Answers & Solutions For JEE MAIN 2017
(Code-C)

Time Duration: 3 hrs. **Maximum Mark : 360**

(Chemistry, Mathematics and Physics)

Important Instructions :

1. The test is of 3 hours duration.
2. The Test Booklet consists of 90 questions. The maximum marks are 360.
3. There are three parts in the question paper A, B, C consisting of Chemistry, Mathematics and Physics, having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for each correct response.
4. Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks of the total marks allotted to the question (i.e. 1 mark) will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
5. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.
6. For writing particulars/marking responses on Side-1 and Side-2 of the Answer Sheet use only Black BallPoint Pen provided in the examination hall.
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination hall/room.

PART–A : CHEMISTRY

1. The freezing point of benzene decreases by 0.45ºC when 0.2 g of acetic acid is added to 20 g of benzene. If acetic acid associates to form a dimer in benzene, percentage association of acetic acid in benzene will be

   (K_f for benzene = 5.12 K kg mol^{-1})

   (1) 94.6% (2) 64.6%

   (3) 80.4% (4) 74.6%

Answer (1)

Sol. $0.45 = i(5.12)\dfrac{0.2/60}{20}\times 1000$

$\Rightarrow$ i = 0.527

$$\underset{1-\alpha}{2CH_3COOH} \rightleftharpoons \underset{\frac{\alpha}{2}}{(CH_3COOH)_2}$$

$\Rightarrow$ $i = 1-\dfrac{\alpha}{2}$

$\Rightarrow$ $0.527 = 1-\dfrac{\alpha}{2}$

$\Rightarrow$ $\dfrac{\alpha}{2} = 0.473$

$\Rightarrow$ $\alpha = 0.946$

$\therefore$ % association = 94.6%

2. On treatment of 100 mL of 0.1 M solution of $CoCl_3 \cdot 6H_2O$ with excess $AgNO_3$; 1.2×10^{22} ions are precipitated. The complex is

   (1) $[Co(H_2O)_5Cl]Cl_2 \cdot H_2O$

   (2) $[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O$

   (3) $[Co(H_2O)_3Cl_3] \cdot 3H_2O$

   (4) $[Co(H_2O)_6]Cl_3$

Answer (1)

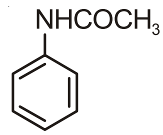
Sol. Millimoles of $AgNO_3$ = $\dfrac{1.2\times10^{22}}{6\times10^{23}}\times1000$ = 20

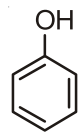
Millimoles of $CoCl_3{\cdot}6H_2O$ = 0.1 × 100 = 10

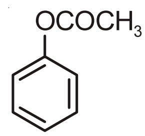
$\therefore$ Each mole of $CoCl_3{\cdot}6H_2O$ gives two chloride ions.

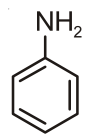
$\therefore$ $[Co(H_2O)_5Cl]Cl_2{\cdot}H_2O$

3. Which of the following compounds will form significant amount of *meta* product during mono-nitration reaction?

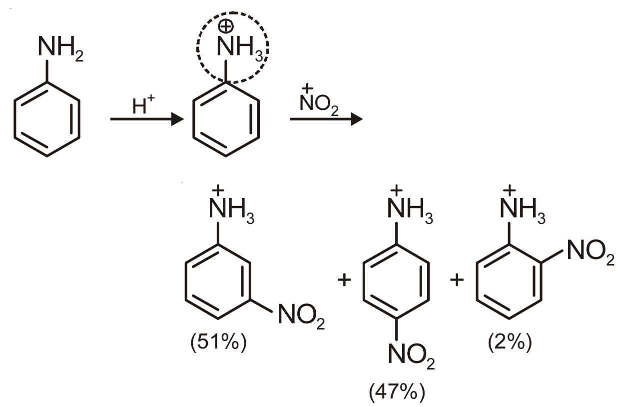
   (1) 

   (2) 

   (3) 

   (4) 

Answer (4)

Sol. 

4. The products obtained when chlorine gas reacts with cold and dilute aqueous NaOH are

   (1) Cl^- and ClO_2^-

   (2) ClO^- and ClO_3^-

   (3) ClO_2^- and ClO_3^-

   (4) Cl^- and ClO^-

Answer (4)

Sol. $Cl_2 + \underset{\text{Cold \& dilute}}{2NaOH} \longrightarrow NaCl + \underset{\text{Sodium hypochlorite}}{NaOCl} + H_2O$

5. Both lithium and magnesium display several similar properties due to the diagonal relationship, however, the one which is incorrect, is

(1) Nitrates of both Li and Mg yield NO_2 and O_2 on heating

(2) Both form basic carbonates

(3) Both form soluble bicarbonates

(4) Both form nitrides

Answer (2)

Sol. Mg forms basic carbonate

$3MgCO_3 \cdot Mg(OH)_2 \cdot 3H_2O$ but no such basic carbonate is formed by Li.

6. A water sample has ppm level concentration of following anions

$F^- = 10;\ SO_4^{2-} = 100;\ NO_3^- = 50$

The anion/anions that make/makes the water sample unsuitable for drinking is/are

(1) Only SO_4^{2-}

(2) Only NO_3^-

(3) Both SO_4^{2-} and NO_3^-

(4) Only F^-

Answer (4)

Sol. Permissible limit of F^- in drinking water is upto 1 ppm. Excess concentration of $F^- > 10$ ppm causes decay of bones.

7. The formation of which of the following polymers involves hydrolysis reaction?

(1) Terylene

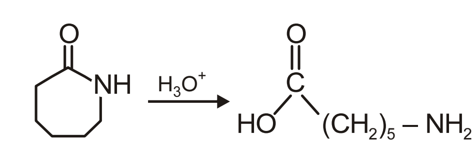
(2) Nylon 6

(3) Bakelite

(4) Nylon 6, 6

Answer (2)

Sol. Caprolactam is hydrolysed to produce caproic acid which undergoes condensation to produce Nylon-6.



(Caprolactam) (Caproic acid)

8. The Tyndall effect is observed only when following conditions are satisfied

(a) The diameter of the dispersed particles is much smaller than the wavelength of the light used.

(b) The diameter of the dispersed particle is not much smaller than the wavelength of the light used

(c) The refractive indices of the dispersed phase and dispersion medium are almost similar in magnitude

(d) The refractive indices of the dispersed phase and dispersion medium differ greatly in magnitude

(1) (b) and (c)
(2) (a) and (d)
(3) (b) and (d)
(4) (a) and (c)

Answer (3)

Sol. For Tyndall effect refractive index of dispersion phase and dispersion medium must differ significantly. Secondly, size of dispersed phase should not differ much from wavelength used.

9. pK_a of a weak acid (HA) and pK_b of a weak base (BOH) are 3.2 and 3.4, respectively, The pH of their salt (AB) solution is

(1) 1.0
(2) 7.2
(3) 6.9
(4) 7.0

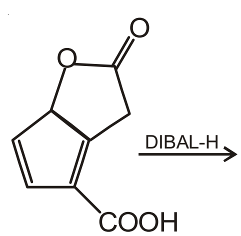
Answer (3)

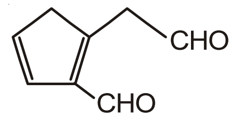
Sol. $pH = 7 + \frac{1}{2}(pK_a - pK_b)$

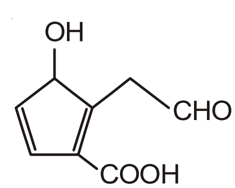
$= 7 + \frac{1}{2}(3.2 - 3.4)$

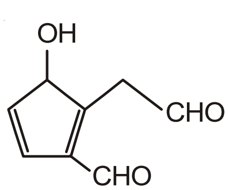
$= 6.9$

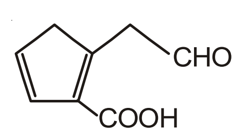
10. The major product obtained in the following reaction is



(1) 

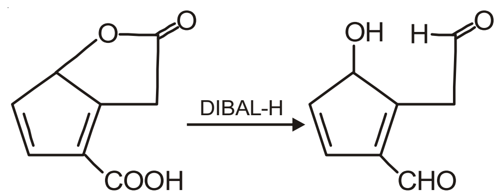
(2) 

(3) 

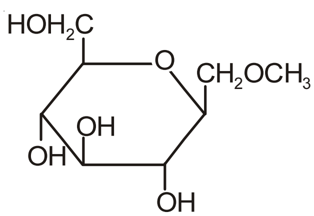
(4) 

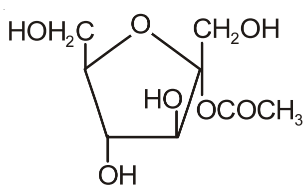
Answer (3)

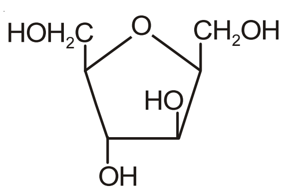
Sol. DIBAL — H reduces esters and carboxylic acids into aldehydes

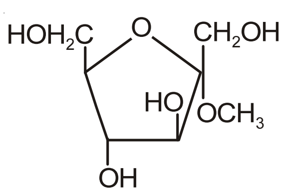


11. Which of the following compounds will behave as a reducing sugar in an aqueous KOH solution?

(1) 

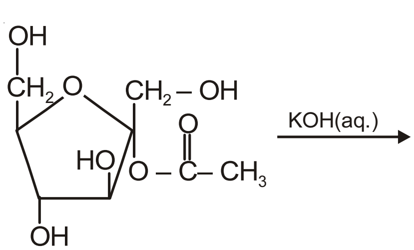
(2) 

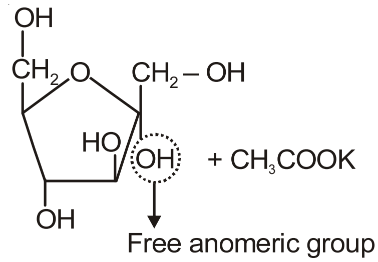
(3) 

(4) 

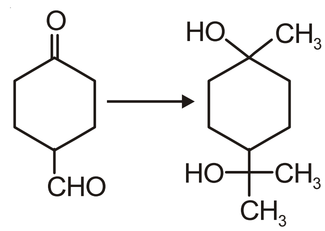
Answer (2)

Sol. Sugars in which there is free anomeric –OH group are reducing sugars





12. The correct sequence of reagents for the following conversion will be



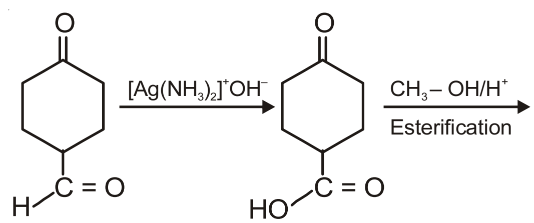
(1) $[Ag(NH_3)_2]^+OH^-$, CH_3MgBr, H^+/CH_3OH

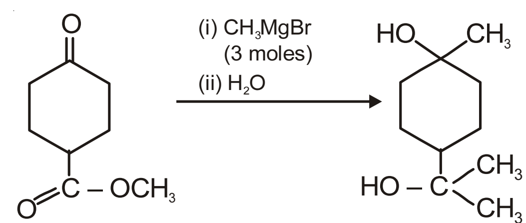
(2) $[Ag(NH_3)_2]^+OH^-$, H^+/CH_3OH, CH_3MgBr

(3) CH_3MgBr, H^+/CH_3OH, $[Ag(NH_3)_2]^+OH^-$

(4) CH_3MgBr, $[Ag(NH_3)_2]^+OH^-$, H^+/CH_3OH

Answer (2)

Sol. 



13. Which of the following species is not paramagnetic?

(1) B_2

(2) NO

(3) CO

(4) O_2

Answer (3)

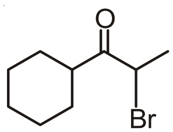
Sol. CO has 14 electrons (even) $\therefore$ it is diamagnetic

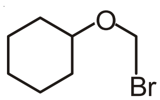
NO has $15e^-$(odd) $\therefore$ it is paramagnetic and has 1 unpaired electron in π^*2p molecular orbital.

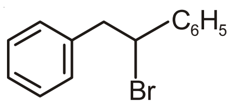
B_2 has $10e^-$ (even) but still paramagnetic and has two unpaired electrons in $\pi 2p_x$ and $\pi 2p_y$ (s-p mixing).

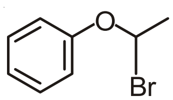
O_2 has 16 e^- (even) but still paramagnetic and has two unpaired electrons in π^*2p_x and π^*2p_y molecular orbitals.

14. Which of the following, upon treatment with *tert*-BuONa followed by addition of bromine water, fails to decolourize the colour of bromine?

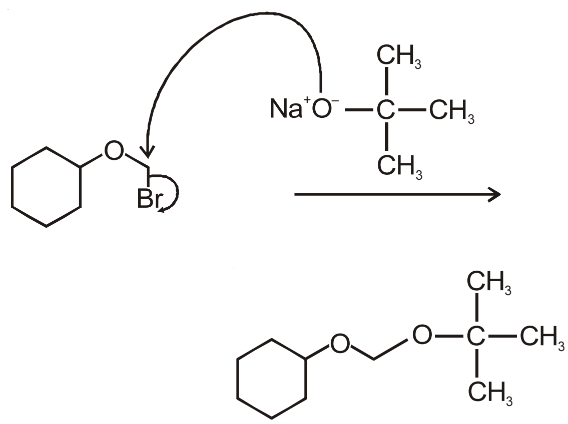
(1) 

(2) 

(3) 

(4) 

Answer (2)

Sol. 

(Product)

The above product does not have any C = C or C ≡ C bond, so, it will not give Br_2-water test.

15. Which of the following reactions is an example of a redox reaction?

(1) $XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$

(2) $XeF_4 + O_2F_2 \rightarrow XeF_6 + O_2$

(3) $XeF_2 + PF_5 \rightarrow [XeF]^+PF_6^-$

(4) $XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$

Answer (2)

Sol. Xe is oxidised from +4(in XeF_4) to +6(in XeF_6)

Oxygen is reduced from +1 (in O_2F_2) to zero (in O_2)

16. ΔU is equal to

(1) Isothermal work (2) Isochoric work

(3) Isobaric work (4) Adiabatic work

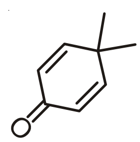
Answer (4)

Sol. For adiabatic process, q = 0

$\therefore$ As per 1st law of thermodynamics,

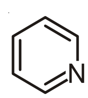
$\Delta U = W$

17. Which of the following molecules is least resonance stabilized?

(1) 

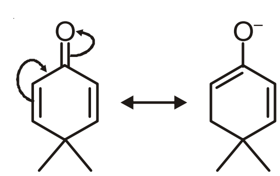
(2) 

(3) 

(4) 

Answer (1)

Sol. However, all molecules given in options are stabilised by resonance but compound given in option (1) is least resonance stabilised (other three are aromatic)



18. The increasing order of the reactivity of the following halides for the S_N1 reaction is

I. $CH_3CH(Cl)CH_2CH_3$

II. $CH_3CH_2CH_2Cl$

III. *p*–$H_3CO - C_6H_4 - CH_2Cl$

(1) (II) < (III) < (I) (2) (III) < (II) < (I)

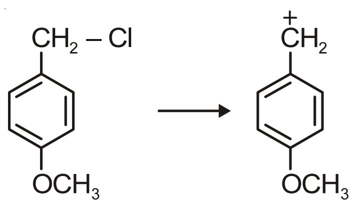
(3) (II) < (I) < (III) (4) (I) < (III) < (II)

Answer (3)

Sol. Rate of S_N1 reaction $\propto$ stability of carbocation

I. $CH_3-CH(Cl)-CH_2-CH_3 \longrightarrow CH_3-\overset{+}{C}H-CH_2-CH_3$

II. $CH_3-CH_2-CH_2-Cl \longrightarrow CH_3-CH_2-\overset{+}{C}H_2$

III. 

So, II < I < III

Increase stability of carbocation and hence increase reactivity of halides.

19. 1 gram of a carbonate (M_2CO_3) on treatment with excess HCl produces 0.01186 mole of CO_2. The molar mass of M_2CO_3 in g mol^{-1} is

(1) 11.86 (2) 1186

(3) 84.3 (4) 118.6

Answer (3)

Sol. $M_2CO_3 + 2HCl \rightarrow 2MCl + H_2O + CO_2$

$$n_{M_2CO_3} = n_{CO_2}$$

$$\frac{1}{M_{M_2CO_3}} = 0.01186$$

$$M_{M_2CO_3} = \frac{1}{0.01186}$$

$$= 84.3 \text{ g/mol}$$

20. Sodium salt of an organic acid 'X' produces effervescence with conc. H_2SO_4. 'X' reacts with the acidified aqueous $CaCl_2$ solution to give a white precipitate which decolourises acidic solution of $KMnO_4$. 'X' is

(1) $Na_2C_2O_4$

(2) C_6H_5COONa

(3) HCOONa

(4) CH_3COONa

Answer (1)

Sol. $\underset{(X)}{Na_2C_2O_4} + \underset{\text{Conc.}}{H_2SO_4} \longrightarrow Na_2SO_4 + \underset{\text{oxalic acid}}{H_2C_2O_4}$

$$H_2C_2O_4 \xrightarrow[\Delta \\ -H_2O]{\text{Conc. } H_2SO_4} \underbrace{CO\uparrow + CO_2\uparrow}_{\text{(effervescence)}}$$

$$\underset{(X)}{Na_2C_2O_4} + CaCl_2 \longrightarrow \underset{\text{white ppt.}}{CaC_2O_4\downarrow} + 2NaCl$$

$$2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \rightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$$

21. The most abundant elements by mass in the body of a healthy human adult are :

Oxygen (61.4%); Carbon (22.9%); Hydrogen (10.0%) and Nitrogen (2.6%).

The weight which a 75 kg person would gain if all ^1H atoms are replaced by ^2H atoms is

(1) 10 kg

(2) 15 kg

(3) 37.5 kg

(4) 7.5 kg

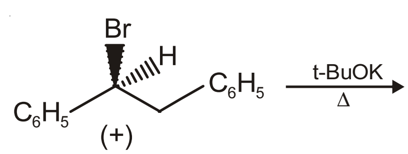
Answer (4)

Sol. Mass of hydrogen = $\frac{10}{100} \times 75$ = 7.5 kg

Replacing ^1H by ^2H would replace 7.5 kg with 15 kg

$\therefore$ Net gain = 7.5 kg

22. The major product obtained in the following reaction is



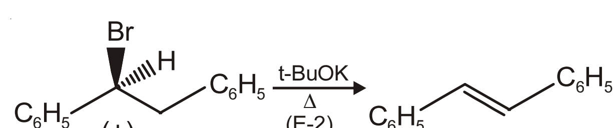
(1) $(-)C_6H_5CH(O^tBu)CH_2C_6H_5$

(2) $(\pm)C_6H_5CH(O^tBu)CH_2C_6H_5$

(3) $C_6H_5CH = CHC_6H_5$

(4) $(+)C_6H_5CH(O^tBu)CH_2C_6H_5$

Answer (3)

Sol. 

23. Given

$C_{(graphite)} + O_2(g) \rightarrow CO_2(g);$

$\Delta_rH^o = -393.5\ kJ\ mol^{-1}$

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l);$

$\Delta_rH^o = -285.8\ kJ\ mol^{-1}$

$CO_2(g) + 2H_2O(l) \rightarrow CH_4(g) + 2O_2(g);$

$\Delta_rH^o = +890.3\ kJ\ mol^{-1}$

Based on the above thermochemical equations, the value of Δ_rH^o at 298 K for the reaction

$C_{(graphite)} + 2H_2(g) \rightarrow CH_4(g)$ will be

(1) $-144.0\ kJ\ mol^{-1}$

(2) $+74.8\ kJ\ mol^{-1}$

(3) $+144.0\ kJ\ mol^{-1}$

(4) $-74.8\ kJ\ mol^{-1}$

Answer (4)

Sol. $C_{(graphite)} + O_2(g) \rightarrow CO_2(g);$

$\Delta_rH° = -393.5\ kJ\ mol^{-1}$ ...(i)

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(l);$

$\Delta_rH° = -285.8\ kJ\ mol^{-1}$ ...(ii)

$CO_2(g) + 2H_2O(l) \rightarrow CH_4(g) + 2O_2(g);$

$\Delta_rH° = 890.3\ kJ\ mol^{-1}$ ...(iii)

By applying the operation

(i) + 2 × (ii) + (iii), we get

$C_{(graphite)} + 2H_2(g) \rightarrow CH_4(g);$

$\Delta_rH° = -393.5 - 285.8 \times 2 + 890.3$

$= -74.8\ kJ\ mol^{-1}$

24. In the following reactions, ZnO is respectively acting as a/an

(a) $ZnO + Na_2O \rightarrow Na_2ZnO_2$

(b) $ZnO + CO_2 \rightarrow ZnCO_3$

(1) Acid and base

(2) Base and acid

(3) Base and base

(4) Acid and acid

Answer (1)

Sol. In (a), ZnO acts as acidic oxide as Na_2O is basic oxide.

In (b), ZnO acts as basic oxide as CO_2 is acidic oxide.

25. The radius of the second Bohr orbit for hydrogen atom is

(Planck's Const. $h = 6.6262 \times 10^{-34}$ Js;

mass of electron $= 9.1091 \times 10^{-31}$ kg;

charge of electron $e = 1.60210 \times 10^{-19}$ C;

permittivity of vacuum

$\varepsilon_0 = 8.854185 \times 10^{-12}\ kg^{-1}\ m^{-3}\ A^2$)

(1) 2.12 Å

(2) 1.65 Å

(3) 4.76 Å

(4) 0.529 Å

Answer (1)

Sol. $r = a_0 \frac{n^2}{Z} = 0.529 \times 4$

$= 2.12$ Å

26. Two reactions R_1 and R_2 have identical pre-exponential factors. Activation energy of R_1 exceeds that of R_2 by 10 kJ mol^{-1}. If k_1 and k_2 are rate constants for reactions R_1 and R_2 respectively at 300 K, then $\ln(k_2/k_1)$ is equal to

($R = 8.314\ J\ mole^{-1}\ K^{-1}$)

(1) 4 (2) 8

(3) 12 (4) 6

Answer (1)

Sol. $k_1 = Ae^{-E_{a_1}/RT}$

$k_2 = Ae^{-E_{a_2}/RT}$

$\frac{k_2}{k_1} = e^{\frac{1}{RT}(E_{a_1} - E_{a_2})}$

$\ln\frac{k_2}{k_1} = \frac{E_{a_1} - E_{a_2}}{RT}$

$= \frac{10 \times 10^3}{8.314 \times 300} \approx 4$

27. A metal crystallises in a face centred cubic structure. If the edge length of its unit cell is 'a', the closest approach between two atoms in metallic crystal will be

(1) $\frac{a}{\sqrt{2}}$

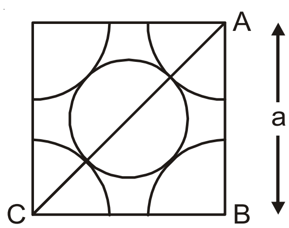
(2) 2a

(3) $2\sqrt{2}a$

(4) $\sqrt{2}a$

Answer (1)

Sol. In FCC, one of the face is like



By ΔABC,

$$2a^2 = 16r^2$$

$$\Rightarrow r^2 = \frac{1}{8}a^2$$

$$\Rightarrow r = \frac{1}{2\sqrt{2}}a$$

Distance of closest approach = $2r = \frac{a}{\sqrt{2}}$

28. The group having isoelectronic species is

(1) O^-, F^-, Na^+, Mg^{2+}

(2) O^{2-}, F^-, Na^+, Mg^{2+}

(3) O^-, F^-, Na, Mg^+

(4) O^{2-}, F^-, Na, Mg^{2+}

Answer (2)

Sol. Mg^{2+}, Na^+, O^{2-} and F^- all have 10 electrons each.

29. Given

$$E^\circ_{Cl_2/Cl^-} = 1.36\ V,\ E^\circ_{Cr^{3+}/Cr} = -0.74\ V$$

$$E^\circ_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33\ V,\ E^\circ_{MnO_4^-/Mn^{2+}} = 1.51\ V$$

Among the following, the strongest reducing agent is

(1) Cl^-

(2) Cr

(3) Mn^{2+}

(4) Cr^{3+}

Answer (2)

Sol. For Cr^{3+}, $E^\circ_{Cr^{3+}/Cr_2O_7^{2-}} = -1.33\ V$

For Cl^-, $E^\circ_{Cl^-/Cl_2} = -1.36\ V$

For Cr, $E^\circ_{Cr/Cr^{3+}} = 0.74\ V$

For Mn^{2+}, $E^\circ_{Mn^{2+}/MnO_4^-} = -1.51\ V$

Positive E° is for Cr, hence it is strongest reducing agent.

30. 3-Methyl-pent-2-ene on reaction with HBr in presence of peroxide forms an addition product. The number of possible stereoisomers for the product is

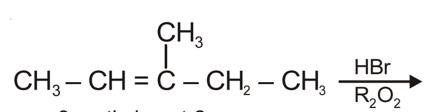
(1) Four

(2) Six

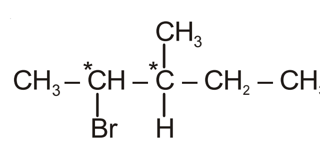
(3) Zero

(4) Two

Answer (1)

Sol. 

3-methyl pent-2-ene



Product (X)

Since product (X) contains two chiral centres and it is unsymmetrical.

So, its total stereoisomers = $2^2 = 4$.

PART–B : MATHEMATICS

31. The integral $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{1+\cos x}$ is equal to

(1) 4 (2) –1

(3) –2 (4) 2

Answer (4)

Sol. $\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{dx}{2\cos^2 \frac{x}{2}} dx = \frac{1}{2}\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \sec^2 \frac{x}{2} dx$

$$= \frac{1}{2}\left[\frac{\tan \frac{x}{2}}{\frac{1}{2}}\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \tan\frac{3\pi}{8} - \tan\frac{\pi}{8}$$

$$\left[\tan\frac{\pi}{8} = \sqrt{\frac{1-\cos\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}} = \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}} = \frac{\sqrt{2}-1}{1}\right.$$

$$\left.\tan\frac{3\pi}{8} = \sqrt{\frac{1-\cos\frac{3\pi}{4}}{1+\cos\frac{3\pi}{4}}} = \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} = \sqrt{2}+1\right]$$

$$= (\sqrt{2}+1)-(\sqrt{2}-1)$$

$$= 2$$

32. Let $I_n = \int \tan^n x dx, (n > 1)$. If $I_4 + I_6 = a \tan^5 x + bx^5 + C$, where C is a constant of integration, then the ordered pair (a, b) is equal to

(1) $\left(\frac{1}{5}, -1\right)$ (2) $\left(-\frac{1}{5}, 0\right)$

(3) $\left(-\frac{1}{5}, 1\right)$ (4) $\left(\frac{1}{5}, 0\right)$

Answer (4)

Sol. $I_n = \int \tan^n x dx,\ n > 1$

$$I_4 + I_6 = \int (\tan^4 x + \tan^6 x) dx$$

$$= \int \tan^4 x \sec^2 x dx$$

Let $\tan x = t$

$\sec^2 x\ dx = dt$

$$= \int t^4 dt$$

$$= \frac{t^5}{5} + C$$

$$= \frac{1}{5}\tan^5 x + C$$

$$a = \frac{1}{5},\ b = 0$$

33. The area (in sq. units) of the region

$\{(x, y) : x \ge 0,\ x + y \le 3,\ x^2 \le 4y \text{ and } y \le 1 + \sqrt{x}\}$

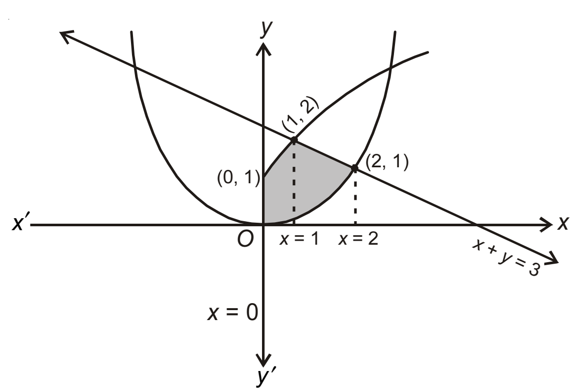
is

(1) $\frac{7}{3}$ (2) $\frac{5}{2}$

(3) $\frac{59}{12}$ (4) $\frac{3}{2}$

Answer (2)

Sol.



Area of shaded region

$$= \int_0^1 \left(\sqrt{x} + 1 - \frac{x^2}{4}\right) dx + \int_1^2 \left((3-x) - \frac{x^2}{4}\right) dx$$

$= \frac{5}{2}$ sq. unit

34. A box contains 15 green and 10 yellow balls. If 10 balls are randomly drawn, one-by-one, with replacement, then the variance of the number of green balls drawn is

(1) 4 (2) $\frac{6}{25}$

(3) $\frac{12}{5}$ (4) 6

Answer (3)

Sol. $n = 10$

p(Probability of drawing a green ball) = $\frac{15}{25}$

$\therefore \quad p = \frac{3}{5}, \; q = \frac{2}{5}$

var(X) = $n.p.q$

$= 10 \cdot \frac{6}{25} = \frac{12}{5}$

35. If $(2+\sin x)\frac{dy}{dx} + (y+1)\cos x = 0$ and $y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to

(1) $-\frac{1}{3}$ (2) $\frac{4}{3}$

(3) $\frac{1}{3}$ (4) $-\frac{2}{3}$

Answer (3)

Sol. $(2+\sin x)\frac{dy}{dx} + (y+1)\cos x = 0$

$y(0) = 1, \; y\left(\frac{\pi}{2}\right) = ?$

$\frac{1}{y+1}dy + \frac{\cos x}{2+\sin x}dx = 0$

$\ln|y+1| + \ln(2+\sin x) = \ln C$

$(y+1)(2+\sin x) = C$

Put $x = 0$, $y = 1$

$(1+1)\cdot 2 = C \Rightarrow C = 4$

Now, $(y+1)(2+\sin x) = 4$

For, $x = \frac{\pi}{2}$

$(y+1)(2+1) = 4$

$y + 1 = \frac{4}{3}$

$y = \frac{4}{3} - 1 = \frac{1}{3}$

36. Let ω be a complex number such that $2\omega + 1 = z$ where $z = \sqrt{-3}$. If

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -\omega^2-1 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = 3k, \text{ then } k \text{ is equal to}$$

(1) –1 (2) 1

(3) $-z$ (4) z

Answer (3)

Sol. $2\omega + 1 = z$, $z = \sqrt{3}\,i$

$\omega = \frac{-1+\sqrt{3}i}{2} \rightarrow$ Cube root of unity.

$C_1 \rightarrow C_1 + C_2 + C_3$

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1-\omega^2 & \omega^2 \\ 1 & \omega^2 & \omega^7 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} = \begin{vmatrix} 3 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix}$$

$= 3(\omega^2 - \omega^4)$

$= 3\left[\left(\frac{-1-\sqrt{3}i}{2}\right) - \left(\frac{-1+\sqrt{3}i}{2}\right)\right]$

$= -3\sqrt{3}i$

$= -3z$

$\therefore \; k = -z$

37. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. Let $\vec{c}$ be a vector such that $|\vec{c} - \vec{a}| = 3, \left|(\vec{a} \times \vec{b}) \times \vec{c}\right| = 3$ and the angle between $\vec{c}$ and $\vec{a} \times \vec{b}$ be 30°. Then $\vec{a} \cdot \vec{c}$ is equal to

(1) 5 (2) $\frac{1}{8}$

(3) $\frac{25}{8}$ (4) 2

Answer (4)

Sol. $|(\vec{a} \times \vec{b}) \times \vec{c}| = 3$ $\quad\quad$ $\vec{a} \times \vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$

$\Rightarrow |\vec{a} \times \vec{b}||\vec{c}|\sin 30° = 3$ $\quad\quad$ $|\vec{a}| = 3 = |\vec{a} \times \vec{b}|$

$\Rightarrow |\vec{c}| = 2$

$|\vec{c} - \vec{a}| = 3$

$\Rightarrow |\vec{c}|^2 + |\vec{a}|^2 - 2(\vec{a} \cdot \vec{c}) = 9$

$\vec{a} \cdot \vec{c} = \frac{9-3-2}{2} = 2$

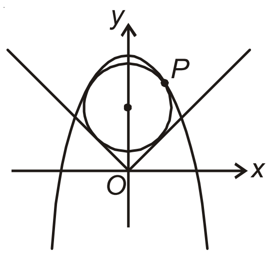
38. The radius of a circle, having minimum area, which touches the curve $y = 4 - x^2$ and the lines, $y = |x|$ is

(1) $4(\sqrt{2}-1)$ (2) $4(\sqrt{2}+1)$

(3) $2(\sqrt{2}+1)$ (4) $2(\sqrt{2}-1)$

Answer (1)

Sol.



$x^2 = -(y-4)$

Let a point on the parabola $P\left(\frac{t}{2},\ 4-\frac{t^2}{4}\right)$

Equation of normal at P is

$$y+\frac{t^2}{4}-4=\frac{1}{t}\left(x-\frac{t}{2}\right)$$

$$\Rightarrow \quad x-ty-\frac{t^3}{4}+\frac{7}{2}t=0$$

It passes through centre of circle, say $(0, k)$

$$-tk-\frac{t^3}{4}+\frac{7}{2}t=0 \qquad \text{...(i)}$$

$t = 0,\ t^2 = 14-4k$

Radius $= r = \left|\frac{0-k}{\sqrt{2}}\right|$ (Length of perpendicular from $(0, k)$ to $y = x$)

$$\Rightarrow \quad r=\frac{k}{\sqrt{2}}$$

Equation of circle is $x^2+(y-k)^2=\frac{k^2}{2}$

It passes through point P

$$\frac{t^2}{4}+\left(4-\frac{t^2}{4}-k\right)^2=\frac{k^2}{2}$$

$$t^4+t^2(8k-28)+8k^2-128k+256=0 \qquad \text{...(ii)}$$

For $t = 0 \Rightarrow k^2-16k+32=0$

$$k=8\pm4\sqrt{2}$$

$\therefore \quad r=\frac{k}{\sqrt{2}}=4(\sqrt{2}-1)$ (discarding $4(\sqrt{2}+1)$) ...(iii)

For $t=\pm\sqrt{14-4k}$

$$(14-4k)^2+(14-4k)(8k-28)+8k^2-128k+256=0$$

$$2k^2+4k-15=0$$

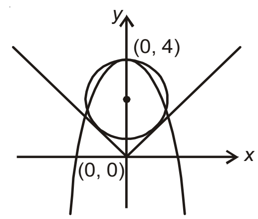
$$k=\frac{-2\pm\sqrt{34}}{2}$$

$\therefore \quad r=\frac{k}{\sqrt{2}}=\frac{\sqrt{17}-\sqrt{2}}{2}$ (Ignoring negative value of r) ...(iv)

From (iii) & (iv),

$$r_{\min}=\frac{\sqrt{17}-\sqrt{2}}{2}$$

But from options, $r=4(\sqrt{2}-1)$



39. If for $x\in\left(0,\ \frac{1}{4}\right)$, the derivative of $\tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}\cdot g(x)$, then $g(x)$ equals

(1) $\frac{3x}{1-9x^3}$ (2) $\frac{3}{1+9x^3}$

(3) $\frac{9}{1+9x^3}$ (4) $\frac{3x\sqrt{x}}{1-9x^3}$

Answer (3)

Sol. $f(x)=2\tan^{-1}(3x\sqrt{x})$ For $x\in\left(0,\ \frac{1}{4}\right)$

$$f'(x)=\frac{9\sqrt{x}}{1+9x^3}$$

$$g(x)=\frac{9}{1+9x^3}$$

40. If two different numbers are taken from the set {0, 1, 2, 3, ..., 10}; then the probability that their sum as well as absolute difference are both multiple of 4, is

(1) $\frac{14}{45}$ (2) $\frac{7}{55}$

(3) $\frac{6}{55}$ (4) $\frac{12}{55}$

Answer (3)

Sol. Total number of ways = ${}^{11}C_2$

= 55

Favourable ways are

(0, 4), (0, 8), (4, 8), (2, 6), (2, 10), (6, 10)

Probability = $\frac{6}{55}$

41. $\lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2\pi)^3}$ equals

(1) $\frac{1}{8}$ (2) $\frac{1}{4}$

(3) $\frac{1}{24}$ (4) $\frac{1}{16}$

Answer (4)

Sol. $\lim_{x \to \frac{\pi}{2}} \frac{\cot x - \cos x}{(\pi - 2x)^3}$

Put, $\frac{\pi}{2} - x = t$

$\lim_{t \to 0} \frac{\tan t - \sin t}{8t^3}$

$= \lim_{t \to 0} \frac{\sin t \cdot 2\sin^2 \frac{t}{2}}{8t^3}$

$= \frac{1}{16}$.

42. The value of

$({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) +$

$({}^{21}C_4 - {}^{10}C_4) + \ldots + ({}^{21}C_{10} - {}^{10}C_{10})$ is

(1) $2^{20} - 2^9$ (2) $2^{20} - 2^{10}$

(3) $2^{21} - 2^{11}$ (4) $2^{21} - 2^{10}$

Answer (2)

Sol. ${}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10} = \frac{1}{2}\left\{{}^{21}C_0 + {}^{21}C_1 + \ldots + {}^{21}C_{21}\right\} - 1$

$= 2^{20} - 1$

$\left({}^{10}C_1 + {}^{10}C_2 + \ldots + {}^{10}C_{10}\right) = 2^{10} - 1$

$\therefore$ Required sum = $(2^{20} - 1) - (2^{10} - 1)$

$= 2^{20} - 2^{10}$

43. For three events A, B and C, P (Exactly one of A or B occurs) = P(Exactly one of B or C occurs)

= P (Exactly one of C or A occurs) = $\frac{1}{4}$ and

P(All the three events occur simultaneously) = $\frac{1}{16}$.

Then the probability that at least one of the events occurs, is

(1) $\frac{7}{64}$ (2) $\frac{3}{16}$

(3) $\frac{7}{32}$ (4) $\frac{7}{16}$

Answer (4)

Sol. $P(A) + P(B) - P(A \cap B) = \frac{1}{4}$

$P(B) + P(C) - P(B \cap C) = \frac{1}{4}$

$P(C) + P(A) - P(A \cap C) = \frac{1}{4}$

$P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) = \frac{3}{8}$

$\because \; P(A \cap B \cap C) = \frac{1}{16}$

$\therefore \; P(A \cup B \cup C) = \frac{3}{8} + \frac{1}{16} = \frac{7}{16}$

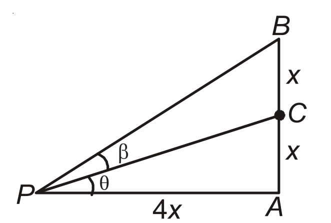
44. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that $AP = 2AB$. If $\angle BPC = \beta$ then $\tan \beta$ is

(1) $\frac{2}{9}$ (2) $\frac{4}{9}$

(3) $\frac{6}{7}$ (4) $\frac{1}{4}$

Answer (1)

Sol. $\tan\theta = \frac{1}{4}$



$\tan(\theta + \beta) = \frac{1}{2}$

$\therefore \; \frac{\frac{1}{4} + \tan\beta}{1 - \frac{1}{4}\tan\beta} = \frac{1}{2}$

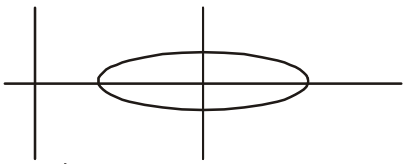
Solving $\tan\beta = \frac{2}{9}$

45. The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directrices is $x = -4$, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is

(1) $4x + 2y = 7$
(2) $x + 2y = 4$
(3) $2y - x = 2$
(4) $4x - 2y = 1$

Answer (4)

Sol.



$x = -4$

$$e = \frac{1}{2}$$

$$\frac{-a}{e} = -4$$

$$-a = -4 \times e$$

$$\boxed{a = 2}$$

Now, $b^2 = a^2(1 - e^2) = 3$

Equation to ellipse

$$\frac{x^2}{4} + \frac{y^2}{3} = 1$$

Equation of normal is

$$\frac{x-1}{\frac{1}{4}} = \frac{y - \frac{3}{2}}{\frac{3}{2\times 3}} \Rightarrow 4x - 2y - 1 = 0$$

46. If, for a positive integer n, the quadratic equation,

$$x(x+1) + (x+1)(x+2) + \ldots + (x + \overline{n-1})(x+n) = 10n$$

has two consecutive integral solutions, then n is equal to

(1) 10
(2) 11
(3) 12
(4) 9

Answer (2)

Sol. Rearranging equation, we get

$$nx^2 + \{1 + 3 + 5 + \ldots + (2n-1)\}x + \{1\cdot 2 + 2\cdot 3 + \ldots + (n-1)n\} = 10n$$

$$\Rightarrow nx^2 + n^2x + \frac{(n-1)n(n+1)}{3} = 10n$$

$$\Rightarrow x^2 + nx + \left(\frac{n^2 - 31}{3}\right) = 0$$

Given difference of roots = 1

$\Rightarrow$ $|\alpha - \beta| = 1$

$\Rightarrow$ $D = 1$

$$\Rightarrow \quad n^2 - \frac{4}{3}(n^2 - 31) = 1$$

So, $n = 11$

47. The following statement $(p \to q) \to [(\sim p \to q) \to q]$ is

(1) Equivalent to $p \to \sim q$
(2) A fallacy
(3) A tautology
(4) Equivalent to $\sim p \to q$

Answer (3)

Sol.

p	q	$p \to q$	$(\sim p \to q)$	$(\sim p \to q) \to q$	$(p \to q) \to [(\sim p \to q) \to q]$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	T	T

(a tautology)

48. The normal to the curve $y(x-2)(x-3) = x + 6$ at the point where the curve intersects the y-axis passes through the point

(1) $\left(\frac{1}{2}, -\frac{1}{3}\right)$
(2) $\left(\frac{1}{2}, \frac{1}{3}\right)$
(3) $\left(-\frac{1}{2}, -\frac{1}{2}\right)$
(4) $\left(\frac{1}{2}, \frac{1}{2}\right)$

Answer (4)

Sol. $y(x-2)(x-3) = x + 6$

At y-axis, $x = 0$, $y = 1$

Now, on differentiation.

$$\frac{dy}{dx}(x-2)(x-3) + y(2x-5) = 1$$

$$\frac{dy}{dx}(6) + 1(-5) = 1$$

$$\frac{dy}{dx} = \frac{6}{6} = 1$$

Now slope of normal = –1

Equation of normal $y - 1 = -1(x - 0)$

$y + x - 1 = 0$ ... (i)

Line (i) passes through $\left(\frac{1}{2}, \frac{1}{2}\right)$

49. For any three positive real numbers a, b and c,

$$9(25a^2+b^2)+25(c^2-3ac)=15b(3a+c).$$

Then

(1) a, b and c are in A.P.

(2) a, b and c are in G.P.

(3) b, c and a are in G.P.

(4) b, c and a are in A.P.

Answer (4)

Sol. $9(25a^2+b^2)+25(c^2-3ac)=15b(3a+c)$

$\Rightarrow\ (15a)^2+(3b)^2+(5c)^2-45ab-15bc-75ac=0$

$\Rightarrow\ (15a-3b)^2+(3b-5c)^2+(15a-5c)^2=0$

It is possible when

$15a-3b=0$ and $3b-5c=0$ and $15a-5c=0$

$15a=3b=5c$

$\frac{a}{1}=\frac{b}{5}=\frac{c}{3}$

$\therefore\ b, c, a$ are in A.P.

50. If the image of the point $P(1, -2, 3)$ in the plane, $2x+3y-4z+22=0$ measured parallel to the line, $\frac{x}{1}=\frac{y}{4}=\frac{z}{5}$ is Q, then PQ is equal to

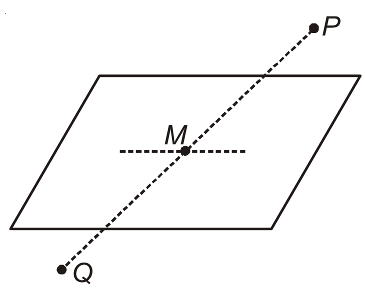
(1) $\sqrt{42}$ (2) $6\sqrt{5}$

(3) $3\sqrt{5}$ (4) $2\sqrt{42}$

Answer (4)

Sol. Equation of PQ, $\frac{x-1}{1}=\frac{y+2}{4}=\frac{z-3}{5}$

Let M be $(\lambda+1, 4\lambda-2, 5\lambda+3)$



As it lies on $2x+3y-4z+22=0$

$\lambda=1$

For Q, $\lambda=2$

Distance $PQ=2\sqrt{1^2+4^2+5^2}=2\sqrt{42}$

51. If $5(\tan^2x-\cos^2x)=2\cos 2x+9$, then the value of $\cos 4x$ is

(1) $\frac{2}{9}$ (2) $-\frac{7}{9}$

(3) $-\frac{3}{5}$ (4) $\frac{1}{3}$

Answer (2)

Sol. $5\tan^2x=9\cos^2x+7$

$5\sec^2x-5=9\cos^2x+7$

Let $\cos^2x=t$

$\frac{5}{t}=9t+12$

$9t^2+12t-5=0$

$t=\frac{1}{3}$ as $t\neq-\frac{5}{3}$

$\cos^2 x=\frac{1}{3}$, $\cos 2x=2\cos^2x-1$

$=-\frac{1}{3}$

$\cos 4x=2\cos^2 2x-1$

$=\frac{2}{9}-1$

$=-\frac{7}{9}$

52. Let $a, b, c\in R$. If $f(x)=ax^2+bx+c$ is such that $a+b+c=3$ and

$f(x+y)=f(x)+f(y)+xy,\ \forall\ x, y\in R,$

then $\sum_{n=1}^{10}f(n)$ is equal to

(1) 190 (2) 255

(3) 330 (4) 165

Answer (3)

Sol. As, $f(x+y)=f(x)+f(y)+xy$

Given, $f(1)=3$

Putting, $x=y=1\ \Rightarrow\ f(2)=2f(1)+1=7$

Similarly, $x=1, y=2\ \Rightarrow\ f(3)=f(1)+f(2)+2=12$

Now, $\sum_{n=1}^{10} f(n) = f(1)+f(2)+f(3)+...+f(10)$

$= 3 + 7 + 12 + 18 + ... = S$ (let)

Now, $S_n = 3+7+12+18+...+t_n$

Again, $S_n = 3+7+12+...+t_{n-1}+t_n$

We get, $t_n = 3+4+5+...n$ terms

$= \dfrac{n(n+5)}{2}$

i.e., $S_n = \sum_{n=1}^{n} t_n$

$= \dfrac{1}{2}\left\{\sum n^2 + 5\sum n\right\}$

$= \dfrac{n(n+1)(n+8)}{6}$

So, $S_{10} = \dfrac{10\times 11\times 18}{6} = 330$

53. The distance of the point (1, 3, –7) from the plane passing through the point (1, –1, –1), having normal perpendicular to both the lines $\dfrac{x-1}{1} = \dfrac{y+2}{-2} = \dfrac{z-4}{3}$ and $\dfrac{x-2}{2} = \dfrac{y+1}{-1} = \dfrac{z+7}{-1}$, is

(1) $\dfrac{5}{\sqrt{83}}$ (2) $\dfrac{10}{\sqrt{74}}$

(3) $\dfrac{20}{\sqrt{74}}$ (4) $\dfrac{10}{\sqrt{83}}$

Answer (4)

Sol. Let the plane be

$a(x-1)+b(y+1)+c(z+1)=0$

It is perpendicular to the given lines

$a - 2b + 3c = 0$

$2a - b - c = 0$

Solving, $a : b : c = 5 : 7 : 3$

$\therefore$ The plane is $5x + 7y + 3z + 5 = 0$

Distance of (1, 3, –7) from this plane $= \dfrac{10}{\sqrt{83}}$

54. If S is the set of distinct values of b for which the following system of linear equations

$x+y+z=1$

$x+ay+z=1$

$ax+by+z=0$

has no solution, then S is

(1) A finite set containing two or more elements

(2) A singleton

(3) An empty set

(4) An infinite set

Answer (2)

Sol.

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & 1 \\ a & b & 1 \end{vmatrix} = 0$$

$\Rightarrow -(1-a)^2 = 0$

$\Rightarrow a = 1$

For $a = 1$

Eq. (1) & (2) are identical *i.e.*, $x + y + z = 1$

To have no solution with $x + by + z = 0$.

$b = 1$

55. If $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to

(1) $\begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$ (2) $\begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$

(3) $\begin{bmatrix} 72 & -84 \\ -63 & 51 \end{bmatrix}$ (4) $\begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$

Answer (4)

Sol. $A = \begin{bmatrix} 2 & -3 \\ -4 & 1 \end{bmatrix}$

$$|A-\lambda I| = \begin{vmatrix} 2-\lambda & -3 \\ -4 & 1-\lambda \end{vmatrix}$$

$= (2 - 2\lambda - \lambda + \lambda^2) - 12$

$f(\lambda) = \lambda^2 - 3\lambda - 10$

$\because A$ satisfies $f(\lambda)$

$\therefore \quad A^2 - 3A - 10I = 0$

$A^2 - 3A = 10I$

$3A^2 - 9A = 30I$

$3A^2 + 12A = 30I + 21A$

$$= \begin{bmatrix} 30 & 0 \\ 0 & 30 \end{bmatrix} + \begin{bmatrix} 42 & -63 \\ -84 & 21 \end{bmatrix}$$

$$= \begin{bmatrix} 72 & -63 \\ -84 & 51 \end{bmatrix}$$

$$\text{adj}(3A^2 + 12A) = \begin{bmatrix} 51 & 63 \\ 84 & 72 \end{bmatrix}$$

56. A hyperbola passes through the point $P(\sqrt{2}, \sqrt{3})$ and has foci at (±2, 0). Then the tangent to this hyperbola at P also passes through the point

(1) $(\sqrt{3}, \sqrt{2})$ (2) $(-\sqrt{2}, -\sqrt{3})$

(3) $(3\sqrt{2}, 2\sqrt{3})$ (4) $(2\sqrt{2}, 3\sqrt{3})$

Answer (4)

Sol. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$a^2 + b^2 = 4$

and $\frac{2}{a^2} - \frac{3}{b^2} = 1$

$\frac{2}{4 - b^2} - \frac{3}{b^2} = 1$

$\Rightarrow \quad b^2 = 3$

$\therefore \quad a^2 = 1$

$\therefore \quad x^2 - \frac{y^2}{3} = 1$

$\therefore$ Tangent at $P(\sqrt{2}, \sqrt{3})$ is $\sqrt{2}x - \frac{y}{\sqrt{3}} = 1$

Clearly it passes through $(2\sqrt{2}, 3\sqrt{3})$

57. Let k be an integer such that the triangle with vertices (k, $-3k$), (5, k) and ($-k$, 2) has area 28 sq. units. Then the orthocentre of this triangle is at the point

(1) $\left(1, -\frac{3}{4}\right)$ (2) $\left(2, \frac{1}{2}\right)$

(3) $\left(2, -\frac{1}{2}\right)$ (4) $\left(1, \frac{3}{4}\right)$

Answer (2)

Sol. Area $= \left| \frac{1}{2} \begin{vmatrix} k & -3k & 1 \\ 5 & k & 1 \\ -k & 2 & 1 \end{vmatrix} \right| = 28$

$$\begin{vmatrix} k-5 & -4k & 0 \\ 5+k & k-2 & 0 \\ -k & 2 & 1 \end{vmatrix} = \pm 56$$

$(k^2 - 7k + 10) + 4k^2 + 20k = \pm 56$

$5k^2 + 13k + 10 = \pm 56$

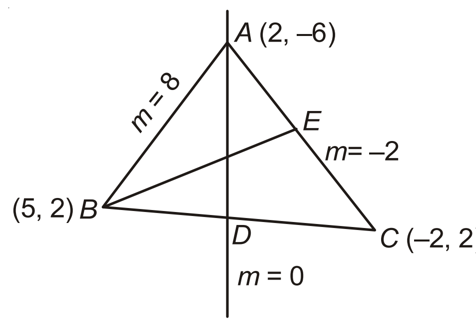
$5k^2 + 13k - 46 = 0$ | $5K^2 + 13K + 66 = 0$

$5k^2 + 13k - 46 = 0$

$k = \frac{-13 \pm \sqrt{169 + 920}}{10}$

= 2, –4.6
reject

For k = 2



Equation of AD,

$x = 2$ ...(i)

Also equation of BE,

$y - 2 = \frac{1}{2}(x - 5)$

$2y - 4 = x - 5$

$x - 2y - 1 = 0$ ...(ii)

Solving (i) & (ii), $2y = 1$

$y = \frac{1}{2}$

Orthocentre is $\left(2, \frac{1}{2}\right)$

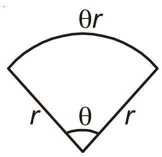
58. Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area (in sq. m) of the flower-bed, is

(1) 25 (2) 30

(3) 12.5 (4) 10

Answer (1)

Sol.



$2r + \theta r = 20$ ... **(i)**

A = area = $\dfrac{\theta}{2\pi} \times \pi r^2 = \dfrac{\theta r^2}{2}$ ... **(ii)**

$A = \dfrac{r^2}{2}\left(\dfrac{20-2r}{r}\right)$

$A = \left(\dfrac{20r - 2r^2}{2}\right) = 10r - r^2$

A to be maximum

$\dfrac{dA}{dr} = 10 - 2r = 0 \Rightarrow r = 5$

$\dfrac{d^2A}{dr^2} = -2 < 0$

Hence for $r = 5$, A is maximum

Now, $10 + \theta \cdot 5 = 20 \Rightarrow \theta = 2$ (radian)

Area = $\dfrac{2}{2\pi} \times \pi (5)^2 = 25$ sq m

59. The function $f : R \to \left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$ defined as $f(x) = \dfrac{x}{1+x^2}$, is

(1) Surjective but not injective

(2) Neither injective nor surjective

(3) Invertible

(4) Injective but not surjective

Answer (1)

Sol. $f(x) = \dfrac{x}{1+x^2}$

$f'(x) = \dfrac{(1+x^2)\cdot 1 - x \cdot 2x}{(1+x^2)^2} = \dfrac{1-x^2}{(1+x^2)^2}$

$f'(x)$ changes sign in different intervals.

$\therefore$ Not injective.

$y = \dfrac{x}{1+x^2}$

$yx^2 - x + y = 0$

For $y \neq 0$

$D = 1 - 4y^2 \geq 0 \Rightarrow y \in \left[-\dfrac{1}{2}, \dfrac{1}{2}\right] - \{0\}$

For, $y = 0 \Rightarrow x = 0$

$\therefore$ Part of range

$\therefore$ Range : $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$

$\therefore$ Surjective but not injective.

60. A man X has 7 friends, 4 of them are ladies and 3 are men. His wife Y also has 7 friends, 3 of them are ladies and 4 are men. Assume X and Y have no common friends. Then the total number of ways in which X and Y together can throw a party inviting 3 ladies and 3 men, so that 3 friends of each of X and Y are in this party, is

(1) 469 (2) 484

(3) 485 (4) 468

Answer (3)

Sol. X(4 L 3 G) Y(3 L 4 G)

3 L 0 G	0 L 3 G
2 L 1 G	1 L 2 G
1 L 2 G	2 L 1 G
0 L 3 G	3 L 0 G

Required number of ways

$= {}^4C_3 \cdot {}^4C_3 + \left({}^4C_2 \cdot {}^3C_1\right)^2 + \left({}^4C_1 \cdot {}^3C_2\right)^2 + \left({}^3C_3\right)^2$

= 16 + 324 + 144 + 1

= 485

61. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light = 3×10^8 ms^{-1})

(1) 12.1 GHz

(2) 17.3 GHz

(3) 15.3 GHz

(4) 10.1 GHz

Answer (2)

Sol. For relativistic motion

$$f = f_0\sqrt{\frac{c+v}{c-v}} \quad ; \quad v = \text{relative speed of approach}$$

$$f = 10\sqrt{\frac{c+\frac{c}{2}}{c-\frac{c}{2}}} = 10\sqrt{3} = 17.3 \text{ GHz}$$

62. The following observations were taken for determining surface tension T of water by capillary method:

diameter of capillary, $D = 1.25 \times 10^{-2}$ m

rise of water, $h = 1.45 \times 10^{-2}$ m.

Using $g = 9.80$ m/s^2 and the simplified relation $T = \frac{rhg}{2} \times 10^3 \text{ N/m}$, the possible error in surface tension is closest to

(1) 1.5%

(2) 2.4%

(3) 10%

(4) 0.15%

Answer (1)

Sol. $\frac{\Delta T}{T} \times 100 = \frac{\Delta D}{D} \times 100 + \frac{\Delta h}{h} \times 100$

$= \frac{0.01}{1.25} \times 100 + \frac{0.01}{1.45} \times 100$

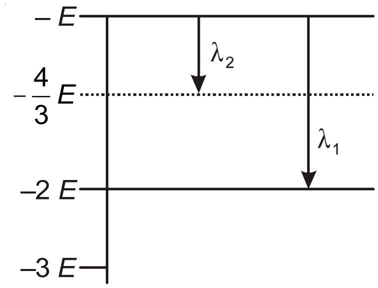
$= \frac{100}{125} + \frac{100}{145}$

$= 0.8 + 0.689$

$= 1.489$

$\simeq 1.5\%$

63. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1/\lambda_2$, is given by



(1) $r = \frac{2}{3}$ (2) $r = \frac{3}{4}$

(3) $r = \frac{1}{3}$ (4) $r = \frac{4}{3}$

Answer (3)

Sol. From energy level diagram

$$\lambda_1 = \frac{hc}{E}$$

$$\lambda_2 = \frac{hc}{\left(\frac{E}{3}\right)}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{1}{3}$$

64. A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10$ ms^{-1}. If, after 10 s, its energy is $\frac{1}{8}mv_0^2$, the value of k will be

(1) 10^{-3} kg s^{-1} (2) 10^{-4} kg m^{-1}

(3) 10^{-1} kg m^{-1} s^{-1} (4) 10^{-3} kg m^{-1}

Answer (2)

Sol. $\frac{k_f}{k_i} = \frac{\frac{1}{8}mv_0^2}{\frac{1}{2}mv_0^2} = \frac{1}{4}$

$$\frac{v_f}{v_i} = \frac{1}{2}$$

$$v_f = \frac{v_0}{2}$$

$$-kv^2 = \frac{mdv}{dt}$$

$$\int_{v_0}^{\frac{v_0}{2}} \frac{dv}{v^2} = \int_0^{t_0} \frac{-kdt}{m}$$

$$\left[-\frac{1}{v}\right]_{v_0}^{\frac{v_0}{2}} = \frac{-k}{m}t_0$$

$$\frac{1}{v_0} - \frac{2}{v_0} = -\frac{k}{m}t_0$$

$$-\frac{1}{v_0} = -\frac{k}{m}t_0$$

$$k = \frac{m}{v_0 t_0}$$

$$= \frac{10^{-2}}{10 \times 10}$$

$= 10^{-4}$ kg m^{-1}

65. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that

$C_p - C_v = a$ for hydrogen gas

$C_p - C_v = b$ for nitrogen gas

The correct relation between a and b is :

(1) $a = b$ (2) $a = 14b$

(3) $a = 28b$ (4) $a = \frac{1}{14}b$

Answer (2)

Sol. Let molar heat capacity at constant pressure = X_p

and molar heat capacity at constant volume = X_v

$$X_p - X_v = R$$

$$MC_p - MC_v = R$$

$$C_p - C_v = \frac{R}{M}$$

For hydrogen; $a = \frac{R}{2}$

For N_2; $b = \frac{R}{28}$

$$\frac{a}{b} = 14$$

$$a = 14b$$

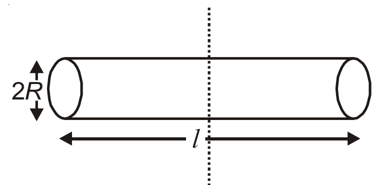
66. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio $\frac{\ell}{R}$ such that the moment of inertia is minimum?

(1) $\frac{\sqrt{3}}{2}$ (2) 1

(3) $\frac{3}{\sqrt{2}}$ (4) $\sqrt{\frac{3}{2}}$

Answer (4)

Sol.



$$I = \frac{mR^2}{4} + \frac{m\ell^2}{12}$$

$$I = \frac{m}{4}\left[R^2 + \frac{\ell^2}{3}\right]$$

$$= \frac{m}{4}\left[\frac{v}{\pi\ell} + \frac{\ell^2}{3}\right]$$

$$\frac{dI}{d\ell} = \frac{m}{4}\left[\frac{-v}{\pi\ell^2} + \frac{2\ell}{3}\right] = 0$$

$$\frac{v}{\pi\ell^2} = \frac{2\ell}{3}$$

$$v = \frac{2\pi\ell^3}{3}$$

$$\pi R^2 \ell = \frac{2\pi\ell^3}{3}$$

$$\frac{\ell^2}{R^2} = \frac{3}{2}$$

$$\frac{\ell}{R} = \sqrt{\frac{3}{2}}$$

67. A radioactive nucleus A with a half life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by

(1) $t = T\frac{\log 1.3}{\log 2}$ (2) $t = T\log(1.3)$

(3) $t = \frac{T}{\log(1.3)}$ (4) $t = \frac{T}{2}\frac{\log 2}{\log 1.3}$

Answer (1)

Sol. $\dfrac{N_0 - N_0 e^{-\lambda t}}{N_0 e^{-\lambda t}} = 0.3$

$\Rightarrow \quad e^{\lambda t} = 1.3$

$\therefore \quad \lambda t = \ln 1.3$

$$\left(\frac{\ln 2}{T}\right) t = \ln 1.3$$

$$t = T.\frac{\ln(1.3)}{\ln 2}$$

$$t = T\frac{\log(1.3)}{\log 2}$$

68. Which of the following statements is false?

(1) In a balanced Wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed

(2) A rheostat can be used as a potential divider

(3) Kirchhoff's second law represents energy conservation

(4) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

Answer (1)

Sol. In a balanced Wheatstone bridge, the null point remains unchanged even if cell and galvanometer are interchanged.

69. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V.

The minimum number of capacitors required to achieve this is

(1) 16

(2) 24

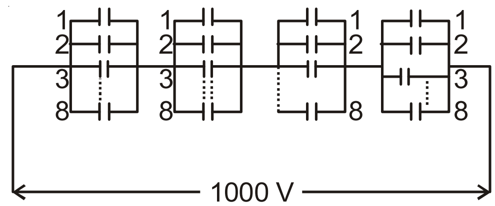
(3) 32

(4) 2

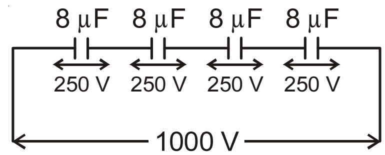
Answer (3)

Sol. Following arrangement will do the needful :

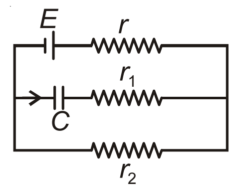
8 capacitors of 1μF in parallel with four such branches in series.



$\equiv$



70. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be :



(1) $CE\dfrac{r_1}{(r_2 + r)}$

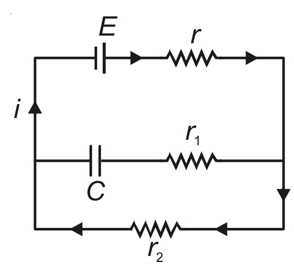
(2) $CE\dfrac{r_2}{(r + r_2)}$

(3) $CE\dfrac{r_1}{(r_1 + r)}$

(4) CE

Answer (2)

Sol. In steady state, flow of current through capacitor will be zero.

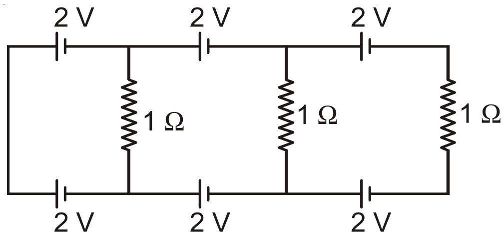


$$i = \frac{E}{r + r_2}$$

$$V_C = i\, r_2 C = \frac{E r_2 C}{r + r_2}$$

$$V_C = CE\,\frac{r_2}{r + r_2}$$

71.



In the above circuit the current in each resistance is

(1) 0.25 A

(2) 0.5 A

(3) 0 A

(4) 1 A

Answer (3)

Sol. The potential difference in each loop is zero.

$\therefore$ No current will flow.

72. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth $(\Delta\omega_m)$ of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave?

(1) ω_c

(2) $\omega_m + \omega_c$

(3) $\omega_c - \omega_m$

(4) ω_m

Answer (4)

Sol. Modulated wave has frequency range.

$\omega_c \pm \omega_m$

$\therefore$ Since $\omega_c >> \omega_m$

$\therefore$ ω_m is excluded.

73. In a common emitter amplifier circuit using an n-p-n transistor, the phase difference between the input and the output voltages will be

(1) 90°

(2) 135°

(3) 180°

(4) 45°

Answer (3)

Sol. In common emitter configuration for n-p-n transistor, phase difference between output and input voltage is 180°.

74. A copper ball of mass 100 gm is at a temperature *T*. It is dropped in a copper calorimeter of mass 100 gm, filled with 170 gm of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. *T* is given by :

(Given : room temperature = 30°C, specific heat of copper = 0.1 cal/gm°C)

(1) 885°C

(2) 1250°C

(3) 825°C

(4) 800°C

Answer (1)

Sol. $100 \times 0.1 \times (t - 75) = 100 \times 0.1 \times 45 + 170 \times 1 \times 45$

$10t - 750 = 450 + 7650$

$10t = 1200 + 7650$

$10t = 8850$

$t = 885°C$

75. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is

(1) 7.8 mm

(2) 9.75 mm

(3) 15.6 mm

(4) 1.56 mm

Answer (1)

Sol. For λ_1

$$y = \frac{m\lambda_1 D}{d}$$

For λ_2

$$y = \frac{n\lambda_2 D}{d}$$

$$\Rightarrow \frac{m}{n} = \frac{\lambda_2}{\lambda_1} = \frac{4}{5}$$

For λ_1

$$y = \frac{m\lambda_1 D}{d}, \lambda_1 = 650 \text{ nm}$$

$= 7.8$ mm

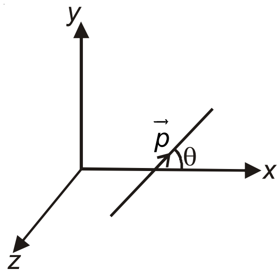
76. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences a torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is

(1) 45° (2) 60°

(3) 90° (4) 30°

Answer (2)

Sol.



$\vec{p} = p\cos\theta\hat{i} + p\sin\theta\hat{j}$

$\vec{E}_1 = E\hat{i}$

$\vec{T}_1 = \vec{p} \times \vec{E}_1$

$= (p\cos\theta\hat{i} + p\sin\theta\hat{j}) \times E(\hat{i})$

$\tau\hat{k} = pE\sin\theta(-\hat{k})$ ...(i)

$\vec{E}_2 = \sqrt{3}E_1\hat{j}$

$\vec{T}_2 = (p\cos\theta\hat{i} + p\sin\theta\hat{j}) \times \sqrt{3}E_1\hat{j}$

$-\tau\hat{k} = \sqrt{3}pE_1\cos\theta\hat{k}$ ...(ii)

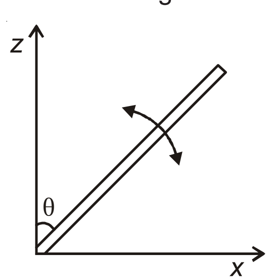
From (i) and (ii)

$pE\sin\theta = \sqrt{3}pE\cos\theta$

$\tan\theta = \sqrt{3}$

$\theta = 60°$

77. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is



(1) $\frac{2g}{3\ell}\sin\theta$

(2) $\frac{3g}{2\ell}\cos\theta$

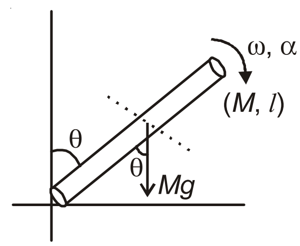
(3) $\frac{2g}{3\ell}\cos\theta$

(4) $\frac{3g}{2\ell}\sin\theta$

Answer (4)

Sol. Torque at angle θ

$\tau = Mg\sin\theta \cdot \frac{\ell}{2}$



$\tau = I\alpha$

$I\alpha = Mg\sin\theta\frac{\ell}{2}$ $\therefore\ I = \frac{M\ell^2}{3}$

$\frac{M\ell^2}{3} \cdot \alpha = Mg\sin\theta\frac{\ell}{2}$

$\frac{\ell\alpha}{3} = g\frac{\sin\theta}{2}$

$\alpha = \frac{3g\sin\theta}{2\ell}$

78. An external pressure P is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by :

(1) $\frac{P}{\alpha K}$

(2) $\frac{3\alpha}{PK}$

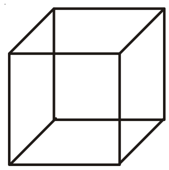
(3) $3PK\alpha$

(4) $\frac{P}{3\alpha K}$

Answer (4)

Sol. $K = \dfrac{\Delta P}{\left(-\dfrac{\Delta V}{V}\right)}$

$\dfrac{\Delta V}{V} = \dfrac{P}{K}$



$\gamma = 3\alpha$

$\therefore\ V = V_0\,(1 + \gamma\Delta t)$

$\dfrac{\Delta V}{V_0} = \gamma\Delta t$

$\therefore\ \dfrac{P}{K} = \gamma\Delta t \Rightarrow \Delta t = \dfrac{P}{\gamma K} = \dfrac{P}{3\alpha K}$

79. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is

(1) Virtual and at a distance of 40 cm from convergent lens

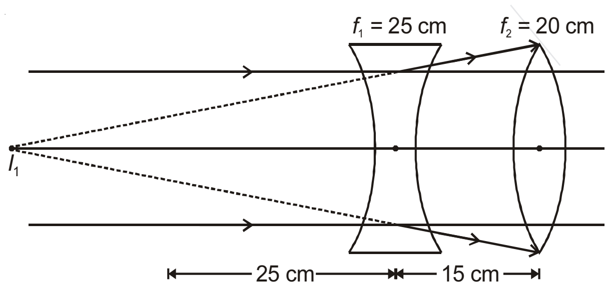
(2) Real and at a distance of 40 cm from the divergent lens

(3) Real and at a distance of 6 cm from the convergent lens

(4) Real and at a distance of 40 cm from convergent lens

Answer (4)

Sol.



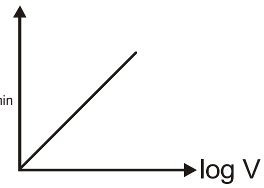
For converging lens

u = –40 cm which is equal to 2f

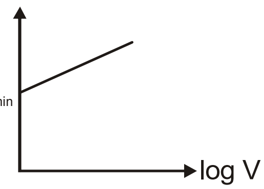
$\therefore$ Image will be real and at a distance of 40 cm from convergent lens.

80. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of log λ_{min} with log V is correctly represented in

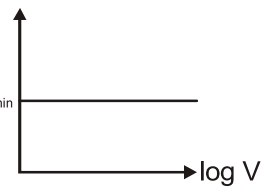
(1) $\log \lambda_{min}$



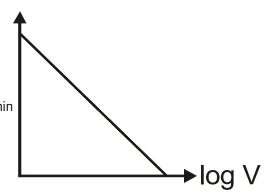
(2) $\log \lambda_{min}$



(3) $\log \lambda_{min}$



(4) $\log \lambda_{min}$



Answer (4)

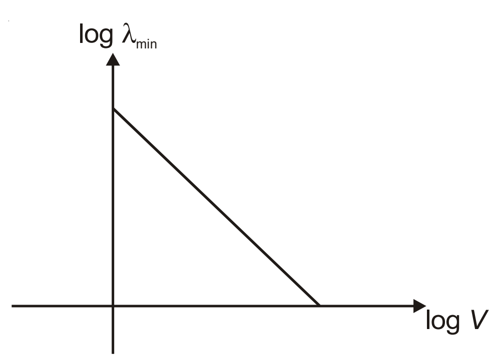
Sol. In X-ray tube

$\lambda_{min} = \dfrac{hc}{eV}$

$\ln\lambda_{min} = \ln\left(\dfrac{hc}{e}\right) - \ln V$

Slope is negative

Intercept on y-axis is positive



81. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be

(1) 1.38×10^{23} (2) 2.5×10^{25}

(3) -2.5×10^{25} (4) -1.61×10^{23}

Answer (3)

Sol. n_1 = initial number of moles

$$n_1 = \frac{P_1V_1}{RT_1} = \frac{10^5 \times 30}{8.3 \times 290} \approx 1.24 \times 10^3$$

n_2 = final number of moles

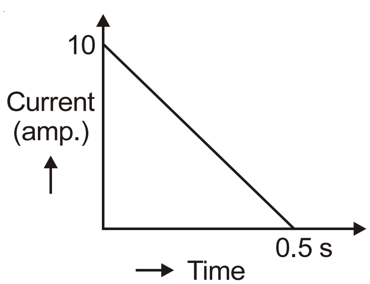
$$= \frac{P_2V_2}{RT_2} = \frac{10^5 \times 30}{8.3 \times 300} \approx 1.20 \times 10^3$$

Change of number of molecules :

$n_f - n_i = (n_2 - n_1) \times 6.023 \times 10^{23}$

$\approx -2.5 \times 10^{25}$

82. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is



(1) 225 Wb (2) 250 Wb

(3) 275 Wb (4) 200 Wb

Answer (2)

Sol. $\varepsilon = \frac{d\phi}{dt}$

$$iR = \frac{d\phi}{dt}$$

$$\int d\phi = R\int i dt$$

Magnitude of change in flux = R × area under current vs time graph

$$= 100 \times \frac{1}{2} \times \frac{1}{2} \times 10$$

= 250 Wb

83. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0-10 V is

(1) 2.045×10^3 Ω

(2) 2.535×10^3 Ω

(3) 4.005×10^3 Ω

(4) 1.985×10^3 Ω

Answer (4)

Sol. $i_g = 5 \times 10^{-3}$ A

$G = 15$ Ω

Let series resistance be R.

$V = i_g (R + G)$

$10 = 5 \times 10^{-3} (R + 15)$

$R = 2000 - 15 = 1985 = 1.985 \times 10^3$ Ω

84. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 second will be

(1) 22 J (2) 9 J

(3) 18 J (4) 4.5 J

Answer (4)

Sol. $6t = 1 \cdot \frac{dv}{dt}$

$$\int_0^v dv = \int 6t\, dt$$

$$v = 6\left[\frac{t^2}{2}\right]_0^1$$

$= 3$ ms^{-1}

$$W = \Delta KE = \frac{1}{2} \times 1 \times 9 = 4.5 \text{ J}$$

85. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is

(1) 8.89 s

(2) 6.98 s

(3) 8.76 s

(4) 6.65 s

Answer (4)

Sol. $T = 2\pi\sqrt{\dfrac{I}{MB}}$

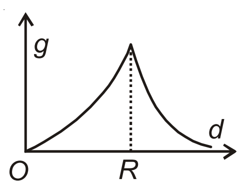
$$= 2\pi\sqrt{\frac{7.5\times10^{-6}}{6.7\times10^{-2}\times0.01}} = \frac{2\pi}{10}\times1.06$$

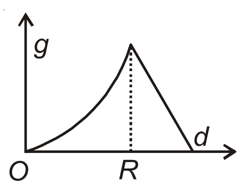
For 10 oscillations,

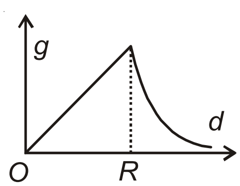
$t = 10T = 2\pi \times 1.06$

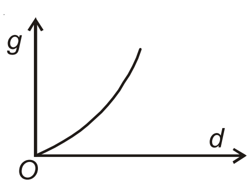
$= 6.6568 \approx 6.65$ s

86. The variation of acceleration due to gravity g with distance d from centre of the earth is best represented by (R = Earth's radius) :

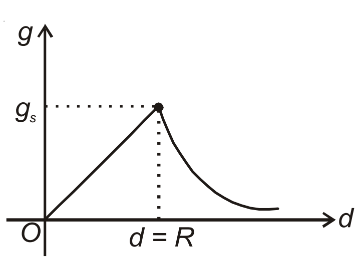
(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. 

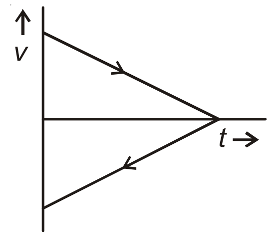
Variation of g inside earth surface

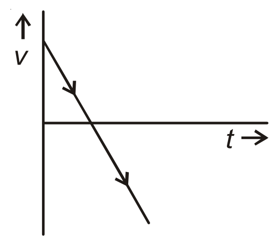
$d < R = g = \dfrac{Gm}{R^2}\cdot d$

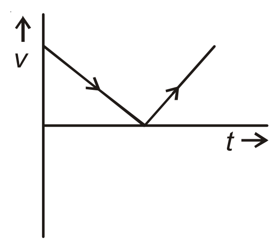
$d = R = g_s = \dfrac{Gm}{R^2}$

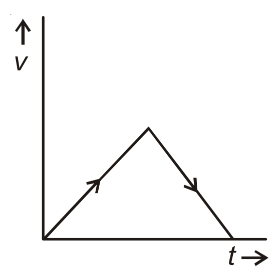
$d > R = g = \dfrac{Gm}{d^2}$

87. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time?

(1) 

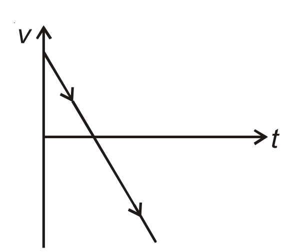
(2) 

(3) 

(4) 

Answer (2)

Sol. Acceleration is constant and negative



88. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is

(1) $\frac{\lambda_A}{\lambda_B} = 2$

(2) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$

(3) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$

(4) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$

Answer (1)

Sol. $v_1 = \frac{(m_1 - m_2)v}{m_1 + m_2} + 0$

$m_1 = m$

$m_2 = \frac{m}{2}$

$= \frac{v}{3}$

$\therefore \quad p_1 = m.\left[\frac{v}{3}\right]$

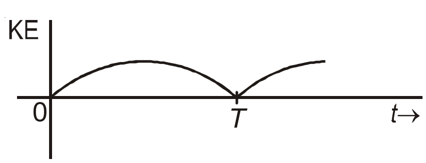
$v_2 = \frac{2m_1 v}{m_1 + m_2} + 0$

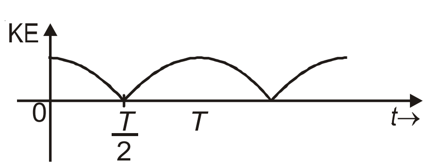
$= \frac{4v}{3}$

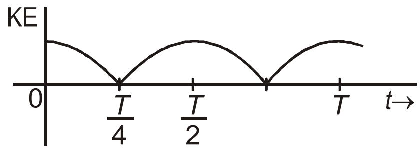
$p_2 = \frac{m}{2}\left[\frac{4v}{3}\right] = \frac{2mv}{3}$

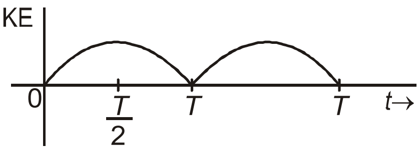
$\therefore$ de-Broglie wavelength $\frac{\lambda_A}{\lambda_B} = \frac{p_2}{p_1} = 2:1$

89. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like :

(1)


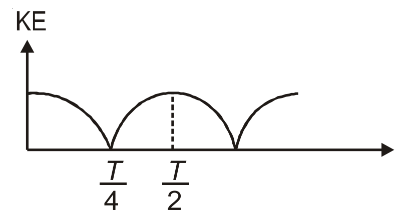
(2)


(3)


(4)


Answer (3)

Sol. $\text{K.E} = \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$



90. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of

(1) $\frac{1}{9}$

(2) 81

(3) $\frac{1}{81}$

(4) 9

Answer (4)

Sol. $\frac{V_f}{V_i} = 9^3$

$\because$ Density remains same

So, mass $\propto$ Volume

$\frac{m_f}{m_i} = 9^3$

$\frac{(\text{Area})_f}{(\text{Area})_i} = 9^2$

$\text{Stress} = \frac{(\text{Mass}) \times g}{\text{Area}}$

$\frac{\sigma_2}{\sigma_1} = \left(\frac{m_f}{m_i}\right)\left(\frac{A_i}{A_f}\right)$

$= \frac{9^3}{9^2} = 9$

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