chapter (4).

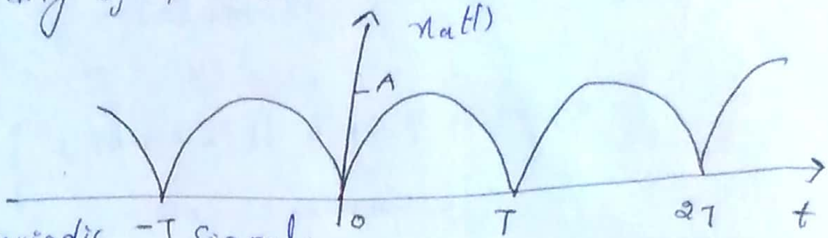
4.1. Consider the full-wave rectified Sinusoid

(a). Determine its Spectrum $X_a(F)$

(b) Compute the power of the signal.

(c) plot the power spectral density

(d) Check the validity of Parseval's relation for this signal.



$x_a(t)$ is a periodic signal.

for appling fourier transform we need to find the fourier Series and Then apply the fourier transform for determining the Spectrum $X_a(f)$.

$$x_a(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{+nj\omega t} \qquad \text{where} \Rightarrow f = 1/T$$

$$= \sum_{k=-\infty}^{\infty} C_k \cdot e^{j2\pi k \cdot f t}$$

$$= \sum_{k=-\infty}^{\infty} C_k \cdot e^{j2\pi k \cdot t/T}$$

$\sin(\pi \cdot x\, t/T)$ where t is from

A Sin odd funct

$1-2k$

$k=1$ cell $\frac{1}{0}$ odd

$$C_k = \frac{1}{T} \int_0^T A \cdot \sin(\pi \cdot t/T) \cdot e^{-j2\pi k \cdot t/T} \, dt$$

$\frac{e^{j\theta} + e^{-j\theta}}{2} = \cos\theta$

$$= \frac{1}{T} \int_0^T A \left[\frac{e^{+j\frac{\pi t}{T}} - e^{-j\frac{\pi t}{T}}}{2} \right] \cdot e^{-j2\pi k \cdot \frac{t}{T}} \, dt$$

$\frac{e^{j\theta} - e^{-j\theta}}{2} = \sin$

$$= \frac{A}{T} \int_0^T \frac{e^{j\frac{\pi t}{T}(1-2k)} - e^{-j\frac{\pi t}{T}(1+2k)}}{2} \, dt$$

e^{j0}

$-e^{j0}$

0

$$= \frac{A}{T} \times \frac{1}{2} \left[\frac{e^{\frac{j\pi(1-2k)}{T} \cdot t}}{j\frac{\pi(1-2k)}{T}} + \frac{e^{-\frac{j\pi t(1+2k)}{T}}}{j\frac{\pi}{T}(1+2k)} \right]_0^T$$

$$= \frac{A}{2T} \left[\frac{-2}{1-2k} + \frac{-2}{1+2k} \right] \times \frac{T}{j\pi}$$

$$C_k = \frac{A}{2j\pi}\left[\frac{1+e^{2/k}+1-e^{2k}}{1-4k^2}\right]\times\frac{1}{j}$$

$$= \frac{-A}{j\pi}\times\frac{2}{1-4k^2}\times\frac{1}{j}$$

$$C_k = \frac{2A}{\pi(1-4k^2)}$$

$$C_k = \frac{2A}{\pi(4k^2-1)}\quad \pi(1-4k^2$$

$$x_a(t) = \sum_{k=-\infty}^{\infty}\frac{A}{\pi(4k^2+1)}\cdot e^{j2\pi k\cdot t/\tau}$$

$$X_a(f) = \int_{-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\frac{2A}{\pi(4k^2-1)}\cdot e^{j2\pi k\cdot t/\tau}\cdot e^{-j2\pi f t}\cdot dt$$

$$X_a(f) = \int_{-\infty}^{\infty}\sum_{k=-\infty}^{\infty}\frac{2A}{\pi(4k^2+1)}\cdot e^{j2\pi(f-k/\tau)\cdot t}\,dt$$

$$e^{j\pi f} \rightleftharpoons \delta(f-f).$$

$$= \sum_{k=-\infty}^{\infty}\frac{2A}{(4k^2+1)\pi}\cdot\delta(f-F+\tfrac{k}{\tau}).\quad f-k/\tau$$

$$e^{j\omega_0 t}\cdot 1 \rightleftharpoons 2\pi\delta(\omega-\omega_0).$$

$$\underbrace{e^{j2\pi(f-k/\tau)\cdot t}}$$

(a) $$\boxed{X_a(f) = \sum_{k=-\infty}^{\infty}\frac{2A}{\pi(4k^2+1)}\delta\left(\cancel{f-F+\tfrac{k}{\tau}}\right)\quad f-k/\tau}.$$

$$\delta(f-\cancel{f+k/\tau})$$
$$\delta(f-k/\tau)$$

(b).

$$P_x = \frac{1}{T}\underbrace{\int_{-\infty}^{\infty} x_a^2 t\cdot dt}_{\text{energy}}/\text{time}$$

$$= \frac{1}{T}\int_0^T A^2\cdot\sin^2(\pi\cdot t/\tau)\cdot dt$$

$$= \frac{A^2}{T2}\int_0^T\left(1-\cos\frac{2\pi t}{\tau}\right)\cdot dt$$

$$\frac{A^2}{2T}\int_0^{T}\left(1-\cancel{\cos 2\pi \cdot t/\tau}\right)dt$$

$$\frac{A^2}{2T}\times T = P_x.$$

Power $= A^2/2$

d). parseval's relation

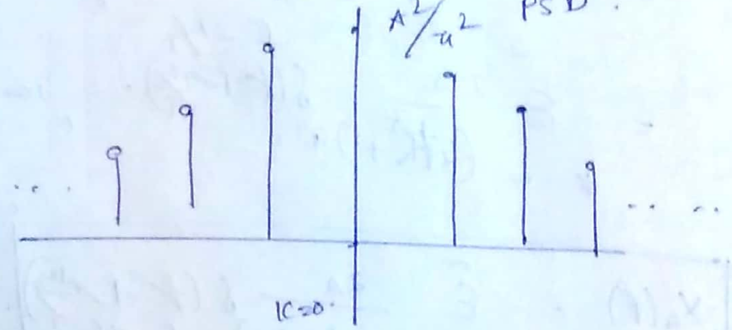
$$P_x = \frac{1}{T}\int_0^{T} x^2 a(t)\,dt$$

as we know that $c_k = \frac{1}{T}\int_0^{T}\left(x_a(t)\cdot e^{-j2\pi kt/T}\right)^2$

$= |c_k|^2$

$$\sum_{k=-\infty}^{\infty}|c_k|^2 = \frac{4A^2}{\pi^2}\sum_{k=-\infty}^{\infty}\frac{1}{(-4k^2+1)^2}$$

$k = 1$ A^2/π^2 PSD.



$k=0$

$$= \frac{4A^2}{\pi^2}\left[\underbrace{1+\frac{2}{3^2}+\frac{2}{15^2}+\frac{2}{35}+\cdots}_{\text{infinit series}}\right]$$

$$= \frac{A^2}{2}$$

4.2 Compute and sketch the magnitude and phase spectra for the following signals $(a>0)$

(a). $x_a(t) = \begin{cases} A\cdot e^{-at}, & t\geq 0. \\ 0, & t<0. \end{cases}$

$x_a(t) = Ae^{-a|t|}$

$$X_a(f) = \int_{-\infty}^{\infty} x_a(t) \cdot e^{-j2\pi f t} \cdot dt$$

$$= \int_{0}^{\infty} A \cdot e^{-at} \cdot e^{-j2\pi f t} \cdot dt$$

$$= A \int_{0}^{\infty} e^{-(a+j2\pi f) \cdot t} \cdot dt$$

$$= A \left[\frac{e^{-(a+j2\pi f)t}}{-(a+j2\pi f)} \right]_0^{\infty}$$

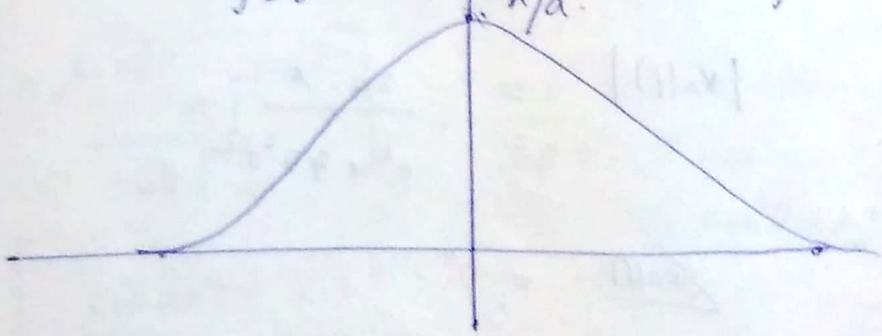
$$= A \left[0 + \frac{1}{a+j2\pi f} \right] = \frac{A}{a+j2\pi f}$$

$$X_a(f) = \frac{A}{a+j2\pi f}$$

$$|x_a(f)| = \frac{\sqrt{A^2}}{\sqrt{a^2+4\pi^2 f^2}} = \frac{A}{\sqrt{a^2+4\pi^2 f^2}}$$

magnitude spectrum

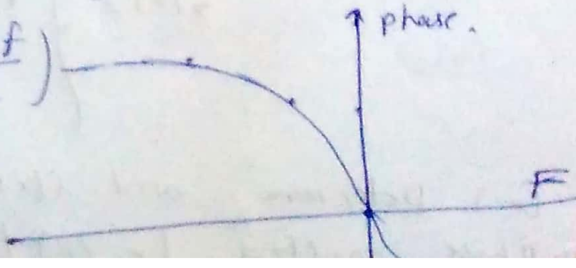
$f = 0$. A/a. $f = \infty$.



$$\angle X_a(f) = \frac{-\tan^{-1}(0/A)}{\tan^{-1}\left(\frac{2\pi f}{a}\right)} = -\tan^{-1}\left(\frac{2\pi f}{a}\right).$$

$a = 2$

$\tan^{-1}(\pi f)$



(b). $x_a(t) = A \cdot e^{-a|t|}$

$$= A \cdot e^{-at} + A \cdot e^{at}$$

$$X_a(f) = \int_{-\infty}^{0} A \cdot e^{at} \cdot e^{-2\pi j f t} \, dt + \int_{0}^{\infty} A \cdot e^{-at} \cdot e^{-j2\pi f t} \, dt$$

$$= A \int_{-\infty}^{0} e^{+(a - j2\pi f)t} \, dt + A \int_{0}^{\infty} e^{-(a - j2\pi f)t} \, dt$$

$$= A \left[\frac{e^{(a - j2\pi f)t}}{a - j2\pi f} \right]_{-\infty}^{0} + A \left[\frac{e^{-(a + j2\pi f)}}{-(a + j2\pi f)} \right]_{0}^{\infty}$$

$$= A \left[\frac{1}{a - j2\pi f} \right] + A \cdot \frac{1}{a + j2\pi f}$$

$$= A \left[\frac{a + \cancel{j2\pi f} + a - \cancel{j2\pi f}}{a^2 + 2\pi^2 f^2} \right]$$

$$= \frac{2a \cdot A}{a^2 + 4\pi^2 f^2}$$

$$|X_a(f)| = \frac{2a \cdot A}{a^2 + 4\pi^2 f^2}$$

$$\angle X_a(f) = 0^\circ.$$

4.3. Consider the signal

$$x(t) = \begin{cases} 1 - |t|/\tau, & |t| \le \tau \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Determine and sketch its magnitude and phase spectra, $|X_a(f)|$ and $\angle X_a(f)$.

$$x_a(f) = \int_{-T}^{0} (1 + t/T)\, e^{-j2\pi f t}\, dt + \int_{0}^{T} (1 - t/T)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-T}^{0} e^{-j2\pi ft} + \frac{t}{T}\cdot e^{-j2\pi ft}\, dt + \int_{0}^{T} e^{-j2\pi ft} - \frac{t}{T}\cdot e^{-j2\pi ft}\, dt$$

$$= \left[\frac{e^{-j2\pi ft}}{-j2\pi f}\right]_{-T}^{0} + \frac{1}{T}\left[-t\cdot\frac{e^{-j2\pi ft}}{j2\pi f} - \frac{e^{-j2\pi ft}}{2\pi f}\right]_{-T}^{0}$$

$t \cdot e^{-j2\pi ft}$

$\frac{e^{-j2\pi ft}}{-j2\pi f}$

1

$\frac{e^{-j2\pi ft}}{2\pi f}$

$$+\left[\frac{e^{-j2\pi ft}}{-j2\pi f}\right]_{0}^{T} - \frac{1}{T}\left[-t\cdot\frac{e^{-j2\pi ft}}{j2\pi f} - \frac{e^{-j2\pi ft}}{2\pi f}\right]_{0}^{T}$$

$$= \cancel{\frac{-1}{j2\pi f}} + \cancel{\frac{e^{j2\pi fT}}{j2\pi f}} - \frac{1}{2\pi f T} - \cancel{1\cdot\frac{e^{j2\pi fT}}{j2\pi f}} + \frac{1}{T}\cdot\frac{e^{+j2\pi fT}}{2\pi f}$$

$$+ \cancel{\frac{e^{-j2\pi fT}}{-j2\pi f}} + \cancel{\frac{1}{j2\pi f}} + \cancel{\frac{e^{-j2\pi fT}}{j2\pi f}} + \frac{1}{T}\cdot\frac{e^{-j2\pi fT}}{2\pi f}$$

$$-\frac{1}{T\cdot 2\pi f T}$$

$$= \frac{1}{T}\left[\frac{e^{j2\pi fT} + e^{-j2\pi fT}}{2\pi f}\right] - \frac{1}{\pi f T}$$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$= \frac{1}{\pi f T}\left[\cos 2\pi f T - 1\right]$$

$\frac{1-\cos 2\theta = 2\sin}{2}$

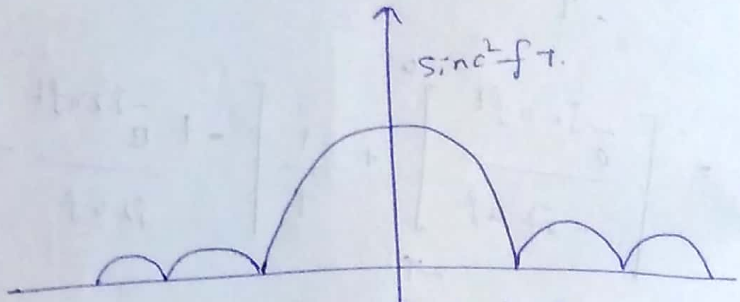
$$= +\frac{-2}{\pi f T} \times \sin^2 2\pi f T = -\frac{2\sin^2\pi f T}{\pi f T} = x_a(f).$$

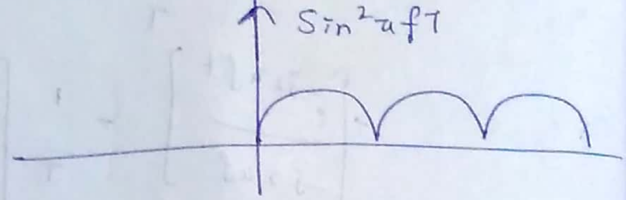
$$x_a(f) = \frac{-2\cdot\sin^2\pi f T}{\pi f T}$$

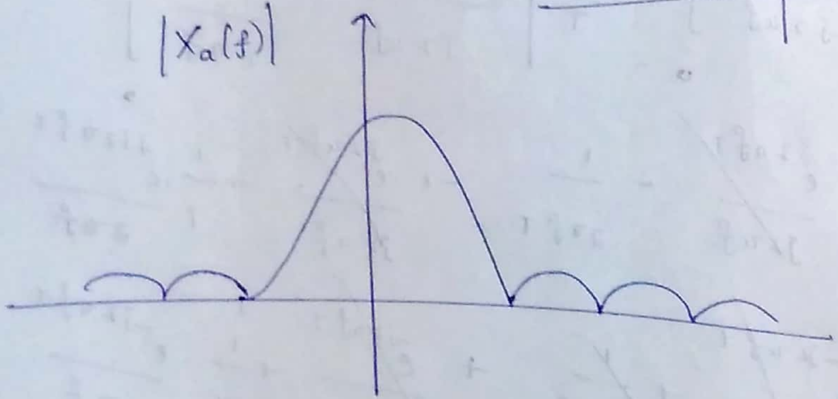
$$|X_a(f)| = \left| -2 \cdot \frac{\sin^2 \pi f \tau}{\pi f \tau} \right|$$

$$= \left| -2 \cdot \frac{\sin \pi f \tau}{\pi f \tau} \cdot \sin \pi f \tau \right|$$

$$|X_a(f)| = 4 \; \mathrm{sinc}^2 f\tau \cdot \sin^2 \pi f \tau .$$







$$\angle X_a(f) = \tan^{-1}\left(0 \Big/ -2 \cdot \frac{\sin^2 \pi f \tau}{\pi f \tau} \right) = 0 .$$

$$\angle X_a(f) = 0 .$$

(b) Create a periodic signal $x_p(t)$ with fundamental period $T_p \geq 2\tau$, so that $x(t) = x_p(t)$ for $|t| < T_p/2$. what are the fourier coefficients C_k for the signal $x_p(t)$?

Sol:-

$$C_K = \frac{1}{T_p} \int_{-T_p/2}^{T_p/2} (x(t)) \cdot e^{-j2\pi f \cdot k t} \cdot dt \qquad P = 2\tau$$

$$= \frac{1}{T_p} \int_{-T_p/2} \qquad \frac{1}{T_p} \int_{-\tau}^{\tau} x(t) \cdot e^{-j2\pi t/T_p} \, dt$$

$$C_k = \frac{1}{T_p}\left[\int_{-\tau}^{0}\left(1+\frac{t}{\tau}\right)\cdot e^{-j2\pi k\cdot t/T_p}\,dt + \int_{0}^{\tau}\left(1-\frac{t}{\tau}\right)\cdot e^{\frac{-j2\pi k t}{T_p}}\,dt\right]$$

$$= \frac{1}{T_b}\cdot -2\,\frac{\sin^2 \pi k\tau/T_p}{\frac{\pi \tau k}{T_p}}$$

$$= \frac{1}{T_p}\cdot -2\cdot \operatorname{sinc}(k\tau/T_p)\cdot \sin(\pi k\cdot \tau/T_p).$$

(c). $C_k = \frac{1}{T_p}\, x_a(t/T_p)$

4.4. Consider the following periodic signal

$$x(n) = \{\ldots, 1, 0, 1, 2, \underset{\uparrow}{3}, 2, 1, 0, 1, \ldots\}$$

(a) Sketch the signal $x(n)$ and its magnitude and phase spectra.

(b) Using the result in part (a), verify Parseval's relation by computing the power in the time and frequency domains.

Sol:- Length $(x(n)) = 6 = N$. as it is a discrete signal.

$$C_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\cdot e^{-j2\pi \frac{k\cdot n}{6}}$$

where $\omega = \frac{2\pi k}{6} = \frac{2\pi k}{N}$, 6 is N

$$= \frac{1}{N}\sum_{n=0}^{N-1} x(n)\cdot e^{-j\omega n}$$

$f = \frac{k}{N}$ {discrete frequency}

$$= \frac{1}{\cancel{6}}\left[3 + \cancel{\left(\frac{e^{\frac{j2\pi k\cdot x}{6_3}} + e^{\frac{-j2\pi k\cdot x}{6_3}}}{2}\right)}\times 2 + 2\times\frac{e^{\frac{j2\pi k}{3}} + e^{\frac{-j2\pi k}{3}}}{2}\right]$$

$X(\omega) =$ Coefficient of C_k.

$$= \frac{1}{6}\sum_{n=0}^{5} x(n)\cdot e^{-\frac{j\pi k\cdot n}{3}}$$

$$C_0 = \frac{1}{6}\sum_{n=0}^{5} x(n).$$

$$= \frac{1}{6}\left[x(0) + x(1) + x(2) + x(3) + x(4) + x(5)\right]$$

$$= 9/6,$$

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$$C_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\cdot e^{\frac{-j2\pi k}{\cancel{6}3}\cdot n}$$

$\frac{\pi}{3}=60 \quad \frac{5\pi}{3}=30$

$$= \frac{1}{6}\left[\underset{x(0)}{3} + x(1)\cdot e^{\frac{-j\pi k}{3}} + x(2)\cdot e^{\frac{-j\pi k 2}{3}} + x(3)\cdot e^{-j\pi k} + x(4)\cdot e^{\frac{-j\pi k\cdot 4}{3}} + x(5)\cdot e^{\frac{-j\pi k\cdot 5}{3}}\right]$$

$$= \frac{1}{6}\left[3 + 2\cdot e^{\frac{-j\pi k}{3}} + e^{\frac{-j\pi k 2}{3}} + 0 + e^{\frac{-j4\pi k}{3}} + 2\cdot e^{\frac{-j5\pi k}{3}}\right]$$

$$C_k = \frac{1}{6}\left[3 + 2\cdot e^{\frac{-j\pi k}{3}} + e^{\frac{-j2\pi k}{3}} + e^{\frac{-j4\pi k}{3}} + 2\cdot e^{\frac{-j5\pi k}{3}}\right]$$

$$= \frac{1}{6}\left[3 + 4 e^{\frac{-j\pi k}{3}} + 2\cdot e^{\frac{-j2\pi k}{3}}\right] \quad \frac{\pi k}{3} = \frac{5\pi k}{3}$$

$$= \frac{1}{6}\left[3 + 4\left[\cos\left(\frac{\pi}{3}k\right) - j\sin\left(\frac{\pi}{3}k\right)\right] + 2\left[\cos\left(\frac{2\pi}{3}k\right) - j\sin\left(\frac{2\pi}{3}k\right)\right]\right]$$

$$= \frac{1}{6}\left[3 + 4\cdot\cos\left(\frac{\pi k}{3}\right) + 2\cdot\cos\left(\frac{2\pi}{k}\right)3\right].$$

$$C_k = \frac{1}{6}\left[3 + 2\cdot\left[\cos\left(\frac{\pi}{3}k\right) - \cancel{j\sin\left(\frac{\pi}{3}k\right)}\right] + \cos\left(\frac{2\pi}{3}k\right) - \cancel{j\sin\left(\frac{2\pi}{3}k\right)} + \cos\left(\frac{4\pi k}{3}\right) - \cancel{j\sin\left(\frac{4\pi k}{3}\right)} + 2\left[\cos\left(\frac{5\pi k}{3}\right) - \cancel{j\sin\left(\frac{5\pi k}{3}\right)}\right]\right]$$

$$C_k = \frac{1}{6}\left[3 + 4\cos\left(\frac{\pi}{3}k\right) + 2\cdot\cos\left(\frac{2\pi}{3}k\right)\right]$$

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$C_0 = \frac{1}{6}[3+4+2]$

$= \frac{9}{6}$

$C_1 = \frac{1}{6}[3+\cancel{4}^{2} \times \frac{1}{\cancel{2}} + \cancel{2} \times \frac{-1}{\cancel{2}}]$

$= \frac{1}{6}[3+2-1] = 4/6$

$C_2 = \frac{1}{6}[3+\cancel{4}^{2} \times \frac{-1}{\cancel{2}} + \cancel{2} \times \frac{-1}{\cancel{2}}$

$= 0$

$C_3 = \frac{1}{6}[3+4\times(-1)+2\times(1)]$

$C_3 = \frac{1}{6}$

$C_4 = \frac{1}{6}[3+\cancel{4}^{2} \times \frac{-1}{\cancel{2}} + \cancel{2} \times \frac{-1}{\cancel{2}}$

$= 0$

$C_5 = \frac{1}{6}[3+\frac{\cancel{4}^{2}\times 1}{\cancel{2}} + \cancel{2} \times \frac{-1}{\cancel{2}}$

$C_5 = 4/6$

(b). power in time domain.

$$P_t = \frac{1}{6}\sum_{n=0}^{5}|x(n)|^2$$

$$= \frac{1}{6}[3^2+2^2+1^2+0^2+1^2+2^2]$$

$$= \frac{1}{6}[19] = \frac{19}{6} = 3.1667 \text{ watts.}$$

Power in frequency domain.

$$P_f = \sum_{n=0}^{5}|c(n)|^2$$

$$= \frac{9^2}{6^2} + \frac{4^2}{6^2} + 0^2 + \frac{1}{6^2} + \frac{4^2}{6^2}$$

$$= \frac{114}{36} = \frac{19}{6}.$$

both are same.

for

4.5. Consider the signal

$$x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$$

(a) determine and sketch its power density spectrum.

(b) Using the result Evaluate the power of the signal.

Sol:- $x(n) = 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}$

$\omega = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}.$ $\quad \frac{\pi}{2} = \omega_0$

$\omega_0 = \frac{2\pi}{T_0}$

$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{\frac{\pi}{4}} = 2\pi \times \frac{2}{\pi} = 8.$

$N = 8.$

we need to take the smaller subperiod which is having more frequency (fundamental frequency).

$$C_k = \frac{1}{8}\sum_{n=0}^{7} x(n) \cdot e^{-j\frac{2\pi k \cdot n}{8}}$$

$$= \frac{1}{8}\Big[x(0) + x(1) \cdot e^{-j\frac{\pi k}{4}} + x(2) \cdot e^{-j\frac{\pi k}{2}} + x(3) e^{-j\frac{3\pi k}{4}} + x(4) \cdot e^{-j\pi k} + x(5) \cdot e^{-j\frac{5\pi k}{4}} + x(6) e^{-j\frac{3\pi k}{2}} + x(7) \cdot e^{-j\frac{7\pi k}{4}}\Big]$$

$x(0) = 2 + 2 + 1 + \frac{1}{2} = \frac{11}{2}$

$x(1) = 2 + 2\cos\left(\frac{\pi}{4}\right) + \cos\frac{\pi}{2} + \frac{1}{2}\cos\frac{3\pi}{4}$

$= 2 + 2 \times \frac{1}{\sqrt{2}} + 0 + \frac{1}{2} \times \frac{-\sqrt{2}}{2}$

$= 3.06$

$x(2) = 2 + 2 \cdot 0 + (-1) + \frac{1}{2}\cos(+0$

$= 2 - 1 = 1.$

$x(3) = 2 + 2 \times \cancel{\frac{\sqrt{2}}{2}} + \frac{\cos(\alpha)}{8} + \frac{\sqrt{2}}{2} \times \cancel{\frac{1}{2}} \quad 2$

$x(4) = \cancel{2} + \cancel{-2} + 1 + \frac{1}{2} \cancel{-\frac{2}{2}(-1)}$

$= 1/2$

$x(5) = 2 + 2\left(-\frac{\sqrt{2}}{2}\right) + 0 + \frac{1}{2} \times \frac{\sqrt{2}}{2}$

$= 0.9393$

$x(6) = 2 + (0) + (-1) + \frac{1}{2} \times \frac{\sqrt{2}}{2}$

$= 1 + \frac{1}{2} \times \frac{\sqrt{2}}{2} = 1.353$

$x(7) = 2 + \frac{\sqrt{2}}{2} \times 2 + 0 + \frac{1}{2} \times \frac{-\sqrt{2}}{2}$

$= 3.06$

$$C_k = \frac{1}{8}\left[\sum_{n=0}^{7} x(n) \cdot e^{-j\frac{\pi k n}{4}}\right]$$

$$= \frac{1}{8}\left[x(0) \cdot \frac{11}{2} + 3.06 \cdot e^{\frac{-j\pi k}{4}} + e^{-\frac{j\pi k}{2}} + 0.93 \cdot e^{\frac{-j3\pi k}{4}} + \frac{1}{2} e^{-j\pi k} + 1.353 \cdot e^{-\frac{j3\pi k}{2}} + 3.06\, e^{-j\frac{7\pi k}{4}}\right]$$

$C_0 = 1.92$

$$C_k = \frac{1}{8}\left[\frac{11}{2} + 2 \times 3.06 \cos\left(\frac{\pi}{4}k\right) + 2.353 \cos\left(\frac{\pi}{2}k\right) + 0.93 \cos\left(\frac{3\pi k}{4}\right) + \frac{1}{2}\cos(\pi k)\right].$$

$$C_1 = \frac{1}{8}\left[\frac{11}{2} + 2 \times 3.06 \times \frac{\sqrt{2}}{2} + 2.353(0) + 0.93\left(-\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\right].$$

$C_1 = 1.208$, $C_2 = C_6 = \frac{1}{2}$, $C_3 = C_5 = \frac{1}{4}$, $C_4 = 0$

(b) $$P = \sum_{i=0}^{7} |c(i)|^2$$

$= 4 + 1 + 1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{16} + \frac{1}{16}.$

$= 53/8.$

4.6. Determine and sketch the magnitude and phase spectra of the following periodic signals.

(a). $x(n) = 4 \cdot \sin \frac{\pi(n-2)}{3}$

Sol:-

$= 4 \cdot \sin \frac{2\pi(n-2)}{6}$

$C_k = \frac{1}{6} \sum_{n=0}^{5} x(n) e^{-\frac{j2\pi kn}{6}}$

$= \frac{4}{6} \sum_{n=0}^{5} \sin \frac{2\pi(n-2)}{6} \cdot e^{-\frac{j2\pi kn}{6}}$

$= \frac{4}{6} \Big[x(0) + x(1) e^{\frac{-j\pi k}{3}} + x(2) \cdot e^{-\frac{j2\pi k}{3}} + x(3) e^{\frac{-j3k\pi}{3}}$

$+ x(4) \cdot e^{-j\frac{4k\pi}{3}} + x(5) \cdot e^{\frac{-j5\pi k}{3}}$

$x(0) = \sin \frac{2\pi(0-2)}{6}$ $\quad x(1) = \sin\left(\frac{2\pi(1-2)}{6}\right) = -\frac{\sqrt{3}}{2}$

$= \sin \frac{-4\pi}{6} = -\frac{\sqrt{3}}{2}$

$x(3) = \sin\left(\frac{2\pi(3-2)}{6}\right) = \frac{\sqrt{3}}{2}$

$x(2) = \sin\left(\frac{2\pi(0)}{6}\right) = 0$

$x(4) = \sin\left(\frac{\pi(4-2)}{3}\right) = \frac{\sqrt{3}}{2}$

$x(5) = \sin\left(\frac{\pi(5-2)}{3}\right) = 0.$

$= \frac{4}{6} \Big[-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cdot e^{\frac{-j\pi k}{3}} + 0 + \frac{\sqrt{3}}{2} e^{-jk\pi} + \frac{\sqrt{3}}{2} e^{\frac{-j4k\pi}{3}} \Big]$

$C_k = \frac{4}{6} \Big[-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cos \frac{\pi k}{3} + j\frac{\sqrt{3}}{2} \sin \frac{\pi k}{3} + \frac{\sqrt{3}}{2} \cos \pi k$

$- \frac{\sqrt{3}}{2} j \sin \pi k + \frac{\sqrt{3}}{2} \cos \frac{4\pi k}{6}$

$- j\frac{\sqrt{3}}{2} \sin \frac{4\pi k}{6} \Big]$

$C_0 = \frac{4}{6} \Big[-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \Big] = 0.$

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$$C_1 = \frac{4}{6}\left[-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\times\frac{1}{2} + j\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}(-1) + 0 + \frac{\sqrt{3}}{2}(-1/2) + j\cdot\frac{\sqrt{3}}{2}\left(+\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{4}{6}\left[-\frac{3\sqrt{3}}{2} + \cancel{2}\cdot j\,\frac{3}{\cancel{4}\,2}\right].$$

$$= -\sqrt{3} + j.$$

$$|C_1| = \sqrt{(-\sqrt{3})^2 + 1} = \sqrt{3+1} = 2.$$

$$C_2 = \frac{4}{6}\left[-\cancel{\frac{\sqrt{3}}{2}} - \cancel{\frac{\sqrt{3}}{2}\times\frac{1}{2}} + j\cancel{\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}} + \cancel{\frac{\sqrt{3}}{2}(1)} + \cancel{\frac{\sqrt{3}}{2}\left(-\frac{1}{2}\right)}.\ \cancel{-j\cdot\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}}\right.$$

$$= 0.$$

$$C_3 = 0.$$

$$C_4 = 0.$$

$$C_5 = \cancel{\frac{4}{6}}\left[-\cancel{\frac{\sqrt{3}}{2}} - \cancel{\frac{\sqrt{3}}{2}}\right. \quad -\sqrt{3} - j$$

$$|C_5| = 2.$$

$$\angle C_1 = \tan^{-1}\left(1/\sqrt{3}\right) = 30 = 180 - 30 = 150^\circ$$

$$\angle C_0 = \angle C_3 = \angle C_2 = \angle C_4 = 0.$$

$$\angle C_5 = \tan^{-1}\left(-1/-\sqrt{3}\right) = 180 + 30 = 210^\circ$$

(b) $x(n) = \cos\frac{2\pi}{3}n + \sin\frac{2\pi}{5}n.$ $\quad \cos(2\pi f_1 n) + \sin 2\pi f_2 n$

$\omega = \frac{2\pi}{3}$ $\quad \omega = \frac{2\pi}{5}.$ $\quad 2\pi f_1 = \frac{2\pi}{3}$ $\quad 2\pi f_2 = \frac{2\pi}{5}$

$f_1 = 1/3$ $\quad f_2 = 1/5$

$f_1 = \frac{k}{N}$, $\quad f_2 = \frac{k}{N}$

$N_1 = 3,\quad N_2 = 5$

$\frac{N_1}{N_2} = \frac{3}{5}$ is also a ration,

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LCM N_1, N_2

$$3 \underline{| 3, 5}$$
$$5 \underline{| 1, 5}$$
$$1$$

15

fundamental period

$N = 15$.

$$C_k = \frac{1}{15} \sum_{n=0}^{14} x(n) \cdot e^{\frac{-j2\pi n k}{N}}$$

or

$$C_k = C_{1k} + C_{2k}.$$

$$\left[\because \cos \frac{2\pi n}{3} = \frac{e^{+\frac{j2\pi n}{3}} + e^{-\frac{j2\pi n}{3}}}{2} \right.$$

$$\left. \sin \frac{2\pi n}{5} = \frac{e^{\frac{j2\pi n}{5}} - e^{-\frac{j2\pi n}{5}}}{2j} \right]$$

$$x(n) = \underbrace{\cos \frac{2\pi}{3} n}_{C_{1k}} + \underbrace{\sin \frac{2\pi}{5} n}_{C_{2k}}.$$

$\omega = 2\pi f$

$$C_{1k} = 3 \sum_{n=0}^{2} x(n)\, e^{\frac{-j2\pi n k}{N}} \qquad f = \frac{k}{N} \qquad 2\pi \times \frac{1}{3}.$$

$N = 3$

$$= 3 \left[x_1(0)\, e^{\frac{-j2\pi 0}{3}} + x_1(1) \cdot e^{\frac{-j2\pi}{3} k} + x_1(2)\, e^{\frac{-j2\pi}{3} 2k} \right]$$

$$x_1(0) = \cos \frac{2\pi (0)}{3} = 1$$

$$x_1(1) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

$$x_1(2) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}$$

$$C_{1k} = 3 \left[1 - \frac{1}{2} \cdot e^{\frac{-j2\pi k}{3}} - \frac{1}{2} \cdot e^{-j\frac{4\pi k}{3}} \right]$$

$$C_{10} = 3 \left[1 - \frac{1}{2} - \frac{1}{2} \right] = 0.$$

$$C_{11} = 3 \left[1 - \frac{1}{2} e^{\frac{-j2\pi}{3}} - \frac{1}{2} \cdot e^{\frac{-j4\pi}{3}} \right.$$

$$= 3 \left[1 - \frac{1}{2} \left[\cos \frac{2\pi}{3} - j \sin \frac{2\pi}{3} \right] - \frac{1}{2} \left[\cos \frac{4\pi}{3} - j \sin \frac{4\pi}{3} \right] \right.$$

$$= 3 \left[1 + \frac{1}{2} \left[\frac{1}{2} \right] + j \cdot \cancel{\frac{\sqrt{3}}{2}} \frac{}{4} - \frac{1}{2}\left(-\frac{1}{2}\right) + j \left(\cancel{-\frac{\sqrt{3}}{2}}\right) \right.$$

$c_1 = \frac{9}{2}$

$c_2 = 3\left[1 - \frac{1}{2}\left[\cos\frac{4\pi}{3} - j\sin\frac{4\pi}{3}\right] - \frac{1}{2}\left[\cos\frac{8\pi}{3} - j\sin\frac{8\pi}{3}\right]\right]$

$= 3 \times \frac{3}{2} = 4.5$

$$C_2 k = 5 \cdot \sum_{n=0}^{4} x(n) \cdot e^{-j\frac{2\pi nk}{5}}$$

$$= 5\left[x(0) + x(1) \cdot e^{-j\frac{2\pi}{5}k} + x(2)\, e^{-j\frac{4\pi}{5}k} + x(3)\, e^{-j\frac{6\pi}{5}k} + x(4) \cdot e^{-j\frac{8\pi k}{5}}\right]$$

$x(0) = 0$ $\quad x(2) = 0.587$ $\quad x(4) = -0.95$

$x(1) = 0.95$ $\quad x(3) = -0.58$

$$= 5\left[0.95\left[\cos\frac{2\pi k}{5} - j\sin\frac{2\pi k}{5}\right] + 0.587\left[\cos\frac{4\pi k}{5} - j\sin\frac{4\pi k}{5}\right] - 0.58\left[\cos\frac{6\pi k}{5} - j\sin\frac{6\pi k}{5}\right] - 0.95\left[\cos\frac{8\pi k}{5} - j\sin\frac{8\pi k}{5}\right]\right]$$

$$C_0 = 5\left[0.95 + 0.587 - 0.58 - 0.95\right]$$

$= 0$

$$c_1 = 5\left[0.95 \times 0.309 - 0.95 j \times 0.95 + 0.587(-0.809) \; 0.587 \times -j \cdot 0.587 - 0.58 \times -0.809 + 0.58 \times j(-0.587) - 0.95 \times 0.309 + 0.95 j(-0.583)\right]$$

Similarly c_1 and c_2 will solved.

(c) $x(n) = \cos\frac{2\pi}{3}n \sin\frac{2\pi}{5}n$

Sol.

$$x(n) = \frac{1}{2}\left[\sin\left(\frac{2\pi}{3}+\frac{2\pi}{5}\right)n + \sin\left(\frac{2\pi}{3}-\frac{2\pi}{5}\right)n\right]$$

cos (A + B),
cos +
sinA cosB

$$\frac{\sin(A+B)+\sin(A-B)}{2}$$

$$= \frac{1}{2}\left[\sin\left(\frac{5\times2\pi+3\times2\pi}{15}\right)n + \sin\left(\frac{5\times2\pi-3\times2\pi}{15}\right)n\right]$$

$$= \frac{1}{2}\left[\sin\frac{16\pi}{15}n + \sin\frac{4\pi}{15}n\right]$$

$f_1 = \frac{8}{15}$ $\quad$ $f_2 = \frac{2}{15}$

$N_1 = 15$ $\quad$ $N_2 = 15$.

$N = 15$

$$C_k = \frac{1}{15}\sum_{n=0}^{14} x(n)\, e^{\frac{-j2\pi kn}{N}}$$

$$C_k = \frac{1}{15}\sum_{n=0}^{14} x(n)\, e^{\frac{-j2\pi k}{15}n}$$

Same procedure is followed as previous one.

(d) $x(n) = \{\ldots, \underbrace{-2, -1, \overset{0}{0}, \overset{1}{1}, \overset{2}{2}}, \overset{3}{-2}, \overset{4}{-1}, 0, 1, 2, \ldots\}$.

Sol.

length of $x(n) = 5$

$N = 5$

$$C_k = \frac{1}{5}\sum_{n=0}^{4} x(n)\cdot e^{\frac{-j2\pi n\cdot k}{5}}$$

$$= \frac{1}{5}\left[x(0) + x(1)\cdot e^{\frac{-j2\pi k}{5}} + x(2)\cdot e^{-j\frac{4\pi k}{5}} + x(3)\cdot e^{\frac{-j6\pi k}{5}} + x(4)\cdot e^{\frac{-j8\pi k}{5}}\right].$$

$$= \frac{1}{5}\left[0 + e^{\frac{-j2\pi k}{5}} + 2\cdot e^{\frac{-j4\pi k}{5}} - 2\cdot e^{-j\frac{6\pi k}{5}} - e^{\frac{-j8\pi k}{5}}\right]$$

$$C_k = \frac{1}{5}\left[\cos\frac{2\pi k}{5} - j\sin\frac{2\pi k}{5} + 2\cdot e^{\frac{4\pi k}{5}} - 2j\sin\frac{4\pi k}{5}\right.$$

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$-2\cos\frac{6\pi k}{5} + 2j\sin\frac{6\pi k}{5} - \cos\frac{8\pi k}{5} + j\cdot\sin\frac{8\pi k}{5}\Big]$

(e). $x(n) = \{ \ldots, -1, 2, \underset{\uparrow}{1}, 2, -1, 0, -1, 2, 1, 2, \ldots \}$.

Sol:

$N = 6$

$$c_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\cdot e^{\frac{-j2\pi n\cdot k}{6}}$$

$$= \frac{1}{6}\Big[x(0)\cdot e^{-j\frac{\pi}{3}(0)} + x(1)\cdot e^{-j\frac{\pi}{3}k} + x(2)\cdot e^{-j\frac{2\pi}{3}k} + x(3)\cdot e^{-j\frac{3\pi}{3}k} + x(4)\cdot e^{\frac{-j4\pi k}{2}} + x(5)\cdot e^{-j\frac{4\pi k}{2}}\Big].$$

Same process will be done as previous.

(f). $x(n) = \{ \ldots 0, 0, 0, \underset{\uparrow}{1}, 1, 0, 0, 0, 1, 1, 0, 0 \ldots \}$.

Sol:

$N = 5$.

$$\underbrace{x(0)\cdot e^{-j2\pi(0)}}_{1} + x(1)\cdot e^{-j\frac{2\pi}{5}k}.$$

$$c_k = \frac{1}{5}\sum_{n=0}^{4}\Big[x(0) + x(1)\cdot e^{-j\frac{2\pi}{5}k}\Big]$$

$$c_k = \frac{1}{5}\Big[1 + \cos\frac{2\pi k}{5} - j\sin\frac{2\pi k}{5}\Big]$$

$$c_0 = \frac{1}{5}[2] = \frac{2}{5}$$

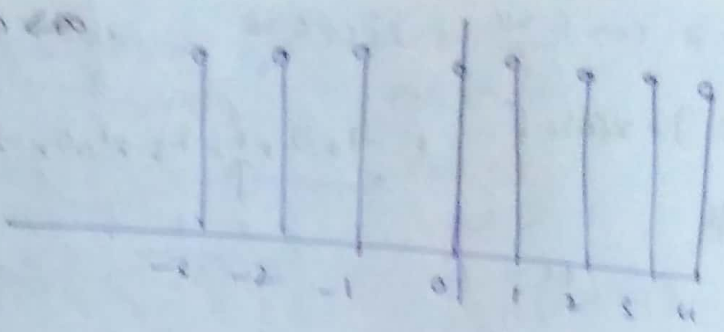
$$c_1 = \frac{1}{5}[1 + 0.309 - j0.95]$$

$$= \frac{1}{5}[1.309 - j0.95].$$

Similarly c_2 and c_3 and c_4 will be found and their magnitude and phase will be taken.

(g). $x(n) = 1 \quad -\infty < n < \infty$

$N = 1$

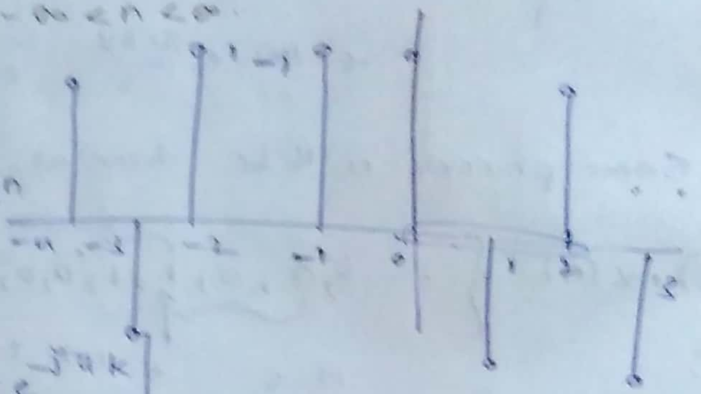


$$\frac{1}{1}\sum_{n=0}^{0} x(n) \cdot \underline{e^{j(0)}}$$

$x(n)$

$c_k = x(0) = 1.$

(h) $x(n) = (-1)^n, \; -\infty < n < \infty$



$$c_k = \frac{1}{2}\sum_{n=0}^{1} x(n) \cdot e^{\frac{-j2\pi k n}{2}}$$

$$c_k = \frac{1}{2}\left[x(0) + x(1) \cdot e^{-j\pi k}\right]$$

$$c_0 = \frac{1}{2}\left[x(0) + x(1)\right]$$

$= 0.$

$$c_1 = \frac{1}{2}\left[1 + (-1) \cdot \underbrace{e^{-j\pi}}_{1}\right]$$

$$= \frac{1}{2} \times 2 = 1.$$

$c_1 = 1.$

4.7 Determine the periodic signals $x(n)$, with fundamental period $N = 8$, if their fourier coefficients are given by

(a) $c_k = \cos\frac{k\pi}{4} + \sin\frac{3k\pi}{4}$

Sol: $$x(n) = \sum_{k=0}^{7} c_k \cdot e^{\frac{j2\pi n k}{8}}$$

$$= \sum_{k=0}^{7} \left[\cos\left(\frac{k\pi}{4}\right) + \sin\left(\frac{3k\pi}{4}\right)\right] \cdot e^{\frac{j\pi n k}{4}}$$

$$c_k = \frac{1}{2}\left[e^{+j\frac{\pi k}{4}} + e^{\frac{-j\pi k}{4}}\right] + \frac{1}{2j}\left[e^{+j\frac{3\pi k}{4}} - e^{\frac{-j3\pi k}{4}}\right]$$

$$= \sum_{k=0}^{7} \frac{1}{2}\left[e^{\frac{j\pi k}{4}} + e^{-\frac{j\pi k}{4}}\right] \cdot e^{\frac{j4\pi n k}{4}} + \frac{1}{2j}\left[e^{\frac{j3\pi k}{4}} - e^{-\frac{j3\pi k}{4}}\right] \cdot e^{\frac{j4\pi n k}{4}}$$

$$= \sum_{k=0}^{7} \frac{1}{2}\left[e^{j(n+1)\cdot\frac{\pi k}{4}} + e^{j\frac{\pi k}{4}(n-1)}\right] + \frac{1}{2j}\left[e^{j\frac{\pi k}{4}(n+3)} - e^{+j\frac{\pi k}{4}(n-3)}\right]$$

$$Ck = \frac{1}{N}\sum_{n=0}^{7} x(n)\cdot e^{\frac{-j2\pi n \cdot k}{N}}$$

$$C_k = \frac{1}{8}\sum_{n=0}^{7} x(n)\cdot e^{\frac{-j2\pi n \cdot k}{N}}$$

$$\frac{1}{2}\left[e^{\frac{j\pi k}{4}} + e^{\frac{-j\pi k}{4}}\right] + \frac{1}{2j}\left[e^{\frac{j3\pi k}{4}} - e^{\frac{-j3\pi k}{4}}\right] = \frac{1}{8}\sum_{n=0}^{7} x(n)\cdot e^{\frac{-j2\pi n \cdot k}{8}}$$

$$= \frac{1}{8}\Big[x(0) + x(1)\cdot e^{\frac{-j\pi k}{4}} + x(2)\cdot e^{\frac{-j2\pi k}{4}} + x(3)\cdot e^{\frac{-j3\pi k}{4}} + x(4)\cdot e^{\frac{-j4\pi k}{4}} + x(5)\cdot e^{\frac{-j5\pi k}{4}} + x(6)\cdot e^{\frac{-j6\pi k}{4}} + x(7)\cdot e^{\frac{-j7\pi k}{4}}$$

$x(0) = 0$ $\qquad x(-1) = \frac{1}{2}\times 8$

$x(1) = \frac{1}{2}\times 8,\ x(2) = 0.$

$x(3) = -\frac{1}{2j}\times 8\ 4 \qquad x(-3) = \frac{1}{2j}\times 8\ 4 =$

$x(1) = 4 \qquad x(-1) = 4.$

$x(3) = 4j \qquad x(-3) = -4j$

$x(n) = 4\delta(n+1) + 4\delta(n-1) - 4j\delta(n-3) + 4j\delta(n+3)$

(b) $C_k = \begin{cases} \sin\frac{k\pi}{3}, & 0 \le k \le 6 \\ 0, & k=7. \end{cases}$

$$C_k = \frac{1}{2i}\left[e^{+j\frac{k\pi}{3}} - e^{\frac{jk\pi}{3}}\right]$$

$$\frac{1}{8}\sum_{n=0}^{7} x(n)\cdot e^{-j\frac{2\pi n\cdot k}{N}} = \frac{1}{2j}\left[e^{\frac{jk\pi}{3}} - e^{\frac{-jk\pi}{3}}\right]$$

(or).

$$x(n) = \sum_{k=0}^{6} C_k \cdot e^{\frac{j2\pi nk}{8}}$$

$C_0 = \sin 0 = 0.$

$C_1 = \sqrt{3}/2 \quad C_2 = \frac{\sqrt{3}}{2}$

$C_3 = 0 \quad C_4 = \frac{-\sqrt{3}}{2}$

$C_5 = -\frac{\sqrt{3}}{2} \quad C_6 = 0$

$$= \sum_{k=0}^{6}$$

$$x(n) = C_0 + C_1\cdot e^{\frac{j2\pi n}{8}} + C_2\cdot e^{\frac{j2\pi\cdot 2n}{8}} + \underbrace{C_3}_{0}\cdot e^{\frac{j2\pi\cdot n\cdot 3}{8}}$$

$$+ C_4\cdot e^{\frac{j2\pi n\cdot 4}{8}} + C_5\cdot e^{\frac{j2\pi n\cdot 5}{8}} + \underbrace{C_6}_{0}\cdot e^{\frac{j2\pi n\cdot 6}{8}}$$

$$= 0 + \frac{\sqrt{3}}{2}\cdot e^{\frac{j\cancel{2}\pi n}{\cancel{8}4}} + \frac{\sqrt{3}}{2}\cdot e^{\frac{j\cancel{4}\pi n}{\cancel{8}2}} - \cancel{\frac{\sqrt{3}}{2}\cdot e^{\frac{j8\pi n}{8}}} - \frac{\sqrt{3}}{2}\cdot e^{\frac{j5\cancel{10}\pi n}{\cancel{8}4}}$$

$$x(n) = \frac{\sqrt{3}}{2}\cdot e^{\frac{j\pi n}{4}} + \frac{\sqrt{3}}{2}\cdot e^{\frac{j\pi n}{2}} - \frac{\sqrt{3}}{2}\cdot e^{j\pi n} - \frac{\sqrt{3}}{2}\cdot e^{\frac{j5\pi}{}}$$

$$= \cancel{\sqrt{3}}\left[\cancel{\frac{1}{2}e^{\frac{j\pi n}{4}}}\right.$$

(c) $\{C_k\} = \{\ldots 0, \frac{1}{4}, \frac{1}{2}, 1, 2, 1, 1, \frac{1}{2}, \frac{1}{4}, 0, \ldots\}$ (with $\uparrow$ under 2; indices 0 1 2 3 4 5 6 7)

Sol.

$$x(n) = \sum_{k=0}^{7} C_k\cdot e^{\frac{j2\pi k\cdot n}{N=8}}$$

$$= \left[C_0\cdot e^{j2\pi(0)} + C_1\cdot e^{j2\frac{\pi n}{8}} + C_2\cdot e^{\frac{j2\pi\cdot 2n}{8}}\right.$$

$$+ C_3\cdot e^{\frac{j2\pi\cdot 3\cdot n}{8}} + C_4\cdot e^{\frac{j2\pi 4\cdot n}{8}} + C_5\cdot e^{\frac{j2\pi\cdot 5}{8}}$$

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$$+ C_6 \cdot e^{\frac{j2\pi 6n}{8}} + C_7 \cdot e^{\frac{j2\pi 7n}{8}}$$

$$= 2 + e^{\frac{j\pi n}{4}} + \frac{1}{2} e^{\frac{j\pi n}{2}} + \frac{1}{4} e^{\frac{j3\pi n}{4}} + 0 + \frac{1}{4} e^{\frac{j5\pi n}{4}} + \frac{1}{2} e^{\frac{j3\pi n}{2}} + e^{\frac{j7\pi n}{4}}$$

$$= 2 + e^{\frac{j\pi n}{4}} + \frac{1}{2} \cdot e^{\frac{j\pi n}{2}} + \frac{1}{4} \cdot e^{\frac{j3\pi n}{4}} + \frac{1}{4} \cdot e^{\frac{j5\pi n}{4}} + \frac{1}{2} \cdot e^{\frac{j3\pi n}{2}}$$

$$+ e^{\frac{j7\pi n}{4}}$$

$$= 2 + \frac{1}{2} 2\cos\frac{\pi n}{2} + \frac{2}{2} 2 \cdot \cos\frac{\pi n}{4} + \frac{2}{4 \cdot 2} \cos\frac{3\pi n}{4}$$

$$x(n) = 2 + \cos\frac{\pi n}{2} + 2 \cdot \cos\frac{\pi n}{4} + \frac{1}{2} \cdot \cos\frac{3\pi n}{4}$$

4.8. Two DT signals, $s_k(n)$ and $s_l(n)$, are said to be orthogonal over an interval $[N_1, N_2]$ if

$$\sum_{n=N_1}^{N_2} s_k(n) s_l^*(n) = \begin{cases} A_k, & k = l \\ 0, & k \neq l \end{cases}$$

If $A_k = 1$, the signals are called orthonormal.

(a) prove the relation.

$$\sum_{n=0}^{N-1} e^{\frac{j2\pi kn}{N}} = \begin{cases} N, & k = 0, \pm N, \pm 2N, \dots \\ 0, & \text{otherwise.} \end{cases}$$

Soln,

$$\sum_{n=0}^{N-1} e^{\frac{j2\pi kn}{N}} = 1 + e^{\frac{j2\pi k}{N}} + e^{\frac{j2\pi 2k}{N}} + \dots$$

$$= \frac{1\left(1 - \left(e^{\frac{j\pi k}{N}}\right)^N\right)}{1 - e^{\frac{j\pi k}{N}}}$$

$$\sum_{n=0}^{N-1} 1 = N$$

$$\sum_{n=0}^{N-1} = N$$

$$= 1 - e^{j\pi k} \quad \text{If } k \neq 0, \pm N$$

(c). Show that the harmonically related signals

$$S_k(n) = e^{j(2\pi/N)kn}.$$

are orthogonal over any interval of length N.

sol:-

$$\sum_{n=0}^{N-1} S_k(n) \cdot S_i^*(n) = \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}kn} \cdot e^{-j\frac{2\pi}{N}in}$$

$$= \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(k-i)n}.$$

$$= N. \qquad k = i$$

$$= 0. \qquad k \neq i$$

$\therefore$ The $\{S_k(n)\}$ are orthogonal.

4.9. Compute the fourier transform of the following signals.

(a). $x(n) = u(n) - u(n-6)$

sol!

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}.$$

$$= \sum_{n=0}^{5} x(n) \cdot e^{-j\omega n}.$$

$$= \sum_{n=0}^{5} e^{-j\omega n}$$

$$= 1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega} + e^{-j5\omega}$$

$$= \frac{1 - (e^{-j\omega})^6}{1 - e^{-j\omega}} = \frac{1 - e^{-6j\omega}}{1 - e^{-j\omega}}.$$

(b) $x(n) = 2^n u(-n)$

sol:

$$x(\omega) = \sum_{-\infty}^{0} 2^n \cdot e^{-j\omega n}$$

$$x(\omega) = \sum_{-\infty}^{0} (2 \cdot e^{-j\omega})^n$$

$$= \sum_{-\infty}^{0} \left(\frac{2}{e^{j\omega}}\right)^n$$

$$= \sum_{m=0}^{\infty} \left(\frac{2^{-n}}{e^{-j\omega n}}\right) = \sum_{m=0}^{\infty} \frac{e^{j\omega n}}{2^n} = \sum_{m=0}^{\infty} \left(\frac{e^{j\omega}}{2}\right)^n$$

$$= 1 + \frac{e^{j\omega}}{2} + \frac{e^{2j\omega}}{4} + \cdots$$

$$= \frac{1 - \left(\frac{e^{j\omega}}{2}\right)^{n=\infty}}{1 - \frac{e^{j\omega}}{2}} = \frac{1}{1 - \frac{e^{j\omega}}{2}} = \frac{2}{2 - e^{j\omega}}$$

(c). $x(n) = \left(\frac{1}{4}\right)^n u(n+4)$

sol:-

$$X(\omega) = \sum_{n=-4}^{\infty} \left(\frac{1}{4}\right)^n \cdot e^{-j\omega n}$$

$m + 4 = n$

$n + 4 = m$

$$= \sum_{n=-4}^{\infty} \left(\frac{1}{4} \cdot e^{-j\omega}\right)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{1}{4} \cdot e^{-j\omega}\right)^n + \left(\frac{1}{4} \cdot e^{-j\omega}\right)^{-4} + \left(\frac{1}{4} \cdot e^{-j\omega}\right)^{-3} + \left(\frac{1}{4} \cdot e^{-j\omega}\right)^{-2} + \left(\frac{1}{4} \cdot e^{-j\omega}\right)^{-1}$$

$$= \frac{1}{1 - \frac{e^{-j\omega}}{4}} = \frac{4}{4 - e^{-j\omega}} +$$

(d). $x(n) = (\alpha^n \sin \omega_0 n)\, u(n) \quad |\alpha| < 1$

sol:- $X(\omega) = \sum_{n=0}^{\infty} \alpha^n \left[\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j} \right] . e^{-j\omega n}$

$= \frac{1}{2j} \sum_{n=0}^{\infty} \alpha^n . e^{j\omega_0 n} . e^{-j\omega n} - \alpha^n . e^{-j\omega_0 n} . e^{-j\omega n}.$

$= \frac{1}{2j} \sum_{n=0}^{\infty} \left(\alpha . e^{j(\omega_0 - \omega)}\right)^n - \left(\alpha . e^{-j(\omega + \omega_0)}\right)^n.$

$= \frac{1}{2j} \left[\frac{1}{1 - \alpha . e^{j(\omega_0 - \omega)}} - \frac{1}{1 - \alpha . e^{-j(\omega + \omega_0)}} \right].$

$= \frac{1}{2j} \left[\frac{\cancel{1} - \alpha . \cos(\cancel{\omega} + \omega_0) + j\alpha \sin(\omega + \omega_0) - \cancel{1} + \cancel{\alpha . \cos(\omega_0 - \omega) + j\alpha \sin(\omega_0 - \omega)}}{\left(1 - \alpha\, e^{j(\omega_0 - \omega)}\right)\left(1 - \alpha . e^{-j(\omega + \omega_0)}\right)} \right]$

$= \frac{\sin(\omega + \omega_0)}{1 - \alpha . e^{j(\omega_0 - \omega)} - \alpha . e^{-j(\omega + \omega_0)} + \alpha^2 . e^{j(\omega_0 - \omega)} . e^{-j(\omega + \omega_0)}}.$

$= \alpha^2 . e^{j(\cancel{\omega_0} - \omega - \omega - \cancel{\omega_0})}$

$\alpha^2 . e^{j(-2\omega)}.$

$$X(\omega) = \frac{\sin(\omega + \omega_0)}{1 - \alpha . \left[\cos(\omega_0 - \omega) + j \sin(\omega_0 - \omega)\right] - \alpha \left[\cos(\omega + \omega_0) - j \sin(\omega + \omega_0)\right] + \alpha^2 . \left[\cos(2\omega) - j \sin(2\omega)\right]}$$

(e). $x(n) = |\alpha|^n \sin \omega_0 n, \quad |\alpha| < 1.$

$\sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=-\infty}^{\infty} |\alpha|^n |\sin \omega_0 n|$

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let $w_0 = \frac{\pi}{2}$, so that $|\sin w_0 n| = 1$

$$\sum_{n=-\infty}^{\infty} |a|^n = \sum_{n=-\infty}^{\infty} |x(n)| \to \infty.$$

There, the fourier transform does not exist.

(f). $x(n) = \begin{cases} 2 - \left(\frac{1}{2}\right)^m & |n| \le 4. \\ 0 & \text{elsewhere.} \end{cases}$

$$X(\omega) = \sum_{n=-4}^{4} x(n) \cdot e^{-j\omega n}$$

$$= \sum_{n=-4}^{4} \left[2 - \left(\frac{1}{2}\right) n\right] \cdot e^{-j\omega n}$$

$$= \sum_{n=-4}^{4} 2 \cdot e^{-j\omega n} - \sum_{n=-4}^{4} \left(\frac{1}{2}\right) n \cdot e^{-j\omega n}$$

here $m = -4$.

$$= e^{j\omega m} + e^{j\omega(m+1)} + e^{j\omega(m+2)} + \cdots$$

$$\underline{e^{j\omega n}} \left(1 + e^{j\omega} + e^{j\omega 2} + \cdots\right).$$

$$e^{-j\omega(n=-4)} \cdot \left(\frac{1+e}{}\right.$$

$$\sum_{n=0}^{4} 2 \cdot e^{-j\omega n}.$$

$$2 \cdot e^{j\omega 4} \cdot \left[\frac{1 - 8^{-j\omega}}{1 - e^{-j\omega}}\right]$$

$$2 \cdot e^{j4\omega} \cdot \left[\frac{1}{1 - e^{-j\omega}}\right] - \frac{1}{2}\left[-4e^{j4\omega} + 4e^{-j4\omega} - 3e^{-j3\omega} + 3 \cdot e^{j3\omega} - 2 \cdot e^{j2\omega} + 2 \cdot e^{-j2\omega} - e^{j\omega} + e^{-j\omega}\right]$$

$$= \frac{2 \cdot e^{j4\omega}}{1 - e^{-j\omega}} + j\left[4 \cdot \sin 4\omega + 3 \cdot \sin 3\omega + 2 \cdot \sin 2\omega + \sin\omega\right].$$

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(g). $x(n) = \{ -2, -1, 0, 1, 2 \}$
$\quad\quad\quad\quad -2 \; -1 \; \underset{0}{\uparrow} \; 1 \; 2$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n}$$

$$= x(-2) \cdot e^{j\omega 2} + x(-1) \cdot e^{j\omega} + x(0) + x(1) e^{-j\omega} + x(2) \cdot e^{-j\omega 2}$$

$$= -2 \cdot e^{j\omega 2} + 1 \cdot e^{j\omega} + 0 + e^{-j\omega} + 2 \cdot e^{-j2\omega}$$

$$= -\frac{2 \times 2j}{2j} \left[e^{+j2\omega} - e^{-j2\omega} \right] - \frac{2j}{2j} \left[e^{j\omega} - e^{-j\omega} \right]$$

$$= -4j \sin 2\omega - 2j \sin\omega$$

$$= -2j \left(2\sin 2\omega + \sin\omega \right).$$

(h) $x(n) = \begin{cases} A(2M+1-|n|), & |n| \le M \\ 0 & |n| > M. \end{cases}$

Sol. $$X(\omega) = \sum_{n=-M}^{M} x(n) \cdot e^{-j\omega n}$$

$$= A \sum_{n=-M}^{M} (2M+1-|n|) \cdot e^{-j\omega n}$$

$$= (2M+1)A + A \sum_{n=1}^{M} (2M+1-k)\frac{(e^{-j\omega k} + e^{j\omega k})}{2}$$

$$\sum_{n=-M}^{0} (2M+1-|n|)$$

$$= (2M+1)A + 2A \sum_{k=1}^{M} (2M+1-k)\cos\omega k.$$

4.10 Determine the signals having the following fourier transforms.

(a). $X(\omega) = \begin{cases} 0, & 0 \le |\omega| \le \omega_0 \\ 1, & \omega_0 < |\omega| \le \pi \end{cases}$

$\omega_0 <$
$|\omega| < \pi$
$-\pi < \omega < \pi$
$-\omega_0 \le \omega \le \pi$

sol.

$$x(n) = \frac{1}{2\pi} \int_{2\pi} X(\omega) \cdot e^{j\omega n} \, d\omega .$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^{-\omega_0} e^{j\omega n} \, d\omega + \int_{\omega_0}^{\pi} e^{j\omega n} \, d\omega \right]$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega n}}{jn} \right]_{-\pi}^{-\omega_0} + \left[\frac{e^{j\omega n}}{jn} \right]_{\omega_0}^{\pi}$$

$$= \frac{1}{2\pi} \left[\frac{e^{-j\omega_0 n} - e^{-j\pi n}}{jn} + \frac{e^{j\pi n}}{jn} - \frac{e^{j\omega_0 n}}{jn} \right]$$

$$= \frac{1}{2\pi} \left[\frac{e^{-j\omega_0 n} - e^{j\omega_0 n}}{jn} + \frac{e^{j\pi n} - e^{-j\pi n}}{jn} \right]$$

$$= \frac{1}{2\pi n} \left[- \left[\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j} \right] + \frac{e^{j\pi n} - e^{-j\pi n} = 0}{jn} \right]$$

$$= \frac{1}{\pi n} \left(-\sin n\omega_0 \right) . = \frac{-\sin n\omega_0}{n\pi} \quad n \neq 0 .$$

$$x(n) = \frac{-\sin n\omega_0}{n\pi} .$$

(b) $X(\omega) = \cos^2 \omega$

$$= \frac{1}{4} \left(e^{j\omega} + e^{-j\omega} \right)^2$$

$$= \frac{1}{4} \left[e^{2j\omega} + e^{-j2\omega} + 2 \right] .$$

$$x(n) = \frac{1}{2\pi} \int_0^{2\pi} X(\omega) . e^{j\omega n} . d\omega$$

$$x(n) = \frac{1}{2\pi}\cdot\frac{1}{4}\int_0^{2\pi}\left(e^{j2\omega} + e^{-j2\omega} + 2\right)e^{j\omega n}\, d\omega$$

$$= \frac{1}{8\pi}\int_0^{2\pi}\left(e^{j2\omega} + e^{-j2\omega} + 2e^{j\omega n}\right)d\omega$$

* inverse fourier transform of $\frac{1}{4}\left[e^{j2\omega} + 2 + e^{-j2\omega}\right]$

$$x(n) = \frac{1}{8\pi}\left[2\pi\,\delta(n+2) + 4\pi\delta(n) + 2\pi\,\delta(n-2)\right]e^{j2\omega}$$

$$x(n) = \frac{1}{4}\left[2\,\delta(n+2) + 4\delta(n) + 2\delta(n-2)\right]$$

$x(n) \rightleftharpoons X(\omega)$

$X(n) \rightleftharpoons 2\pi x(-\omega)$

(c) $X(\omega) = \begin{cases} 1, & \omega_0 - \frac{\delta\omega}{2} \le |\omega| \le \omega_0 + \frac{\delta\omega}{2} \\ 0, & \text{elsewhere.} \end{cases}$

sol:

$$x(n) = \frac{1}{2\pi}\int_0^{2\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{\omega_0 - \frac{\delta\omega}{2}}^{\omega_0 + \frac{\delta\omega}{2}} e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\omega_0 - \frac{\delta\omega}{2}}^{\omega_0 + \frac{\delta\omega}{2}}$$

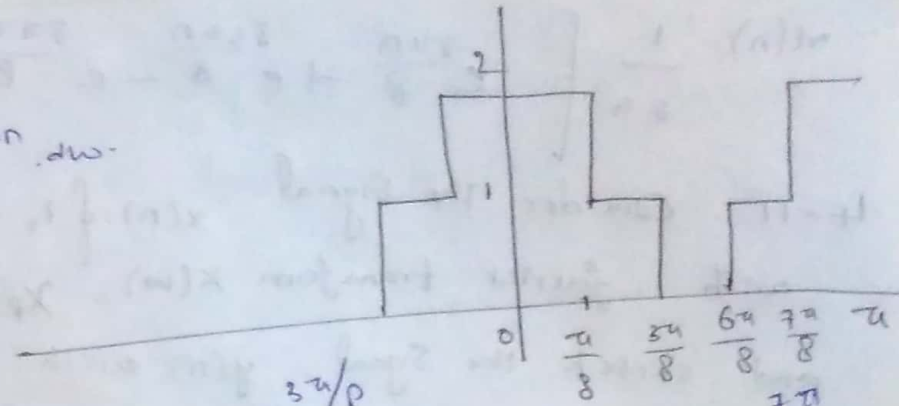
$$= \frac{1}{2\pi}\left[\frac{e^{j\left(\omega_0 + \frac{\delta\omega}{2}\right)n} - e^{j\left(\omega_0 - \frac{\delta\omega}{2}\right)}}{jn}\right]$$

$$= \frac{1}{2\pi}\left[\frac{\cancel{e^{j\omega_0 n}} + e^{j\frac{\delta\omega n}{2}} \quad \cancel{e^{j\omega_0}} - e^{-j\frac{\delta\omega}{2}}}{jn}\right]$$

$$= \frac{1}{2\pi n}\,\sin\left(\frac{\delta\omega n}{2}\right)\times\frac{2\delta\omega}{\delta\omega\; 2} = \frac{\delta\omega}{2\pi}\;\frac{\sin\left(\frac{\delta\omega n}{2}\right)}{\delta\omega\, n/2}$$

(d).

$$X(\omega) = \frac{1}{2\pi}\int_{-2\pi}^{2\pi} x(n)\, e^{j\omega n}\, d\omega$$



$$= \frac{1}{2\pi}\left[\int_0^{\pi/8} 2\cdot e^{j\omega n}\cdot d\omega + \int_{\pi/8}^{3\pi/8} 1\cdot e^{j\omega n}\cdot d\omega + \int 0 + \int_{6\pi/8}^{7\pi/8} e^{j\omega n}\, d\omega + \int_{7\pi/8}^{\pi} 2\cdot e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[2\left[\frac{e^{j\omega n}}{jn}\right]_0^{\pi/8} + \left[\frac{e^{j\omega n}}{jn}\right]_{\pi/8}^{3\pi/8} + \left[\frac{e^{j\omega n}}{jn}\right]_{6\pi/8}^{7\pi/8} + 2\left[\frac{e^{j\omega n}}{jn}\right]_{7\pi/8}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[2\cdot\cancel{\frac{e^{j\frac{\pi}{8}n}}{jn}} - \frac{2}{jn} + \frac{e^{j\frac{3\pi}{8}n}}{jn} - \cancel{\frac{e^{j\frac{\pi}{8}n}}{jn}} + \frac{e^{j\frac{7\pi}{8}n}}{jn} - \frac{e^{j\frac{6\pi}{8}n}}{jn} + 2\cdot\frac{e^{j\pi n}}{jn} - 2\cdot\frac{e^{j\frac{7\pi}{8}n}}{jn}\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\frac{\pi}{8}n}}{jn} - \cancel{\frac{2}{jn}} + \frac{e^{j\frac{3\pi}{8}n}}{jn} - \frac{e^{j\frac{7\pi}{8}n}}{jn} - \frac{e^{j\frac{6\pi}{8}n}}{jn} + \cancel{2\cdot\frac{e^{j\pi n}}{jn}}\right]$$

$$x(n) = \frac{1}{2\pi}\left[\frac{e^{j\frac{\pi}{8}n} + e^{j\frac{3\pi}{8}n} - e^{j\frac{7\pi}{8}n} - e^{j\frac{6\pi}{8}n}}{jn}\right]$$

$$x(n) = \frac{1}{2\pi}\left[e^{\frac{j\pi n}{8}} + e^{\frac{j3\pi n}{8}} - e^{\frac{j\pi n}{8}} - e^{\frac{j6\pi n}{8}} \right]$$

4.11. Consider the signal $x(n) = \{1, 0, -1, \underset{\uparrow}{2}, 3\}$ with fourier transform $X(\omega) = X_R(\omega) + j\, X_I(\omega)$. determine and sketch the signal $y(n)$ with fourier transform

$$Y(\omega) = X_I(\omega) + X_R(\omega)\cdot e^{j2\omega}.$$

Sol:

$$x(n) = \{1, 0, -1, \underset{\uparrow}{2}, 3\}$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\cdot e^{-j\omega n}$$

$$= \sum_{n=-3}^{1} x(n)\cdot e^{-j\omega n}$$

$$X(\omega) = \left[x(-3)\cdot e^{3j\omega} + x(-2)\cdot e^{2j\omega} + x(-1)\cdot e^{j\omega} + x(0) + x(1)\cdot e^{-j\omega} \right]$$

$$= e^{j3\omega} - e^{j\omega} + 2 + 3\cdot e^{-j\omega}$$

$$= e^{j3\omega} - e^{j\omega} + 2 \cdot 3 e^{-j\omega}$$

$$= \cos 3\omega + j\sin 3\omega - \cos\omega - j\sin\omega + 2 + 3\cos\omega - 3j\sin\omega$$

$$= \cos 3\omega + j\sin 3\omega + 2 + 2\cdot\cos\omega - 4j\sin\omega.$$

$$= \underbrace{\cos 3\omega + 2 + 2\cdot\cos\omega}_{X_R(\omega)} + j\cdot\underbrace{(\sin 3\omega - 4\sin\omega)}_{X_I(\omega)}.$$

$$Y(\omega) = \sin 3\omega - 4\sin\omega + (\cos 3\omega + 2 + 2\cdot\cos\omega)\cdot e^{j2\omega}$$

$$= \sin 3\omega - 4\sin\omega + (\cos 3\omega + 2 + 2\cdot\cos\omega)(\cos 2\omega + j\sin 2\omega)$$

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continuation of 11th problem.

$y(\omega) = \sin 3\omega - 4\sin\omega + \cos 3\omega \cdot \cos 2\omega + 2\cdot\cos 2\omega + 2\cdot\cos\omega$

$\cos 2\omega + j\cdot\sin 2\omega\cdot\cos 3\omega + 2j\cdot\sin 2\omega$

$+ 2\cdot j\cdot\sin 2\omega\cdot\cos\omega.$

$= \sin 3\omega - 4\sin\omega + \frac{1}{2}\cos 5\omega + \frac{1}{2}\sin\omega + 2\cdot\cos 2\omega$

$+ \cos 3\omega + \cos\omega + \frac{j}{2}\left[\sin 5\omega - \sin\omega\right]$

$+ 2j\cdot\sin\omega + j\left[\sin 3\omega + \sin\omega\right].$

$= \sin 3\omega - 4\sin\omega + \frac{1}{2}\cos 5\omega + \frac{1}{2}\sin\omega + 2\cdot\cos 2\omega$

$+ \cos 3\omega + \cos\omega + \frac{j}{2}\cdot\sin 5\omega - \frac{j}{2}\cdot\sin\omega + 2j\sin\omega$

$+ j\sin 3\omega + j\cdot\sin\omega.$

$= \sin 3\omega - \frac{7}{2}\sin\omega + \frac{1}{2}\cos 5\omega + 2\cdot\cos 2\omega$

$+ \cos\omega + \cos 3\omega + j\left[\frac{\sin 5\omega}{2} + \cancel{\frac{2}{2}\sin\omega} + \frac{5}{2}\sin\omega + \sin 3\omega\right]$

$y(n) = \frac{1}{2\pi}\int_{2\pi} Y(\omega)\, e^{j\omega n}\, d\omega.$

$= \frac{1}{2\pi}\left[\int_0^{2\pi}\sin 3\omega - \frac{7}{2}\sin\omega + \frac{1}{2}\cos 5\omega + 2\cos 2\omega + \cos\omega + \cos 3\omega\right]$

$+ \frac{1}{2\pi}j\left[\int_0^{2\pi}\left(\frac{\sin 5\omega}{2} + \frac{5}{2}\sin\omega + \sin 3\omega\right)\cdot e^{j\omega n}\, d\omega\right]$

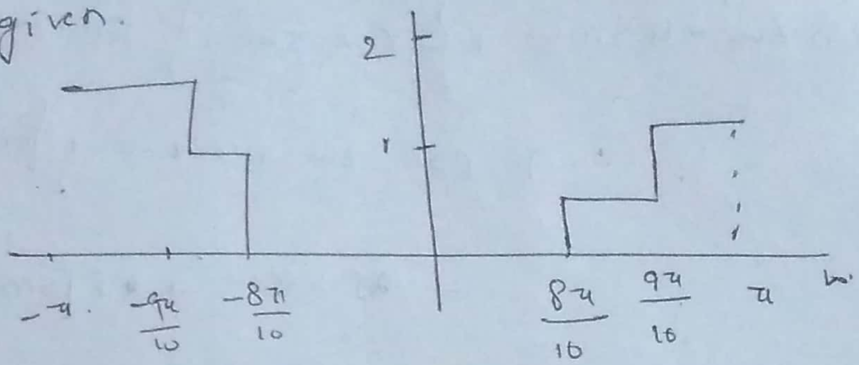
$= \frac{1}{2\pi}\left[\left[-\cancel{\frac{\cos 3\omega}{3}}\right]_0^{2\pi} + \frac{7}{2}\left[\cancel{\cos\omega}\right]_0^{2\pi} + \frac{1}{2}\left[\cancel{\frac{\sin 5\omega}{5}}\right]_0^{2\pi}\right.$

$+ 2\left[\cancel{\frac{\sin 2\omega}{2}}\right]_0^{2\pi} + \left[\cancel{\sin\omega}\right]_0^{2\pi}$

$$\rightarrow \left[\frac{\sin 3\omega}{3}\right]_0^{2\pi}$$

$$y[n] = 0.$$

4.12. determine the signal $x(n)$ if its fourier transform is as given.



Sol:

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{+j\omega n}\, d\omega.$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\frac{9\pi}{10}} 2\cdot e^{j\omega n}\, d\omega + \int_{-\frac{9\pi}{10}}^{-\frac{8\pi}{10}} e^{j\omega n}\, d\omega + \int_{\frac{8\pi}{10}}^{9\pi/10} e^{j\omega n}\, d\omega + \int_{\frac{9\pi}{10}}^{\pi} 2\cdot e^{j\omega n}\, d\omega\right]$$

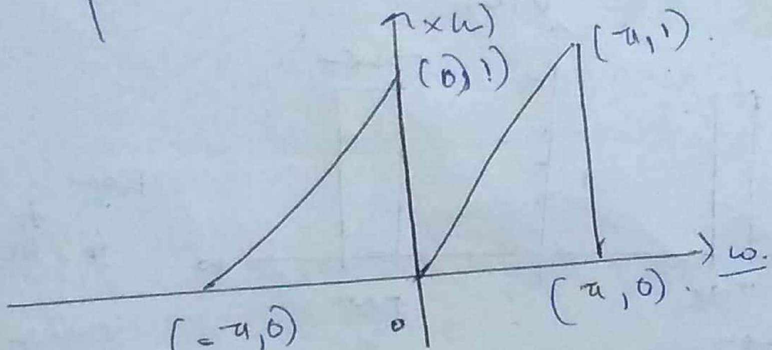
$$= \frac{1}{2\pi}\left[\left[\frac{2\cdot e^{j\omega n}}{jn}\right]_{-\pi}^{-\frac{9\pi}{10}} + \left[\frac{e^{j\omega n}}{jn}\right]_{-\frac{9\pi}{10}}^{-\frac{8\pi}{10}} + \left[\frac{e^{j\omega n}}{jn}\right]_{\frac{8\pi}{10}}^{\frac{9\pi}{10}} + \left[\frac{2\cdot e^{j\omega n}}{jn}\right]_{\frac{9\pi}{10}}^{\pi}\right]$$

$$= \frac{1}{2\pi jn}\left[2\cos\left(\frac{9\pi}{10}\right) + j\sin\left(\frac{9\pi}{10}\right) + \cos\left(\frac{8\pi}{10}\right) + j\sin\left(+\frac{8\pi}{10}\right)\right.$$

$$-\cancel{\cos\frac{9\pi}{10}} + j\cdot\sin\frac{9\pi}{10} + \cancel{\cos\frac{9\pi}{10}} + j\sin\frac{9\pi}{10} - \cancel{\cos\frac{8\pi}{10}} - j\sin\frac{8\pi}{10} - \cancel{2} - \cancel{2\cos\frac{9\pi}{10}} + j\sin\frac{9\pi}{10}\Big]$$

$$= \left[-2j\sin\left(\frac{8\pi}{10}\right) + 2j\cdot\sin\frac{9\pi}{10} - \cancel{2}\right] \times \frac{1}{2\pi jn}$$

$$X(\omega) = \left[-\sin\frac{8\pi}{10} + \sin\frac{9\pi}{10}\right]\cdot\frac{1}{\pi n}$$



$$\frac{1-0}{\pi}(\omega) = \frac{\omega}{\pi} = y$$

$$\omega\ 0: (x+\pi) = y$$

π

$$+ = \frac{\omega}{\pi}(\omega+\pi)$$

$$= \frac{\omega}{\pi} + 1$$

Sol:-

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\cdot e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0}\left(\frac{\omega}{\pi}+1\right)e^{j\omega n}\,d\omega + \int_{0}^{\pi}\left(\frac{\omega}{\pi}\right)e^{j\omega n}\,d\omega\right]$$

$$\boxed{\begin{array}{l} \omega \quad e^{j\omega n} \\ 1 \quad - \quad \frac{e^{j\omega n}}{jn} \\ 0 \quad + \quad \frac{e^{j\omega n}}{-n} \end{array}}$$

$$= \frac{1}{2\pi}\left[\frac{1}{\pi}\cdot\left[\frac{\omega\cdot e^{j\omega n}}{jn} + \frac{e^{j\omega n}}{n}\right]_{-\pi}^{0} + \left[\frac{e^{j\omega n}}{jn}\right]_{-\pi}^{0} + \frac{1}{\pi}\left[\frac{\omega\cdot e^{j\omega n}}{jn} + \frac{e^{j\omega n}}{n}\right]_{0}^{\pi}\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{\pi}\left[\cancel{\frac{1}{n}} - \cancel{\frac{\pi\cos(n\pi)}{jn}} - \cancel{\frac{\cos(n\pi)}{n}}\right] + \frac{1}{jn} - \frac{\cos(n\pi)}{jn}\right.$$

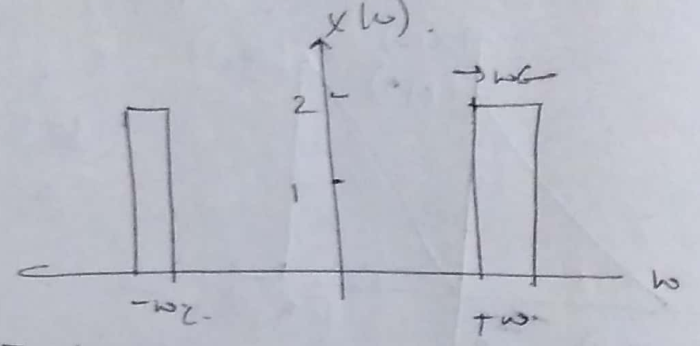
$$+ \frac{1}{\pi}\left[\cancel{\frac{\pi\cdot\cos n\pi}{jn}} + \cancel{\frac{\cos n\pi}{n}} - \cancel{\frac{1}{n}}\right]$$

$$x(n) = \left[\frac{1}{jn} - \frac{\cos n\pi}{jn}\right] \times \frac{1}{2\pi}$$

$$\frac{1-\cos 2\theta}{2} = \sin^2\theta$$

$$= \left[\frac{1-\cos\frac{n\pi}{2}\cdot 2}{2}\right] \times \frac{1}{jn}$$

$$x(n) = \frac{\sin^2\frac{n\pi}{2}}{jn}$$



$B\omega =$

$f_H = BW + \frac{f_c}{2}$

$f_L = BW - \frac{f_c}{2}$

$$x(n) = \frac{1}{2\pi}\left[\int_{-\omega_c - \frac{\omega}{2}}^{-\omega_c + \frac{\omega}{2}} 2\cdot e^{j\omega n}\, d\omega + \int_{\omega_c - \frac{\omega}{2}}^{\omega_c + \frac{\omega}{2}} 2\cdot e^{j\omega n}\, d\omega\right]$$

$$x(n) = \frac{1}{\pi}\left[\left[\frac{e^{j\omega n}}{jn}\right]_{-\omega_c - \frac{\omega}{2}}^{-\omega_c + \frac{\omega}{2}} + \left[\frac{e^{j\omega n}}{jn}\right]_{\omega_c - \frac{\omega}{2}}^{\omega_c + \frac{\omega}{2}}\right]$$

$$= \frac{1}{jn\pi}\left[e^{-j\left(\omega_c - \frac{\omega}{2}\right)n} - e^{-j\left(\omega_c + \frac{\omega}{2}\right)n} + e^{j\left(\omega_c + \frac{\omega}{2}\right)n} - e^{j\left(\omega_c - \frac{\omega}{2}\right)}\right]$$

$$= \frac{1}{jn\pi}\left[-2\left[\frac{e^{j\left(\omega_c - \frac{\omega}{2}\right)n} - e^{-j\left(\omega_c - \frac{\omega}{2}\right)n}}{2}\right] + 2\left[\frac{e^{j\left(\omega_c + \frac{\omega}{2}\right)n} - e^{-j\left(\omega_c + \frac{\omega}{2}\right)n}}{2}\right]\right]$$

$$x(n) = \frac{-2}{n\pi}\cdot \sin\left(\omega_c - \frac{\omega}{2}\right)n + \frac{2}{n\pi}\sin\left(\omega_c + \frac{\omega}{2}\right)n$$

4-13. fourier transform of the signal

$$x(n) = \begin{cases} 1, & -M \le n \le M \\ 0, & \text{otherwise.} \end{cases}$$

$$X(\omega) = 1 + 2 \sum_{n=1}^{M} \cos \omega n.$$

Show that the fourier transform of

$$x_1(n) = \begin{cases} 1, & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

and $x_2(n) = \begin{cases} 1, & -M \le n \le -1. \\ 0, & \text{otherwise} \end{cases}$

$$X_1(\omega) = \frac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}} \qquad X_2(\omega) = \frac{e^{j\omega} - e^{j\omega(M+1)}}{1 - e^{+j\omega}}$$

$N = M+1$

$$X_1(\omega) = \sum_{n=0}^{M} 1 \cdot e^{-j\omega n} = \frac{a(1-r^N)}{1-r} = \frac{(1 - (e^{-j\omega})^N)}{1 - e^{-j\omega}}$$

$a = 1$.

$r = e^{-j\omega}$

$$X_1(\omega) = \frac{1 - e^{-j\omega(M+1)}}{1 - e^{-j\omega}}.$$

$$X_2(\omega) = \sum_{n=-M}^{-1} 1\, e^{-j\omega n}$$

$$\sum_{1}^{M} e^{j\omega n}$$

$$= \sum_{n=1}^{M} e^{j\omega n} = e^{j\omega} + \sum_{n=0}^{M} e^{j\omega n}$$

$n = M.$ $\quad = e^{j\omega}$ or.

$$e^{j\omega M} + e^{j\omega(M+1)} + \cdots e^{j\omega(M)}$$

$$e^{j\omega M}\left(1 + e^{j\omega M} + e^{j\omega(M+1)} \cdots e^{j-(M)}\right)$$

$$e^{j\omega}\left(\frac{1 - e^{j\omega M}}{1 - e^{j\omega}}\right)$$

$$X_2(\omega) = \frac{1 - e^{j\omega m}}{1 - e^{j\omega}} e^{j\omega}$$

$$e^{j\omega m} \cdot e^{j\omega} \cdot e^{j\omega}$$

$$X(\omega) = X_1(\omega) + X_2(\omega).$$

$$= \frac{1 - e^{-j\omega(m+1)}}{1 - e^{-j\omega}} + \frac{(1 - e^{j\omega m}) \cdot e^{j\omega}}{1 - e^{j\omega}}$$

$$= \frac{\cancel{1} - \cancel{e^{j\omega}} - e^{-j\omega(m+1)} + e^{-j\omega m} + \cancel{e^{j\omega}} - e^{j\omega(m+1)} \cancel{1} + e^{j\omega m}}{1 - e^{-j\omega} - e^{j\omega} + 1}$$

$$= \frac{- \cancel{e^{j\omega}} - \cancel{e^{j\omega(m+1)}} + \cancel{e^{j\omega m}} - \left(\frac{e^{-j\omega(m+1)} + e^{j\omega(m+1)}}{2}\right)2}{2 - e^{-j\omega} - e^{j\omega}}$$

$$\frac{2 \cdot \cos \omega m}{2}$$

$$X(\omega) = \frac{-\cancel{e^{j\omega}} - e^{j\omega m + j\omega} \cdot e + e^{j\omega m}}{2 - e^{-j\omega} - e^{j\omega}}$$

$$= \frac{e^{j\omega m}(1 - e^{j\omega}) - e^{j\omega}}{2 - e^{-j\omega} - e^{j\omega}} \text{ (or)}$$

$$\hookrightarrow 2 - 2(\cos \omega).$$

$$\cancel{1} - e^{j\omega} - \cancel{e^{j\omega}} + e^{-j\omega(m+1)} \cdot e^{-j\omega}$$

$$X(\omega) = \frac{2 \cdot \cos \omega m - 2 \cdot \cos \omega(m+1)}{2 - 2\cos \omega}$$

$$= \frac{2\left[\cos \omega m - \left[\cos \omega m \cdot \cos \omega - \sin \omega m \sin \omega\right.\right.}{2(1 - \cos \omega)}$$

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$$= \frac{\cancel{2}\left[\cos wm(1-\cos w) + \sin wm \sin w\right]}{\cancel{2}\, 2\sin^2 w/2}$$

$$= \frac{\cos wm\, 2\sin^2 w/2 + \sin wm \cdot \sin w}{2 \cdot \sin^2 w/2}$$

(or)

$$\cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A+B}{2}\right).$$

$A = wm \quad B = w(M+1).$

$$= \frac{wm + wm + w}{2} \qquad \frac{2wm + w}{2}$$

$$wm + \frac{w}{2}$$

$$w\left(m + \frac{1}{2}\right).$$

$$\frac{\cancel{wm} - \cancel{wm} - w}{2}$$

$$-\frac{w}{2}$$

$$= \frac{\cancel{2}\sin\left(m+\frac{1}{2}\right) \cdot w \cdot \cos\frac{w}{2}}{\cancel{2}\sin\left(\frac{w}{2}\right)}. = \frac{\sin\left(m+\frac{1}{2}\right) \cdot w \cdot \cos\frac{w}{2}}{\sin\left(\frac{w}{2}\right) \cdot \sin\left(\frac{w}{2}\right).}$$

$$\frac{\cos(w/2)}{\sin(w/2)}. = \frac{\cot(w/2)}{\sec(}. = 1$$

$$X(w) = \frac{\sin\left(M+\frac{1}{2}\right)w}{\sin\left(\frac{w}{2}\right)}$$

4.16 Consider the signal $x(n) = \{-1, 2, \underset{\uparrow}{-3}, 2, -1\}$ with fourier transform $X(w)$. compute the following quantities without explicitly computing $X(w)$.

(a) $X(0)$.

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$$X(\omega) = \sum_{n=-2}^{2} x(n) e^{j(0)} = 1$$

$$X(0) = -1+2-3+2-1 = -1$$

(b) $\angle X(\omega)$

$\cancel{X(\omega)} = \pi$ for all ω

(c) normally

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$n = 0$

$$x(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) \, d\omega$$

$$2\pi \cdot x(0) = -3 \times 2\pi = -6\pi$$

(d) $X(\pi)$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$X(\pi) = \left[x(0) \cdot e^{-j\pi(0)} + 2x(1) e^{-j\pi} + 2(x(2)) e^{-j2\pi} \right]$$

$$= -3 + 2 \cdot 2(-1) + 2(-1)$$

$$= -3 - 4 - 2 = -9.$$

(e)
$$\int_{-\pi}^{\pi} |X(\omega)|^2 \, d\omega = 2\pi \sum |x(n)|^2 \, dt$$

$$= 2(1) + 2 \times 4 + 9$$

$$= (2+8+9)(2\pi) = 38\pi$$

4.15. The center of gravity of a signal $x(n)$ is defined as

$$c = \frac{\sum_{n=-\infty}^{\infty} n x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$$

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and provides a measure of the "time delay" of the signal.

(a). Express c in terms of $X(\omega)$.

Sol:-
$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n}.$$

$$X(0) = \sum_{n=-\infty}^{\infty} x(n).$$

$$\frac{d\,X(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} x(n) \cdot \frac{d}{d\omega}\left(e^{-j\omega n}\right).$$

$$= \sum_{n=-\infty}^{\infty} x(n) \cdot -jn \cdot e^{-j\omega n}.$$

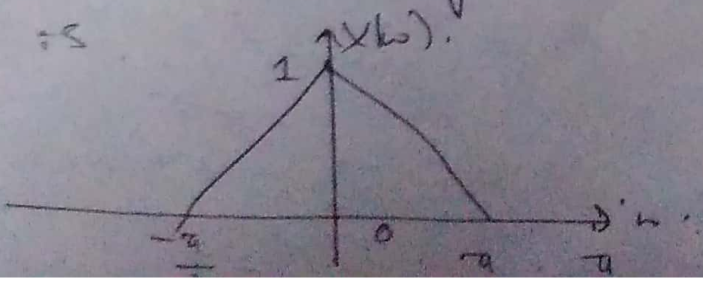
$$\frac{d\,X(\omega)}{d\omega} \cdot \frac{1}{-j} = \sum_{n=-\infty}^{\infty} x(n) \cdot n \cdot e^{-j\omega n}.$$

$$j\,\frac{d\,X(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} n\,x(n) \cdot e^{-j\omega n},$$

$$c = \frac{j\,\frac{d}{d\omega}(X(\omega))\Big|_{\omega=0}}{X(\omega)\Big|_{\omega=0}}.$$

$$\boxed{c = \frac{j \cdot \frac{d}{d\omega}(X(\omega))}{X(\omega)}\Bigg|_{\omega=0}}$$

(b). Compute c for the signal $x(n)$. whose fourier transform is



Sol:

$$x(0) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} X(\omega)\, e^{j\omega n}\, d\omega$$

$$x(0) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} X(\omega)\, d\omega$$

$$2\pi\, x(0) = \int_{-\pi/2}^{\pi/2} X(\omega)\, d\omega$$

$(0,1)$

$(-\pi/2, 0)$ $\quad \frac{1}{\frac{\pi}{2}}\left(x+\frac{\pi}{2}\right) = y$

$(0,1)$ $\quad (\pi/2, 0)$ $\quad \frac{2}{\pi}\left(\omega+\frac{\pi}{2}\right)$

$$\frac{0-1}{\frac{\pi}{2}}(x-0) = y(-1) \qquad \frac{\omega \times 2}{\pi} + 1$$

$$-\frac{2}{\pi}\omega + 1 = y$$

$$-\frac{2\omega}{\pi} + 1$$

$$2\pi\, x(0) = \int_{-\pi/2}^{0} \left(\frac{2\omega}{\pi}+1\right) d\omega + \int_{0}^{\pi/2} \left(-\frac{2\omega}{\pi}+1\right)$$

$$= \left[\frac{\cancel{2}\cdot\omega^2}{\cancel{2}\pi} + \omega\right]_{-\pi/2}^{0} + \left[-\frac{2\,\omega^2}{2\pi} + \omega\right]_{0}^{\pi/2}$$

$$= -\frac{\pi^2}{4\cancel{\pi}} + \frac{\pi}{2} - \frac{\pi^2}{4\cancel{\pi}} + \frac{\pi}{2}$$

$$x(0) = \frac{-\frac{\pi}{4} + \frac{\pi}{2} - \frac{\pi}{4} + \frac{\pi}{2}}{2\pi}$$

$$= \frac{-\frac{2}{4}\pi + \pi}{2\pi} = \frac{\frac{1}{2}\cancel{\pi}}{2\cancel{\pi}}$$

$$= \frac{1}{4}$$

5.46 Consider the Fourier transform pair

$$a^n u(n) \xleftrightarrow{F} \frac{1}{1 - a e^{-j\omega}} \qquad |a| < 1$$

Use the differentiation in frequency theorem and induction to show that

$$x(n) = \frac{(n+l-1)!}{n!\,(l-1)!} a^n u(n) \xleftrightarrow{F} X(\omega) = \frac{1}{(1 - a e^{-j\omega})^l}$$

Sol

$$a^n u(n) \xleftrightarrow{F} \frac{1}{1 - a e^{-j\omega}}$$

$$x_k(n) = \frac{(n+k-1)!}{n!\,(k-1)!} a^n u(n) \xleftrightarrow{F} \frac{1}{(1 - a e^{-j\omega})^k}$$

$$x_{k+1}(n) = \frac{(n+k)!}{(n+k)!\,(k)!} a^n u(n) \xleftrightarrow{F} ?$$

$$x_{k+1}(n) = \frac{(n+k)!}{n!\,k!} a^n u(n)$$

$$= \frac{(n+k-1)!\,(n+k)}{n!\,(k-1)!\,k} a^n u(n)$$

$$= \frac{(n+k-1)!}{n!\,(k-1)!} \times \frac{n+k}{k} a^n u(n)$$

$$x_k(n) \; \frac{n+k}{k}$$

$$X_{k+1}(\omega) = \frac{1}{k} \sum_n n\, x_k(n)\, e^{-j\omega n} + \sum_n e^{-j\omega n} \, [illegible]$$

$$\frac{1}{k} \; j \, \frac{dX_k(\omega)}{d\omega} + X_k(\omega)$$

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$$X_k(\omega) = \frac{1}{(1-a\cdot e^{-j\omega})^k}$$

$$\frac{dX_k(\omega)}{d\omega} = \frac{(1-a\cdot e^{-j\omega})^k (0) - k(1-a\cdot e^{-j\omega})^{k-1} (+aj\cdot e^{-j\omega})}{(1-a\cdot e^{-j\omega})^{2k}}$$

$$= \frac{1}{\cancel{k}} \times \frac{-\cancel{k}\,\cancel{(1-a\cdot e^{-j\omega})^k}\cdot(aj\cdot e^{-j\omega})}{(1-a\cdot e^{-j\omega})^{k+1}}$$

$$X_{k+1}(\omega) = \frac{1}{k} j \cdot \frac{dX_k(\omega)}{d\omega} + X_k(\omega)$$

$$= \frac{a\cdot e^{-j\omega}}{(1-a\cdot e^{-j\omega})^{k+1}} + \frac{1}{(1-a\cdot e^{-j\omega})^k}$$

4.17. let $x(n)$ be an arbitrary signal, not necessarily real valued, with fourier transform $X(\omega)$. Express the fourier transforms of the following signals in terms of $X(\omega)$.

(a) $x^*(n)$.

Sol. $\sum_{n=-\infty}^{\infty} x^*(n)\cdot e^{-j\omega n} = \sum_{n=-\infty}\left(x^*(n)\cdot e^{j(-\omega)}\right)^*$

(b). $x^*(-n)$. $= X^*(-\omega)$.

$$\sum_{n=-\infty}^{\infty} x^*(-n)\cdot e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x^*(n)\cdot e^{j\omega n} = \left(\sum_{n=-\infty}^{\infty} x(n)\cdot e^{-j\omega n}\right)^*$$

$= X^*(\omega)$

(c). $y(n) = x(n) - x(n-1)$

sol. $\sum_{n=-\infty}^{\infty} y(n) \cdot e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n} - \sum_{n=-\infty}^{\infty} x(n-1) \cdot e^{-j\omega n}$

$$Y(\omega) = X(\omega) - X(\omega) \cdot e^{-j\omega}$$

$$= (1 - e^{-j\omega}) X(\omega).$$

(d) $y(n) = \sum_{k=-\infty}^{n} x(k)$ $\quad y(n) = x(-\infty) + \dots + x(n-1) + x(n)$

$y(n-1) = x(-\infty) + \dots + x(n-1)$

sol. $y(n) - y(n-1) = x(n)$ $\quad y(n) - y(n-1)$

$\Rightarrow y(n) = x(n) + y(n-1)$

$Y(\omega) = X(\omega)$

$$X(\omega) = Y(\omega) - e^{-j\omega} Y(\omega)$$

$$= (1 - e^{-j\omega}) Y(\omega).$$

$$Y(\omega) = \frac{X(\omega)}{1 - e^{-j\omega}}$$

(e) $y(n) = x(2n)$.

sol. $Y(\omega) = \sum_{n=-\infty}^{\infty} x(2n) \cdot e^{-j\omega n}$.

$2n = k$

$$= \sum_{k=-\infty}^{\infty} x(k) e^{-j\omega \cdot \frac{k}{2}} \quad n = k/2$$

$$= \sum_{k=-\infty}^{\infty} x(k) \cdot e^{-j\left(\frac{\omega}{2}\right) k}$$

$$= X(\omega/2).$$

(f) $y(n) = \begin{cases} x(n/2), & n \text{ even} \\ & n \text{ odd} \end{cases}$

a)

$$Y(\omega) = \sum_{n=-\infty}^{\infty} x(n/2) \cdot e^{-j\omega n}$$

here n = 2k

$n/2 = k$

$n = 2k$

$$= \sum_{n=-\infty}^{\infty} x(k) \cdot e^{-j\omega(2k)}$$

$Y(\omega) = X(2\omega)$.

4.18. Determine and sketch the fourier transform $X_1(\omega)$, $X_2(\omega)$ and $X_3(\omega)$ of the following signals.

a) $\Rightarrow x_1(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$

-2 -1 0 1 2

$$X_1(\omega) = \sum_{n=-2}^{2} x(n) \cdot e^{-j\omega n}$$

$$X_1(\omega) = x(-2) \cdot e^{2j\omega} + x(-1)\, e^{j\omega} + x(0) + x(1)\, e^{-j\omega} + x(2) \cdot e^{-j2\omega}$$

$$= e^{j2\omega} + e^{-j2\omega} + e^{j\omega} + 1 + e^{-j\omega}$$

$$= 2 \cdot \cos 2\omega + 2 \cos \omega + 1$$

(b) $x_2(n) = \{1, 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1\}$

-4 -3 -2 -1 0 1 2 3 4

$$X_2(\omega) = \sum_{n=-4}^{4} x(n)\, e^{-j\omega n}$$

$X_2(\omega) = x(-4)\cdot e^{+j4\omega} + x(-3)\,e^{j3\omega} + x(-2)\cdot e^{j2\omega} + x(-1)\,e^{j\omega} + x(0) + x(1)\cdot e^{-j\omega} + x(2)\cdot e^{-j2\omega} + x(3)\cdot e^{-j3\omega}$

$+ x(4)\cdot e^{-j4\omega}$

$= e^{j4\omega} + e^{j2\omega} + x(0) + e^{-j2\omega} + e^{-j4\omega}$

$= 2\cdot \cos 4\omega + 2\cdot \cos 2\omega + 1$

(c) $x_3(n) = \{1, 0, 0, 1, 0, 0, \underset{\uparrow}{1}, 0, 0, 1, 0, 0, 1\}$

Sol:

$X_3(\omega) =$

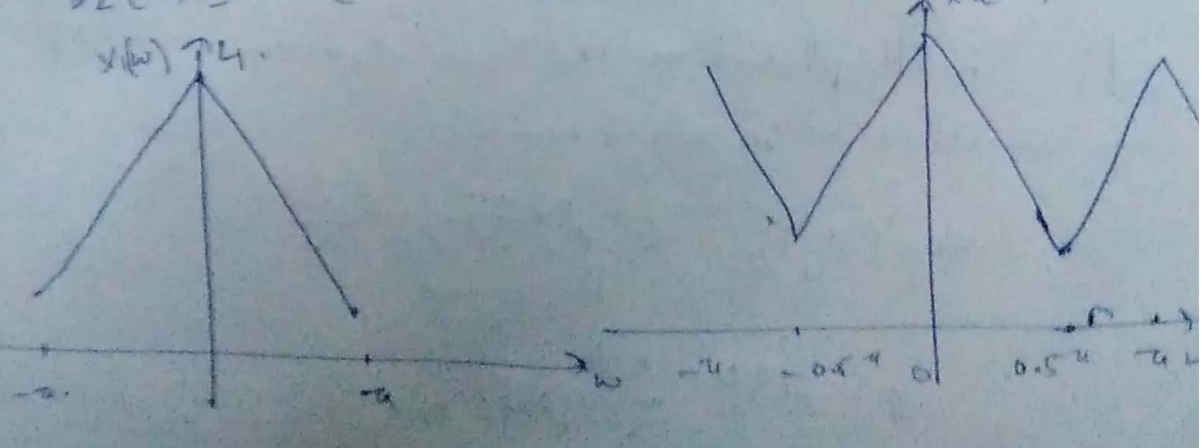
$x(-6)\cdot e^{j6\omega} + x(-3)\cdot e^{+j3\omega} + x(0) + x(3)\cdot e^{-j3\omega}$

$+ x(6)\,e^{-j6\omega}$

$= 1 + 2\cos 3\omega + 2\cdot \cos 6\omega.$

(d) Is there any relation between $X_1(\omega)$, $X_2(\omega)$, and $X_3(\omega)$? What is its physical meaning?

Sol: $X_2(\omega) = X_1(2\omega)$ $\quad X_3(\omega) = X_1(3\omega)$.



Similarly $X_3(\omega)$.

(e). Show that if

$$x_k(n) = \begin{cases} x\left(\frac{n}{k}\right), & \text{if } n/k \text{ integer} \\ 0, & \text{otherwise.} \end{cases}$$

then $X_k(\omega) = X(k\omega)$.

Sol:

$$X_k(\omega) = \sum_{n=-\infty}^{\infty} x_k(n) \cdot e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x\left(\frac{n}{k}\right) \cdot e^{-j\omega n} \qquad \frac{n}{k} = l$$

$$= \sum_{l=-\infty}^{\infty} x(l)\, e^{-j(k\cdot\omega) l}, \qquad n = kl.$$

$$\boxed{X_k(\omega) = X(k\omega)}$$

4.19. Let $x(n)$ be a signal with fourier transform as shown in figure. Determine and sketch the fourier transforms of the following signals.

(a) $x_1(n) = x(n) \cdot \cos(\pi n/4)$.

Sol: multiplication in one domain corresponds to convolution in other domain.

$$\cos\left(\frac{\pi n}{4}\right) = \frac{e^{+j\frac{\pi n}{4}} + e^{-j\frac{\pi n}{4}}}{2}.$$

$$1.\ e^{j\frac{\pi n}{4}} \overset{F.T}{\rightleftharpoons} 2\pi\delta\left(\omega - \frac{\pi}{4}\right).$$

$$e^{-j\frac{\pi n}{4}} \rightleftharpoons 2\pi\delta\left(\omega + \frac{\pi}{4}\right).$$

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$$\cos\frac{\pi n}{4} \xleftrightarrow{F.T} \pi\left[\delta\left(\omega-\frac{\pi}{4}\right)+\delta\left(\omega+\frac{\pi}{4}\right)\right].$$

$$x(n) * \cos\frac{\pi n}{4} \xleftrightarrow{F.T} X(\omega) * \pi\left[\delta\left(\omega-\frac{\pi}{4}\right)+\delta\left(\omega+\frac{\pi}{4}\right)\right]$$

$$\Rightarrow \quad \pi\left[X(\omega) * \delta\left(\omega-\frac{\pi}{4}\right)+X(\omega) * \delta\left(\omega+\frac{\pi}{4}\right)\right]$$

$$\Rightarrow \quad X_1(\omega) = \pi\left[X\left(\omega-\frac{\pi}{4}\right)+X\left(\omega+\frac{\pi}{4}\right)\right].$$

(b) $x_2(n) = x(n)\cdot\sin(\pi n/2).$

$$\sin(\pi n/2) = \frac{e^{j\frac{\pi n}{2}} - e^{-j\frac{\pi n}{2}}}{2j}.$$

$$X_2(\omega) = \frac{\pi}{j}\left[X\left(\omega-\frac{\pi}{2}\right) - X\left(\omega+\frac{\pi}{2}\right)\right].$$

(c). $x_3(n) = x(n)\cdot\cos(\pi n/2).$

$$X_3(\omega) = \pi\left[X\left(\omega-\frac{\pi}{2}\right)+X\left(\omega+\frac{\pi}{2}\right)\right].$$

(d). $x_4(n) = x(n)\cdot\cos\pi n.$

$$X_4(\omega) = \pi\left[X(\omega-\pi)+X(\omega+\pi)\right].$$

$$= 2\pi X(\omega-\pi$$

4.20 Consider an aperiodic signal $x(n)$ with fourier transform $X(\omega)$, show that the fourier series coefficient C_k^y of the periodic signal.

$$y(n) = \sum_{l=-\infty}^{\infty} x(n-lN).$$

are given by

$$C_k^y = \frac{1}{N} X\left(\frac{2\pi}{N}k\right), \quad k=0,1,\dots N-1$$

Sol:

$$C_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\cdot e^{-j\frac{2\pi}{N}kn}.$$

$$C_k = \frac{1}{N}\sum_{n=0}^{N-1}\left[\sum_{l=-\infty}^{\infty} x(n-lN)\right]e^{-j\frac{2\pi k}{N}\cdot n}$$

$$= \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{n=0}^{N-1} x(n-lN)\cdot e^{-j\frac{2\pi k}{N}\cdot n}$$

$$n-lN = m.$$

$$n = m+lN.$$

$$= \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m=-lN}^{N-1-lN} x(m)\cdot e^{-j\frac{2\pi}{N}k(m+lN)}.$$

$$= \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m=-lN}^{N-1-lN} x(m)\cdot e^{-j\frac{2\pi k}{N}\cdot m}\cdot e^{-j\frac{2\pi k}{\cancel{N}}l\cancel{N}}$$

o.f.

$$= \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m=-lN}^{N-1-lN} x(m)\cdot e^{-j\omega(m+lN)}.$$

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$C_k{}^{y} = \frac{1}{N} X\left(\frac{2\pi k}{N}\right)$.

4.21 prove that

$$X_N(\omega) = \sum_{n=-N}^{N} \frac{\sin \omega_c n}{\pi n} \cdot e^{-j\omega n}$$

may be expressed as

$$X_N(\omega) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} \frac{\sin\left[(2N+1)(\omega - \theta/2)\right]}{\sin\left[(\omega-\theta)/2\right]} \cdot d\theta .$$

Sol:

$$x_N(n) = \frac{\sin \omega_c n}{\pi n} \qquad -N \le n \le N.$$

$$= x(n) \cdot w(n)$$

$$x(n) = \frac{\sin \omega_c n}{\pi n} \qquad -\infty \le n \le \infty$$

$$w(n) = 1 \qquad -N \le n \le N$$

$$\quad\;\; 0 , \quad \text{otherwise}$$

$$\frac{\sin \omega_c n}{\pi n} \rightleftharpoons X(\omega).$$

$$= 1 , \quad |\omega| \le \omega_c$$

$$\quad 0 , \quad \text{otherwise}$$

$$X_N(\omega) = X(\omega) * W(\omega)$$

$$= \int_{-\pi}^{\pi} X(\tau) \cdot W(\omega - \tau) \cdot d\tau .$$

$$= \int_{-\omega_c}^{\omega_c} \frac{\sin\left[(2N+1)(\omega - \frac{\theta}{2})\right]}{\sin\left[(\omega-\theta)/2\right]} \cdot d\theta$$

4.22. A signal $x(n)$ has the following fourier transform

$$X(\omega) = \frac{1}{1 - a e^{-j\omega}}$$

determine the fourier transform of the following signals:

(a) $x(2n+1)$

sol. $$X_1(\omega) = \sum_{n=-\infty}^{\infty} x(2n+1) \cdot e^{-j\omega n}$$

$2n+1 = l$, $n = \frac{l-1}{2}$

$$= \sum_{n=\infty}^{\infty} x(l) \cdot e^{-j\omega \frac{(l-1)}{2}}$$

$$= \sum_{n=-\infty}^{\infty} x(l) \cdot e^{\frac{-j\omega l}{2}} \cdot e^{\frac{j\omega}{2}}$$

$$= e^{\frac{j\omega}{2}} \sum_{n=-\infty}^{\infty} x(l) \cdot e^{-j\left(\frac{\omega}{2}\right) l}$$

$$X_1(\omega) = e^{\frac{j\omega}{2}} \cdot X\left(\frac{\omega}{2}\right)$$

$$= \frac{e^{j\omega/2}}{1 - a e^{-j\omega/2}}$$

(b) $e^{\pi n/2} x(n+2)$

sol. $$X_2(\omega) = \sum_{n=-\infty}^{\infty} e^{\pi n/2} \cdot x(n+2) \cdot e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} e^{-j \cdot j\pi n/2} \cdot x(n+2)\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} e^{-jn\left(j\frac{\pi}{2} + \omega\right)} x(n+2)$$

let $n+2 = l$

$n = l-2$

$$= \sum_{l=-\infty}^{\infty} e^{-j(l-2)\left(j\frac{\pi}{2} + \omega\right)} \cdot x(l)$$

$$= \sum_{l=-\infty}^{\infty} x(l)\, e^{-jl\left(\frac{j\pi}{2}+\omega\right)} \cdot e^{j\pi\frac{\pi}{4}} \cdot e^{j2\omega}$$

$e^{j\pi}$ $\cos(-\pi)$

$$= -1 \sum_{l=-\infty}^{\infty} x(l) \cdot e^{-jl(\omega + j\pi/2)}\, e^{j2\omega}$$

$$= -X\left(\omega + j\frac{\pi}{2}\right) \cdot e^{j2\omega}.$$

(c) $x(-2n)$

$$X_3(\omega) = \sum_{n=-\infty}^{\infty} x(-2n)\, e^{-j\omega n} \qquad -2n = l$$

$$n = -\frac{l}{2}$$

$$= \sum_{n=-\infty}^{\infty} x(l)\, e^{-j\omega(-l/2)}.$$

$$= \sum_{n=-\infty}^{\infty} x(l) \cdot e^{-j\left(\frac{-\omega}{2}\right)l}$$

$$= X\left(-\frac{\omega}{2}\right).$$

(d) $x(n) \cdot \cos(0.3\pi n)$.

sol: $X_4(\omega) = \pi\left[X(\omega - 0.3\pi) + X(\omega + 0.3\pi)\right]$.

(e) $x(n) * x(n-1)$ $\qquad x(n-1) \rightleftharpoons e^{-j\omega l} \cdot X(\omega)$.

$x(n) \rightleftharpoons X(\omega)$

~~$X(\omega) \cdot X(\omega - 1)$~~.

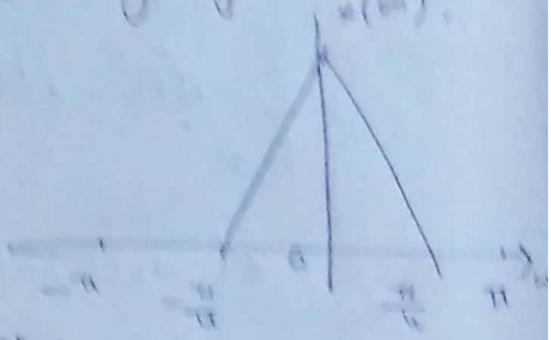
$X(\omega) \cdot X(\omega) \cdot e^{-j\omega}$

$X^2(\omega)\, e^{-j\omega}$.

(f) $x(n) * x(-n)$.

$$X_6(\omega) = X(\omega) \cdot X(-\omega) = \frac{1}{(1 - a e^{-j\omega})(1 - a e^{j\omega})}$$

4.23 from a discrete - time signal $x(n)$ with fourier transform $X(\omega)$, shown in fig determine and sketch the fourier transform of the following signals.



(a) $y_1(n) = \begin{cases} x(n), & n\ \text{even} \\ 0, & n\ \text{odd} \end{cases}$

Sol.

$$Y_1(\omega) = \sum_{n=n,\ \text{even}} x(n) \cdot e^{-j\omega n}$$

(b) $y_2(n) = x(2n)$.

$$Y_2(\omega) = \sum_{n=-\infty}^{\infty} x(2n)\, e^{-j\omega n}$$

$2n = l$

$n = l/2$

$$= \sum_{n=-\infty}^{\infty} x(l)\, e^{-j\omega \frac{l}{2}}$$

$$= \sum_{n=-\infty}^{\infty} x(l) \cdot e^{-j\left(\frac{\omega}{2}\right) l}$$

$$Y_2(\omega) = X\left(\frac{\omega}{2}\right)$$

(c) $y_3(n) = \begin{cases} x(n/2), & n,\ \text{even} \\ 0, & n,\ \text{odd} \end{cases}$

Soll.

$$Y_3(\omega) = \sum_{n=-\infty}^{\infty} x(n/2) \cdot e^{-j\omega n}$$

$n/2 = l$

$n = 2l$

$$= \sum_{n=-\infty}^{\infty} x(l)\, e^{-j\omega 2l}$$

$$= \sum_{n=-\infty}^{\infty} x(l) \cdot e^{-j2\omega l}$$

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$Y_s(\omega) = X(2\omega)$.

Given the note that $y_1(n) = x(n) \cdot s(n)$

where $s(n) = \{ \ldots 0, 1, 0, \underset{\uparrow}{1}, 0, 1, 0, 1 \ldots \}$

$\qquad -2 \quad 0 \quad 2 \quad 4$

only even terms.

$$y_1(n) = \begin{cases} y_2(n/2) & , n \text{ even} \\ 0 & n \text{ odd.} \end{cases}$$

$$\boxed{Y_1(\omega) = Y_2(2\omega)}$$

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