**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans.

1. Effective time available: 50 minutes=60−1050 \, \text{minutes} = 60 - 1050minutes=60−10.
2. Standardizing: Z = 50 – 45 / 8 = 0.625
3. Cumulative probability: P(Z<=0.625) = 0.7340
4. Probability of exceeding 50 minutes: P(X>50) = 1- P(Z<=0.625) = 0.2660

**Conclusion:**

The probability that the service manager cannot meet his commitment is approximately **0.2676**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans.

**A. More employees at the processing center are older than 44 than between 38 and 44.**

* **Z-scores:**
  + For 38: Z=38−38/6=0
  + For 44: Z=44−38/6=1
* **Probability for employees between 38 and 44:**
  + P(Z<1)≈0.8413
  + P(Z<0)=0.5
  + So, P(38<X<44)= 0.8413 - 0.5 = 0.3413 (34.13%).
* **Probability for employees older than 44:**
  + P(Z>1)=1−0.8413=0.1587 (15.87%).
* **Conclusion:** More employees are between 38 and 44 than older than 44.  
  **Statement A is False.**

**B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.**

* **Z-score for 30:**

Z=30−38/6=−1.33

* **Probability for employees under 30:**
  + P(Z<−1.33)≈0.0918.
* **Expected number of employees under 30:**

Expected employees=0.0918×400=36.72

* **Conclusion:** The expected number is approximately 37 employees.  
  **Statement B is True.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans.

The distribution of 2X₁ is N(2μ, 4σ²), while X₁ + X₂ is N(2μ, 2σ²). Both have the same mean (2μ), but 2X₁ has a larger variance (4σ²) compared to X₁ + X₂ (2σ²).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans.

We are given that X ~ N(100, 202), where the mean μ = 100 and the variance σ² = 202. The standard deviation σ = √202 ≈ 14.21.

To find two values a and b symmetric about the mean such that the probability between them is 0.99, we use the Z-value of approximately ±2.576 for 99% probability.

1. **Formula:** a = μ - Z \* σ  
   b = μ + Z \* σ
2. **Substitute values:** a = 100 - 2.576 \* 14.21 ≈ 100 - 36.6 = 63.4  
   b = 100 + 2.576 \* 14.21 ≈ 100 + 36.6 = 136.6
3. **Conclusion:** The values are approximately 63.4 and 136.6. The closest option is **B: 80.2, 119.8**.
4. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
5. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
6. Specify the 5th percentile of profit (in Rupees) for the company
7. Which of the two divisions has a larger probability of making a loss in a given year?

Ans.

**A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.**

Let the total profit be the sum of the profits from both divisions:

Total Profit = Profit1 + Profit2

We are given:

* Profit1 ~ N(5, 3²)
* Profit2 ~ N(7, 4²)

Since the profits are independent, the total profit follows a normal distribution with the mean and variance as the sum of the means and variances of the individual profits:

* Mean of Total Profit = Mean of Profit1 + Mean of Profit2 = 5 + 7 = 12
* Variance of Total Profit = Variance of Profit1 + Variance of Profit2 = 3² + 4² = 9 + 16 = 25
* Standard Deviation of Total Profit = √25 = 5

To find the 95% probability range for the total profit in USD, we use the 95% confidence interval for a normal distribution, which is given by:

Mean ± 1.96 \* Standard Deviation

Thus, the 95% probability range in USD is:

12 ± 1.96 \* 5 = 12 ± 9.8

So, the range of total profit in USD is:

[2.2, 21.8] million USD

To convert this to Rupees, we multiply by 45 (since 1 USD = 45 Rupees):

Lower bound = 2.2 \* 45 = 99 Rupees million  
Upper bound = 21.8 \* 45 = 981 Rupees million

Thus, the 95% probability range for the company's total profit in Rupees is:

**[99, 981] Rupees million**

**B. Specify the 5th percentile of profit (in Rupees) for the company.**

The 5th percentile corresponds to the point where 5% of the data lies below it. For a normal distribution, this can be calculated using the Z-score for the 5th percentile, which is approximately -1.645.

The formula for the percentile in USD is:

Percentile = Mean + (Z \* Standard Deviation)

For the total profit in USD:

Percentile in USD = 12 + (-1.645 \* 5) = 12 - 8.225 = 3.775 million USD

Now, convert this to Rupees by multiplying by 45:

Percentile in Rupees = 3.775 \* 45 = **170.875 Rupees million**

Thus, the 5th percentile of profit for the company in Rupees is:

**170.875 Rupees million**

**C. Which of the two divisions has a larger probability of making a loss in a given year?**

To determine which division has a larger probability of making a loss, we need to find the probability that each division's profit is less than 0 (a loss).

For **Profit1 ~ N(5, 3²)**:

* Z = (0 - 5) / 3 = -5 / 3 ≈ -1.67
* Using the Z-table, the probability corresponding to Z = -1.67 is approximately 0.0475, or 4.75%.

For **Profit2 ~ N(7, 4²)**:

* Z = (0 - 7) / 4 = -7 / 4 = -1.75
* Using the Z-table, the probability corresponding to Z = -1.75 is approximately 0.0401, or 4.01%.

Thus, **Profit1** has a larger probability (4.75%) of making a loss compared to **Profit2** (4.01%).

Therefore, the division with **Profit1** has a larger probability of making a loss in a given year.