Mathematical Models (Group 2)

BASICS

N = Population ($N1$ = Population of species 1 etc.)

B = TOTAL births (of a specific species)

D = TOTAL deaths (of a specific species)

b = births PER CAPITA (of a specific species) = B/N

d = deaths PER CAPITA (of a specific species) = D/N

$r = b - d$

b and d are ASSUMED TO BE CONSTANT. They do not change (in our model), they are the “rate an animal would procreate/die” if there was unlimited resources and space! Accounting for space comes later.

NEXT STEP

Now we need to calculate the “real growth”, which is a derivative:

dN/dt = change in N / change in t

$dN/dt = (b' - d')N$

b’ and d’ **<u>IS NOT THE SAME AS</u>** b and d

b’ and d’ are the “real” growth rates, which is the ones we can observe.

The higher the population, the lower birth rates become (and the opposite for deaths).

FORMULA

$b' = b - aN$

$d' = d + cN$

a and c are the STRENGTH OF DENSITY DEPENDENCE, which means that they are the rate that births/deaths decrease/increase PER ADDED INDIVIDUAL.

Say I add 1 individual so N becomes $(N+1)$, then $d' = d + cN$ becomes $d' = d + c(N+1)$

Which is $d' = d + cN + c$

REMEMBER THAT MEASUREMENTS ARE b’ AND d’ ALREADY, AND NOT THE REAL DEAL

JUST LIKE b and d, OUR MODEL ASSUMES a and c AS CONSTANT

CARRYING CAPACITY (K)

This is the "ideal amount" of a species that the environment can support (so refers to a specific species in a specific space).

OUR MODEL ASSUMES CARRYING CAPACITY = CONSTANT

$K = (b-d)/(a+c)$

NOTE THAT THESE ARE b and d (NOT b' and d'!)

LOGISTIC MODEL

$$dN1/dT = r1 * N1 * (1 – (N1/K1)$$

r * N is the "unchecked growth"

(1 – (N1/K1)) also written as ((K1 – N1)/K1)

N1/K1 is the percentage that the population is at carrying capacity.

1 – (N1/K1) is the "unused" percentage of carrying capacity

If N = K (pop at carrying capacity) then (1 – (N1/K1)) = 1 – 1 = 0

So dN1/dT = r1 * N1 * (1 – (N1/K1) = r1 * N1 * 0 = 0

If N is OVER carrying capacity (N > K) then (1 – (N1/K1)) = NEGATIVE #

So dN1/dT = r1 * N1 * (1 – (N1/K1) = r1 * N1 * NEGATIVE # = DECREASING!

COMPETITION MODEL

Logistic model explains slower growth due to increase in number: (1 – (N1/K1)

But the growth is also decreased by the population of competing species N2: (1-((N1-α*N2)/K1)

The carrying capacity is the same, but is also taken up by the competing species in a factor α, which is the COMPETITION COEFFICIENT.

COMPETITION COEFFICIENT = α (alpha, not the same as a)

Multiple equations will use β or alpha with numbers to indicate different coefficients.

This is where consumption comes into play: if species 2 consumes 4 times as much, then every individual of species 2 will count as 4 times species 1 individual towards the carrying capacity.

This means that α = 4.00 in this example.

COMPLETE FORMULA:

dN1/dt = r1 * N1 * ((K1 – N1 – α*N2)/K1)

dN2/dt = r2 * N2 * ((K2 – N2 – β*N1)/K2)