Time Duration : 3 hrs. Maximum Marks: 360

Solutions For

JEE MAIN 2016 (Code-F)

(Chemistry, Mathematics and Physics)

Important Instructions :

1. The test is of 3 hours duration. The CODE for this Booklet is F.
2. The Test Booklet consists of 90 questions. The maximum marks are 360.
3. There are three parts in the question paper A, B, C consisting of Chemistry, Mathematics and Physics having 30 questions in each part of equal weightage. Each question is allotted 4 (four) marks for each correct response.
4. Candidates will be awarded marks as stated above in Instructions No. 3 for correct response of each question. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
5. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 4 above.
6. For writing particulars/marking responses on Side-1 and Side-2 of the Answer Sheet use only Blue/Black Ball Point Pen provided by the Board.
7. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admin Card inside the examination hall/room.

PART–A : CHEISTRY

1. A stream of electrons from a heated filament was passed between two charged plates kept at a potential difference V esu. If e and m are charge and mass of an electron, respectively, then the value of $\frac{h}{\lambda}$ (where λ is wavelength associated with electron wave) is given by:

   (1) 2meV (2) $\sqrt{meV}$

   (3) $\sqrt{2meV}$ (4) meV

Answer (3)

Sol. Kinetic energy of electron is = e × V

As per de-Broglie's equation

$$\lambda = \frac{h}{\sqrt{2mE_k}} = \frac{h}{\sqrt{2meV}}$$

$$\therefore \quad \frac{h}{\lambda} = \sqrt{2meV}$$

2. 2-chloro-2-methylpentane on reaction with sodium methoxide in methanol yields:

   (a) $C_2H_5CH_2\overset{\displaystyle CH_3}{\underset{\displaystyle CH_3}{\overset{|}{\underset{|}{C}}}}-OCH_3$

   (b) $C_2H_5CH_2\underset{\displaystyle CH_3}{\underset{|}{C}}=CH_2$

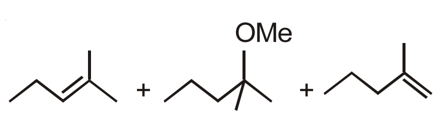
   (c) $C_2H_5CH=\underset{\displaystyle CH_3}{\underset{|}{C}}-CH_3$

   (1) (a) and (c) (2) (c) only

   (3) (a) and (b) (4) All of these

Answer (4)

Sol. $CH_3-CH_2-CH_2-\overset{\displaystyle CH_3}{\underset{\displaystyle Cl}{\overset{|}{\underset{|}{C}}}}-CH_3 \xrightarrow[S_N1\ \&\ E_1\ \text{path}]{\text{MeONa}/\text{MeOH}}$



3. Which of the following compounds is metallic and ferromagnetic?

   (1) CrO_2 (2) VO_2

   (3) MnO_2 (4) TiO_2

Answer (1)

Sol. CrO_2 is strongly attracted towards magnetic field so it is ferromagnetic.

4. Which of the following statements about low density polythene is **FALSE**?

   (1) It is a poor conductor of electricity

   (2) Its synthesis requires dioxygen or a peroxide initiator as a catalyst

   (3) It is used in the manufacture of buckets, dust-bins etc.

   (4) Its synthesis requires high pressure

Answer (3)

Sol. Low density polyethene is synthesised by

$$nH_2C = CH_2 \xrightarrow[\substack{\text{trace of peroxide}\\ \text{(as initiator) catalyst}}]{O_2/1000\text{–}2000\ \text{atm}} \left[CH_2 = CH_2 \right]$$

It is poor conductor of electricity, hence option (3) is false regarding it.

5. For a linear plot of $\log\left(\frac{x}{m}\right)$ 'versus log p in a Freundlich adsorption isotherm, which of the following statements is correct? (k and n are constants)

   (1) $\frac{1}{n}$ appears as the intercept

   (2) Only $\frac{1}{n}$ appears as the slope

   (3) $\log\left(\frac{1}{n}\right)$ appears as the intercept

   (4) Both k and $\frac{1}{n}$ appear in the slope term

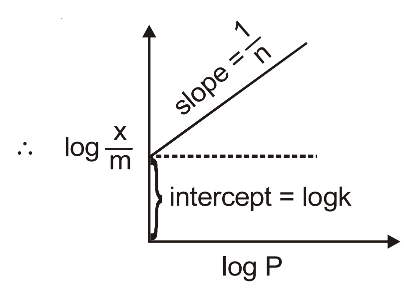
Answer (2)

Sol. According to Freundlich adsorption isother :

$$\frac{x}{m} = k \times P^{\frac{1}{n}}$$

$$\therefore \log\frac{x}{m} = \frac{1}{n}\log P + \log k$$

$$y = m \cdot x + C$$



6. The heats of combustion of carbon and carbon monoxide are –393.5 and –283.5 kJ mol^{-1}, respectively. The heat of formation (in kJ) of carbon monoxide per mole is

(1) 676.5 (2) –676.5

(3) –110.5 (4) 110.5

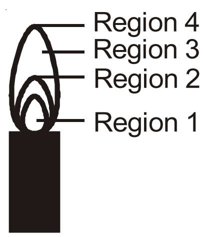
Answer (3)

Sol. $C(s) + O_2(g) \rightarrow CO_2(g)\ \Delta H = -393.5\ kJ\ mol^{-1}$

$$\underset{-}{CO(g)} + \underset{-}{\tfrac{1}{2}}O_2(g) \rightarrow \underset{-}{CO_2(g)}\ \Delta H = \underset{+}{-}\ 283.5\ kJ\ mol^{-1}$$

$$C(s) + \frac{1}{2}O_2(g) \longrightarrow CO(g)\ \Delta H = -110.5\ kJ\ mol^{-1}$$

7. The hottest region of Bunsen flame shown in the figure below is



(1) Region 2

(2) Region 3

(3) Region 4

(4) Region 1

Answer (1)

Sol. Region "2" is the hottest region of Bunsen flame.

8. Which of the following is an anionic detergent?

(1) Sodium lauryl sulphate

(2) Cetyltrimethyl ammonium bromide

(3) Glyceryl oleate

(4) Sodium stearate

Answer (1)

Sol. Sodium lauryl sulphate → Anionic detergent; cetyltrimethyl ammonium bromide → (Cationic surfactant)

Glycerol oleate → [oil]

sodium stearate → [soap]

9. 18 g glucose ($C_6H_{12}O_6$) is added to 178.2 g water. The vapor pressure of water (in torr) for this aqueous solution is

(1) 76.0

(2) 752.4

(3) 759.0

(4) 7.6

Answer (2)

Sol. $$\frac{p^o - p_s}{p_s} = \frac{n_{solute}}{n_{solvent}} = \frac{\frac{18}{180}}{\frac{178.2}{18}} = \frac{18}{17.82}$$

At normal boiling point of water V.P. = p° = 760 torr

$$\therefore\ \frac{760 - p_s}{p_s} = \frac{18}{1782}$$

or, 1800 p_s = 760 × 1782

p_s = 752.4 torr

10. The distillation technique most suited for separating glycerol from spent-lye in the soap industry is

(1) Fractional distillation

(2) Steam distillation

(3) Distillation under reduced pressure

(4) Simple distillation

Answer (3)

Sol. As glycerol decomposes before reaching its boiling point under 1 atm pressure. So, in order to prevent its deomposition, it is distilled under reduced pressure.

11. The species in which the N atom is in a state of *sp* hybridization is

(1) NO_2^-

(2) NO_3^-

(3) NO_2

(4) NO_2^+

Answer (4)

Sol.

Molecule/ion	Hybridization
NO_2^-	sp^2
NO_3^-	sp^2
NO_2	sp^2
NO_2^+	sp

12. Decomposition of H_2O_2 follows a first order reaction. In fifty minutes, the concentration of H_2O_2 decreases from 0.5 to 0.25 M in one such decomposition. When the concentration of H_2O_2 reaches 0.05 M, the rate of formation of O_2 will be

(1) 6.93×10^{-4} mol min^{-1}

(2) 2.66 L min^{-1} at STP

(3) 1.34×10^{-2} mol min^{-1}

(4) 6.93×10^{-2} mol min^{-1}

Answer (1)

Sol. Rate $= K[H_2O_2] = \frac{0.693}{25} \times 0.05$

Rate of formation of $H_2O_2 = \frac{1}{2} \times \frac{0.693}{25} \times 0.05$

$= 6.93 \times 10^{-4}$ mol min^{-1}

13. The pair having the same magnetic moment is

[At. No.: Cr = 24, Mn = 25, Fe = 26, Co = 27]

(1) $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$

(2) $[Mn(H_2O)_6]^{2+}$ and $[Cr(H_2O)_6]^{2+}$

(3) $[CoCl_4]^{2-}$ and $[Fe(H_2O)_6]^{2+}$

(4) $[Cr(H_2O)_6]^{2+}$ and $[CoCl_4]^{2-}$

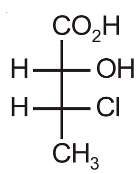
Answer (1)

Sol. Identical the number of unpaired electrons higher the magnetic moment

	Metal ion	Unpaired electrons
$[Cr(H_2O)_6]^{2+}$	Cr^{2+}	4
$[Fe(H_2O)_6]^{2+}$	Fe^{2+}	4

$\therefore$ $[Cr(H_2O)_6]^{2+}$ and $[Fe(H_2O)_6]^{2+}$ have identical magnetic moment.

14. The absolute configuration of



is

(1) (2S, 3R)

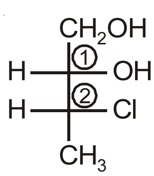
(2) (2S, 3S)

(3) (2R, 3R)

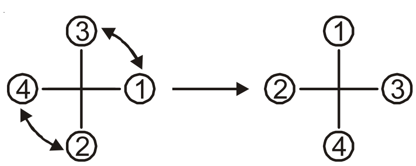
(4) (2R, 3S)

Answer (1)

Sol.

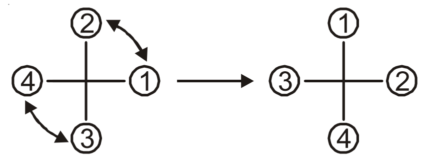


At (1),



It is 'S' configurated.

At (2),



It is 'R' configurated.

15. The equilibrium constant at 298 K for a reaction $A + B \rightleftharpoons C + D$ is 100. If the initial concentration of all the four species were 1 M each, then equilibrium concentration of D (in mol L^{-1}) will be

(1) 0.818 (2) 1.818

(3) 1.182 (4) 0.182

Answer (2)

Sol. $A + B \rightleftharpoons C + D$

Initially : 1 1 1 1 $[D]_{eq} = 1.818$ M

Q = 1

$Q < k_{eq}$

$\therefore$ Equilibrium is forward shifted.

$A + B \rightleftharpoons C + D$

Equilibrium : 1–x 1–x 1+x 1+x

$$\frac{(1+x)^2}{(1-x)^2} = 10^2$$

$$\Rightarrow \frac{1+x}{1-x} = 10$$

$\Rightarrow$ 1 + x = 10 – 10x

$\Rightarrow$ 11x = 9

$\therefore$ $x = \frac{9}{11} = 0.818$

So, equilibrium concentration of 'D' = 1.818 M.

16. Which one of the following ores is best concentrated by froth floatation method?

(1) Siderite (2) Galena

(3) Malachite (4) Magnetite

Answer (2)

Sol. Sulphide ores are concentrated by froth floatation.

Galena (PbS) is concentrated by froth floatation.

17. At 300 K and 1 atm, 15 mL of a gaseous hydrocarbon requires 375 mL air containing 20% O_2 by volume for complete combustion. After combustion the gases occupy 330 mL. Assuming that the water formed is in liquid form and the volumes were measured at the same temperature and pressure, the formula of the hydrocarbon is

(1) C_3H_8 (2) C_4H_8

(3) C_4H_{10} (4) C_3H_6

Answer (1)

Sol. $C_3H_8(g) + 5O_2(g) \rightarrow 3CO_2(g) + 4H_2O(l)$

So, volume of O_2 required for the combustion of 1 mL hydrocarbon = 5 mL.

So, volume of O_2 requierd for the combustion of 15 mL of hydrocarbon = 75 mL (*i.e.*, 20% of 375 mL air)

NOTE : But for this, the total volume of gases after combustion should be 345 mL, rather than 330 mL.

18. The pair in which phosphorous atoms have a formal oxidation state of +3 is

(1) Pyrophosphorous and hypophosphoric acids

(2) Orthophosphorous and hypophosphoric acids

(3) Pyrophosphorous and pyrophosphoric acids

(4) Orthophosphorous and pyrophosphorous acids

Answer (4)

Sol. The phosphorous atoms of orthophosphorous acid H_3PO_3 and pyrophosphorous $H_4P_2O_5$ have a formal oxidation state +3.

19. Which one of the following complexes shows optical isomerism?

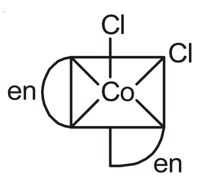
(1) cis$[Co(en)_2Cl_2]Cl$ (2) trans$[Co(en)_2Cl_2]Cl$

(3) $[Co(NH_3)_4Cl_2]Cl$ (4) $[Co(NH_3)_3Cl_3]$

(en = ethylenediamine)

Answer (1)

Sol.



has no plane of symmetry, so it is optically active.

20. The reaction of zinc with dilute and concentrated nitric acid, respectively, produces

(1) NO_2 and NO (2) NO and N_2O

(3) NO_2 and N_2O (4) N_2O and NO_2

Answer (4)

Sol. $4Zn + 10HNO_3(dil) \longrightarrow 4Zn(NO_3)_2 + N_2O + 5H_2O$

$Zn + 4HNO_3(conc.) \longrightarrow Zn(NO_3)_2 + 2NO_2 + 2H_2O$

21. Which one of the following statements about water is FALSE?

(1) Water can act both as an acid and as a base

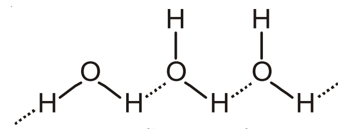
(2) There is extensive intramolecular hydrogen bonding in the condensed phase

(3) Ice formed by heavy water sinks in normal water

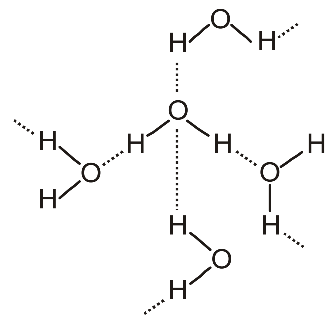
(4) Water is oxidized to oxygen during photosynthesis

Answer (2)

Sol. There is intermolecular hydrogen bonding in water



(in water)



(in ice)

22. The concentration of fluoride, lead, nitrate and iron in a water sample from an underground lake was found to be 1000 ppb, 40 ppb, 100 ppm and 0.2 ppm, respectively. This water is unsuitable for drinking due to high concentration of

(1) Lead (2) Nitrate

(3) Iron (4) Fluoride

Answer (2)

Sol. The maximum limit of nitrate in drinking water is 50 ppm.

Excess nitrate in drinking water can cause blue baby syndrome.

The prescribed limit for fluoride is 10 ppm.

The prescribed limit for lead is 50 ppb.

The prescribed limit for iron is 0.2 ppm.

23. The main oxides formed on combustion of Li, Na and K in excesss of air are, respectively:

(1) LiO_2, Na_2O_2 and K_2O

(2) Li_2O_2, Na_2O_2 and KO_2

(3) Li_2O, Na_2O_2 and KO_2

(4) Li_2O, Na_2O and KO_2

Answer (3)

Sol. Li mainly gives oxide, Na gives peroxide and K gives superoxide.

24. Thiol group is present in

(1) Cystine (2) Cysteine

(3) Methionine (4) Cytosine

Answer (2)

Sol. (– SH) Thiol group is present in cysteine.

$$HS-CH_2-\underset{\displaystyle NH_2}{\underset{|}{CH}}-COOH$$

25. Galvanization is applying a coating of

(1) Cr (2) Cu

(3) Zn (4) Pb

Answer (3)

Sol. Galvanisation is coating a layer of Zn.

26. Which of the following atoms has the highest first ionization energy?

(1) Na (2) K

(3) Sc (4) Rb

Answer (3)

Sol. Sc is d-block element having high Z_{eff} hence high ionisation enthalpy.

27. In the Hofmann bromamide degradation reaction, the number of moles of NaOH and Br_2 used per mole of amine produced are

(1) Four moles of NaOH and two moles of Br_2

(2) Two moles of NaOH and two moles of Br_2

(3) Four moles of NaOH and one mole of Br_2

(4) One mole of NaOH and one mole of Br_2

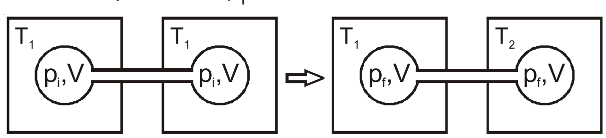
Answer (3)

Sol. $R-\underset{\underset{O}{\|}}{C}-NH_2 + Br_2 + 4NaOH \longrightarrow$

$$RNH_2 + Na_2CO_3 + 2NaBr + 2H_2O$$

From the reaction it can easily be said 4 moles of NaOH and 1 mole of Br_2 is used in this reaction.

28. Two closed bulbs of equal volume (V) containing an ideal gas initially at pressure p_i and temperature T_1 are connected through a narrow tube of negligible volume as shown in the figure below. The temperature of one of the bulbs is then raised to T_2. The final pressure p_f is



(1) $2p_i\left(\dfrac{T_1}{T_1+T_2}\right)$ (2) $2p_i\left(\dfrac{T_2}{T_1+T_2}\right)$

(3) $2p_i\left(\dfrac{T_1T_2}{T_1+T_2}\right)$ (4) $p_i\left(\dfrac{T_1T_2}{T_1+T_2}\right)$

Answer (2)

Sol. No. of moles initially = no. of moles finally.

$$\Rightarrow \frac{p_i \times V}{RT_1} + \frac{p_iV}{RT_1} = \frac{p_fV}{RT_1} + \frac{p_fV}{RT_2}$$

$$\Rightarrow \frac{2p_i}{T_1} = p_f\left(\frac{1}{T_1} + \frac{1}{T_2}\right)$$

$$\Rightarrow \frac{2p_i}{T_1}\left(\frac{T_1T_2}{T_1+T_2}\right) = p_f$$

$$\Rightarrow \boxed{p_f = 2p_i\left(\frac{T_2}{T_1+T_2}\right)}$$

29. The reaction of propene with HOCl ($Cl_2 + H_2O$) proceeds through the intermediate

(1) $CH_3 - CH^+ - CH_2 - Cl$

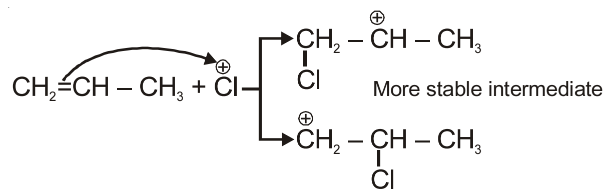
(2) $CH_3 - CH(OH) - CH_2^+$

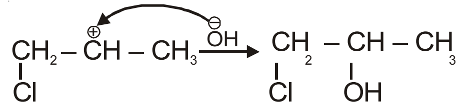
(3) $CH_3 - CHCl - CH_2^+$

(4) $CH_3 - CH^+ - CH_2 - OH$

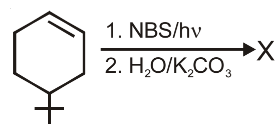
Answer (1)

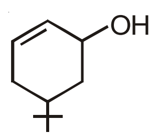
Sol.

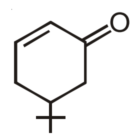


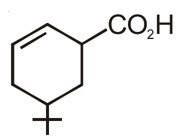


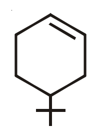
30. The product of the reaction given below is



(1) 

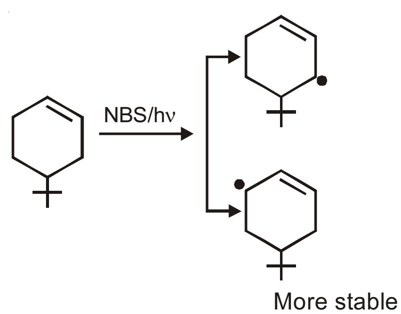
(2) 

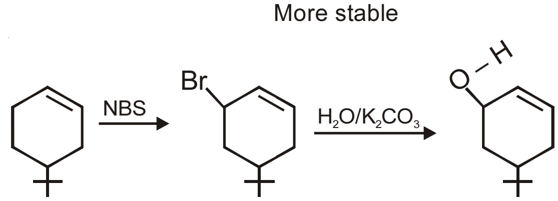
(3) 

(4) 

Answer (1)

Sol.





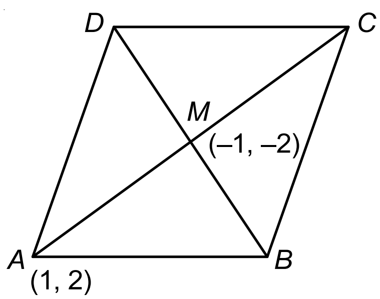
31. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus?

(1) $(-3, -8)$ (2) $\left(\frac{1}{3}, -\frac{8}{3}\right)$

(3) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (4) $(-3, -9)$

Answer (2)

Sol. Point of intersection of sides



$x - y + 1 = 0$

and $7x - y - 5 = 0$

$\therefore$ $x = 1, y = 2$

Slope of $AM = \frac{4}{2} = 2$

$\therefore$ Equation of BD : $y + 2 = -\frac{1}{2}(x+1)$

$\Rightarrow$ $x + 2y + 5 = 0$

Solving $x + 2y + 5 = 0$ and $7x - y - 5 = 0$

$x = \frac{1}{3}, y = -\frac{8}{3} \Rightarrow \left(\frac{1}{3}, -\frac{8}{3}\right)$

32. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is

(1) $\frac{4}{3}$ (2) 1

(3) $\frac{7}{4}$ (4) $\frac{8}{5}$

Answer (1)

Sol. $a + d$, $a + 4d$, $a + 8d$ are in G.P.

$(a + 4d)^2 = (a + d)(a + 8d)$

$\Rightarrow$ $a^2 + 8ad + 16d^2 = a^2 + 9ad + 8d^2$

$\Rightarrow$ $8d^2 = ad \Rightarrow \frac{a}{d} = 8$

$\therefore$ Common ratio $= \frac{a+4d}{a+d}$

$= \frac{8+4}{8+1} = \frac{4}{3}$

33. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y + 6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is

(1) $x^2 + y^2 - x + 4y - 12 = 0$

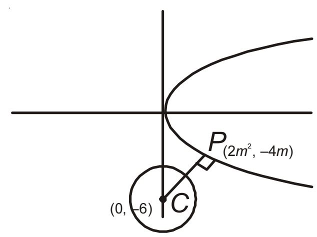
(2) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$

(3) $x^2 + y^2 - 4x + 9y + 18 = 0$

(4) $x^2 + y^2 - 4x + 8y + 12 = 0$

Answer (4)

Sol. Let the normal of parabola be



$y = mx - 4m - 2m^3$

$(0, -6)$ lies on it

$\therefore$ $-6 = -4m - 2m^3$

$\Rightarrow$ $m^3 + 2m - 3 = 0$

$(m - 1)(m^2 + m + 3) = 0$

$m = 1$

$\therefore$ Point P: $(2m^2, -4m)$

$= (2, -4)$

$\therefore$ Equation of circle is

$(x - 2)^2 + (y + 4)^2 = (4 + 4)$

$\Rightarrow$ $x^2 + y^2 - 4x + 8y + 12 = 0$

34. The system of linear equations

$x + \lambda y - z = 0$

$\lambda x - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for

(1) Exactly one value of λ

(2) Exactly two values of λ

(3) Exactly three values of λ

(4) Infinitely many values of λ

Answer (3)

Sol. $\begin{vmatrix} 1 & \lambda & -1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$1(\lambda + 1) - \lambda(-\lambda^2 + 1) + 1(-\lambda - 1) = 0$

$\lambda^3 - \lambda + \lambda + 1 - \lambda - 1 = 0$

$\lambda^3 - \lambda = 0$

$\lambda(\lambda^2 - 1) = 0$

$\lambda = 0, \lambda = \pm 1$

Exactly three values of λ

35. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x,\ x \neq 0,$ and

$S = \{x \in R : f(x) = f(-x)\}$; then S

(1) Contains exactly one element

(2) Contains exactly two elements

(3) Contains more than two elements

(4) Is an empty set

Answer (2)

Sol. $f(x) + 2f\left(\frac{1}{x}\right) = 3x$

$\Rightarrow \quad f\left(\frac{1}{x}\right) + 2f(x) = \frac{3}{x}$

$\therefore \quad 3f(x) = \frac{6}{x} - 3x$

$\therefore \quad f(x) = \left(\frac{2}{x} - x\right)$

$f(-x) = -\frac{2}{x} + x$

$f(x) = f(-x)$

$\Rightarrow \quad \frac{2}{x} - x = -\frac{2}{x} + x$

$\Rightarrow \quad 2x - \frac{4}{x} = 0$

$\Rightarrow \quad x = \pm\sqrt{2}$

36. Let $p = \lim\limits_{x \to 0+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to

(1) 1

(2) $\frac{1}{2}$

(3) $\frac{1}{4}$

(4) 2

Answer (2)

Sol. $p = \lim\limits_{x \to 0^+} \left(1 + \tan^2 \sqrt{x}\right)^{\frac{1}{2x}}$

$= e^{\lim\limits_{x \to 0} \frac{1}{2x} \tan^2 \sqrt{x}}$

$= e^{\lim\limits_{x \to 0} \frac{1}{2} \left(\frac{\tan\sqrt{x}}{\sqrt{x}}\right)^2}$

$= e^{\frac{1}{2}}$

$\log p = \frac{1}{2}$

37. A value of θ for which $\frac{2 + 3i\sin\theta}{1 - 2i\sin\theta}$ is purely imaginary, is

(1) $\frac{\pi}{6}$

(2) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$

(3) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(4) $\frac{\pi}{3}$

Answer (3)

Sol. $\frac{2 + 3i\sin\theta}{1 - 2i\sin\theta} \times \frac{1 + 2i\sin\theta}{1 + 2i\sin\theta}$ = purely in imaginary

$\Rightarrow \quad 2 - 6\sin^2\theta = 0 \Rightarrow \sin^2\theta = \frac{1}{3}$

$\therefore \quad \sin\theta = \pm\frac{1}{\sqrt{3}}$

38. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is

(1) $\frac{4}{\sqrt{3}}$

(2) $\frac{2}{\sqrt{3}}$

(3) $\sqrt{3}$

(4) $\frac{4}{3}$

Answer (2)

Sol. Given $\frac{2b^2}{a} = 8,\ 2b = ae$

$\frac{b}{a} = \frac{e}{2}$

We know that $b^2 = a^2(e^2 - 1)$

$\frac{b^2}{a^2} = e^2 - 1$

$$\frac{e^2}{4} = e^2 - 1, \; e^2 = \frac{4}{3}$$

$$e = \frac{2}{\sqrt{3}}$$

39. If the standard deviation of the number 2, 3, a and 11 is 3.5, then which of the following is true?

(1) $3a^2 - 32a + 84 = 0$

(2) $3a^2 - 34a + 91 = 0$

(3) $3a^2 - 23a + 44 = 0$

(4) $3a^2 - 26a + 55 = 0$

Answer (1)

Sol. $Var = \sigma^2 = \frac{\Sigma x_1^2}{n} - (\bar{x})^2$

Standard Deviation =

$$\sqrt{\frac{2^2 + 3^2 + a^2 + 11^2}{4} - \left(\frac{2 + 3 + a + 11}{4}\right)^2} = 3.5$$

$$\Rightarrow \frac{134 + a^2}{4} - \left(\frac{16 + a}{4}\right)^2 = (3.5)^2$$

$$\frac{4(134 + a^2)}{16} - \frac{(16^2 + a^2 + 32a)}{16} = (3.5)^2$$

$536 + 4a^2 - 256 - a^2 - 32a = 196$

$3a^2 - 32a + 84 = 0$

40. The integral $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3}$ is equal to

(1) $\frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$

(2) $\frac{x^5}{2(x^5 + x^3 + 1)^2} + C$

(3) $\frac{-x^{10}}{2(x^5 + x^3 + 1)^2} + C$

(4) $\frac{-x^5}{(x^5 + x^3 + 1)^2} + C$

where C is an arbitrary constant.

Answer (1)

Sol. $\int \frac{2x^{12} + 5x^9}{(x^2 + x^3 + 1)} dx$

$$= \int \frac{\left(\frac{2}{x^3} + \frac{5}{x^6}\right) dx}{\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3}$$

$$1 + \frac{1}{x^2} + \frac{1}{x^5} = t$$

$$\left(-\frac{2}{x^3} - \frac{5}{x^6}\right) dx = \text{dt}$$

$$= \int \frac{-dt}{t^3} = \frac{1}{2t^2} + C$$

$$= \frac{1}{2\left(1 + \frac{1}{x^2} + \frac{1}{x^5}\right)^3} + C = \frac{x^{10}}{2(x^5 + x^3 + 1)^2} + C$$

41. If the line, $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane, $lx + my - z = 9$, then $l^2 + m^2$ is equal to

(1) 18 (2) 5

(3) 2 (4) 26

Answer (3)

Sol. Line is perpendicular to normal of plane

$$\Rightarrow (2\hat{i} - \hat{j} + 3\hat{k}) \bullet (l\hat{i} + m\hat{j} - \hat{k}) = 0$$

$2l - m - 3 = 0$ ...(i)

(3, –2, –4) lies on the plane

$3l - 2m + 4 = 9$

$3l - 2m = 5$ ...(ii)

Solving (i) and (ii)

$l = 1, m = -1$

$l^2 + m^2 = 2$

42. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is

(1) 5 (2) 7

(3) 9 (4) 3

Answer (2)

Sol. $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$$2\cos\frac{5x}{2} \cdot \cos\frac{3c}{2} + 2\cos\frac{5x}{2} \cdot \cos\frac{x}{2} = 0$$

$$2\cos\frac{5x}{2} \times 2\cos \cdot x \cos\frac{x}{2} = 0$$

$x = \frac{(2n+1)\pi}{5}, \frac{(2k+1)\pi}{2}, (2r+1)\pi,$

where $n, k, \in \mathbb{Z}$ $0 \le x < 2\pi$

Hence $x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{2\pi}{5}, \frac{9\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{2}$

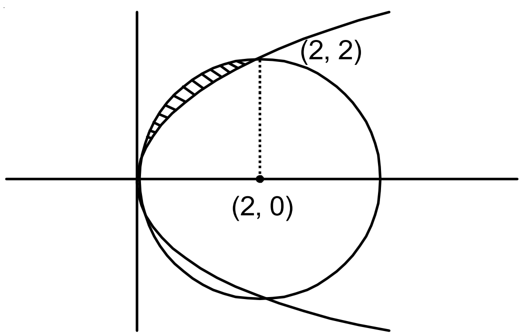
43. The area (in sq. units) of the region $\{(x,y) : y^2 \ge 2x \text{ and } x^2 + y^2 \le 4x, x \ge 0, y \ge 0\}$ is

(1) $\pi - \frac{8}{3}$ (2) $\pi - \frac{4\sqrt{2}}{3}$

(3) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (4) $\pi - \frac{4}{3}$

Answer (1)

Sol.



$$\text{Area} = \frac{\pi \cdot 2^2}{4} - \int_0^2 \sqrt{2x}\,dx$$

$$= \pi - \sqrt{2} \cdot \frac{2}{3} x^{\frac{3}{2}} \Big|_0^2$$

$$= \pi - \frac{8}{3}$$

44. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is

(1) $\frac{\pi}{2}$ (2) $\frac{2\pi}{3}$

(3) $\frac{5\pi}{6}$ (4) $\frac{3\pi}{4}$

Answer (3)

Sol. $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$ and after comparing

$$\vec{a} \cdot \vec{b} = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos\theta = -\frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = \frac{5\pi}{6}$$

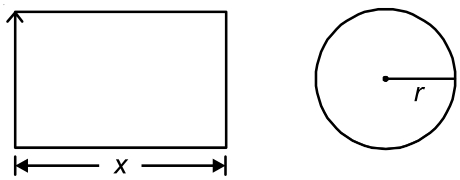
45. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then :

(1) $(4-\pi)x = \pi r$ (2) $x = 2r$

(3) $2x = r$ (4) $2x = (\pi + 4)r$

Answer (2)

Sol. Length of wire = 2



Given $4x + 2\pi r = 2$

$\Rightarrow 2x + \pi r = 1$ ...(i)

$$A = x^2 + \pi r^2 = \left(\frac{1-\pi r}{2}\right)^2 + \pi r^2$$

$$\Rightarrow \frac{dA}{dr} = 2\left(\frac{1-\pi r}{2}\right)\left(-\frac{\pi}{2}\right) + 2\pi r$$

For max and min $\Rightarrow \frac{dA}{dr} = 0$

$\pi(1 - \pi r) = 4\pi r$

$1 = 4r + \pi r$ ...(ii)

from (i) and (ii)

$2x + \pi r = 4r + \pi r$

$x = 2r$

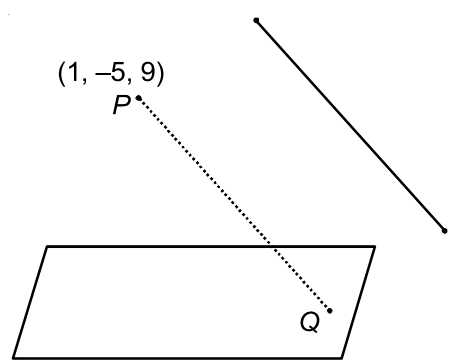
46. The distance of the point (1, –5, 9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is

(1) $10\sqrt{3}$ (2) $\frac{10}{\sqrt{3}}$

(3) $\frac{20}{3}$ (4) $3\sqrt{10}$

Answer (1)

Sol.



L: $x = y = z$

equation of line PQ:

An x point Q on the line PQ is $(\lambda + 1, \lambda - 5. \lambda + 9)$

$\because$ Point Q lies on the plane : $x - y + z = 5$

$(\lambda + 1) - (\lambda - 5) + \lambda + 9 = 5$

$\lambda + 10 = 0$

$\lambda = -10$

Point Q is $(-9, -15, -1)$

$PQ = \sqrt{(1+9)^2 + (-5+15)^2 + (9+1)^2} = 10\sqrt{3}$

47. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)\, dx = x\, dy$, then $f\left(-\frac{1}{2}\right)$ is equal to :

(1) $-\frac{4}{5}$ (2) $\frac{2}{5}$

(3) $\frac{4}{5}$ (4) $-\frac{2}{5}$

Answer (3)

Sol. $ydx - xdy = -y^2xdx$

$$\Rightarrow \frac{ydx - xdy}{y^2} = -xdx$$

$$\Rightarrow d\left(\frac{x}{y}\right) = -xdx$$

On integrating both sides

$$\frac{x}{y} = \frac{-x^2}{2} + c$$

it passes through $(1, -1)$

$$\Rightarrow -1 = \frac{1}{2} + c \Rightarrow c = \frac{-1}{2}$$

So, $\frac{x}{y} = \frac{-x^2}{2} - \frac{1}{2}$

$$\Rightarrow y = \frac{-2x}{x^2+1} \text{ i.e., } f\left(-\frac{1}{2}\right) = \frac{4}{5}$$

48. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n, x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is

(1) 2187 (2) 243

(3) 729 (4) 64

Answer (3)

Sol. Number to terms is $2n + 1$ which is odd but it is given 28. If we take $(x + y + z)^n$ then number of terms is ${}^{n+2}C_2 = 28$

Hence $n = 6$

$$\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^6 = a_0 + a_1x + a_2x^2 + ... + a_6x^6$$

Sum of coefficients can be obtained by $x = 1$

$(1 - 2 + 4)^6 = 3^6 = 729$

So according to what the examiner is trying to ask option 3 can be correct.

49. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$.

A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ also passes through the point

(1) $\left(0, \frac{2\pi}{3}\right)$ (2) $\left(\frac{\pi}{6}, 0\right)$

(3) $\left(\frac{\pi}{4}, 0\right)$ (4) $(0, 0)$

Answer (1)

Sol. $f(x) = \tan^{-1}\sqrt{\frac{1+\sin x}{1-\sin x}}$

$$= \tan^{-1}\sqrt{\frac{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)^2}}$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right)$$

$$= \frac{\pi}{4} + \frac{x}{2}$$

$$\Rightarrow f'(x) = \frac{1}{2} \text{ and at } x = \frac{\pi}{6}, f(x) = \frac{\pi}{3}$$

So, equation of normal is

$$y-\frac{\pi}{3}=-2\left(x-\frac{\pi}{6}\right) \Rightarrow y+2x=\frac{2\pi}{3}$$

50. For $x\in R, f(x)=|\log 2-\sin x|$ and $g(x)=f(f(x))$, then :

(1) $g'(0)=\cos(\log 2)$

(2) $g'(0)=-\cos(\log 2)$

(3) g is differentiable at x = 0 and $g'(0) = -\sin(\log 2)$

(4) g is not differentiable at x = 0

Answer (1)

Sol. $g(x)=f(f(x))=|\log 2-\sin|\log 2-\sin x||$

$g(x) = f(f(x)) = \log 2 - \sin(\log 2 - \sin x)$

$g'(x) = \cos(\log 2 - \sin x)x - \cos x$

$g'(0) = \cos(\log 2)$

51. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true?

(1) E_2 and E_3 are independent

(2) E_1 and E_3 are independent

(3) E_1, E_2 and E_3 are independent

(4) E_1 and E_2 are independent

Answer (3)

Sol. $P(E_1)=\frac{1}{6}$

$P(E_2)=\frac{1}{6}$

$P(E_1 \cap E_2) = P(A$ shows 4 and B shows 2)

$=\frac{1}{36}=P(E_1).P(E_2)$

So E_1, E_2 are independent

Also as $E_1 \cap E_2 \cap E_3 = \phi$

So $P(E_1 \cap E_2 \cap E_3) \neq P(E_1. P(E_2). P(E_3)$

So E_1, E_2, E_3 are not independent.

52. If $A=\begin{bmatrix}5a & -b\\ 3 & 2\end{bmatrix}$ and A.adj $A = A A^T$, then $5a + b$ is equal to :

(1) 5 (2) 4

(3) 13 (4) –1

Answer (1)

Sol. A - adj A = IAI = A.A^T

$\Rightarrow$ adj $A = A^T$

$$\begin{bmatrix}2 & b\\ -3 & 5a\end{bmatrix}=\begin{bmatrix}5a & 3\\ -b & 2\end{bmatrix}$$

$\Rightarrow$ $5a = 2$, $b = 3$

So, $5a + b = 5$

53. The Boolean expression $(p\wedge \sim q)\vee q\vee(\sim p\wedge q)$ is equivalent to :

(1) $p\wedge q$

(2) $p\vee q$

(3) $p\vee \sim q$

(4) $\sim p\wedge q$

Answer (2)

Sol. $(p\wedge \sim q)\vee q\vee(\sim p\wedge q)$

$= ((p\vee q)\wedge(\sim q\vee q))\vee(\sim p\wedge q)$

$= ((p\vee q)\wedge t)\vee(\sim p\wedge q)$

$= (p\vee q)\vee(-p\wedge q)$

$= (p\vee q\vee -p)\wedge(p\vee q\vee q)$

$= t\wedge(p\vee q)$

$= p\vee q$

54. The sum of all real values of x satisfying the equation $(x^2-5x+5)^{x^2+4x-60}=1$ is

(1) – 4 (2) 6

(3) 5 (4) 3

Answer (4)

Sol. $x^2 - 5x + 5 = 1$

$\Rightarrow x = 1, 4$

or $x^2 - 5x + 5 = -1$

$\Rightarrow x = 2, 3$

or $x^2 + 4x - 60 = 0$

$\Rightarrow x = -10, 6$

$\therefore$ $x = 3$ will be rejected as L.H.S. becomes –1

So, sum of value of x = 1 + 4 + 2 – 10 + 6 = 3

55. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on

(1) An ellipse which is not a circle

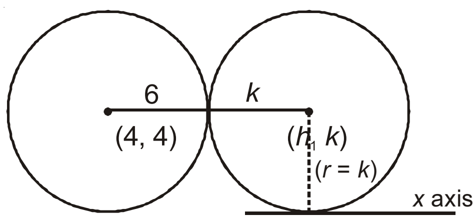
(2) A hyperbola

(3) A parabola

(4) A circle

Answer (3)

Sol.



Radius = $\sqrt{16+16+4} = 6$

$(6 + k)^2 = (h - 4)^2 + (k - 4)^2$

Replace $h \to x$, $k \to y$

$(y + 6)^2 - (y - 4)^2 = x^2 - 8x + 16$

$(2y + 2)(10) = x^2 - 8x + 16$

$20y + 20 = x^2 - 8x + 16$

$x^2 - 8x - 20y - 4 = 0$

Centre lies on parabola

56. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is :

(1) 59th (2) 52nd

(3) 58th (4) 46th

Answer (3)

Sol. Words starting with $A = \frac{4!}{2!} = 12$

Words starting with L = 4! = 24

Words starting with $M = \frac{4!}{2!} = 12$

Words starting with $SA = \frac{3!}{2!} = 3$

Words starting with SL = 3! = 6

Next words is SMALL

$\therefore$ Rank = 12 + 24 + 12 + 3 + 6 + 1 = 58

57. $\lim\limits_{n\to\infty}\left(\frac{(n+1)(n+2)\ldots 3n}{n^{2n}}\right)^{1/n}$ is equal to :

(1) $\frac{27}{e^2}$

(2) $\frac{9}{e^2}$

(3) 3 log3 – 2

(4) $\frac{18}{e^4}$

Answer (1)

Sol. $p = \lim\limits_{n\to\infty}\left[\frac{(n+1)(n+2)(n+3)\ldots\ldots(n+2n)}{n.\,n.\,\ldots\ldots n}\right]^{\frac{1}{n}}$

$$\log p = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\log\left(\frac{n+r}{n}\right)$$

$$= \int_0^2 \log(1+x)dx$$

$$= [\log(1+x)dx]_0^2 - \int_0^2 \frac{1.x}{1+x}dx$$

$$= 2\log 3 - \left[\int_0^2\left(1-\frac{1}{1+x}\right)dx\right]$$

$$= 2\log 3 - [x - \log(1+x)]_0^2$$

= 2log3 – (2 – log3]

log$p$ = 3 log3 – 2

$$p = e^{3\log 3 - 2} = \frac{e^{\log 27}}{e^2} = \frac{27}{e^2}$$

58. If the sum of the first ten terms of the series

$$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots\ldots,$$

is $\frac{16}{5}m$, then m is equal to :

(1) 101 (2) 100

(3) 99 (4) 102

Answer (1)

Sol. $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + ... = \frac{16}{5}m$

$\Rightarrow \left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right) + ...10 \text{ tens} = \frac{16}{6}m$

$\Rightarrow \left(\frac{4}{5}\right)^2 [2^2 + 3^2 + 4^2 + 5^2 ...... 10 \text{ terms}] = \frac{16}{5}m$

$\Rightarrow \left(\frac{4}{5}\right)^2 [2^2 + 3^2 + 4^2 .... + 11^2] = \frac{16}{5}m$

$\Rightarrow \left(\frac{4}{5}\right)^2 [1^2 + 2^2 + 3^2 .... + 11^2 - 1^2] = \frac{16}{5}m$

$\Rightarrow \left(\frac{4}{5}\right)^2 \left[\frac{11.12.23}{6} - 1\right] = \frac{16}{5}m$ (given)

$\Rightarrow \frac{16}{25}[22.23 - 1] = \frac{16}{5}m$

$\Rightarrow \frac{1}{5}(505) = m$

$\Rightarrow m = 101$

59. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at $(-3, 2)$, then the radius of S is

(1) $5\sqrt{3}$

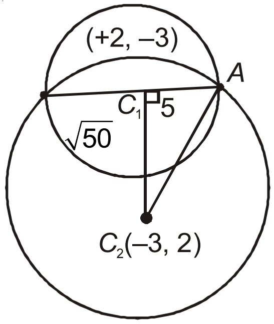
(2) 5

(3) 10

(4) $5\sqrt{2}$

Answer (1)

Sol.



Eq. $x^2 + y^2 - 4x + 6y - 12 = 0$

C_1; $(2, -3)$, $r_1 = \sqrt{4+9+12} = 5$

$C_2 = (-3, 2)$

$C_1C_2 = \sqrt{5^2 + 5^2} = \sqrt{50}$

Then, $C_2A = \sqrt{5^2 + (\sqrt{50})^2} = \sqrt{75} = 5\sqrt{3}$

60. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, from B to reach the pillar, is

(1) 10

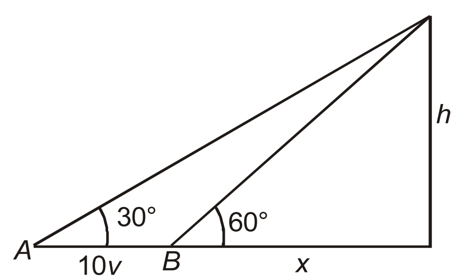
(2) 20

(3) 5

(4) 6

Answer (3)

Sol.



let speed = v units/min

$\frac{h}{10v + x} = \tan 30°$

$\frac{h}{x} = \tan 60°$

$\Rightarrow \frac{x}{10v + x} = \frac{1}{3} \Rightarrow x = 5v$

So, time = 5 minutes.

PART–C : PHYSICS

61. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is

(take g = 10 ms^{-2})

(1) 2 s (2) $2\sqrt{2}$ s

(3) $\sqrt{2}$ s (4) $2\pi\sqrt{2}$ s

Answer (2)

Sol. $v = \sqrt{\frac{T}{\mu}}$

$$\frac{dx}{dt} = \sqrt{\frac{\mu xg}{\mu}}$$

$$\int_0^L \frac{dx}{\sqrt{xg}} = \int_0^t dt$$

$\Rightarrow$ $t = 2\sqrt{2}$ s

62. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8 × 10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take g = 9.8 ms^{-2}

(1) 6.45 × 10^{-3} kg (2) 9.89 × 10^{-3} kg

(3) 12.89 × 10^{-3} kg (4) 2.45 × 10^{-3} kg

Answer (3)

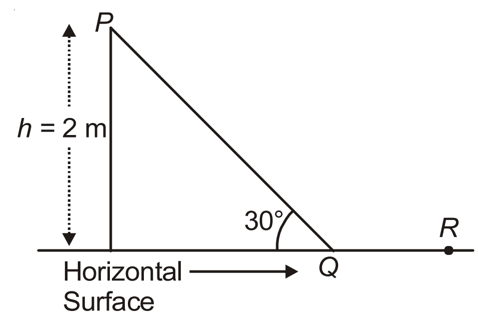
Sol. Let x kg of fat is burned then

$$x \times 3.8 \times 10^7 \times \frac{20}{100} = 10 \times 9.8 \times 1000$$

$\Rightarrow$ x = 12.89 × 10^{-3} kg

63. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ. The particle is released, from rest from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR

The values of the coefficient of friction μ and the distance x(= QR), are, respectively close to



(1) 0.2 and 3.5 m (2) 0.29 and 3.5 m

(3) 0.29 and 6.5 m (4) 0.2 and 6.5 m

Answer (2)

Sol. Work done by friction along PQ

= work done by friction along QR

$$\mu mg \cos\theta \frac{h}{\sin 30°} = \mu mg\, x$$

$\Rightarrow$ x = 3.5 m

Now, according to work energy theorem

$mgh = w_f(PQ) + w_f(QOR)$

Since, $w_f(PQ) = w_f(QR)$

$$mg(2) = 2 \times \mu mg \cos 30° \frac{h}{\sin 30°}$$

$\Rightarrow$ μ = 0.29

64. Two identical wires A and B, each of length 'l' carry the same current I. Wire A bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is

(1) $\frac{\pi^2}{16\sqrt{2}}$ (2) $\frac{\pi^2}{16}$

(3) $\frac{\pi^2}{8\sqrt{2}}$ (4) $\frac{\pi^2}{8}$

Answer (3)

Sol. For A For B

$2\pi R = L$ $4a = L$

$\Rightarrow R = \dfrac{L}{2\pi}$ $\Rightarrow a = \dfrac{L}{4}$

$$B_A = \frac{\mu_0 i}{2R} \quad B_B = 4\left[\frac{\mu_0 i}{4\pi a/2}\left(\sin\frac{\pi}{4}+\sin\frac{\pi}{4}\right)\right]$$

Now $\dfrac{B_A}{B_B} = \dfrac{\pi^2}{8\sqrt{2}}$

65. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is

(1) 2 Ω (2) 0.1 Ω

(3) 3 Ω (4) 0.01 Ω

Answer (4)

Sol. $I_g = 10^{-3}$ A

$R_g = 100\ \Omega$

R_S (shunt resistance) $= \dfrac{I_g(R_g)}{I - I_g} = 0.01\ \Omega$

66. An observer looks at a distant tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears

(1) 10 times nearer (2) 20 times taller

(3) 20 times nearer (4) 10 times taller

Answer (3)

Sol. By defination of magnification in telescope object will appear 20 times nearer to the observer.

67. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by

(1) Linear increase for Cu, exponential increase for Si

(2) Linear increase for Cu, exponential decrease for Si

(3) Linear decrease for Cu, linear decrease for Si

(4) Linear increase for Cu, linear increase for Si

Answer (2)

Sol. For metals, resistance increases upon increase in temperature. For undoped Si, resistance decreases upon decrease in temperature.

68. Choose the correct statement:

(1) In amplitude modulation the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(2) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

(3) In frequency modulation the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.

(4) In amplitude modulation the amplitude of the high frequency carrier wave is made to vary proportion to the amplitude of the audio signal.

Answer (4)

Sol. By definition of amplitude modulation.

69. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes, the ratio of decayed numbers of A and B nuclei will be

(1) 4 : 1 (2) 1 : 4

(3) 5 : 4 (4) 1 : 16

Answer (3)

Sol. $T_{\frac{1}{2}}$ of A = 20 minutes

$T_{\frac{1}{2}}$ of B = 40 minutes

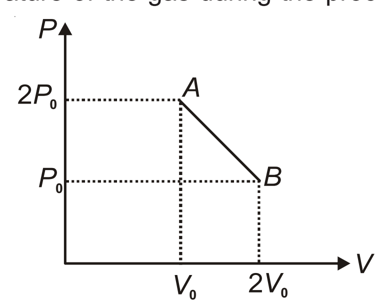
After 80 minutes

$$\frac{N_A(\text{decayed})}{N_0} = 1-\left(\frac{1}{2}\right)^4 \quad \ldots(1)$$

$$\frac{N_B(\text{decayed})}{N_0} = 1-\left(\frac{1}{2}\right)^2 \quad \ldots(2)$$

$$\frac{N_A}{N_B} = \frac{5}{4}$$

70. 'n' moles of an ideal gas undergoes a process $A \rightarrow B$ as shown in the figure. The maximum temperature of the gas during the process will be



(1) $\dfrac{3P_0V_0}{2nR}$ (2) $\dfrac{9P_0V_0}{2nR}$

(3) $\dfrac{9P_0V_0}{nR}$ (4) $\dfrac{9P_0V_0}{4nR}$

Answer (4)

Sol. $P = \dfrac{-P_0}{V_0}V + 3P_0$

$$T = \frac{-P_0 v^2}{V_0 nR} + \frac{3P_0 v}{nR} \qquad \ldots(1)$$

For T_{max}

$$\frac{dT}{dv} = 0$$

$$\Rightarrow \quad v = \frac{3}{2} v_0 \qquad \ldots(2)$$

Using (1) & (2)

$$T_{max} = \frac{9}{4} \frac{P_0 V_0}{nR}$$

71. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to

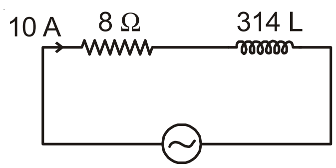
(1) 0.08 H (2) 0.044 H

(3) 0.065 H (4) 80 H

Answer (3)

Sol. $R = \frac{V}{I} = 8\,\Omega$

P = 800 W



$(220)^2 = (10 \times 8)^2 + (314 \times L \times 10)^2$

$\Rightarrow$ L = 0.065 H

72. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequency of the air column is now

(1) $\frac{3f}{4}$ (2) $2f$

(3) f (4) $\frac{f}{2}$

Answer (3)

Sol. Before dipping

$$f = \frac{v}{2L}$$

After dipping

$$f' = \frac{v}{4\frac{L}{2}} = f$$

73. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction. The spot would then have its minimum size (say b_{min}) when

(1) $a = \sqrt{\lambda L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

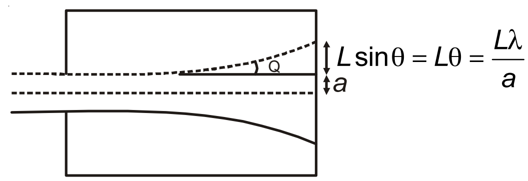
(2) $a = \sqrt{\lambda L}$ and $b_{min} = \sqrt{4\lambda L}$

(3) $a = \frac{\lambda^2}{L}$ and $b_{min} = \sqrt{4\lambda L}$

(4) $a = \frac{\lambda^2}{L}$ and $b_{min} = \left(\frac{2\lambda^2}{L}\right)$

Answer (2)

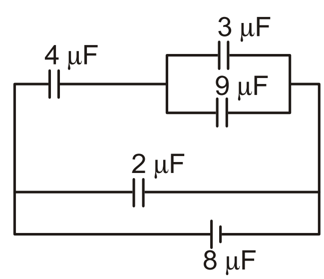
Sol.



$$b = a + \frac{L\lambda}{a}$$

For b to be min, $a = \sqrt{\lambda L}$ $b_{min} = 2\sqrt{L\lambda}$

74. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point distant 30 m from it, would equal :



(1) 360 N/C (2) 420 N/C

(3) 480 N/C (4) 240 N/C

Answer (2)

Sol. C_{net} = 5 μF

Q_{net} = 5 × 8 = 40

$Q_{4\,\mu F}$ = 24

$Q_{9\,\mu F}$ = 18

$Q = Q_{4\,\mu F} + Q_{9\,\mu F}$ = 42 μC

$$E = \frac{kQ}{r^2} = \frac{9\times10^4 \times 42\times10^{-6}}{30\times30} = 420 \text{ N/C}$$

75. Arrange the following electromagnetic radiations per quantum in the order of increasing energy:

A : Blue light　　*B* :Yellow light

C : X-ray　　*D* : Radiowave

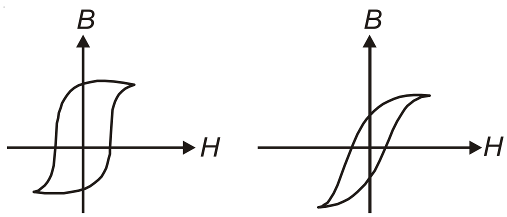
(1) *A, B, D, C*　　(2) *C, A, B, D*

(3) *B, A, D, C*　　(4) *D, B, A, C*

Answer (4)

Sol. According to electromagnetic spectrum *D*, *B*, *A*, *C*

76. Hysteresis loops for two magnetic materials *A* and *B* are given below:



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use:

(1) *A* for electromagnets and *B* for electric generators

(2) *A* for transformers and *B* for electric generators

(3) *B* for electromagnets and transformers

(4) *A* for electric generators and transformers

Answer (3)

Sol. For electromagnets, electric generators and transformers energy loss should be less.

Hence, (B) should be used

77. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the co-efficient of linear expansion (α) of the metal of the pendulum shaft are respectively:

(1) 60°C, $\alpha = 1.85 \times 10^{-4}$/°C

(2) 30°C, $\alpha = 1.85 \times 10^{-3}$/°C

(3) 55°C, $\alpha = 1.85 \times 10^{-2}$/°C

(4) 25°C, $\alpha = 1.85 \times 10^{-5}$/°C

Answer (4)

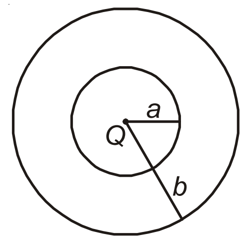
Sol. $\frac{1}{2}\alpha(40-T)\times 86400 = 12\,\text{s}$　　...(1)

$\frac{1}{2}\alpha(T-20)\times 86400 = 4$　　...(2)

On dividing (1) by (2) and solving

T = 25°C

78. The region between two concentric spheres of radii '*a*' and '*b*', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where *A* is a constant and *r* is the distance from the centre. At the centre of the spheres is a point charge *Q*. The value of *A* such that the electric field in the region between the spheres will be constant, is :

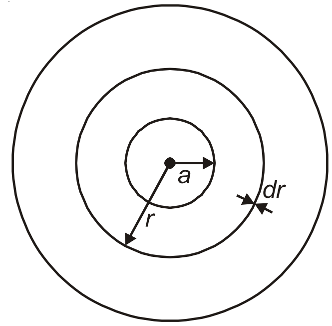


(1) $\frac{Q}{2\pi(b^2-a^2)}$　　(2) $\frac{2Q}{\pi(a^2-b^2)}$

(3) $\frac{2Q}{\pi a^2}$　　(4) $\frac{Q}{2\pi a^2}$

Answer (4)

Sol. At $r = a$



$E_a = \frac{kQ}{a^2}$

Take a shell at $r = r$

$(a \le r \le b)$

$dq = 4\pi r^2 dr \frac{A}{r}$

$\therefore$ q from $r = a$ to $r = r$

$q = 4\pi A \int_a^r r\,dr = 2\pi A[r^2 - a^2]$

$\therefore$ Charge from $r = a$ to $r = b$

$q = 2\pi A[b^2 - a^2]$

Now, field at $r = b$ is $E_b = \frac{2\pi A[b^2-a^2]+Q}{\varepsilon_0 \times 4\pi b^2}$

Now, $E_a = E_b$ gives, $A = \frac{Q}{2\pi a^2}$

79. In an experiment for determination of refractive index of glass of a prism by $i - \delta$, plot, it was found that a ray incident at angle 35°, suffers a deviation of 40° and that it emerges at angle 79°. In that case which of the following is closest to the maximum possible value of the refractive index?

(1) 1.6　　(2) 1.7

(3) 1.8　　(4) 1.5

Answer (4)

Sol. From the given data, A = $i + e - \delta = 74°$, $\delta = 40°$

Now, $\mu = \dfrac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)} < \dfrac{\sin\left(\dfrac{A+\delta}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$

$\Rightarrow \quad \mu < \dfrac{\sin 57°}{\sin 37°} \Rightarrow \mu < 1.39°$

Nearest value is 1.5

80. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90 s, 91 s, 95 s and 92 s. If the minimum division in the measuring clock is 1 s, then the reported mean time should be:

(1) 92 ± 5.0 s

(2) 92 ± 1.8 s

(3) 92 ± 3 s

(4) 92 ± 2 s

Answer (4)

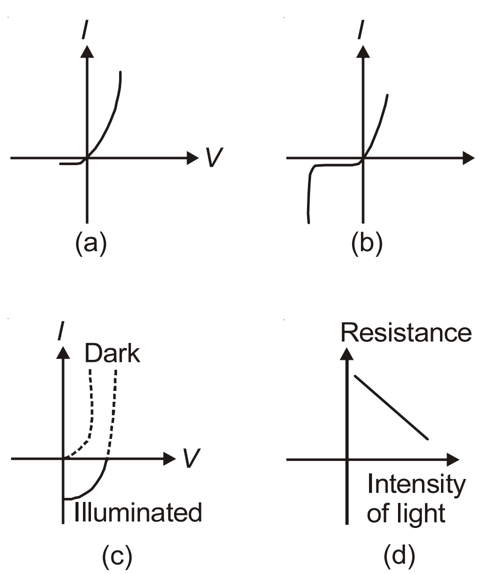
Sol. $T = \dfrac{T_1 + T_2 + T_3 + T_4}{4}$

$\Delta T = \dfrac{|T_1 - T| + |T_2 - T| + |T_3 - T| + |T_4 - T|}{4}$

= 1.5 s $\approx$ 2 s (Since least count is 1 s)

$\Rightarrow$ final answer is 92 ± 2 s

81. Identify the semiconductor devices whose characteristics are given below, in the order (a), (b), (c), (d) :



(1) Zener diode, Simple diode, Light dependent resistance, Solar cell

(2) Solar cell, Light dependent resistance, Zener diode, Simple diode

(3) Zener diode, Solar cell, Simple diode, Light dependent resistance

(4) Simple diode, Zener diode, Solar cell, Light dependent resistance

Answer (4)

Sol. Information based question.

82. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $\dfrac{3\lambda}{4}$, the speed of the fastest emitted electron will be

(1) $< v\left(\dfrac{4}{3}\right)^{\frac{1}{2}}$

(2) $= v\left(\dfrac{4}{3}\right)^{\frac{1}{2}}$

(3) $= v\left(\dfrac{3}{4}\right)^{\frac{1}{2}}$

(4) $> v\left(\dfrac{4}{3}\right)^{\frac{1}{2}}$

Answer (4)

Sol. $\dfrac{hc}{\lambda} - \phi = \dfrac{1}{2}mv^2$ ...(i)

$4\dfrac{hc}{3\lambda} - \phi = \dfrac{1}{2}mv_1^2$ ...(ii)

(ii) – $\dfrac{4}{3}\times$(i) gives, $\dfrac{\phi}{3} = \dfrac{1}{2}mv^2 - \dfrac{1}{2}mv_1^2 \times \dfrac{4}{3}$

$\Rightarrow \quad V_1^2 > V^2 \times \dfrac{4}{3} \Rightarrow V_1 > \left(\dfrac{4}{3}\right)^{\frac{1}{2}} V$

83. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\dfrac{2A}{3}$ from equilibrium position. The new amplitude of the motion is

(1) $3A$ (2) $A\sqrt{3}$

(3) $\dfrac{7A}{3}$ (4) $\dfrac{A}{3}\sqrt{41}$

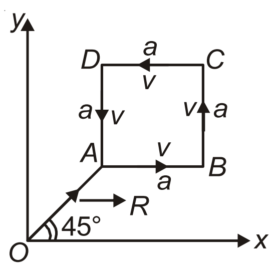
Answer (3)

Sol. $\nu = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2} = \frac{\omega\sqrt{5}A}{3}$

Now, $V' = 3 \times \frac{\omega\sqrt{5}A}{3}$

Now, $V' = \omega\sqrt{A'^2 - \left(\frac{2A}{3}\right)^2} \Rightarrow A' = \frac{7A}{3}$

84. A particle of mass m is moving along the side of a square of side a, with a uniform speed v in the x-y plane as shown in the figure



Which of the following statements is false for the angular momentum $\vec{L}$ about the origin?

(1) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D

(2) $\vec{L} = mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C

(3) $\vec{L} = \frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from D to A

(4) $\vec{L} = -\frac{mv}{\sqrt{2}}R\hat{k}$ when the particle is moving from A to B

Answer (1, 3)

Sol. In option A, correct $\vec{L}$ should be $mv\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from C to D.

In option 3, correct $\vec{L}$ should be $\frac{-mv}{\sqrt{2}}R\hat{k}$

85. An ideal gas undergoes a quasi-static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here C_p and C_v are molar specific heat at constant pressure and constant volume, respectively) :

(1) $n = \frac{C - C_p}{C - C_v}$

(2) $n = \frac{C_p - C}{C - C_v}$

(3) $n = \frac{C - C_v}{C - C_p}$

(4) $n = \frac{C_p}{C_v}$

Answer (1)

Sol. $C = C_V + \frac{R}{1-n} \Rightarrow C - C_V = \frac{R}{1-n}$

or $C = C_P - R - \frac{R}{1-n} \Rightarrow C - C_P = \frac{nR}{1-n}$

$\Rightarrow \frac{C - C_P}{C - C_V} = n$

86. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of Aluminium. Before starting the measurement, its is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet if the main scale reading is 0.5 mm and the 25th deivision coincides with the main scale line?

(1) 0.80 mm

(2) 0.70 mm

(3) 0.50 mm

(4) 0.75 mm

Answer (1)

Sol. Zero error = –5 division of circular scale

1 division of circular scale

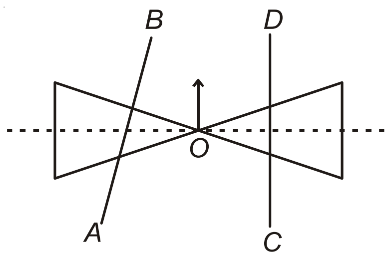
$= \frac{0.5}{50} = 10^{-2}$ mm = 0.01 mm

$\therefore$ Zero error = –5 × 10^{-2} = –0.05 mm

Zero correction = + 0.05 mm

Reading = 0.5 + 25 × 0.01 + 0.05 = 0.80 mm

87. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to



(1) Turn right

(2) Go straight

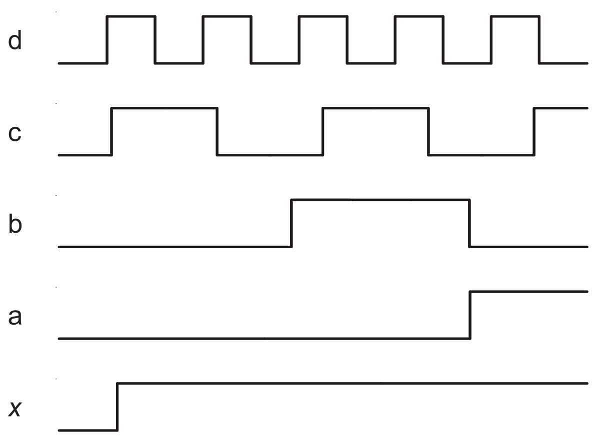
(3) Turn left and right alternately

(4) Turn left

Answer (4)

Sol. The roller will turn left as a force of friction will develop on rail AB in the backward direction.

88. If a, b, c, d are inputs to a gate and x is its output, then, as per the following time graph, the gate is :



(1) AND

(2) OR

(3) NAND

(4) NOT

Answer (2)

Sol. OR gate as output is 1 when any of the input is 1.

89. For a common emitter configuration, if α and β have their usual meanings, the **incorrect** relationship between α and β is :

(1) $\alpha = \frac{\beta}{1-\beta}$

(2) $\alpha = \frac{\beta}{1+\beta}$

(3) $\alpha = \frac{\beta^2}{1+\beta^2}$

(4) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$

Answer (1, 3)

Sol. $I_e = I_b + I_c$

$$\frac{I_e}{I_c} = \frac{I_b}{I_c} + 1$$

$$\frac{1}{\alpha} = \frac{1}{\beta} + 1$$

or

$$\alpha = \frac{\beta}{1+\beta}$$

90. A satellite is revolving in a circular orbit at a height 'h' from the earth's surface (radius of earth R; $h<<R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the earth's gravitational field, is close to : (Neglect the effect of atmosphere.)

(1) $\sqrt{gR}$

(2) $\sqrt{gR/2}$

(3) $\sqrt{gR}(\sqrt{2}-1)$

(4) $\sqrt{2gR}$

Answer (3)

Sol. $V_0 = \sqrt{gR}$

$$V_e = \sqrt{2gR}$$

$$\Delta V = (\sqrt{2}-1)\sqrt{gR}$$

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