} 400 &= 4.44 \times 80 \times 50 \times \phi_m \\ \phi_m &= 22.5mWb \end{aligned}$$

2. A single-phase transformer has 480 turns on the primary and 90 turns on the secondary. The mean length of the flux path in the core is 1.8 m and the joints are equivalent to an airgap of 0.1 mm. The value of the magnetic field strength for 1.1 T in the core is 400 A/m, the corresponding core loss is 1.7 W/kg at 50 Hz and the density of the core is 7800 kg/m^3 . If the maximum value of the flux density is to be 1.1 T when a p.d. of 2200 V at 50 Hz is applied to the primary, calculate:
   (a) the cross-sectional area of the core;
   (b) the secondary voltage on no load;
   Solution
   (a) From eqn. 2.16,

$$\begin{aligned} 2200 &= 4.44 \times 480 \times 50 \times \phi_m \\ \phi_m &= 0.0206Wb \end{aligned}$$

   and cross-sectional area of core is

$$0\ .\ 0206\ /\ 1.1 = 0.0187\ m^2$$

This is the net area of the core; the gross area of the core is about 10 per cent greater than this value to allow for the insulation between the laminations.

(b) Secondary voltage on no load is

$$2200 \times 90\ /480 = 413 \text{ V}$$

3. The primary and secondary windings of a 500 kVA transformer have resistances of 0.42 Ω and 0.0019 Ω respectively. The primary and secondary voltages are 11000 V and 400 V respectively and the core loss is 2.9 kW, assuming the power factor of the load to be 0.8. Calculate the efficiency on (a) full load; (b) half load.

Solution

(a) Full-load secondary current is

$$\frac{500 \times 1000}{400} = 1250A$$

and Full-load primary current is

$$\frac{500 \times 1000}{11000} = 45.5A$$

Therefore secondary I^2R loss on full load is $(1250)^2 \times 0.0019 = 2969W$

and primary I^2R loss on full load is $(45.5)^2 \times 0.42 = 870W$

$\therefore$ Total I^2R loss on full load= 3839 W = 3.84 kW

and Total loss on full load = 3.84 + 2.9 = 6.74 kW

Output power on full load = 500 0.8 = 400 kW

$\therefore$ Input power on full load = 400 + 6.74 = 406.74 kW

Efficiency on full load is

$$\left(1 - \frac{6.74}{406.74}\right) = 0.983 \text{per unit} = 98.3 \text{per cent}$$

(b) Since the I^2R loss varies as the square of the current, $\therefore$ Total I^2R loss on half load= 3.84 × $(0.5)^2$= 0.96 kW

and Total loss on half load = 0.96 + 2.9 = 3.86 kW

Output power on half load = 0.5 × 500 0.8 = 200 kW

$\therefore$ Input power on half load = 200 + 3.86 = 203.86 kW

$\therefore$ Efficiency on full load is

$$\left(1 - \frac{3.86}{203.86}\right) = 0.981\text{per unit} = 98.1\text{per cent}$$

2.7 Auto-transformer

A transformer in which part o the winding is common to both primary and secondary circuits, is known as an auto-transformer. The primary is electrically, as well as magnetically coupled to it; thus, in figure 2.15 winding AB has a tapping at C, the load being connected across CB and the supply voltage applied across AB.

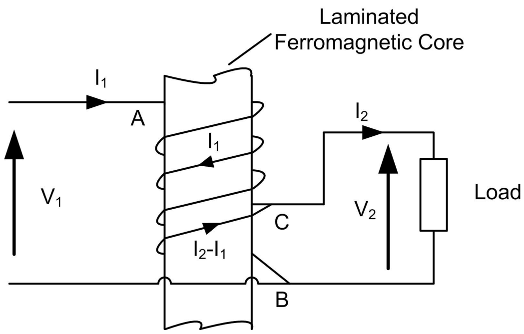


Figure 2.15: An auto-transformer

I_1 and I_2	=	primary and secondary currents respectively
N_1	=	no. of turns between A and B
N_2	=	no. of turns between C and B
n	=	ratio of the smaller voltage to the larger voltage

Neglecting the losses, the leakage reactance and the magnetizing current, we have for

$$n = \frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

The nearer the ratio of transformation is to unity, the greater is the economy of conductor material. Also, for the same current density in the windings and the same peak values of the flux and of the flux density, the Iˆ2R loss in the auto-transformer is lower and the efficiency higher than in the two- winding transformer.

Advantages of auto-transformer:

An auto-transformer has the following advantages:

1. Higher efficiency.
2. small size.
3. Lower cost.
4. Better voltage regulation when compared with a conventional two-winding transformer of same rating.

Auto-transformers are mainly used for the following applications:

1. As a regulating transformer.
2. A continuously variable auto-transformer finds useful applications in electrical testing laboratory
3. To obtain partial line voltages for starting induction and synchronous motors with squirrel-cage windings.
4. To give a small boost to a distribution cable to correct for the voltage drop.
5. As furnace transformers for getting a convenient supply to suit the furnace winding from a 230 V supply

2.8 DC Machines

DC generators and motors are collectively known as DC machines. A DC generator converts mechanical energy into electrical energy. It works on the principle of Faradays laws of electro magnetic induction. A DC motor converts electrical energy into mechanical energy. It works on the principle that a current carrying conductor kept in a magnetic field experiences a force. DC motors have a wide range of applications such as electric locomotives, textile mills, cranes etc.

2.8.1 Force on current carrying conductor

Let a straight conductor, carrying a current is placed in a magnetic field as shown in the Fig.2.16(a).
Now, current carrying conductor also produces its own magnetic field around it. Assuming current direction away from observer i.e. into the paper, the direction of its flux can be determined by right hand thumb rule. This is clockwise as shown in the Fig, 2.16(b). [For simplicity, flux only due to current carrying conductor is shown in the Fig. 2.16 (b).]

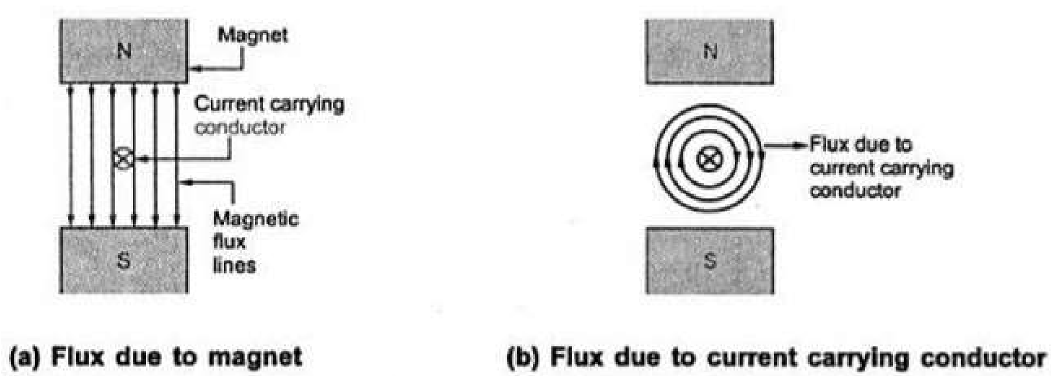


Figure 2.16: Current carrying conductor placed in a magnetic field

Now there is presence of two magnetic fields namely due to permanent magnet and due to current carrying conductor. These two fluxes interact with each other. Such interaction is shown in the Fig, 2.17 (a).
This interaction as seen is in such a way that on one side of the conductor the two lines help each other, while on other side the two try to cancel each other. This means on left hand side of the conductor shown in the Fig. 2.17 the two fluxes are in the same direction and hence assisting each other , As against this, on the right hand side of the conductor the two fluxes are in opposite direction hence trying to cancel each other. Due to such interaction on one side of the conductor, there is accumulation of

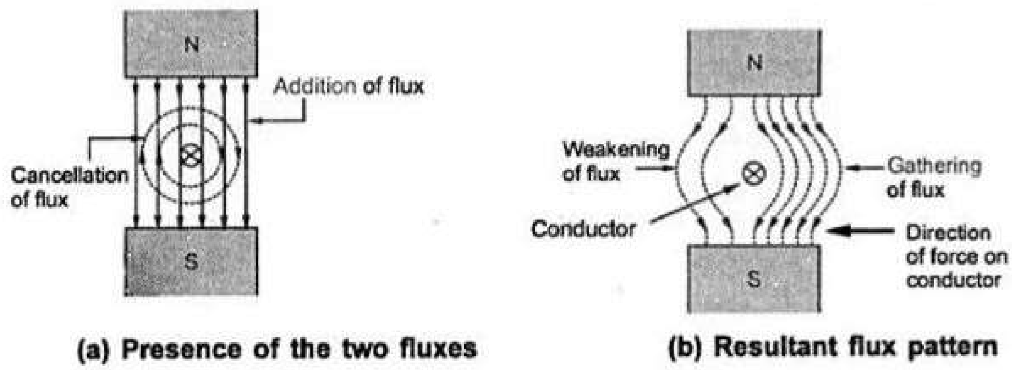


Figure 2.17: Interaction of two flux lines

flux lines (gathering of the flux lines) while on the other side there is weakening of the flux lines.

The resultant flux pattern around the conductor is shown in the Fig. 2.17(b).

According to properties of flux lines, these flux lines will try to shorten themseleves. While doing so, flux lines which gathered will exert force on conductor. So conductor experiences a mechanical force from high flux lines area towards low flux lines area i.e. from left to right for a conductor shown in Fig 2.17.

The force F on the conductor is given by

$$F \propto BlI \tag{2.18}$$

where B is the magnetic field density, l is the length of the conductor and I is the current through the conductor.

This is the basic principle on which DC electric motors work.

2.8.2 Fleming's Rules:

Fleming's Right Hand Rule:

When the thumb, the fore finger and the middle finger of the right hand are held perpendicular to each other, the first finger of the right hand is pointed in the direction of the magnetic flux and if the thumb is pointed in the direction of motion of the conductor relative to the magnetic field, then the second finger represents the direction of the e.m.f.

Fleming's Left Hand Rule

: If the first three fingers of the left hand are held mutually at right angles to each other, with First finger pointing the direction of the flux, second finger pointing the

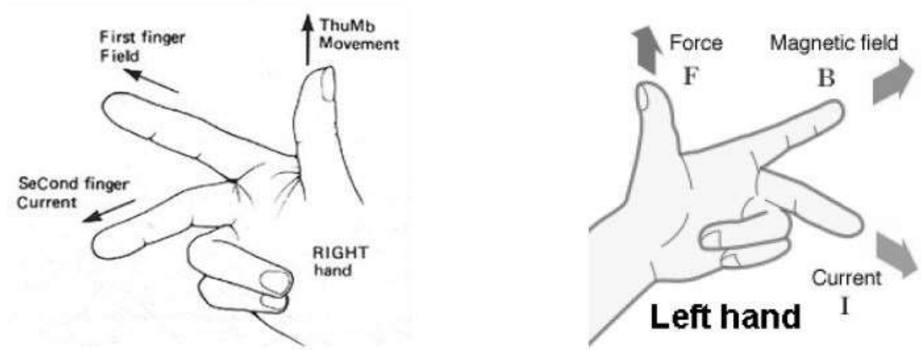


Figure 2.18: a. Fleming's right hand rule b. Fleming's left hand rule

direction of the current, then thumb indicates the direction of the Mechanical force exerted by the conductor.

2.8.3 Construction of DC Machines

The d.c. generators and d.c. motors have the same general construction. In fact, when the machine is being assembled, the workmen usually do not know whether it is a d.c. generator or motor. Any d.c. generator can be run as a d.c. motor and vice-versa.
A D.C generator mainly consists of two parts:

1. Armature which is the rotating part which converts mechanical energy into electrical energy.
2. Field which is stationary part which produces the magnetic flux. They are separated by a small air gap

All d.c. machines have following principal components viz.,

1. **Yoke or Magnetic Frame:** Yoke is usually made up of cast iron or cast steel. Yoke provides the mechanical support to the poles and serves as a cover. It is cylindrical in shape. It also provides the path for magnetic flux.
2. **Poles:** The field magnets consist of pole cores and pole shoes. The pole shoes serve two purposes:
(i) They spread out the flux in the air gap and also reduce the reluctance of the magnetic path.
(ii) They support the exciting coils (field coils) The poles are made of an alloy steel of high permeability. The pole core is laminated to reduce the eddy current

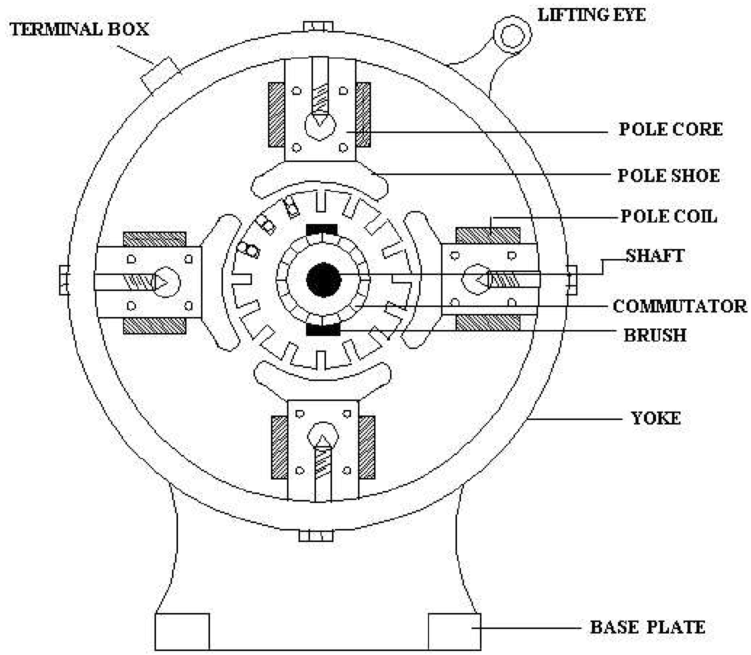


Figure 2.19: Construction of DC machine

losses. The pole core supports the field coils. The function of field windings (or coils) is to provide the number of ampereturns for excitation required to give the flux through the armature.

3. **Armature:** The armature consists of armature core and armature winding. The armature core is made of high permeability silicon steel laminations which are insulated from one another by varnish. It supports the armature conductor. It causes the conductor to rotate between the magnetic field and it provides low reluctance path for the magnetic field produced by the field coils. The conductors placed in the slots are not only insulated from one another but also from the slots of the armature core. The armature conductors are connected together either as a lap winding or wave winding.

   Armature windings: There are two type of armature windings: (i) Lap windings and ii) Wave windings

   Lap windings: In Lap winding the finishing point of one coil is connected to the starting point of next coil as shown in fig2.20(a). In this winding the number of parallel paths is equal to the number of poles and the total armature current divides equally among the parallel paths. These are used in low voltage high current machines.

   Wave Windings: In wave winding the finishing point of one coil is connected to the starting point of another coil, which is well away from the first coil as

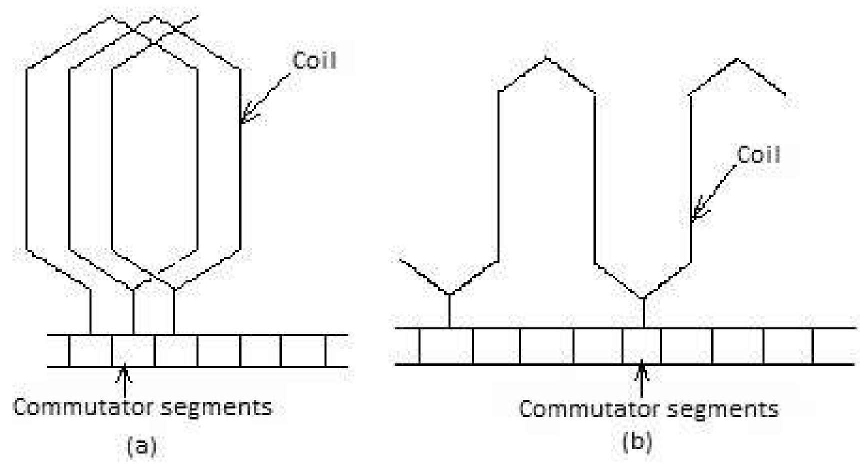


Figure 2.20: a. Lap winding b. Wave winding

shown in fig2.20(b). In this winding there are only two parallel paths irrespective of the number of poles. Wave windings are used in high voltage low current machines.

4. **Commutator:** The commutator converts the alternating E.M.F. generated in the armature winding into direct current voltage in the external circuit. The commutator is cylindrical in shape and is built of wedge shaped segment made of hard drawn copper which are insulated from one another and from the shaft by mica strips. The segments are connected to the armature conductors.

5. **Shaft and Bearings:** The shaft of a DC generator is rotated by a prime mover to which the armature fixed to it also rotates.

6. **Brushes:** They are made of carbon. Brushes are fixed in brush holders and with the help of springs, are made to contact the commutator segments. DC output voltage is taken out through these brushes.

2.8.4 Working principle of DC generator

A DC generator works on the principle of Faradays laws of electro magnetic induction. A coil 'ABCD' is rotated in a magnetic field produced by a permanent magnet or an electro magnet. When the plane of the coil is at right angles to the direction of the lines of flux, the coil will be moving parallel to the lines of flux. Hence the coil does not cut any flux and the induced e.m.f. is zero. When the plane of the coil is parallel to the lines of flux, flux cut is maximum and hence maximum e.m.f is induced in the coil. Since the coil sides alternately come under north and south poles, the direction of the induced voltage in the coil reverses at regular intervals and thus we get an a.c voltage across the terminals of the coil. To convert this a.c. voltage into unidirectional voltage, a copper drum is mounted on the same shaft as that of the coil. The drum is split into two halves and insulation is placed between them. The two ends of the coil are connected to the two halves of the drum. Two fixed carbon brushes B˙1 and B˙2 make contact with the surface of the drum. The voltage between the brushes becomes unidirectional as brush B˙1 always makes contact with the coil side under North Pole and the brush B˙2 always makes contact with the coil side under South Pole.

The wave forms of the voltage induced in the coil and the voltage between the

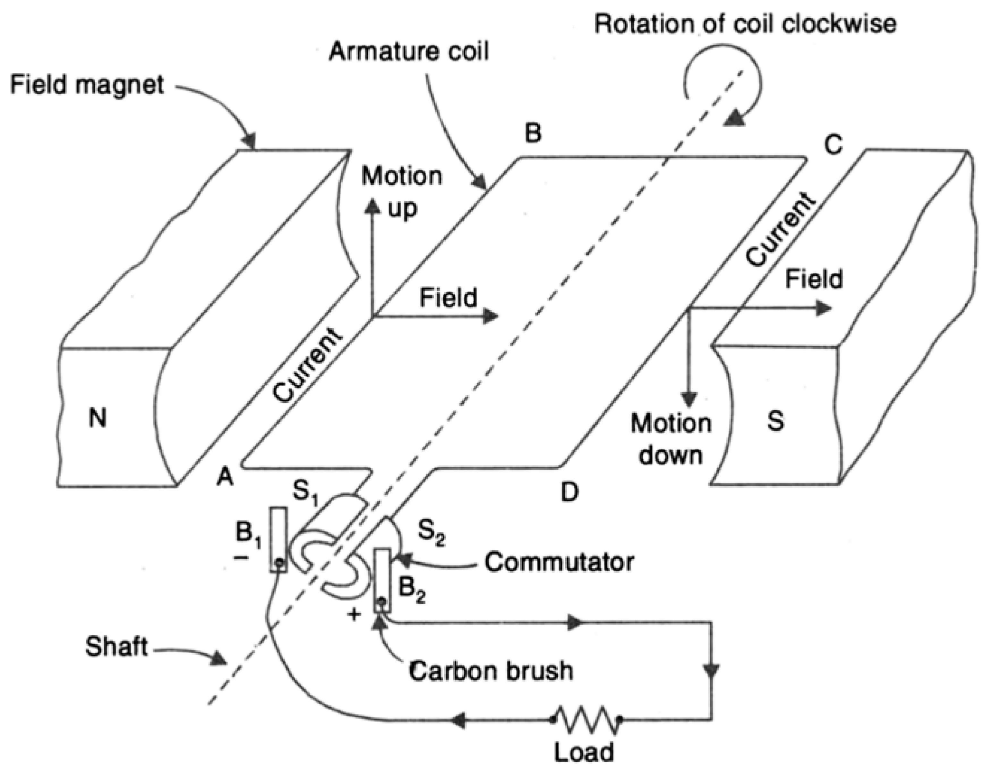


Figure 2.21: Working principle of DC generator

brushes B˙1 and B˙2 are as shown in Fig 2.22.

2.8.5 E.M.F. equation of DC generator

Let

$$\phi = \text{Flux/pole in Wb}$$
$$Z = \text{Total number of armature conductors or coil sides on armature}$$
$$= \text{Number of slots} \times \text{Number of conductors/slot.}$$
$$P = \text{No. of poles}$$

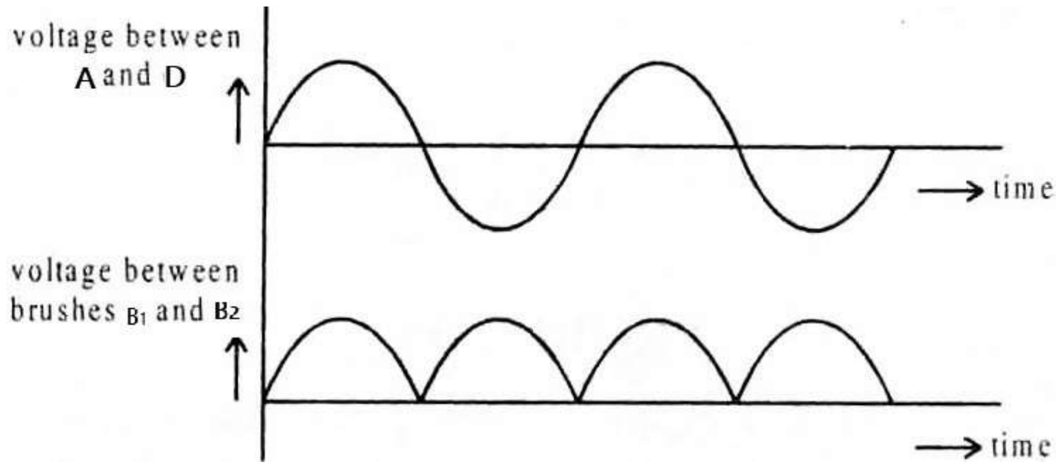


Figure 2.22: Voltage waveforms

A = Number of parallel paths.

N = Speed of armature in r.p.m.

The E.M.F. induced in a conductor when rotated in a magnetic field is directly proportional to the rate of change of flux.
The flux cut by a conductor in one revolution, $d\phi = P\phi$ Wb.
The time taken by the conductor to make one revolution $= dt = \dfrac{60}{N}$ sec.
According to Faraday's laws of electromagnetic induction,

$$\text{the E.M.F. induced in one conductor} = \frac{d\phi}{dt} = \frac{P\phi}{\left(\frac{60}{N}\right)} = \frac{\phi PN}{60} \tag{2.19}$$

Generated e.m.f.E_g = e.m.f. generated/ parallel path.

= E.M.F. induced/conductor × number of conductors /parallel path

$$E_g = \frac{\phi ZNP}{60A}$$

2.9 DC Motors

A DC motor is a machine which converts electrical energy into mechanical energy. It is similar in construction to a DC generator. A DC machine can work both as a generator and a motor. As a matter of fact any DC generator will run as motor when its field and armature windings are connected to a source of direct current. The field winding produces the necessary magnetic field and the flow of current through the armature conductor produces a force which rotates the armature.

2.9.1 Principle of Operation of DC Motor

"Whenever a current carrying conductor is placed in a magnetic field, it experiences a force whose direction is given by Fleming's Left Hand Rule".

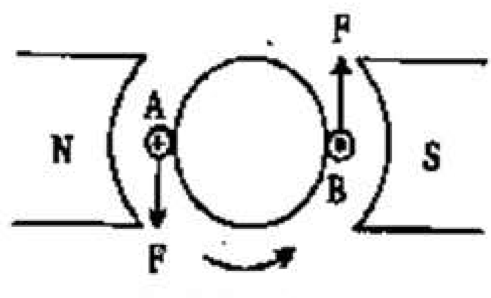


Figure 2.23: Working principle of DC Motor

A coil AB is kept around the armature and the current is passed through it. The side A experiences a force F downward and the side B experiences a force F upward, hence there is net turning moment in the anticlockwise direction. When coils are placed throughout the surface of armature and current passed through them, the armature will experience a continuous force and start rotating.

2.9.2 Back E.M.F. or Counter E.M.F

In a DC motor when the armature rotates, the conductors on it cut the magnetic field in which they revolve, so that an E.M.F. is induced in the armature. The induced E.M.F. acts in opposition to the current in the machine and therefore, to the applied voltage, so that it is customary to refer to this voltage as the back E.M.F. (As per Lenzs Law). The magnitude of this back E.M.F. can be calculated by using formula for the induced E.M.F. in a generator, proportional to the product of the flux and the speed, denoted by Eb.

$$E_b = \frac{\phi ZNP}{60A}$$

The value of this E.M.F. is always lesser than the applied voltage. This difference actually drives current through resistance of the armature circuit. If this resistance is represented by Ra, the back E.M.F. by Eb and the applied voltage by V, then

$$V = E_b + I_a R_a$$

$$\text{Armature current} I_a = \frac{V - E_b}{R_a}$$

The voltage V applied across the motor armature has to

1. Overcome the back emf Eb and
2. Supply the armature ohmic drop I˙aR˙a

Therefore V= E˙b +I˙a R˙a This is known as the voltage equation of a motor. Multiplying both sides by Ia, we get

$$VI_a = E_b I_a + I_a^2 R_a$$

Here, V Ia=Electrical input to the motor, Eb Ia= Electrical equivalent of mechanical power developed in the armature, Ia^2 Ra= Cu loss in the armature

2.9.3 Armature torque of a motor

The torque is meant the turning moment or twisting moment of a force about an axis. Let Ta be the torque developed by the armature of a motor in N-m, N is speed in r.p.m,

$$\text{Then power developed} = T_a \omega$$

Here omega (w) is the angular velocity in radian / second

$$\text{If N is in rpm} = \omega = \frac{2\pi N}{60} \text{rad/sec}$$

Then power developed =2NTa /60 watts (work done /second).

$$\text{Then power developed} = \omega = \frac{2\pi N T_a}{60} \text{workdone/sec}$$

But Electrical power converted into mechanical power in the armature = E˙bI˙a watts. Comparing two expressions,

$$\frac{2\pi N T_a}{60} = E_b I_a$$

$$\text{substituting for back emf}, E_b = \frac{\phi ZNP}{60A}$$

$$T_a = 0.159\phi Z I_a \frac{P}{A}$$

2.9.4 Shaft torque of a DC motor

The armature torque is the gross torque, which is developed by the armature. A certain percentage of torque is developed by the armature is lost to overcome the iron and friction losses. Net torque (gross torque torque lost in iron and friction losses) is known as shaft torque.

1. If Ta is torque developed by armature in N-m.,
2. T is torque lost in iron and friction losses.
3. T˙sh is the shaft torque or useful torque.

$$T_a = \frac{E_b I_a}{\frac{2\pi N}{60}}$$

$$T = \frac{W_i}{\frac{2\pi N}{60}}$$

$$T_{sh} = \frac{E_b I_a - W_i}{\frac{2\pi N}{60}}$$

$$T_{sh} = \frac{P_{out}}{\frac{2\pi N}{60}} = \frac{BHP \times 735.5}{\frac{2\pi N}{60}}$$

$$T_{sh} = \frac{BHP \times 735.5}{\frac{2\pi N}{60}}$$

2.10 Types of DC Motors

Depending on how the field winding is connected to the armature, dc motors can be classified as:

1. Shunt motors
2. Series motors
3. Compound motors
   (a) Cumulatively compound
   (b) Differentially compound

Shunt motors:

In a shunt motor the field winding is connected in parallel with the armature as shown in figure given below.

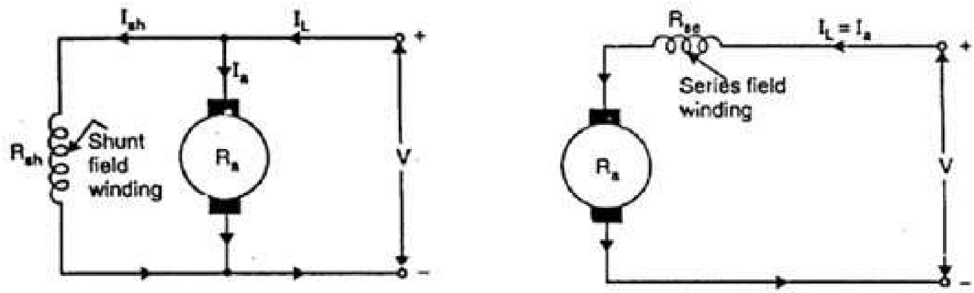


Figure 2.24: DC shunt Motor and series motor

I_L is the line current, I_f is the field current. I_f=V/Rsh where Rsh is the resistance of shunt field winding.

Armature current I_a=I_L-I_f and Back emf $E_b = V - I_aR_a$,

Brushes which are made of carbon will have a resistance hence voltage drop Back emf $E_b = V - I_aR_a - 2$(voltage drop per brush) ,

Series motors:

In a Series motor the field winding is connected in series with the armature as shown in Fig.b

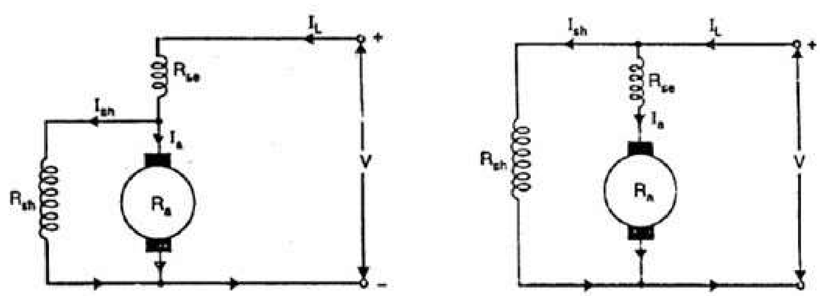


Figure 2.25: DC compound Motor (long shunt and short shunt)

It is observed that the line current I_L, Field current I_{se} and the armature current I_a are one and the same.

Back emf $E_b = V - I_a(R_a + R_{se}) - 2$(voltage drop per brusg) ,
where Rsc is the resistance of the series field windings.

2.10.1 Compound Motor

A compound motor has both the series and the shunt field windings as shown in Figure above. That may be a short shunt or long shunt.

If the flux produced by series field winding is in the same direction as that of the flux produced by the shunt field winding, then it is called a cumulatively compound motor. If the series flux opposes the shunt field flux, then it is called a differentially compound motor. These two are again classified in to long shunt and short shunt. In long shunt compound motor, the shunt field is in parallel with armature and series field in series. In short shunt, the shunt field is in parallel with armature alone

2.11 Characteristics of DC shunt motors

Ta/Ia Characteristics (Electrical characteristics):

$T_a = 0.159\phi Z I_a \frac{P}{A}$
We know that for shunt motor $T_a \propto \phi I_a$
Since flux is almost remans practically constant
$T_a \propto I_a$

The plot is a straight line as shown. The shaft torque is shown dotted. Since a heavy starting load will need a heavy starting current, shunt motor should never be started on heavy load.

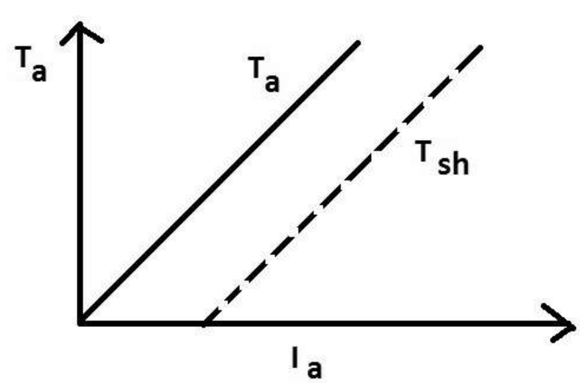


Figure 2.26: Working principle of DC generator

N/Ia Characteristics

From the expression, $E_b = \phi ZNP/(60A)$
$E_b \propto \phi N$

If flux is assumed constant, then N $\propto$ Eb. As Eb is also practically constant, speed is constant. But strictly speaking, both Eb and ϕ decrease with increase in load. Therefore there is some decrease in speed shown in dotted lines. The speed remains almost constant with increase in load current. Therefore DC shunt motor is also called as a constant speed motor.

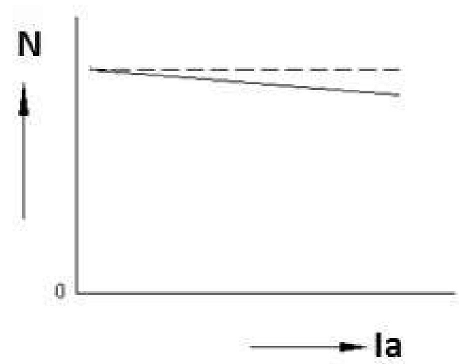


Figure 2.27: Speed v/s atmature current characteristic

N/Ta Characteristics

As Ta proportional to Ia , N /Ta plot will be same as N/ Ia plot.

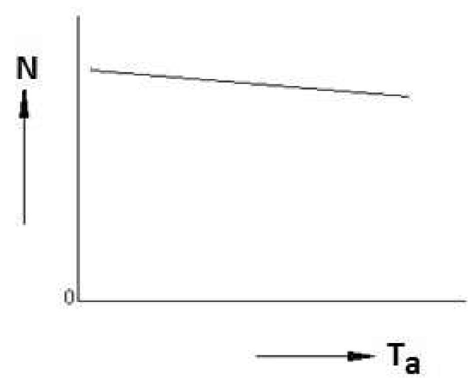


Figure 2.28: Speed v/s torque characteristic

2.12 Characteristics of DC series Motor

2.12.1 Ta /Ia Characteristics

We know that $T_a = 0.159\phi Z I_a \frac{P}{A}$
$T_a \propto \phi I_a$
$\phi \propto I_a$

$T_a \propto I_a^2$

As Ia increases, Ta increases as the square of the current Ia. Once the saturation is reached flux remains constant, Ta $\propto$ Ia. Thus it is known that the starting torque of a series motor is high.

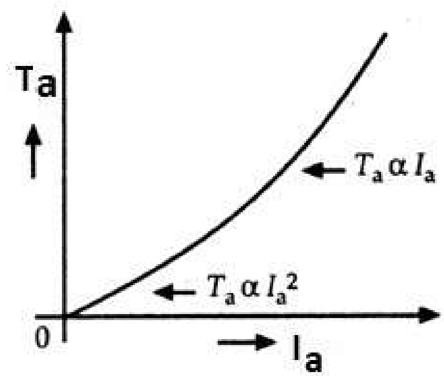


Figure 2.29: Speed v/s torque characteristic

2.12.2 N /Ia Characteristics

$N \propto Eb/\phi$

$N \propto V - Ia(Ra + Rse)/\phi$

Change in Eb for various load currents is small and hence may be neglected. Hence speed varies inversely as armature current. When load is heavy, Ia is large. Hence speed is low. But when load is light, Ia is low, speed becomes dangerously high. Hence a series motor should never be started without the mechanical load on it.

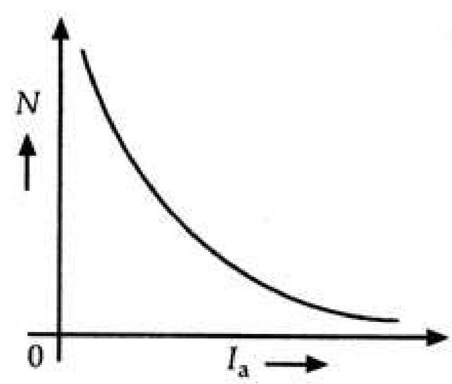


Figure 2.30: Speed v/s torque characteristic

2.12.3 N /Ta Characteristics

W.K.T. $N \propto \frac{V - I_a R_a}{\phi}$

$Ta \propto Ia^2$

$T_a \propto \sqrt{Ia}$
$N \propto \frac{1}{\sqrt{T}}$

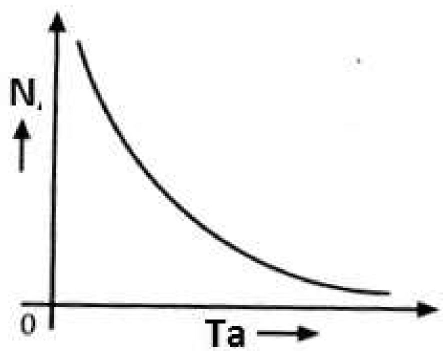


Figure 2.31: Speed v/s torque characteristic

2.13 Applications of DC motors

2.13.1 DC Shunt motor

Employed for constant speed applications; may be used for adjustable speed not greater than 2:1 range. Field of applications include lathes, centrifugal pumps, fans and blowers, machine tools, wood working machines, reciprocating pumps, spinning and weaving machines, printing presses etc.

2.13.2 DC Series motor

Suitable for drives requiring high starting torque and where adjustable, varying speed is satisfactory. Field of applications includes cranes, hoists, trolley cars, conveyors, electric locomotives etc.

2.13.3 Cumulatively Compound DC motor

Suitable for drives requiring high starting torque and only fairly constant speed, pulsating loads with fly wheel action. Field of applications includes shears, punches, elevators, conveyors, rolling mills, heavy planes etc.

2.13.4 Differentially Compound DC motor

suitable for drives requiring wide variation in speed.

2.14 Need for a starter

From the equation for the back e.m.f of a motor,Eb = V-IaRa , armature current is given by Ia = (V-Eb)/Ra where V is the supply voltage, Eb is the back Ra is the armature resistance. When the motor is at rest, there is no back E.M.F developed in the armature. Hence full supply voltage is applied across the stationary armature. It draws a very large current since armature resistance is small. Thus the armature windings get heated up severely and hence get damaged. Therefore at starting a resistance R is connected in series with armature so that the starting current is limited to a safe value. Once the motor picks up speed, back E.M.F is built up and Ia automatically reduces. Now the external resistance R can be gradually reduced to zero as the motor picks up speed.

2.15 Synchronous Machines

2.15.1 Introduction

An electrical machine, which converts mechanical energy into electrical energy of alternating current nature, is called an ALTERNATOR or AC GENERATOR. It is also called a SYNCHRONOUS GENERATOR as its operation is in synchronous with other generators or other AC sources when it is operated along with them. It operates on the fundamental principle of Faraday's Laws of electromagnetic induction, i.e. whenever the magnetic flux linking the armature circuit changes an Electro Motive Force is induced in the circuit, compared to the DC generators, the major differences in construction of AC generators are:

1. In a DC generator the armature rotates and the field system is stationary whereas in an alternator, the armature winding is mounted on a stationary frame and the field winding on a rotating frame.

2. The armature of an alternator is connected to produce AC supply and the field winding is connected to DC supply. Hence no commutator is required in an alternator, which makes the construction simpler.

2.15.2 Construction

An alternator consists of two parts stator and rotor.

1. Stator: It consists of stator frame, stator core and windings.

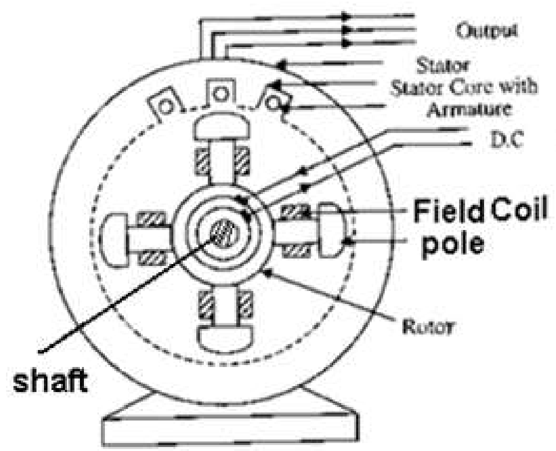


Figure 2.32: Construction of an alternator

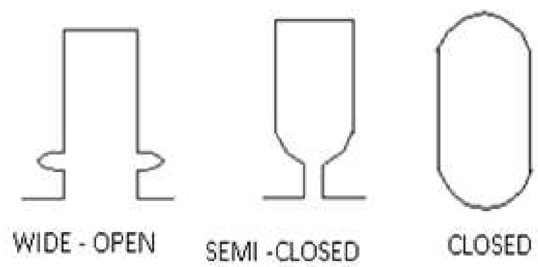


Figure 2.33: Stator slots in the alternator

(a) Stator Frame: It is a cast iron or welded steel protective frame and gives support to the entire machine assembly. In small machines it is made of a single piece of cast iron. In large sized machines, the frame is fabricated by sections of cast iron sheet steel welded together to form a cylindrical drum.

(b) Stator Core: It is made of special magnetic iron or steel alloy laminations. They are laminated to minimize the core losses. These laminations are insulated from one another and pressed together to form the core. Slots are provided on its inner periphery to house the stator conductors. Slots provided on the stator are of three types: Wide Open, Semi-Closed, Wholly closed as shown in Fig.2.33. The wide open type slots are more commonly used because the defective coils can be easily removed and replaced. The laminations also have openings which make axial and radial ventilations for the purpose of cooling.

(c) Stator Windings (armature windings):These are insulated copper conductors housed in stator slots in some specific manner of inter connections.

2. Rotor: It is the rotating part, with North and South poles attached to it. Poles carry field windings which are supplied with direct current through two slip rings and brushes. Rotors are of two types:

   (a) Salient Pole Type: In this the rotor is like a magnetic fly wheel made of cast iron or steel and a number of alternate North and South poles are bolted to it as shown in Fig. 2.34. The Salient or projecting poles are made of thick steel laminations, riveted together and fixed to the rotor poles. The ends of the field windings are connected to the DC supply through slip rings carrying brushes. Such rotors have large diameter and small axial length.

   Advantages:

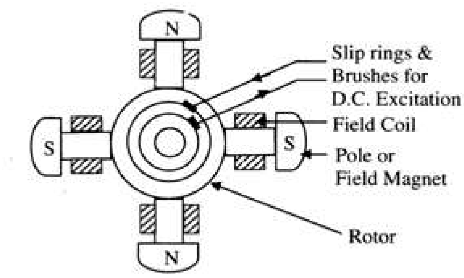


Figure 2.34: Salient Pole Rotor

   i. Less expensive
   ii. It provides sufficient space for field coils

   Disadvantages:

   i. If the salient poles are driven at high speed, it will cause excessive windage losses and would tend to produce noise.
   ii. As the rotor is not robust in construction, it cannot withstand mechanical stress if driven at high speed.

   Application: Used for low and medium speed alternators Examples: Hydraulic Power Plants, Diesel Power Plants and Gas Turbine Power Plants.

   (b) Smooth Cylindrical or Non-salient Pole Type: These rotors are cylindrical in construction and are made from solid forged steel alloy having a number of slots on its outer periphery at regular intervals for accommodating field coils as shown in Fig. 2.35 Field coils are connected to a DC supply by means of slip rings and brushes for excitation purpose. The regions forming the central polar areas are left unslotted. The field coils are so arranged around these polar areas that flux density is maximum on the central polar areas. These types of rotors are characterized by small diameter and very long axial length.

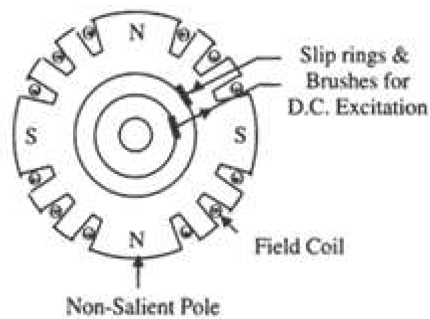


Figure 2.35: Non-Salient Pole Rotor

Advantages:

i. Gives better balance
ii. Noiseless operation
iii. Less windage loss
iv. Better E.M.F. waveform

Application: Used in very high speed turbo alternators Examples: Steam turbines

Operating Principle:

Alternators operate on the fundamental principle of Faraday's Laws of electromagnetic induction, i.e. whenever the magnetic flux linking the armature circuit changes an E.M.F. is induced in the circuit. The field windings of alternators require direct current for excitation. Excitation is supplied by a DC generator called an exciter which is mechanically and electrically coupled with the rotor shaft. As the prime mover rotates, the rotor also rotates and the stator conductors being stationary are cut by magnetic flux of rotor poles, hence an E.M.F. is induced in the stator conductors. As the rotor magnetic poles are alternatively North and South, they induce an alternating E.M.F. and hence current in the stator conductors. The frequency of alternating E.M.F. depends on the number of North and South poles moving past the conductors in one second. The direction of induced E.M.F. is given by Fleming's Right-hand rule.

Frequency of the induced E.M.F. or Relationship between Speed and Frequency:

Consider an alternator whose rotor is being driven at a constant speed of N r.p.m.
Let P = Number of poles
f = frequency of induced E.M.F.

In one complete revolution of the rotor, each of the North and South poles move past all the stator conductors. When one pair of North and South poles moves past the armature conductor, the induced E.M.F. undergoes one full cycle. Therefore in a P-pole machine, in one complete revolution of the field system, the induced E.M.F. in the armature conductors will complete P/2 cycles of waveform. Number of cycles of EMF induced per second= No. of cycles per revolution x No. of revolutions per sec

$$i.e. Frequency in cycles/second = \frac{P}{2} \times \frac{N}{60} \tag{2.20}$$

$$f = \frac{PN}{120} \tag{2.21}$$

or

$$N = \frac{120f}{P} \tag{2.22}$$

Note: In order to keep the frequency constant, the speed N must remain unchanged. Therefore a synchronous generator runs at a constant speed known as synchronous speed.

E.M.F. Equation:

Consider a 3-Phase alternator with P-Poles driven at a constant speed of N r.p.m.
Let Z = Number of conductors or coil sides in series per phase= 2T
where T is the number of coils or turns per phase.
ϕ= useful flux/pole in Webers
f = frequency in Hz
K_d = distribution factor or winding factor or breadth factor
K_p = pitch factor

Consider a conductor on the alternator. Let the alternator rotor move through one revolution in t=60/N seconds. In one revolution of the rotor, all the P poles on the rotor move past each of the stator conductors.
flux cut by the conductor in one revolution = $p.\phi$
According to Faraday's second law of electromagnetic induction,

The average emf induced in the conductor is:

$$E_{av} = \frac{d\phi}{dt} \tag{2.23}$$

where $d\phi$ is the flux cut by the rotor in one revolution of the rotor=$P.\phi$
dt is time taken for one revolution of rotor =60/N
So the average emf induced per conductor

$$E_{av} = \frac{P \times \Phi}{\frac{60}{N}} = \frac{N.P.\Phi}{60} \tag{2.24}$$

The emf induced in Z number of conductors is:

$$E_{av} = \frac{N.P.\Phi.Z}{60} \tag{2.25}$$

Substituting equation 1.3 in the above equation we get

$$E_{av} = \frac{P.\Phi.Z}{60}(\frac{120.f}{P}) \tag{2.26}$$

RMS value of emf/phase =

$$E_{rms} = 1.11 \times 2f\Phi Z == 2.22f\Phi Z = 2.22f\Phi(2T) = 4.44f\Phi T volt \tag{2.27}$$

The above equation of induced E.M.F./phase is true only if the winding is concentrated in one slot. But practically it is not possible as the winding for each phase under each pole is distributed and for such cases Kp and Kd are considered.Thus, E.M.F./phase will be:

$$E_{rms/phase} = 4.44f\Phi T K_p K_d Volts. \tag{2.28}$$

If the alternator is star connected, then the line voltage is

$$E_l = \sqrt{3}.E_{rms/phase} \tag{2.29}$$

$$E_l = \sqrt{3} \times 4.44f\Phi T K_p K_d Volts. \tag{2.30}$$

Note: For full pitched and concentrated windings $K_p = 1 and K_d = 1$.

Voltage Regulation of an Alternator:

The voltage regulation of an alternator is defined as the percentage rise in its terminal voltage when full-load at the specified power-factor is switched off, the excitation being adjusted initially to give normal voltage.

Thus

$$\% = \frac{E_0 - V}{V}.100 \tag{2.31}$$

where E_0= no load terminal voltage
V=full load terminal voltage

Problems:

1. A 6-Pole, 3-Phase, Star-connected alternator has an armature with 90 slots and 8 conductors per slot. It revolves at 1000r.p.m. The flux/pole being 0.05Wb. Calculate the E.M.F. generated if the winding factor 0.97 and all the conductors in phase are in series.
   Solution:
   Given:

   Alternator is STAR connected
   Number of poles: P = 6
   Number of slots = 90
   Number of conductors per slot = 8
   Speed N = 1000r.p.m.
   Flux/pole:ϕ= 0.05Wb
   Distribution factor: K_d = 0.97
   Pitch Factor K_p= 1

   E.M.F induced per phase is:

   $$E_{phase} = 4.44 f \Phi T_p h K_p K_d \tag{2.32}$$

   frequency of the induced E.M.F is

   $$f = \frac{P.N}{120} = \frac{6 \times 1000}{120} = 50Hz \tag{2.33}$$

   T_{ph}=No. of turns per phase

   $$T_{ph} = \frac{Z_{ph}}{2} \tag{2.34}$$

   where

   Z_{ph}=No.of conductors per phase=No. of conductors per slot × No. of slots per phase

$$Z_{ph} = 8 \times \frac{90}{3} = 240 \tag{2.35}$$

$$T_{ph} = \frac{240}{2} = 120 \tag{2.36}$$

Thus

$$E_{ph} = 4.44 \times 50 \times 0.05 \times 120 \times 1 \times 0.97 = 1292.04 Volts \tag{2.37}$$

Since the alternator is star connected the line voltage is

$$E_l = \sqrt{3} \times E_{ph} = \sqrt{3} \times 1292.04 = 2273.4 Volts \tag{2.38}$$

2. A Star connected three phase, 4-pole, 50Hz alternator has winding in 24 slots. There are 50 conductors per slot and the flux/pole is 0.05Wb. Find the E.M.F. generated. Assume $K_d = 0.966, K_p = 1$
   Solution:
   Given:

   (a) Alternator is STAR connected
   Number of poles: P = 6
   Number of slots = 24
   Number of conductors/slot=50
   Flux/pole: Φ= 0.05Wb
   requency: f=50Hz
   Distribution factor: $K_d = 0.97$
   Pitch factor: $K_p = 1$

   Total number of conductors= No of conductors per slot x No. of slots/ 2

$$= 50 \times \frac{24}{2} = 600 \tag{2.39}$$

$$T_{ph} = \frac{600}{3} = 200 \tag{2.40}$$

$$E_{phase} = 4.44 f \Phi T_p h K_p K_d \tag{2.41}$$

$$E_{phase} = 4.44 \times 50 \times 0.05 \times 200 \times 1 \times 0.97 \tag{2.42}$$

$$= 2144.52 Volts \tag{2.43}$$

3. A 3 phase star-connected alternator with 12 poles generates1100Volts on open circuit at a speed of 500r.p.m. Assuming 180turns/phase, a distribution factor of 0.96 and full pitched coils, find the useful flux/pole. Solution: Given:

   (a) Alternator is star connected.
   Number of poles: P=12
   Speed: N=500r.p.m.
   Number of turns/phase: $T_ph = 180$
   Distribution factor Kd = 0.96
   Open circuit voltage = Line voltage generated: E_L=1100V

   Frequency of induced emf is:

$$f = \frac{PN}{120} = \frac{12 \times 500}{120} = 50Hz \tag{2.44}$$

   Induced E.M.F per phase=

$$E_{ph} = \frac{E_L}{\sqrt{3}} = \frac{1100}{\sqrt{3}} = 635.09Volts \tag{2.45}$$

   WKT

$$E_{phase} = 4.44f\Phi T_phK_pK_d \tag{2.46}$$

   Thus useful flux per pole can be found as:

$$\Phi = \frac{E_{phase}}{4.44fT_phK_pK_d} = \frac{635.09}{4.44 \times 50 \times 180 \times 1 \times 0.86} = 0.0166Wb \tag{2.47}$$

2.16 Synchronous Motor

2.16.1 Introduction

A synchronous motor is electrically identical with an alternator or a.c. generator. In fact, a given synchronous machine may be used, at least theoretically, as an alternator, when driven mechanically or as a motor, when driven electrically, just as in the case of d.c. machines. Most synchronous motors are rated between 150 kW and 15 MW and run at speeds ranging from 150 to 1800 r.p.m. Some characteristic features of a synchronous motor are worth noting :

1. It runs either at synchronous speed or not at all i.e. while running it maintains a constant speed. The only way to change its speed is to vary the supply frequency because
$$N_s = \frac{120 \times f}{P} \tag{2.48}$$

2. It is not inherently self-starting. It has to be run upto synchronous (or near synchronous) speed by some means, before it can be synchronized to the supply.

3. It is capable of being operated under a wide range of power factors, both lagging and leading. Hence, it can be used for power correction purposes, in addition to supplying torque to drive loads.

2.16.2 Principle of Operation

When a 3-phase winding is fed by a 3-phase supply, then a magnetic flux of constant magnitude but rotating at synchronous speed, is produced. Consider a two-pole stator of Fig.2.36, in which are shown two stator poles (marked NS and SS) rotating at synchronous speed, say, in clockwise direction. With the rotor position as shown, suppose the stator poles are at that instant situated at points A and B. The two similar poles, N (of rotor) and NS (of stator) as well as S and SS will repel each other, with the result that the rotor tends to rotate in the anticlockwise direction.

But half a period later, stator poles, having rotated around, interchange their positions i.e. NS is at point B and SS at point A. Under these conditions, NS attracts S and SS attracts N. Hence, rotor tends to rotate clockwise (which is just the reverse of the first direction). Hence, we find that due to continuous and rapid rotation of stator poles, the rotor is subjected to a torque which is rapidly reversing i.e., in quick succession, the rotor is subjected to torque which tends to move it first in one direction and then in the opposite direction. Owing to its large inertia, the rotor cannot instantaneously respond to such quickly-reversing torque, with the result that

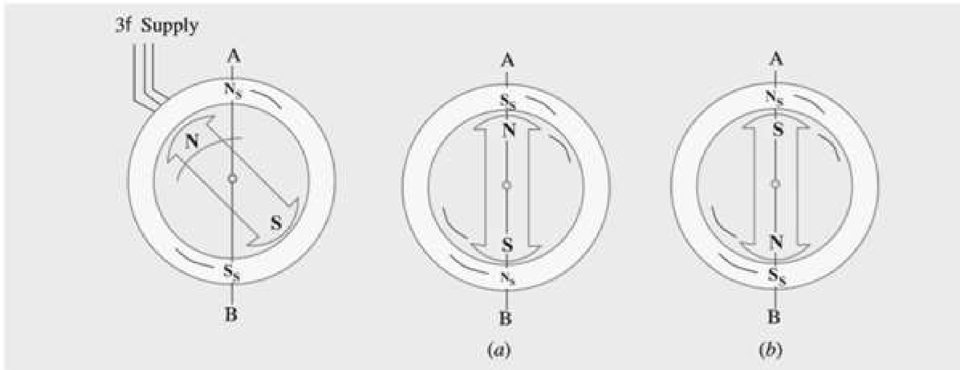


Figure 2.36: Principle of operation

it remains stationary. Now, consider the condition shown in Fig.2.36.a. The stator and rotor poles are attracting each other. Suppose that the rotor is not stationary, but is rotating clockwise, with such a speed that it turns through one pole-pitch by the time the stator poles interchange their positions, as shown in Fig.2.36.b. Here, again the stator and rotor poles attract each other. It means that if the rotor poles also shift their positions along with the stator poles, then they will continuously experience a unidirectional torque i.e., clockwise torque, as shown in Fig.2.36.

2.16.3 Synchronous Motor Applications

Synchronous motors find extensive application for the following classes of service :

1. Power factor correction: Overexcited synchronous motors having leading power factor are widely used for improving power factor of those power systems which employ a large number of induction motors and other devices having lagging p.f. such as welders and flourescent lights etc.

2. Constant-speed applications: Because of their high efficiency and high-speed, synchronous motors (above 600 r.p.m.) are well-suited for loads where constant speed is required such as centrifugal pumps, belt-driven reciprocating compressors, blowers, line shafts, rubber and paper mills etc. Low-speed synchronous motors (below 600 r.p.m.) are used for drives such as centrifugal and screw-type pumps, ball and tube mills, vacuum pumps, chippers and metal rolling mills etc.

3. Voltage regulation: The voltage at the end of a long transmission line varies greatly especially when large inductive loads are present. When an inductive load is disconnected suddenly, voltage tends to rise considerably above its normal value because of the line capacitance. By installing a synchronous motor

with a field regulator (for varying its excitation), this voltage rise can be controlled. When line voltage decreases due to inductive load, motor excitation is increased, thereby raising its p.f. which compensates for the line drop. If, on the other hand, line voltage rises due to line capacitive effect, motor excitation is decreased, thereby making its p.f. lagging which helps to maintain the line voltage at its normal value.

UNIT-III

3.1 Induction Motors

3.1.1 Introduction

Three phase Induction Motors are the most widely used ac motors. This type of motor converts alternating current electrical energy into mechanical energy.

3.1.2 Construction

It mainly consists of two parts (i)Stator (ii) Rotor.The rotor is the rotating part. The stator is the stationary part. They are separated by a small air gap.

Stator

The Stator of an Induction Motor is very similar in construction to the stator of an Alternator. It is a hollow and cylindrical core having slots in its inner surface to house windings. It consists of a set of silicon steel laminations attached to the yoke as shown in figure below. In the slots of the laminations stator conductors are placed with proper insulation. These conductors are properly interconnected to form a balanced star or delta connected winding.

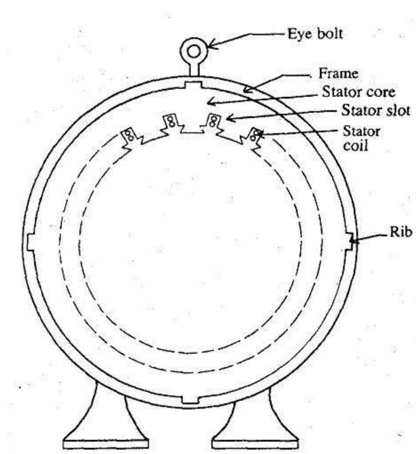


Figure 3.1: Three phase Induction Motor-Stator

Rotor

There are two types : (i) Squirrel cage rotor (ii) Phase wound rotor (Slip ring rotor)

Squirrel cage rotor

The copper or aluminum heavy bars form the rotor conductors. One bar is placed in each slot. Slots are made of steel laminations. All the bars are welded at both ends to two copper end rings thus short circuiting them at both ends. Since they are short circuited on both ends, no external resistance can be connected to it. This type of rotor has low starting torque. To look at, it resembles a cage, hence the name. The motor with this type of rotor is named as Squirrel cage Induction motor.

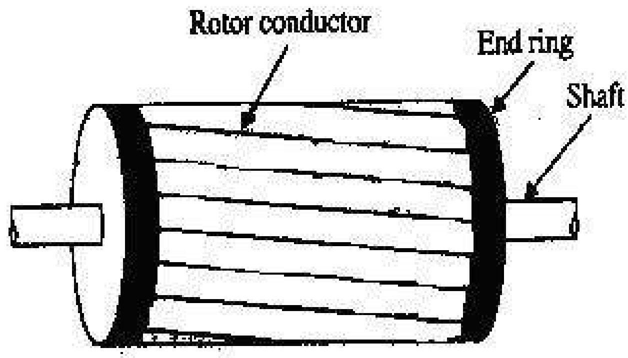


Figure 3.2: Squirrel cage rotor

Phase wound rotor (Slip ring rotor)

This rotor is a laminated, cylindrical core having uniform slots on its outer periphery. A three phase winding which is star connected is placed in these slots. The open ends of the star windings are brought out and connected to three insulated slip rings, mounted on the shaft of this rotor with carbon brushes resting on them. The rotor winding can be shorted through external variable resistance.The motor with this type of rotor is termed as Slip ring Induction motor.

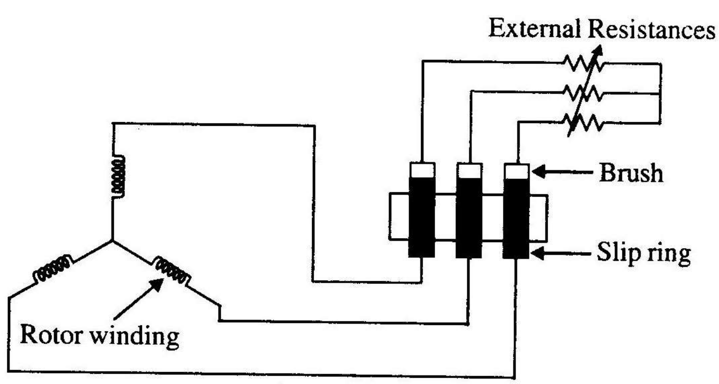


Figure 3.3: Slip ring rotor

3.1.3 Rotating Magnetic Field

When a three phase supply is given to the three windings of the stator, three fluxes are produced in the three windings. The assumed positive directions of fluxes are shown in figure below. The equations for three fluxes are

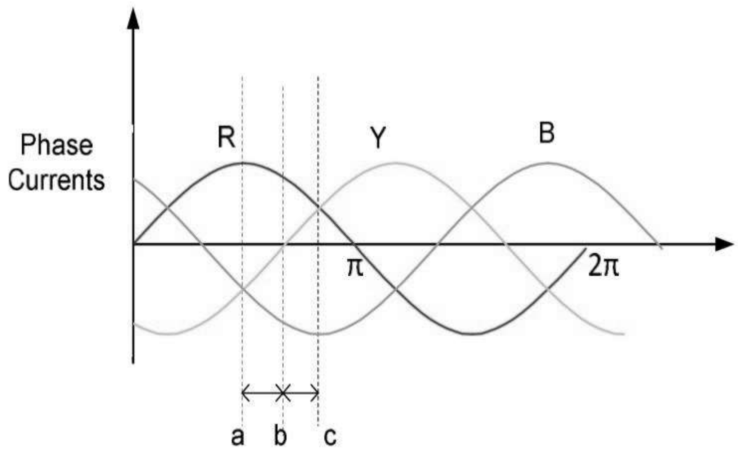


Figure 3.4

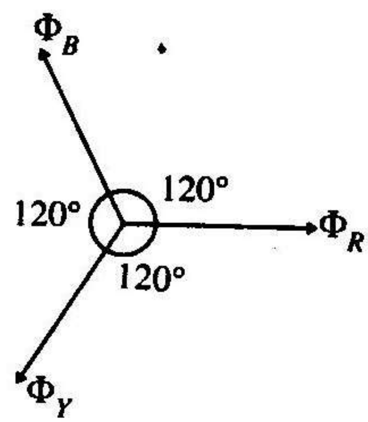


Figure 3.5

$\phi_R = \phi_m \sin \omega t$
$\phi_Y = \phi_m \sin(\omega t - 120°)$
$\phi_B = \phi_m \sin(\omega t - 240°)$
The resultant flux ϕ_r of these three fluxes at any instant is given by the vector sum of the individual fluxes ϕ_R, ϕ_Y and ϕ_B
Case (i) : At $\omega t = 0$

$\phi_R = 0$

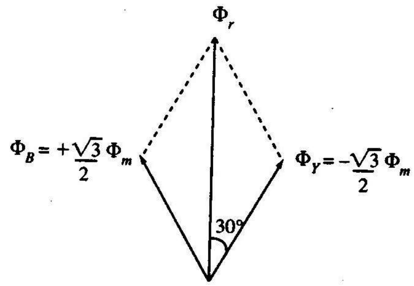


Figure 3.6

$\phi_Y = \phi_m \sin(-120°) = -\frac{\sqrt{3}}{2}\phi_m$
$\phi_B = \phi_m \sin(-240°) = \frac{\sqrt{3}}{2}\phi_m$

$$\begin{aligned}
\phi_r &= \sqrt{\phi_B^2 + \phi_Y^2 + 2\phi_B\phi_Y \cos 60°} \\
&= \sqrt{[\frac{\sqrt{3}}{2}\phi_m]^2 + [\frac{\sqrt{3}}{2}\phi_m]^2 + \frac{\sqrt{3}}{2}\phi_m \frac{\sqrt{3}}{2}\phi_m \frac{1}{2} \times 2} \\
&= \sqrt{\frac{3}{4}\phi_m^2 + \frac{3}{4}\phi_m^2 + \frac{3}{4}\phi_m^2} \\
\phi_r &= \frac{3}{2}\phi_m = 1.5\phi_m
\end{aligned}$$

The resultant flux lies along Y axis.
Case (ii) : When $\omega t = 60°$

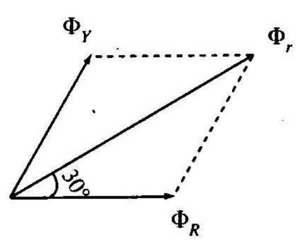


Figure 3.7

$\phi_R = \frac{\sqrt{3}}{2}\phi_m$
$\phi_Y = -\frac{\sqrt{3}}{2}\phi_m$
$\phi_B = 0$
$\phi_r = \frac{3}{2}\phi_m = 1.5\phi_m$
The resultant flux has rotated by 60° in the clockwise direction.
Case (iii) : When $\omega t = 120°$

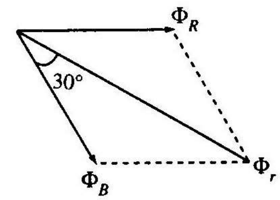


Figure 3.8

$\phi_R = \frac{\sqrt{3}}{2}\phi_m$
$\phi_Y = 0$
$\phi_B = -\frac{\sqrt{3}}{2}\phi_m$
$\phi_r = \frac{3}{2}\phi_m = 1.5\phi_m$
The resultant flux is further moved by 60° in the clockwise direction where as the magnitude of the resultant flux remains the same.
Case (iv) : When $\omega t = 180°$

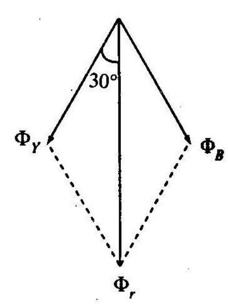


Figure 3.9

$\phi_R = 0$
$\phi_Y = \frac{\sqrt{3}}{2}\phi_m$
$\phi_B = -\frac{\sqrt{3}}{2}\phi_m$
$\phi_r = \frac{3}{2}\phi_m = 1.5\phi_m$
Here resultant flux is rotated by 180 degree from its original position.
From above analysis we can say that as ωt varies from 0 to 360°, the resultant flux also rotates with the same angular velocity ω and having a constant magnitude of $1.5\phi_m$.
Thus when 3ϕ supply is given to the stator windings of 3ϕ induction motor, a rotating magnetic field of constant magnitude and rotating with synchronous speed is produced.
The synchronous speed is given by $N_s = \frac{120f}{P}$
Where f=supply frequency P=number of poles.

3.1.4 Principle of Operation

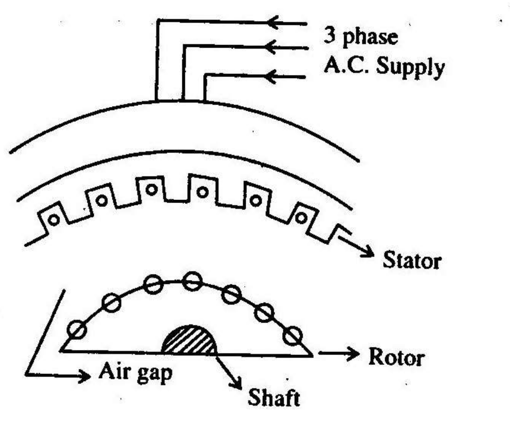


Figure 3.10

When a three-phase supply is given to the three phase stator winding, a magnetic field of constant magnitude $1.5\phi_m$ and rotating with the synchronous speed N_s is produced. The same field links with the rotor conductors. The rotor conductors cut this magnetic field and an emf is induced in these conductors in accordance with the faraday's laws of electromagnetic induction. The direction of the induced emf is to oppose the very cause of it i.e, the relative speed between the rotating magnetic field and the static rotor. As the rotor conductors are short circuited, the induced emf sets up a current in the rotor conductors in such a direction as to produce a torque which rotates the rotor in the same direction as the magnetic field so that the relative speed decreases. The speed of the rotor gradually increases and tries to catch up with the speed of the rotating magnetic field. But it fails to reach the synchronous speed because if it catches up with the speed of the magnetic field, the relative speed becomes zero and hence no emf will be induced in the rotor conductors. The torque becomes zero. Hence the rotor will not be able to catch up with the speed of the magnetic field, but rotates at a speed slightly less than the synchronous speed. The difference between the synchronous speed Ns of the magnetic field and the actual speed of the rotor N is called as the slip speed.

Slip speed = $N_S - N$

The slip(S) of an induction motor is defined as the ratio of slip speed to synchronous speed.

$s = \frac{N_s - N}{N_s}$

When slip becomes unity, rotor speed will be zero.

3.1.5 Expression for Frequency of Rotor Current

When the motor is stationary, frequency of the rotor current is same as the supply frequency. But when the rotor starts rotating, the frequency depends on relative speed or slip speed. Let at any speed, the frequency of the rotor current be fr.

Then $N_s - N = \frac{120f_r}{P} - - - - - - - - - - (i)$

Also $N_s = \frac{120f}{P} - - - - - - - - - - (ii)$

Dividing $(i)\, by\, (ii)$

$f_r = sf$

Torque slip characteristics:

The torque equation of the induction motor is given by

$$T = \frac{k\phi s E_2 R_2}{R_2^2 + (sX2)^2}$$

Where,

T is the motor torque

s is the motor slip.

E_2 is the standstill induced emf per phase in the rotor,

R_2 is the rotor resistance per phase,

X_2 is standstill rotor reactance per phase

It is clear that when s = 0, T = 0, hence the curve starts from point O.

At normal speeds, close to synchronism, the term (sX_2) is small and hence negligible w.r.t. R_2

$$\because T \propto \frac{s}{R_2}$$

or $T \propto s$ if R_2 is constant

Hence, for low values of slip, the torque/slip curve is approximately a straight line.

As slip increases (for increasing load on the motor), the torque also increases and becomes maximum when s = R_2/X_2. This torque is known as pull-out or breakdown torque T_b or stalling torque.

As the slip further increases (i.e. motor speed falls) with further increase in motor load, then R_2 becomes negligible as compared to (sX_2). Therefore, for large values of slip

$T \propto \frac{s}{(sX_2)^2} \propto \frac{1}{s}$

Hence, the torque/slip curve is a rectangular hyperbola. So, we see that beyond the point of maximum torque, any further increase in motor load results in decrease of torque developed by the motor. The result is that the motor slows down and eventually stops. The circuit-breakers will be tripped open if the circuit has been so protected. In fact, the stable operation of the motor lies between the values of s = 0 and that corresponding to maximum torque. The operating range is shown shaded in Fig below.

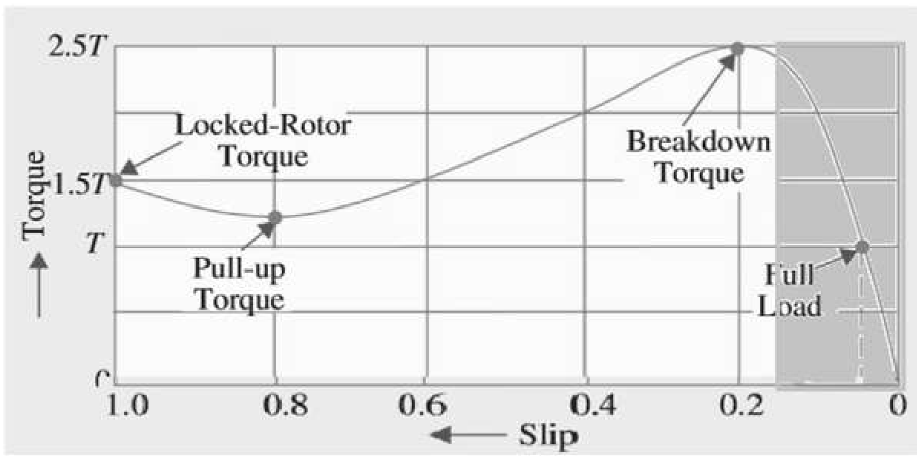


Figure 3.11

3.1.6 Necessity for Starter

When a 3- phase motor of higher rating is switched on directly from the mains it draws a starting current of about 4 -7 times the full load (depending upon on the design) current. This will cause a drop in the voltage affecting the performance of other loads connected to the mains. Hence starters are used to limit the initial current drawn by the 3 phase induction motors. The starting current is limited by applying reduced voltage in case of squirrel cage type induction motor and by increasing the impedance of the motor circuit in case of slip ring type induction motor. This can be achieved by the following methods:1. Star delta starter 2. Auto transformer starter.

3.1.7 Applications

Squirrel cage induction motor

Squirrel cage induction motors are simple and rugged in construction, are relatively cheap and require little maintenance. Hence, squirrel cage induction motors are pre-

ferred in most of the industrial applications such as in
i) Lathes
ii) Drilling machines
iii) Agricultural and industrial pumps
iv) Industrial drives.

Slip ring induction motors

Slip ring induction motors when compared to squirrel cage motors have high starting torque, smooth acceleration under heavy loads, adjustable speed and good running characteristics. They are used in
i) Lifts
ii) Cranes
iii) Conveyors , etc.,

3.1.8 Single-Phase Induction Motor

This motor is similar to a 3 phase induction motor, except that i) its stator is provided with a single-phase winding ii) a centrifugal switch is used in some types of motors, in order to diconnect a winding, which is used only for starting purpose. It has a distributed stator winding and a squirrel cage rotor. When fed from a single-phase supply, its stator winding produces a flux which is only alternating. It is not a revolving flux, as in the case of a three-phase winding. Now an alternating flux acting on a stationary squirrel-cage rotor cannot produce rotation. That is why a single-phase motor is not self-starting. But if the rotor of such a machine is given an initial start by hand then immediately a torque arises and the motor accelerates to its final speed.
To overcome this drawback and make the motor self-starting, it is temporarily converted into a two phase motor during starting period. For this purpose the stator of a single-phase motor is provided with an extra winding, known as starting (or auxillary) winding in addition to the main or running winding. In a capacitor-start induction-run motors a capacitor is connected in series with the starting winding. The capacitor is of the electrolytic type and is mounted on the outside of the motor as a separate unit. The capacitor provides the phase difference of 80° between Is and Im. Hence the motor behaves like a two phase motor. These two currents produce a revolving flux and hence makes the motor self-starting. When the motor reaches about 75 per cent of full speed, the centrifugal switch S opens and disconnects both

the starting winding and the capacitor from the supply, thus leaving only the running winding across the lines.

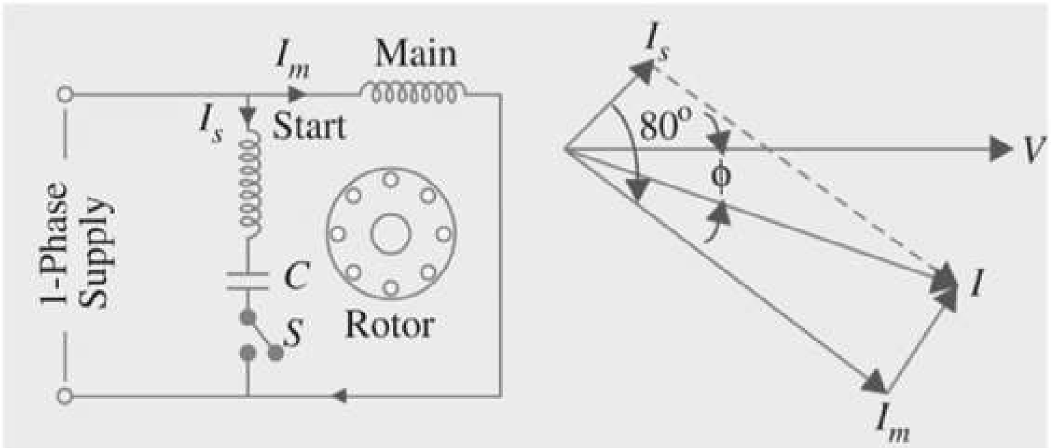


Figure 3.12

The capacitor-start capacitor-run motor is similar to the capacitor-start induction-run motor except that the starting winding and capacitor are connected in the circuit at all times. Centrifugal switch is absent in this type of motors. The advantage of leaving the capacitor permanently in the circuit are i) improvement of over-load capacity of the motor ii)a higher power factor iii)higher efficiency iv)quieter running of the motor which is desirable for small power drives in offices and homes.

3.2 Domestic Wiring

A network of wires drawn connecting the meter board to the various energy consuming loads (lamps, fans, motors etc) through control and protective devices for efficient distribution of power is known as electrical wiring. Electrical wiring done in residential and commercial buildings to provide power for lights, fans, pumps and other domestic appliances is known as domestic wiring.

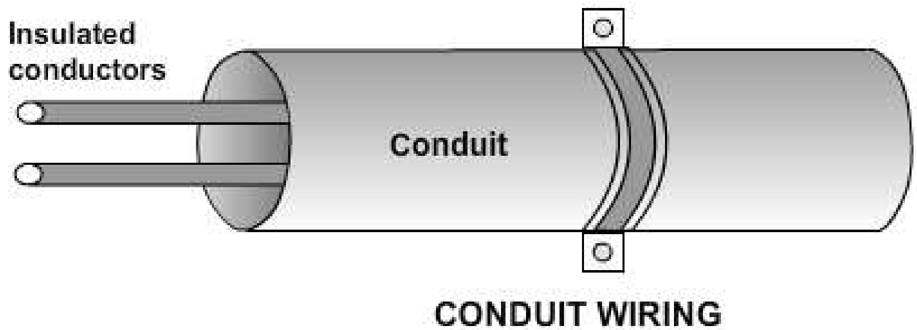


Figure 3.13: Conduit wiring

3.2.1 Advantages:

1. No risk of fire and good protection against mechanical injury.
2. The lead and return wires can be carried in the same tube.
3. Earthing and continuity is assured.
4. Waterproof and trouble shooting is easy.
5. Shock- proof with proper earthing and bonding
6. Durable and maintenance free
7. Aesthetic in appearance

3.2.2 Disadvantages:

1. Very expensive system of wiring.
2. Requires good skilled workmanship.
3. Erection is quiet complicated and is time consuming.
4. Risk of short circuit under wet conditions (due to condensation of water in tubes)

3.3 Two-way control of lamp

In big halls, corridors, bed rooms and stair case, it is necessary to control the lamp from two points. In such cases, a two-way control lamp circuit is use for wiring. This is also called Staircase wiring. It consists of two-way switches. A two-way switch operates in one of the two possible positions.

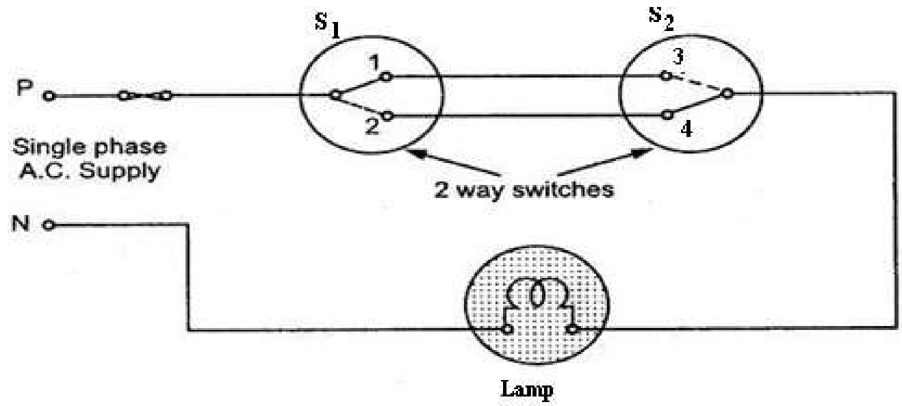


Figure 3.14: Two-way control of Lamp

Table 3.1: Two way control of lamp

Position of S1	Position of S2	Condition of lamp
1	3	on
1	4	off
2	3	off
2	4	on

- Switches S1 and S2 are two-way switches with a pair of terminals 1&2, and 3&4 respectively.
- When the switch S1 is in position1 and switch S2 is in position 4, the circuit does not form a closed loop and there is no path for the current to flow and hence the lamp will be OFF.
- When S1 is changed to position 2 the circuit gets completed and hence the lamp glows or is ON.
- Now if S2 is changed to position 3 with S1 at position 2 the circuit continuity is broken and the lamp is off.
- Thus the lamp can be controlled from two different points

3.4 Three-way control of lamps

In case of very long corridors it may be necessary to control the lamp from 3 different points. In such cases, the circuit connection requires two; two-way switches S1and S2 and an intermediate switch S3. An intermediate switch is a combination of two, two way switches coupled together. It has 4 terminals ABCD. It can be connected in two ways a) Straight connection b) Cross connection In case of straight connection, the terminals or points AB and CD are connected as shown in figure 3. In case of cross connection, the terminals AB and CD is connected as shown in figure 4. As explained in two way control the lamp is ON if the circuit is complete and is OFF if the circuit does not form a closed loop.

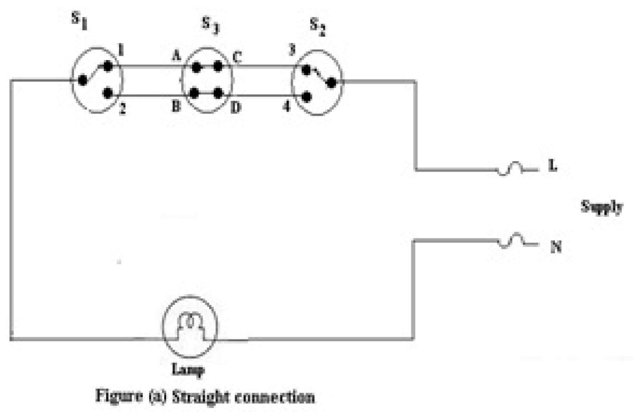


Figure 3.15: Straight connection

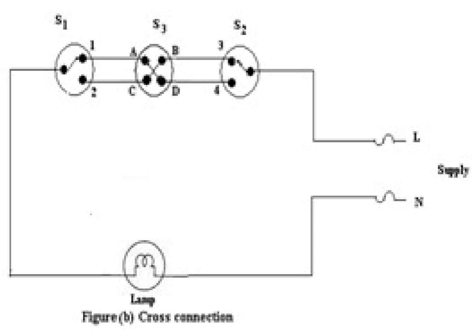


Figure 3.16: Cross connection

Table 3.2: Three way control of lamp

Position of S3	Position of S1	Position of S2	Condition of lamp
Straight connection	1	3	on
	1	4	off
	2	3	off
	2	4	on
Cross connection	1	3	off
	1	4	on
	2	3	on
	2	4	off

3.5 Fuse

The electrical equipments are designed to carry a particular rated value of current under normal circumstances. Under abnormal conditions such as short circuit, overload or any fault the current raises above this value, damaging the equipment and sometimes resulting in fire hazard. Fuses are pressed into operation under such situations. Fuse is a safety device used in any electrical installation, which forms the weakest link between the supply and the load. It is a short length of wire made of lead / tin /alloy of lead and tin/ zinc having a low melting point and low ohmic losses. Under normal operating conditions it is designed to carry the full load current. If the current increases beyond this designed value due any of the reasons mentioned above, the fuse melts (said to be blown) isolating the power supply from the load as shown in the following figure 5 and figure 6.

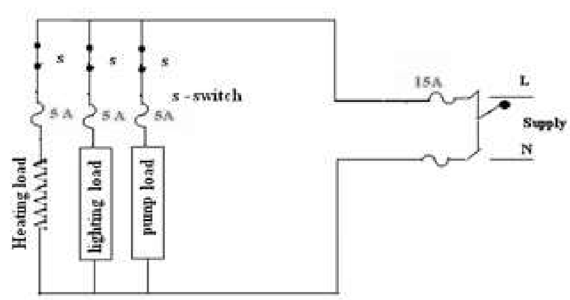


Figure 3.17: Fuse connection

3.5.1 Characteristics of Fuse materials

The material used for fuse wires must have the following characteristics

1. Low melting point

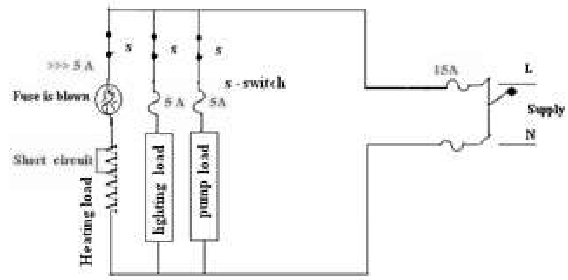


Figure 3.18: Fuse blown out

2. Low ohmic losses
3. High conductivity
4. Lower rate of deterioration

Selection of range:

- The selection of proper fuse is very important.
- An improper blowing of fuse results in an unnecessary stoppage of flow of power.
- This results in loss of time.

The following factors are considered while selecting a fuse:

- Nature of load: This includes consideration of the nature of load whether it is a steady load or a fluctuating load.
- Nature of protection required. This includes factors such as overload or short circuit protection to be provided.
- Fault current: The fault currents are generally high and hence proper peak current, fusing factor etc must be considered.

3.6 Miniature Circuit Breaker (MCB)

Miniature Circuit Breaker is a relatively small circuit breaker fitted in consumer units and small distribution boards. MCBs are fitted in consumer units in place of fuses. They have the advantage that they can be manually reset without having to replace wire as in the case of a traditional fuse. The MCBs have a button or lever that can be flicked to reset it. MCB tripping is an indication that, either the circuit is overloaded or there may be short circuit somewhere in the system. To identify a faulty circuit causing fuses to blow or MCB to reset can be a tricky process. Initially try to remember what appliances were switched on or what happened at the time of fault occurred. First check for overloading, this problem can be overcome by either reducing the load or increasing the rating of MCB and wiring system. If the circuit is not being overloaded next step is to inspect all the outlets, lights, flexes and appliances using the blower circuit. Look for the damage that could have caused a short circuit and repair it.

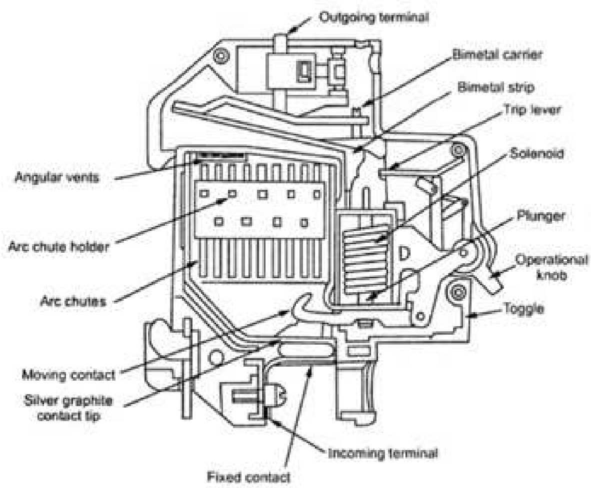


Figure 3.19: Miniature Circuit Breaker

3.7 Electric Shock

A sudden agitation of the nervous system of a body, due to the passage of an electric current is called an electric shock. The factors affecting the severity of the shock are,

1. Magnitude of current passed through the body.
2. Path of the current passed through the body.
3. Time for which the current is passed through the body.
4. Frequency of the current.
5. Physical and psychological condition of the affected person.

3.8 Safety Precautions while Working with Electricity

It is necessary to observe same safety precautions while using the electric supply to avoid the serious problems like shocks and fire hazards. Some of the safety precautions are listed below :

1. Insulation of the conductors used must be proper and in good condition. If it is not so the current carried by the conductors may leak out. The person coming in contact with such faulty insulated conductors may receive a shock.
2. Megger tests should be conducted and insulation must be checked. With the help of megger all the tests discussed above must be performed, on the new wiring before starting use of it.
3. Earth connection should be always maintained in proper condition.
4. Make the mains supply switch off and remove the fuses before starting work with any installation.
5. Fuses must have correct ratings.
6. Use rubber soled shoes while working. Use some wooden slipper under the feet. This removes the contact with the earth.
7. Use rubber gloves while touching any terminals or removing insulation layer from a conductor.
8. Use a line tester to check whether a 'live' terminal carries any current still better method is to use a test lamp.
9. Always use insulated screw drivers, pilers, line testers etc.
10. Never touch two different terminals at the same time.
11. Never remove the plug by pulling the wires connected to it.
12. The sockets should be fixed at a height beyond the reach of the children.

3.9 Necessity and types of Earthing

Earthing is required to save life from danger of shock or death. Earthing effectively blows out the fuse of any apparatus which becomes leaky. It protects large building and all machines fed from overhead lines from atmospheric lightning by taking all voltage of lighting through the lightning arrester. Good earthing is one which gives very low resistance to the flow of heavy current of a given circuit. Earth potential is always as zero for all practical purposes. A wire coming from the ground 2.5 to 3 meters deep from an electrode is called earthing. Double earth is used for 3 phase machine and equipment. When double earth is used, there is an advantage of redundancy. Pipe earthing, plate earthing etc are the different types of earthing. In summer, resistance of the earth increases. It can be reduced by pouring water, increasing the plate area, increasing the depth or by keeping the electrodes in parallel.

3.9.1 Plate Earthing

For good earthing in electric substations, plate earthing is used. Here the looping earth wire is bolted effectively with the earth plate made up of copper of size ($60 \times 60 \times 0.318$) cm (or GI plate of size not less than $60 \times 60 \times 6.35$ mm) and embedded 3 meters in ground. A schematic diagram of plate earthing is shown in figure 8. Copper plates are found to be most effective earth electrodes and are not affected by soil moisture. But due to high material cost galvanized iron plates are preferred and used for normal works. The plate is kept vertical and so arranged that it is embedded in an alternate layer of coke and salt for a minimum thickness of around 15cm. Bolts and nuts should be of copper for copper plates and of galvanized iron for GI plate. Earthing efficiency increases with the increase of plate area and depth of embedding.

3.9.2 Pipe Earthing

In this method of earthing a G.I. pipe of 38 mm diameter and 2 meter (7 feet) length is embedded vertically into the ground. This pipe acts as an earth electrode. The depth depends on the condition of the soil. The earth wires are fastened to the top section of the pipe above the ground level with nut and bolts. The pit area around the pipe is filled with salt and coal mixture for improving the condition of the soil and earthing efficiency. The schematic arrangement of pipe earthing system is shown in the Fig 9. The contact surface of G.I. pipe with the soil is more as compared to the plate due to its circular section and hence can handle heavier leakage current for the same electrode size. According to Indian standard, the pipe should be placed at a depth of 4.75 m. Impregnating the coke with salt decreases the earth resistance. Generally alternate layers of salt and coke are used for best results. In summer season, soil becomes dry, in such case salt water is poured through the funnel connected to the main G.I. pipe through 19 mm diameter pipe. This keeps the soil wet. The earth wires are connected to the G.I. pipe above the ground level and can be physically inspected from time to time. These connections can be checked for performing continuity tests. This is the important advantage of pipe earthing over the plate earthing. The earth lead used must be G.I. wire of sufficient cross-sectional area to carry fault current safely. It should not be less than electrical equivalent of

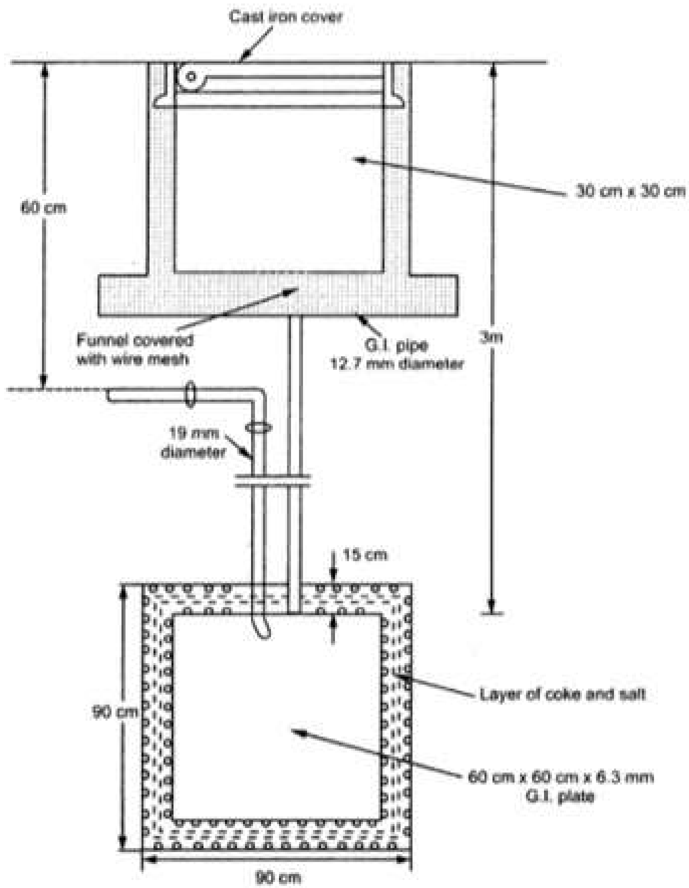


Figure 3.20: Plate Earthing

copper conductor of 12.97 mmˆ2 cross-sectional area. The only disadvantage of pipe earthing is that the embedded pipe length has to be increased sufficiently in case the soil specific resistivity is of high order. This increases the excavation work and hence increased cost. In ordinary soil condition the range of the earth resistance should be 2 to 5 ohms. In the places where rocky soil earth bed exists, horizontal strip earthing is used. This is suitable as soil excavation required for plate or pipe earthing is difficult in such places. For such soils earth resistance is between 5 to 8 ohms.

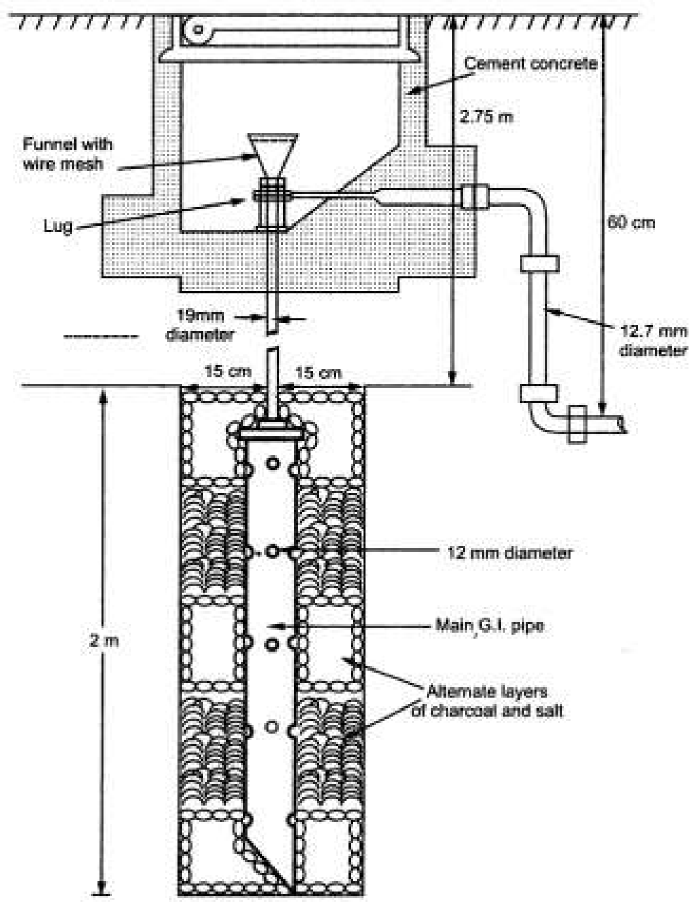


Figure 3.21: Pipe Earthing