2.1. A discrete time Signal $x(n)$ is defined as

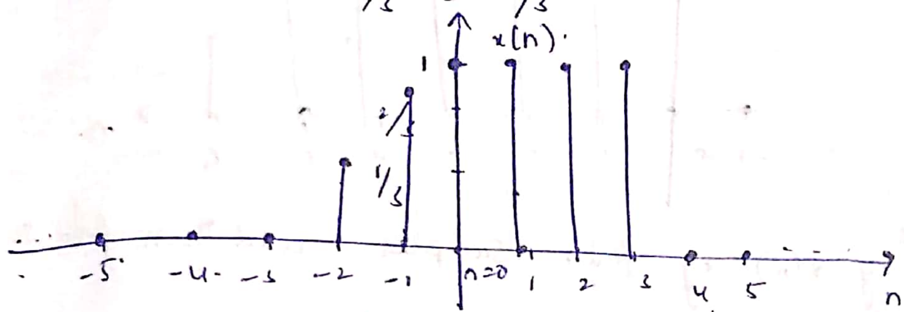
$$x(n) = \begin{cases} 1 + n/3, & -3 \le n \le -1 \\ 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Determine its values and sketch the signal $x(n)$.

$n = -3$ $\quad 1 - 3/3 = 0.$

$n = -2$ $\quad 1 - 2/3 = \frac{3-2}{3} = 1/3$

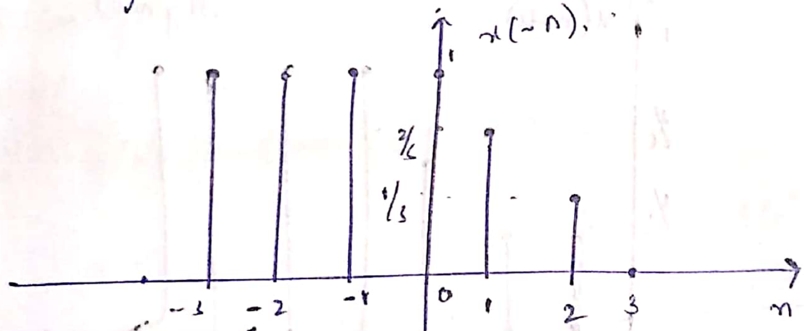
$n = -1$ $\quad 1 - 1/3 = 2/3.$



(b) sketch the signals that result if we:

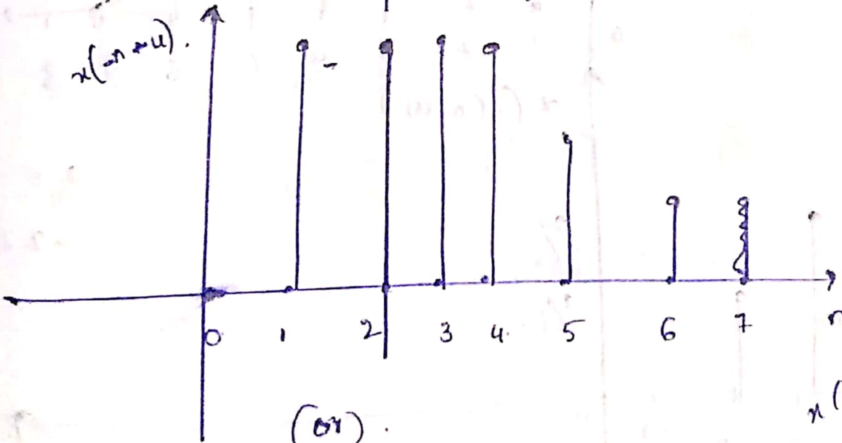
1. First fold $x(n)$ and then delay the resulting signal by four samples.

fold $x(n)$ means $x(-n)$.



$x(-n-4) =$

$x(-(n+4))$.

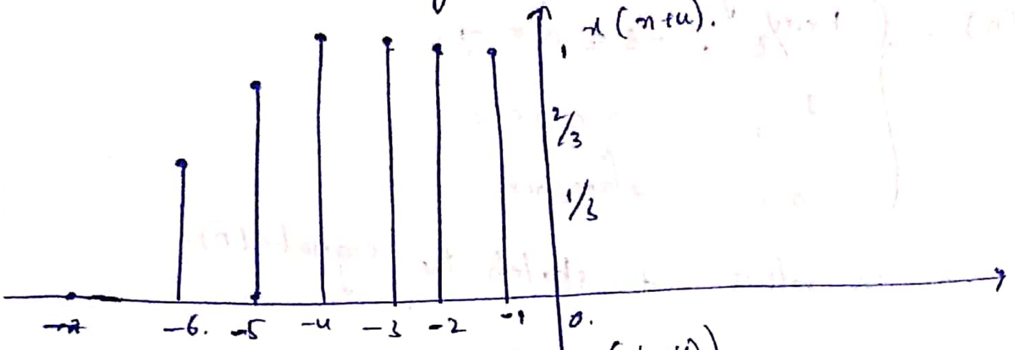


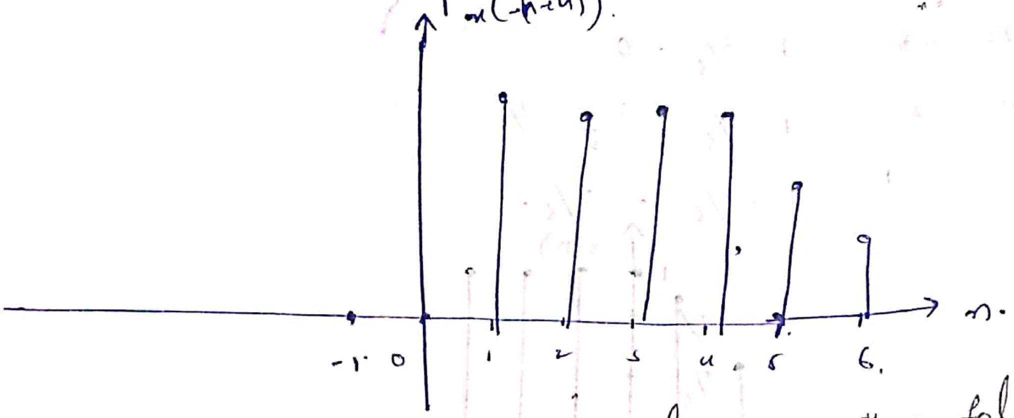
$x(-(n-4))$.

(or)

$x(-n-4) = x(-(n+4))$

Signal advanced by 4 and then reversed.





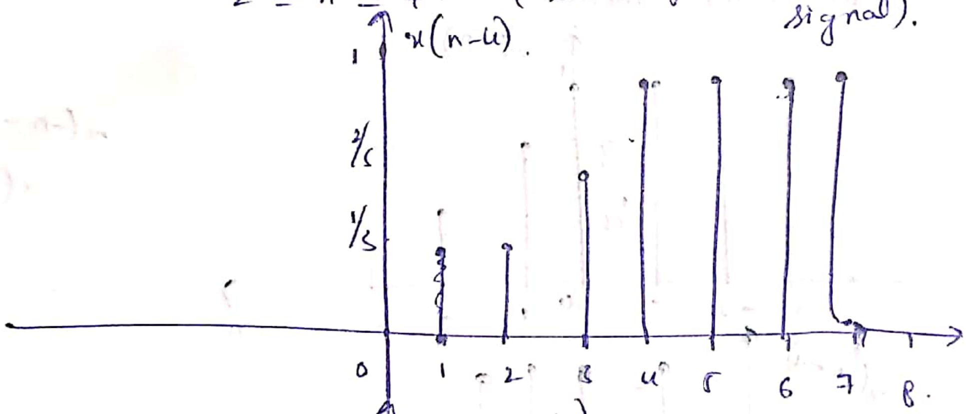
2. First delay x(n) by four samples and then fold the resulting signal.

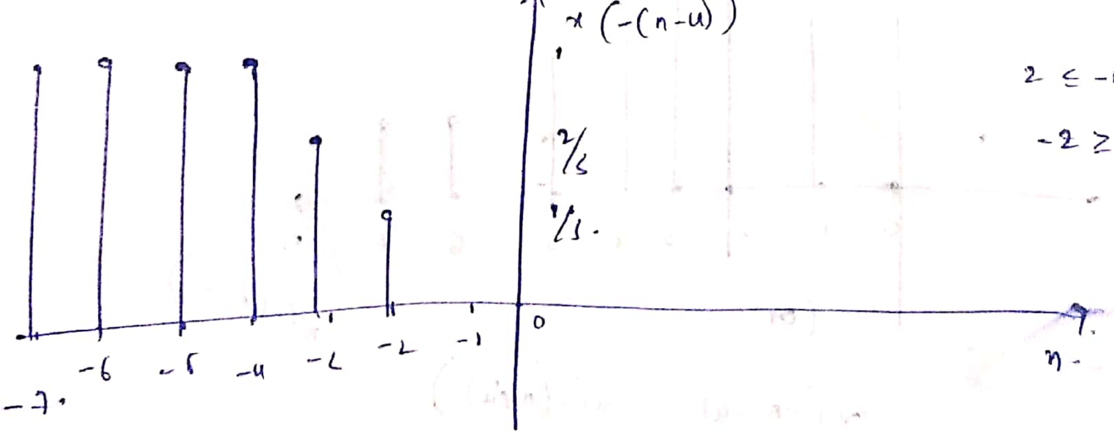
$n - 4 = 3$

$n = 7.$

$n - 4 = -2 \quad n = +4 - 2 = 2.$

$2 \leq n \leq 7.$ (limits for first delayed signal).





$2 \leq -n \leq 7$

$-2 \geq n \geq 7$

$-n = 7$

$n = -7.$

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(c) Sketch the signal $x(-n+4)$.

$x(-(n-4))$.

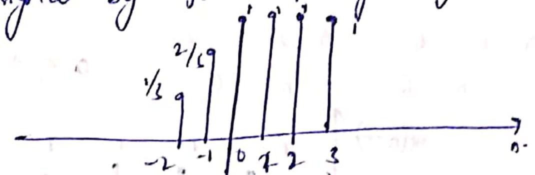
it is like shifting the signal be four samples and reversing

it is 2.

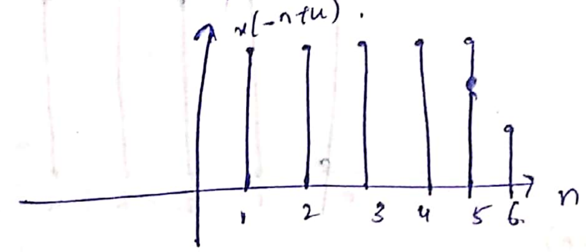
(d) Compare the results in parts (b) and (c) and derive a rule for obtaining the signal $x(-n+k)$ from $x(n)$.

$x(-n+k)$ can be obtained as. $x(-(n-k))$

first shifting the signal by four sampling right and the reversing the signal.



$x(-n+4)$



$-n+4 = 3.$

$n = 4-3 = 1$

$-n+4 = -2$

$n = 4+2 = 6.$

$-n+4 = 1$

$4-1 = n = 3.$

(e) Can you express the signal $x(n)$ in terms of signals $\delta(n)$ and $u(n)$?

Sol:- $x(n)$ in terms of $\delta(n)$ is.

$$x(n) = \tfrac{1}{3}\delta(n+2) + \tfrac{2}{3}\delta(n+1) + \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3)$$

This is only combination of $\delta(n)$.

$x(n)$ interms of both $\delta[n]$ and $u[n]$. is

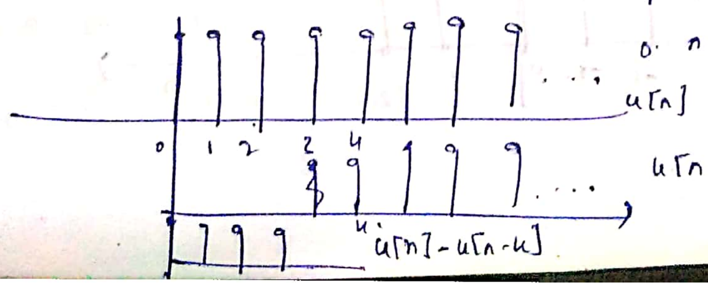
$$x(n) = \tfrac{1}{3}\delta[n+2] + \tfrac{2}{3}\delta[n+1] + u[n] - u[n-4]$$

$u[n]$ is 1 $n \geq 0$.

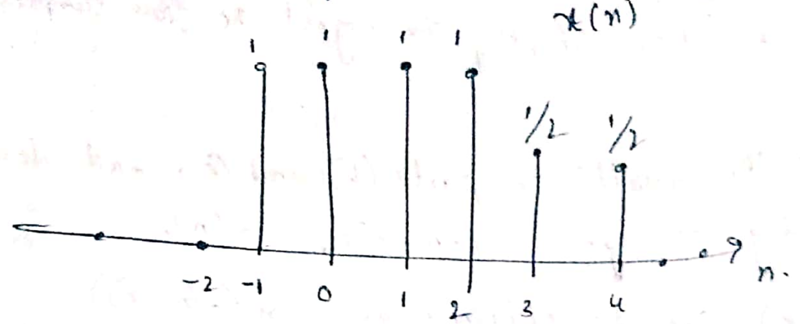
0 $n < 0$.

$u[n-3]$ is 1 $n \geq 3$.

0. $n < 3$.



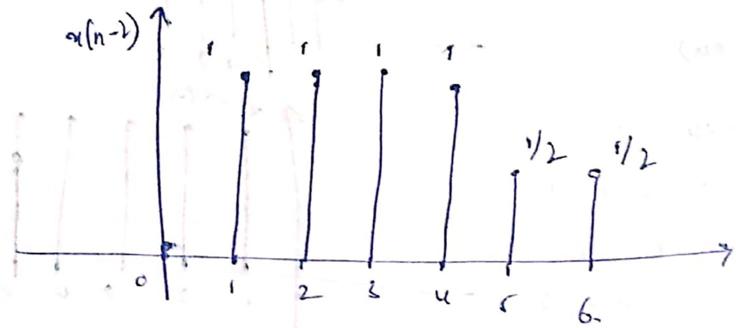
2.2. A discrete-time signal $x(n)$ is shown in fig P2.2. Sketch and label carefully each of the following signals.



(a) $x(n-2)$

$n-2=4$ $\quad$ $n-2=-2$

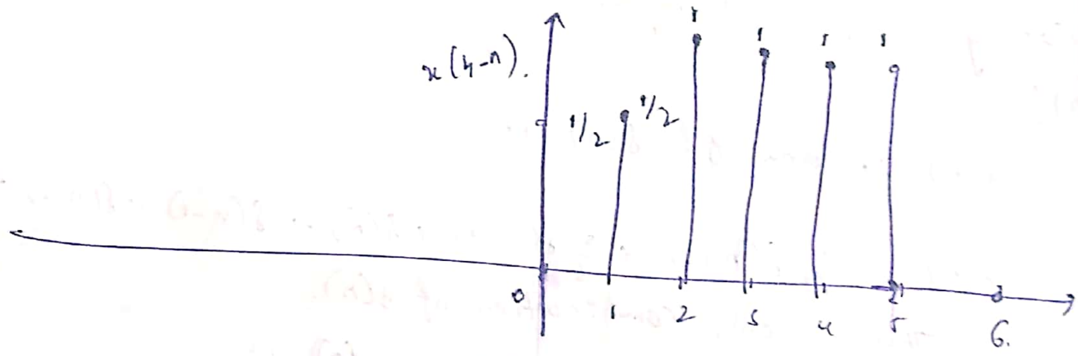
$n=6$ $\quad$ $n=0.$ $\quad$ $0 \le n \le 6.$



(b) $x(4-n)$.

$4-n=4$ $\quad$ $4-n=-2$

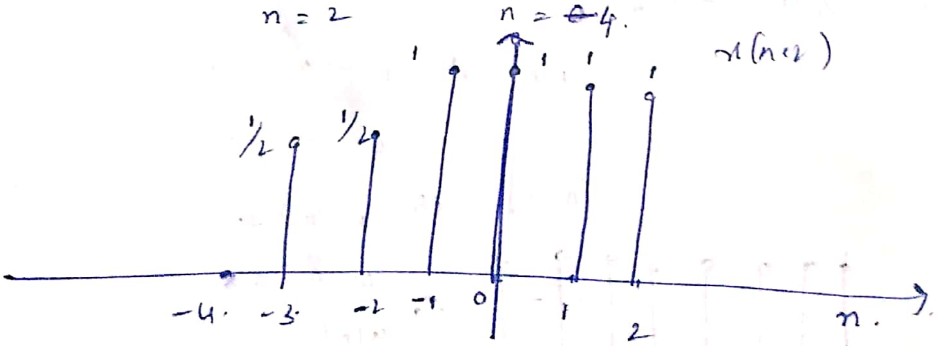
$4-4=n \Rightarrow n=0.$ $\quad$ $4+2=n. \Rightarrow n=6.$



(c) $x(n+2)$.

$n+2=4.$ $\quad$ $n+2=-2$

$n=2$ $\quad$ $n=\cancel{0}4.$



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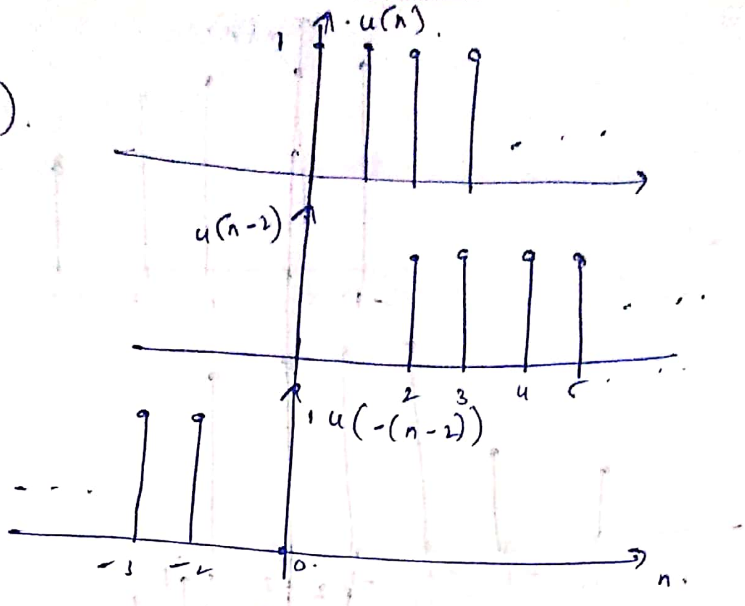
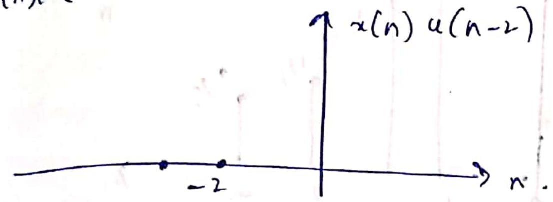
(d) $x(n)u(2-n)$.

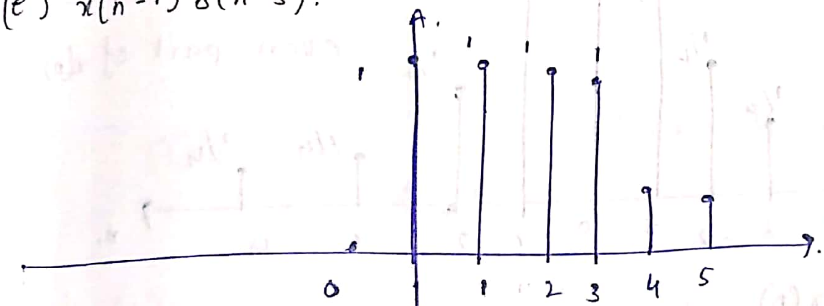
$u(-(n-2))$.

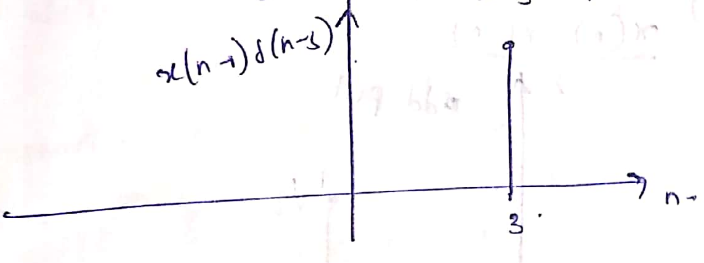


$x(n)(u(n-2))$



(e) $x(n-1)\delta(n-3)$.





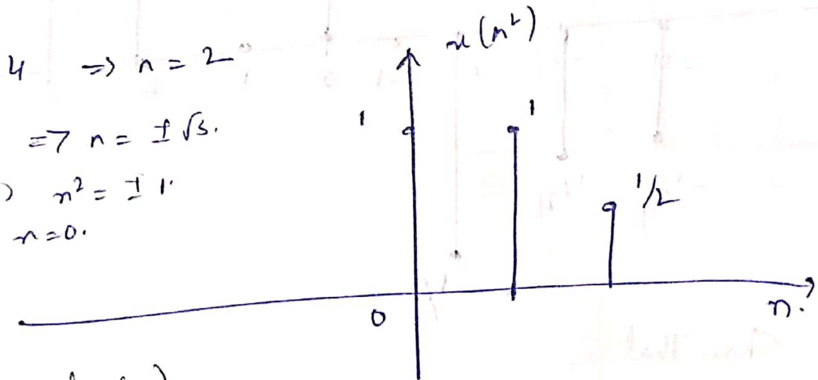
(f) $x(n^2)$.

$n^2 = 4 \Rightarrow n = 2$

$n^2 = 3 \Rightarrow n = \pm\sqrt{3}$.

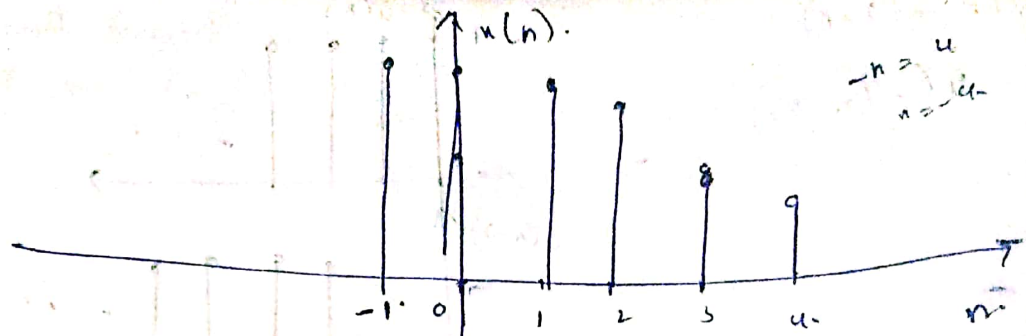
$n^2 = 1 \Rightarrow n^2 = \pm 1$.

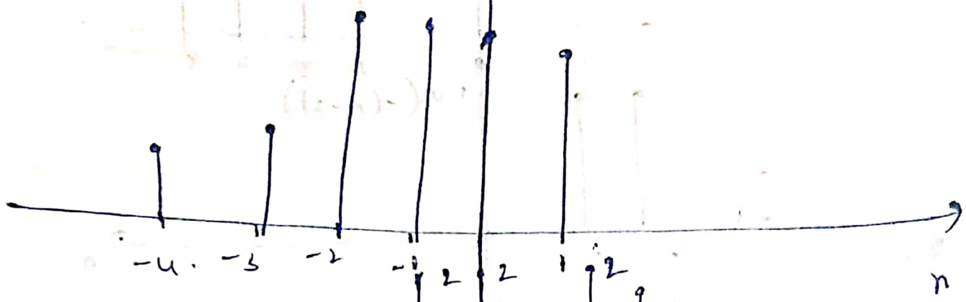
$n^2 = 0 \Rightarrow n = 0$.

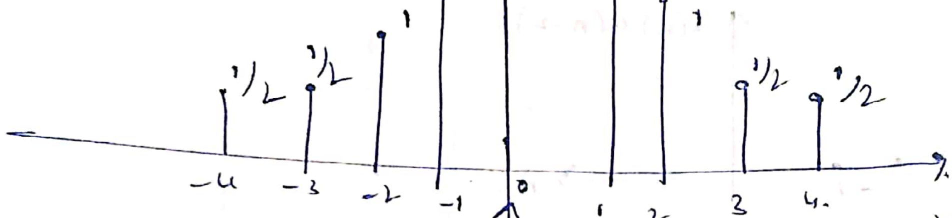


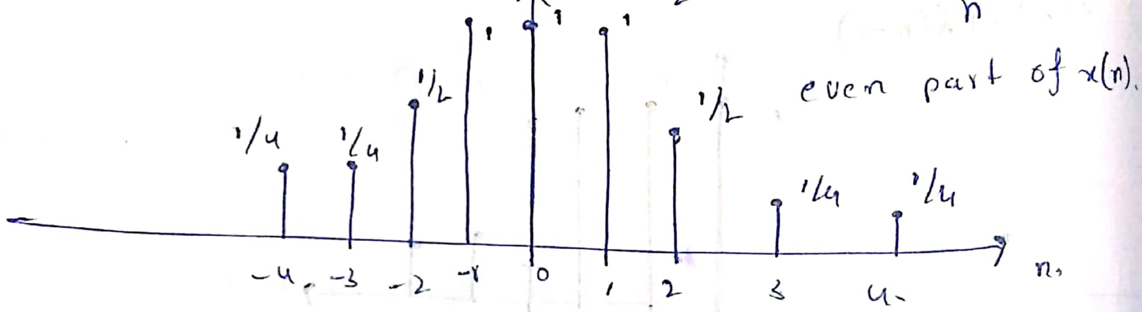
(g) even part of $x(n)$.

$$\text{evenpa } x(n) = \frac{x(n) + x(-n)}{2}$$

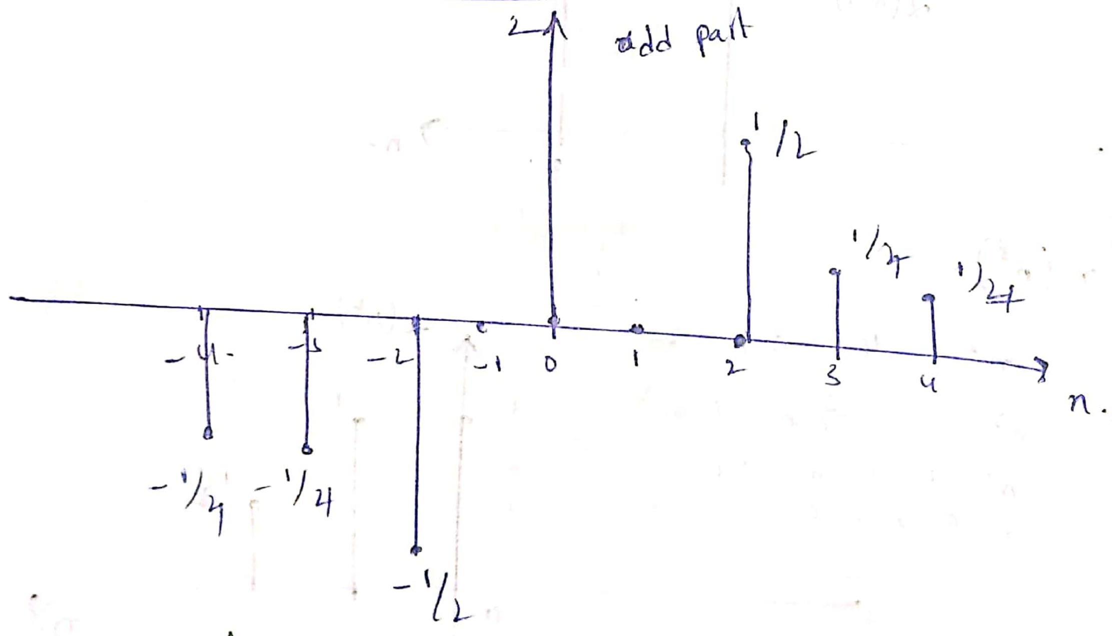






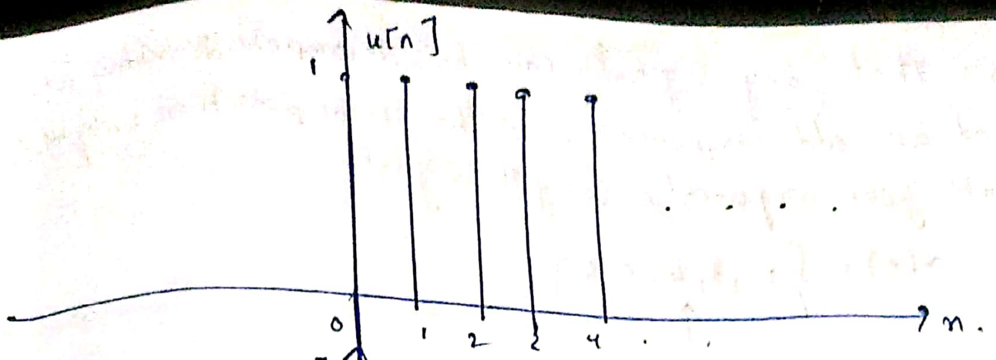


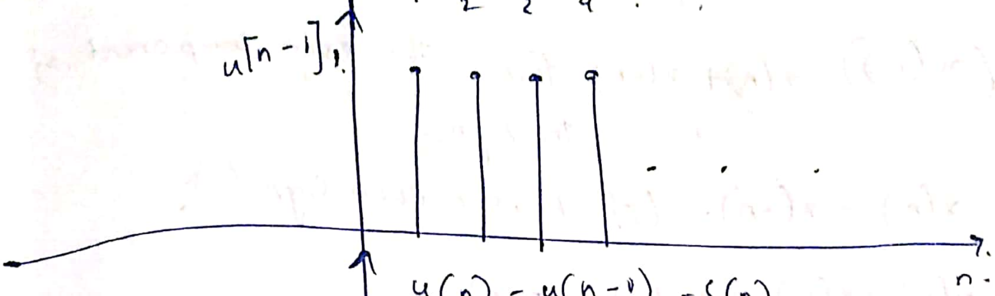
(b) odd part of x(n). $\frac{x(n) - x(-n)}{2}$

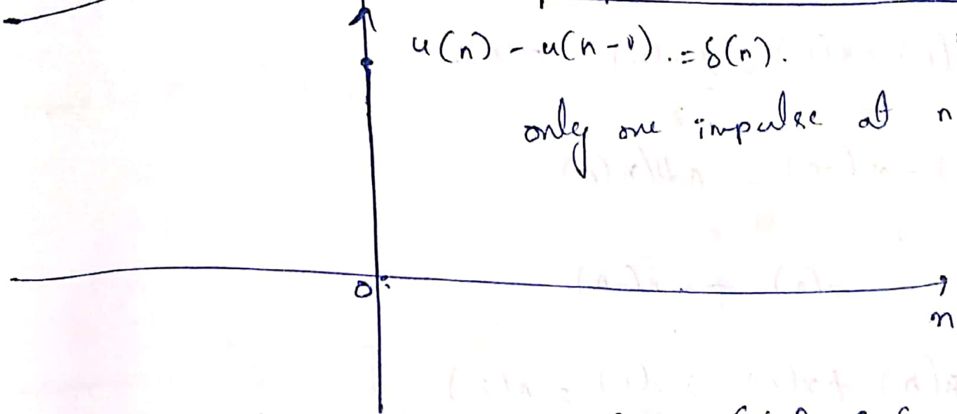


Q. 3. Show That

(a) $\delta(n) = u(n) - u(n-1)$.







$u(n) - u(n-1) = \delta(n)$.

only one impulse at $n=0$.

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0. \end{cases} \qquad \delta(n) = u(n) - u(n-1).$$

(b). $u(n) = \sum_{k=-\infty}^{n} \delta(k) = \sum_{k=0}^{\infty} \delta(n-k)$.

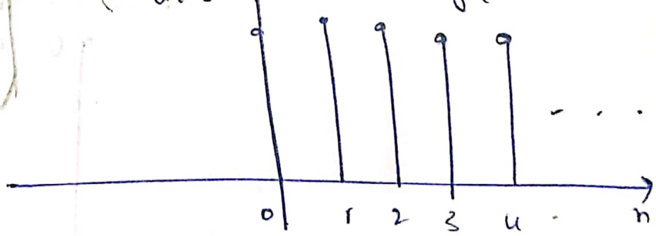
expanding the terms.

$$\sum_{k=-\infty}^{n} \delta(k).$$

$$\boxed{\sum_{k=-\infty}^{0} \delta(k) + \sum_{k=0}^{n} \delta(k).}$$

$$\sum_{k=0}^{\infty} \delta(n-k) = \delta(n) + \delta(n-1) + \delta(n-2) + \delta(n-3) + \cdots \delta(n-\infty).$$

$u[n]$



2.4 Show that any signal can be decomposed into an even and an odd component. Is the decomposition unique, Illustrate your arguments using the signal.

$$x(n) = \{2, 3, \underset{\uparrow}{4}, 5, 6\}$$

Sol:-

$ev(x(n)) = \dfrac{x(n) + x(-n)}{2}$ this is the even component of th signal.

$x(n) = x(-n)$. (if it is a even signal).

$\dfrac{x(n) + x(n)}{2} = \dfrac{\cancel{2}\, x(n)}{\cancel{2}} = x(n).$

$\dfrac{x(n) - x(-n)}{2} = odd(x(n)).$

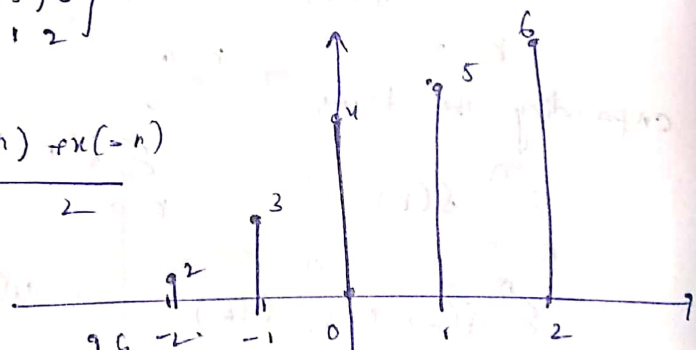
$x(n) = -x(-n).$

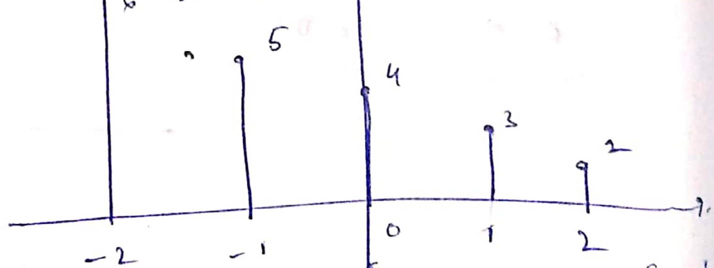
$\dfrac{x(n) + x(n)}{2} = \dfrac{\cancel{2} x(n)}{\cancel{2}} = x(n).$

any signal can be decomposed into an even and odd component

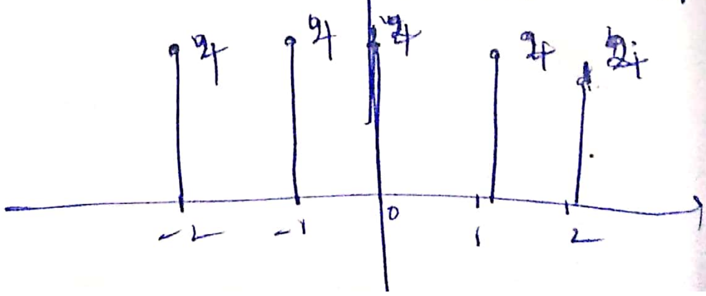
$$x(n) = \{\underset{-2}{2}, \underset{-1}{3}, \underset{\underset{0}{\uparrow}}{4}, \underset{1}{5}, \underset{2}{6}\}$$

$even(x(n)) = \dfrac{x(n) + x(-n)}{2}$



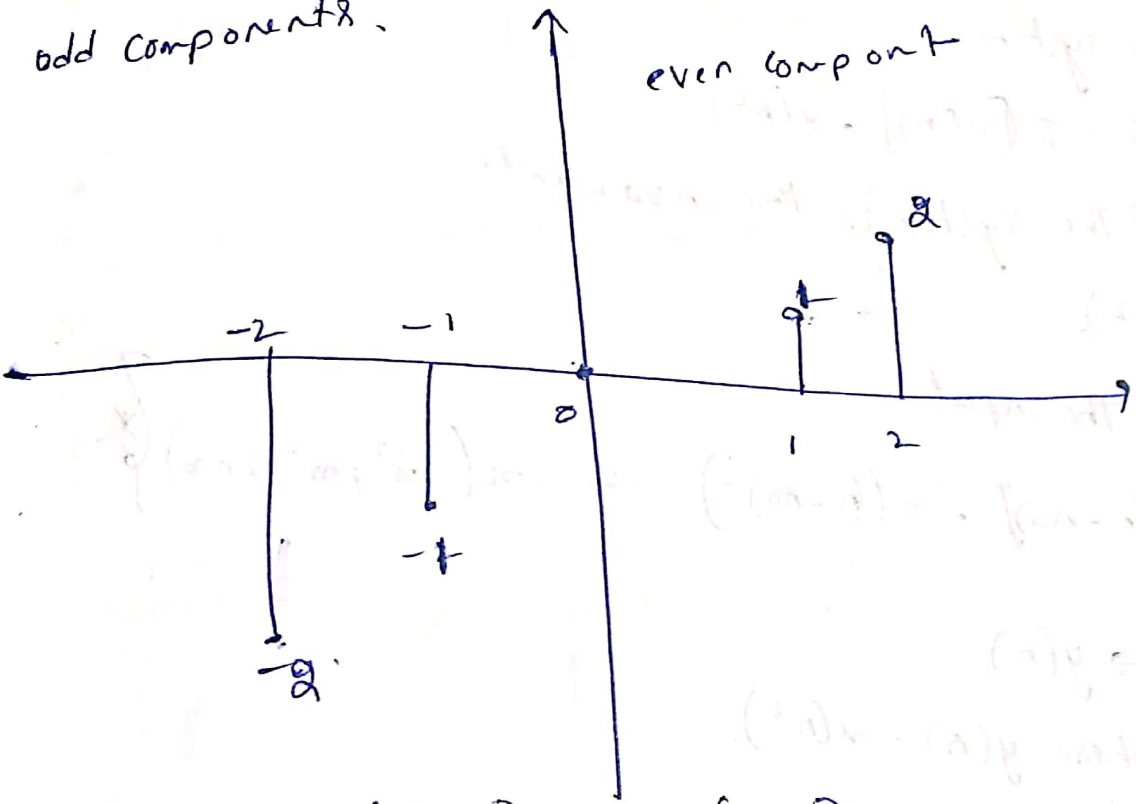


even component of signal

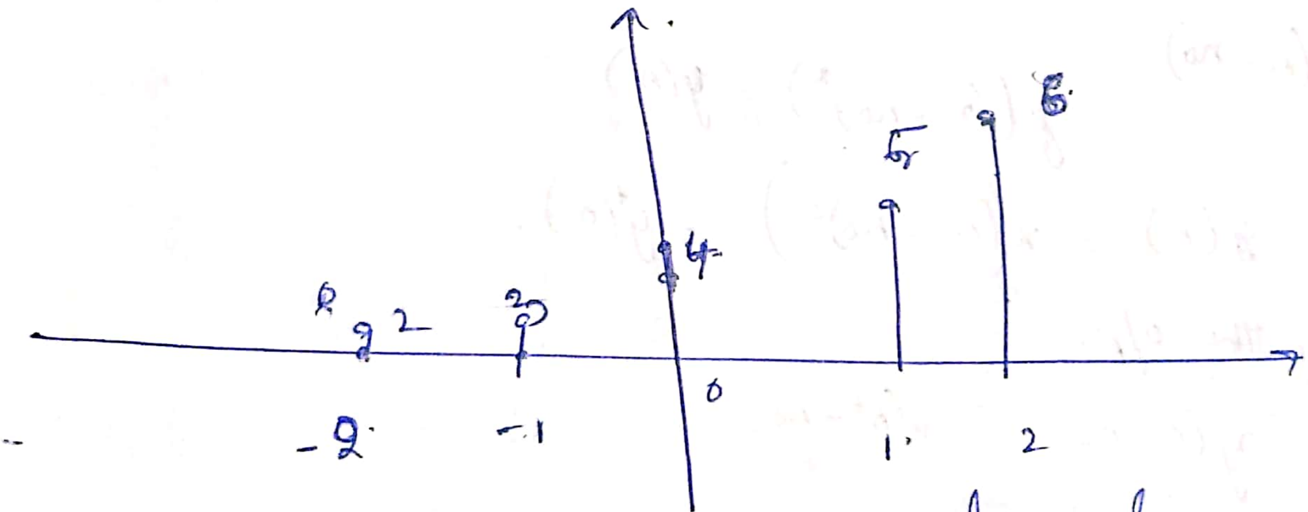


odd components.

even compont



$x(n) = \text{even}(x(n)) + \text{odd}(x(n))$.



This is ~~not~~ the original signal.

Yes the decomposition is unique.

real valued energy (power) si

2.6. Consider the system

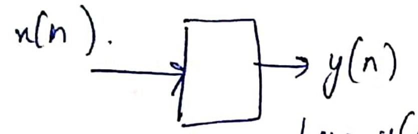
$$y(n) = T[x(n)] = x(n^2)$$

(a) Determine if the system is time invariant

$$y(n) = x(n^2).$$

delaying the input

$$T[x(n-n_0)] = x((n-n_0)^2) = x(n^2+n_0^2-2nn_0).$$



where $y(n) = x(n^2)$.

delay the input

$x(n-n_0)$.

$$y((n-n_0)^2) = y(n).$$

$$y(n) = x((n-n_0)^2) = y'(n).$$

delay the o/p.

$$y(n-n_0) = x(n^2-n_0).$$

$$y(n) \ne y(n-n_0).$$

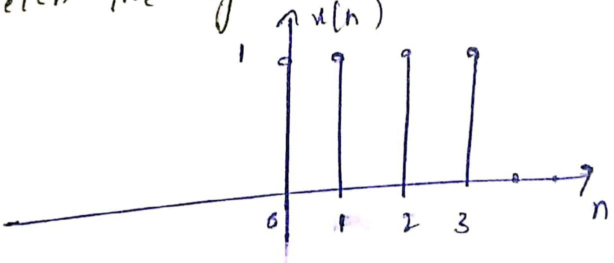
So the system is a time variant system

(b) To clarify the result in part (a) assume that the signal.

$$x(n) = \begin{cases} 1, & 0 \le n \le 3 \\ 0, & \text{elsewhere.} \end{cases}$$

is applied into the system.

(1) sketch the signal $x(n)$.



(2). Determine and sketch the signal $y(n) = T[x(n)]$

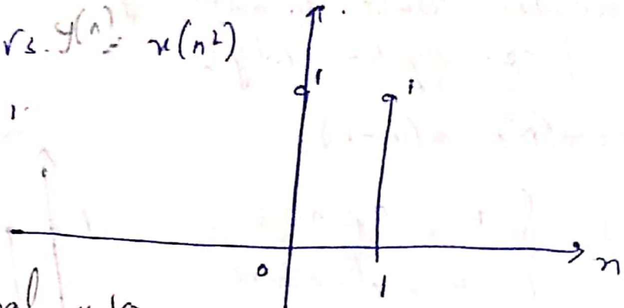
$y(n) = T[x(n)] = x(n^2)$.

$x(n^2)$

$n^2 = 3 \Rightarrow n = \pm\sqrt{3}$. $y(n) = x(n^2)$

$n^2 = 1 \Rightarrow n = \pm 1$

$n^2 = 0 \Rightarrow n = 0$.

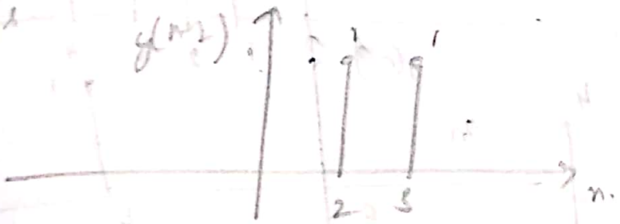


(3) sketch the signal $y_2'(n) = y(n-2)$

$y(n-2) = x((n-2)^2)$.

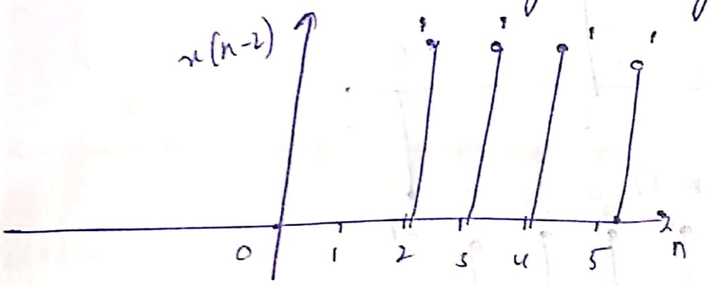
n replaced with $n-2$.

$y(n-2)$ means the signal is shifted right side by 2 units



(4). Determine and sketch the signal $x_2(n) = x(n-2)$.

$x(n)$ is shifted right side by 2 units.



$n-2=3$
$n=5$

$n-2=0$
$n=2$

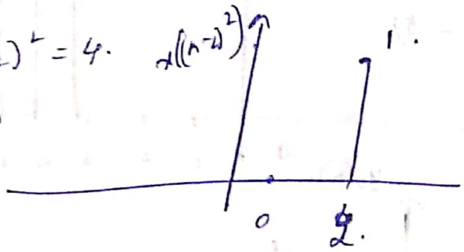
(5) determine and sketch the signal $y_2(n) = T[x_2(n)]$

$y_2(n) = T[x_2(n)] = x((n-2)^2)$.

$= x(n^2+4-2n)$ or.

$(n-2)^2 = 4$.

n



(6) compare the signal $y_2(n)$ and $y(n-2)$. what is your conclusion?

sol:- $y_2(n)$ and $y(n-2)$ are not equal so the given system is time variant system.

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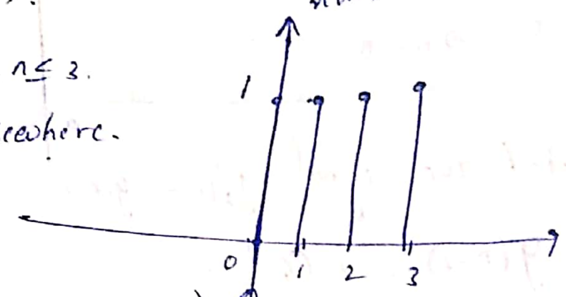
(c) Repeat part (b) for the system

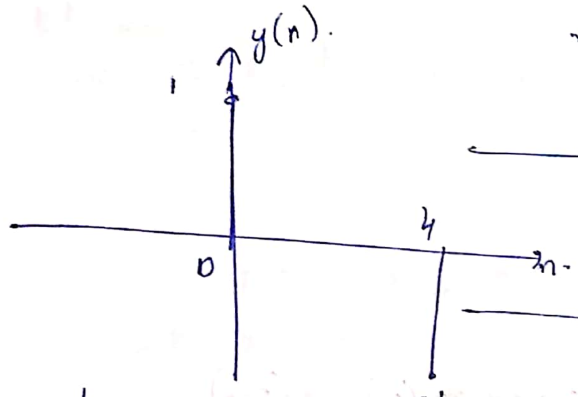
$$y(n) = x(n) - x(n-1)$$

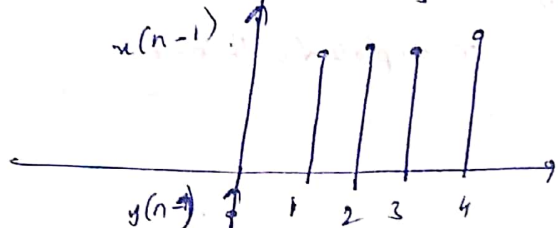
Can you use this result to make any statement about the time invariance of this system? why?

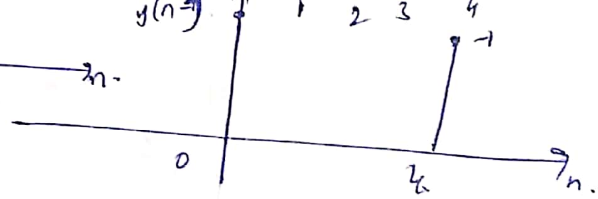
Sol: $y(n) = x(n) - x(n-1)$.

$$x(n) = \begin{cases} 1, & 0 \le n \le 3. \\ 0, & \text{elsewhere.} \end{cases}$$

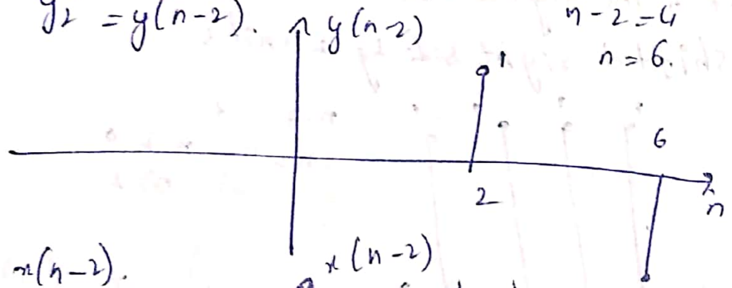








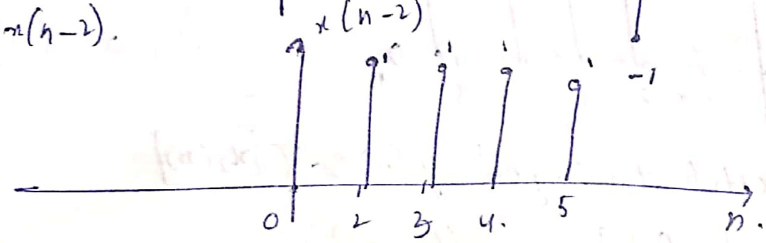
$y_2' = y(n-2)$.



$n-2=4$

$n=6$.

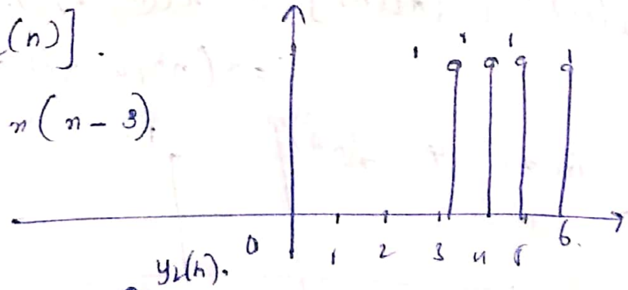
$x(n-2)$.

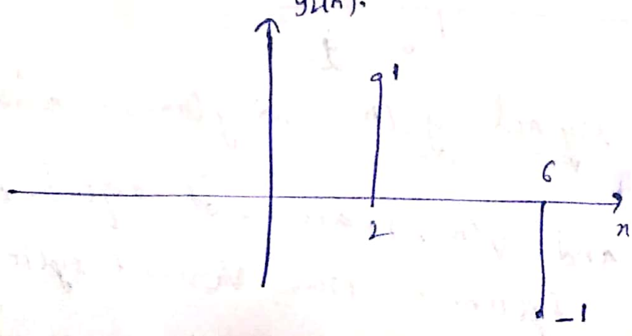


$y_2(n) = T[x_2(n)]$.

$x(n-2) - x(n-3)$.

$n-3=3$



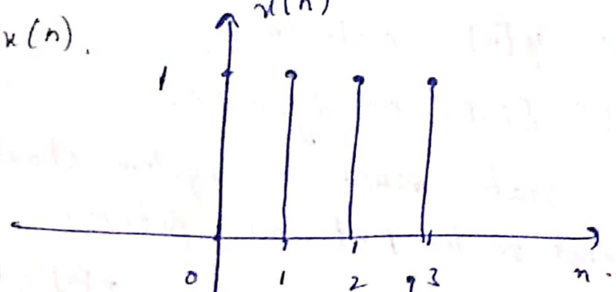


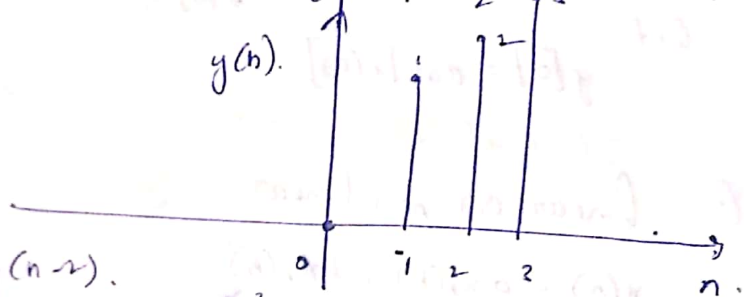
$y_2'(n) = y_2(n)$.
means that the given system is a time invariant system. if we delay the o/p. The result will be equal to delayed i/p o/p means that there is no change it, The system is independent of time. So it is time invariant system.

(d) Repeat parts (b) and (c) for the system.

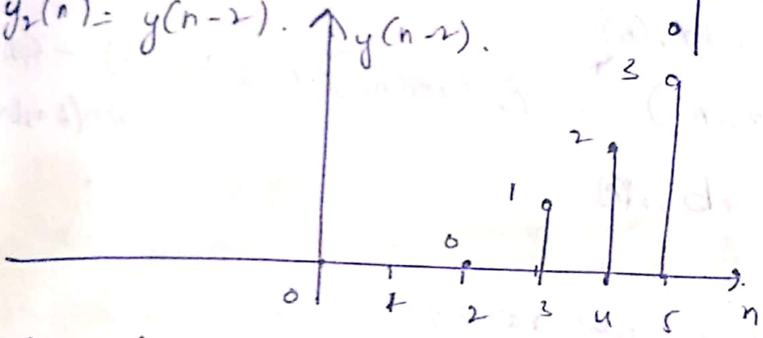
$y(n) = T[x(n)] = n x(n)$.

$x(n) = \begin{cases} 1, & 0 \le n \le 3. \\ 0, & \text{elsewhere}. \end{cases}$



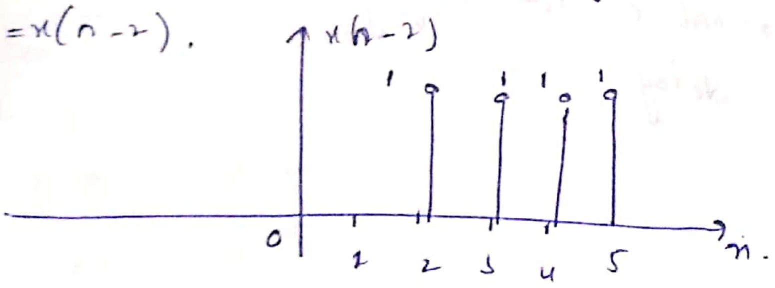


$y_2'(n) = y(n-2)$.

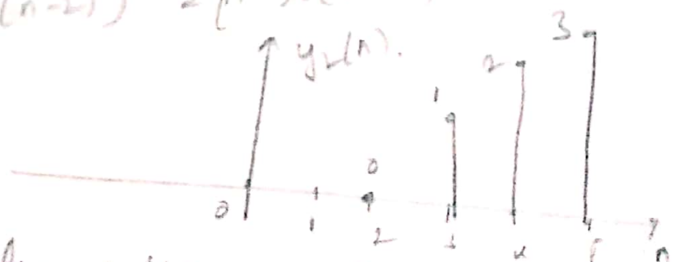


$n - 2 = 3$
$n = 5$

$x_2(n) = x(n-2)$.



$y_2(n) = T(x_2(n))$.
$= T(x(n-2)) = (n-2)x(n-2)$



here also $y_2'(n) = y_2(n)$.
it is also a time invariant system

Q.4. A discrete-time system can be

1. Static or dynamic
2. Linear or non linear.
3. Time invariant or time varying
4. causal or noncausal
5. stable or unstable.

Examine the following systems with respect to the properties a

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$y(n) = \cos[x(n)]$.

1. Static or dynamic.

let $n = 0$

$y(0) = \cos[x(0)]$.

present o/p depends only depends on present i/p. So it is a static system.

2. Linear or nonlinear.

$ax_1[n] + bx_2[n] = ay_1[n] + by_2[n]$.

$y_1[n] = \cos[x_1(n)]$ $\quad y_2[n] = \cos[x_2[n]]$.

$a\cos[x_1(n)] + b\cos[x_2(n)]$.

$\cos[ax_1(n) + bx_2(n)] \neq a\cos[x_1(n)] + b\cos[x_2(n)]$

so the given system is non linear.

3. Time invariant or time varying

let $y(n - n_0) = \cos[x(n - n_0)]. = y'(n)$.

$x[n - n_0]$

$y(n) = \cos[x(n - n_0)]$.

$y'(n) \neq y(n)$

so the given s/m is timeinvariant

4. Causal or non causal.

* as we know that every static s/m is a causal system. But every causal s/m is not a static s/m.

$y(n)$ is a causal system

5. stable or unstable

for $n \geq 0$.

let consider.

stable.

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(b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$.

1.

$y(0) = \sum_{k-\infty}^{n+1} x(k)$.

$= x(-\infty) + \cdots x(-1) + x(0) + x(1) + \cdots x(n) + x(n+1)$.

consider $n=0$.

$y(0) = x(0) + x(0+1)$.

The given system depends on past i/ps and also future i/p.

The given s/m is a dynamic s/m.

2. Linear or non-linear.

$y(n) = \sum_{k=-\infty}^{n+1} x(k)$

let $x(k) = a x_1(k) + b x_2(k)$.

$a y_1(n) + b y_2(n)$.

$b y_2(n) = b \sum_{k=-\infty}^{n+1} x_1(k)$.

$a y_1(n) = a \sum_{k=-\infty}^{n+1} x_1(k)$

$a y_1(n) + b y_2(n) = a \sum_{k=-\infty}^{n+1} x_1(k) + b \cdot \sum_{k=-\infty}^{n+1} x_1(k)$

$y(n) = \sum_{k=-\infty}^{n+1} a x_1(k) + b x_2(k) = a \sum_{k=-\infty}^{n+1} a x_1(k) + b \sum_{k=-\infty}^{n+1}$

So the given s/m is linear s/m.

3. Time invariant or time varying

time invariant s/m.

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4. Causal or non causal :-

It is a non causal s/m.

5. stable or unstable.

unstable.

(c) $y(n) = x(n) \cos(w_0 n)$.

1) static or dynamic :-

$y(0) = x(0) \cos(0)$.

static system.

2) linear or non linear :-

$x(n) = a x_1(n) + b x_2(n)$.

$y(n) = a y_1(n) + b y_2(n)$ $\quad b y_2(n) = b x_2(n) \cos(w_0 n)$.

$a y_1(n) = a x_1(n) \cos(w_0 n)$.

$y'(n) = a x_1(n) \cos(w_0 n) + b x_2(n) \cos(w_0 n)$

$y(n) = a x_1(n) \cos(w_0 n) + b x_2(n) \cos(w_0 n)$

It is a linear s/m.

3) Time invariant or time varying :-

$y'(n) = y(n-n_0) = x(n-n_0) \cdot \cos(w_0(n-n_0))$

$y(n) = x(n-n_0) \cdot \cos(w_0 n)$.

So the given s/m is a time varying s/m

4). Causal or non causal :-

It is a causal s/m

5). Stable or unstable :-

unstable

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(d) $y(n) = x(-n+2)$

1). Static or dynamic :-

$y(0) = x(0+2)$

$y(1) = x(-1+2) = x(1)$

$y(-1) = x(1+2) = x(3)$.

dynamic system

2). Linear or non-linear :-

linear system.

3). Time invariant or time varying :-

$y(n) = x(-n+2)$ $x(-(n-2))$ $x(-(n-2-n_0))$.

$y(n-n_0) = x(-(n-n_0)+2)$.

$y'(n) = x(-n+n_0+2)$.

$y(n) = x(-n+2+n_0)$.

time in varying.

4). Causal or non causal :-

non causal s/m.

5). Stable or unstable :-

stable.

(e) $y(n) =$ Trun $[x(n)]$, where Trun$[x(n)]$ denotes the integer part of $x(n)$, obtained by truncation.

(f) $y(n) =$ Round $[x(n)]$, where Round $[x(n)]$ denotes the integer part of $x(n)$ obtained by truncation.

Remark: The system in parts (e) and (f) are quantizers that perform truncation and rounding, respectively.

(f) 1. static.

$a\,y_1(n) + b\,y_2(n) \neq$ Round $[a x_1(n) + b x_2(n)]$

2. non linear.

3. $y(n) = y(n-n_0)$ = Round $[x(n-n_0)]$ $y'(n) = y(n)$.

$x(n-n_0)$. $y(n) =$ Round $[x(n-n_0)]$ Time invariant S/m.

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4. Casual s/m.
5. stable or unstable.
similar for f also.

(g) $y(n) = |x(n)|$

1. $y(0) = |x(0)|$ (static s/m)

2. $a_1(y_1(n) + b\, y_2(n)) = |a_1 x_1(n) + b\, x_2(n)|$

$a_1 |x_1(n)| + b |x_2(n)| = a_1 |x_1(n)| + b |x_2(n)|$

non linear.

3. time in variant

4. causal.

5.

(h) $y(n) = x(n) . u(n)$

1. static

2. $a\, x_1(n) . u(n) + b x_2\, u(n) = a x_1(n) . u(n) + b . x_2(n) . u(n)$.

linear.

3. $y'(n) = y(n - n_0) = x(n - n_0) . u(n - n_0)$.

$x(n - n_0) . u(n - n_0) = y(n)$.

time in variant

4. Casual

5. stable

i) $y(n) = x(n) + n\, x(n+1)$

1. $y(0) = x(0) + 0 . x(0+1)$.

$y(1) = x(1) + 1 . x(2)$.

Dynamic.

2. linear

3. time variant $y(n) = x(n - n_0) + n\, x(n + 1 - n_0)$.

$y'(n) = y(n - n_0) = x(n - n_0) + (n - n_0) . x(n + 1 - n_0)$.

$y'(n) \neq y(n)$.

4. non. causal s/m.

5. Stable

(j) $y(n) = x(2n)$

1. $y(1) = x(2)$.
Dynamic S/m.

2. $a y_1(n) + b \cdot y_2(n) = [a x_1(2n) + b \cdot x_2(2n)]$
linear.

3. $y'(n)$ $y(n-n_0) = x(2(n-n_0))$
$y'(n) = x(2n - n_0)$.
time variant S/m.

4. non-causal.

5. stable

k. $y(n) = \begin{cases} x(n), & \text{if } x(n) \geq 0 \\ 0, & \text{if } x(n) < 0. \end{cases}$

1. $y(0) = x(0)$.
Static S/m.

2. non-linear s/m $x(n) \geq 0$
$x(n) \leq 0$ $\rightarrow$ it is non linera

3. $y(n-n_0) = x(n-n_0)$.
time invariant S/m.

4. causal s/m.

5. stable

l. $y(n) = x(-n)$.

1. $y(0) = x(-0)$
$y(1) = x(-1)$
dynamic s/m.

2. linear s/m

3. $y'(n)$ $y(n-n_0) = x(-(n-n_0)) = x(-n+n_0)$
doubt
$y(n) = x(-(n-n_0))$.

4. causal s/m

5. stable.

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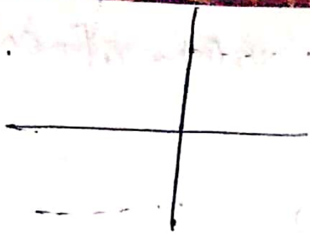
m. $y(n) = \text{Sign}[x(n)]$.

1. $y(0) = \text{Sign}[x(0)]$.



static S/m.

2. non linear
3. time invariant
4. causal S/m
5. stable.

n. The ideal sampling system with i/p $x_a(t)$ and o/p $x(n) = x_a(nT)$

$-\infty < n < \infty$ $\qquad$ $x(n) = x_a(nT)$.

1. $x(n) = x_a(nT)$.

2. linear

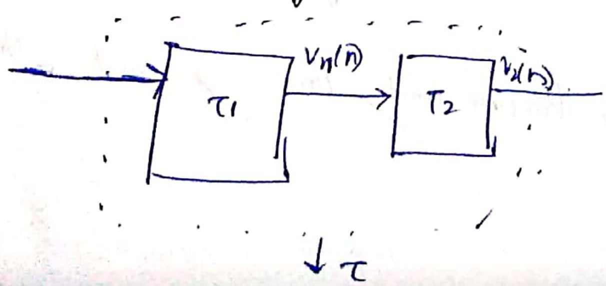
3. $x(n) = x_a(nT - n_0)$.

$x(n)$ = time invariant

4. Causal.
5. Stable.

Two discrete-time S/ms T_1 and T_2 are connected in cascade to form a new system T as shown in Fig. P2.8. Prove or disprove the following statements.

(a) If T_1 and T_2 are linear, the T is linear (i.e., The cascade connection of two linear system is linear).



$v_1(n) = T_1[x_1(n)]$ and $v_2(n) = T_1[x_2(n)]$

Then

$\alpha_1 x_1(n) + \alpha_2 x_2(n)$

yields

$\alpha_1 v_1(n) + \alpha_2 v_2(n)$.

$y_1(n) = T_2[v_1(n)]$ and

$y_2(n) = T_2[v_2(n)]$,

$y(n) = a\,y_1(n) + b\,y_2(n)$.

$= a\,v_1(n) + b\,v_2(n)$.

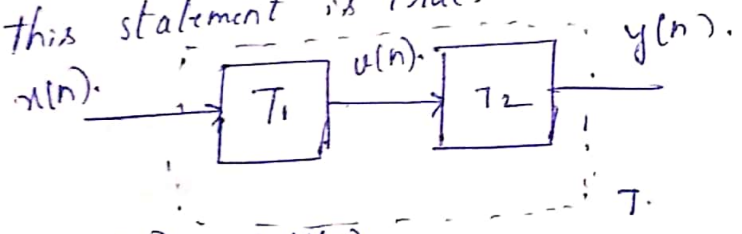
5

$a\,T[x_1(n)] + b\,T[x_2(n)]$ $\boxed{T = T_1 T_2}$

So The overall s/m is linear.

(b) If T_1 and T_2 are time invariant, then T is time invariant

this statement is true.



$x(n) \longrightarrow v(n)$.

$x(n-k) \longrightarrow v(n-k)$. for T_1

$v(n) \longrightarrow \underbrace{y(n). \quad v(n-k)}_{\text{for } T_2} \longrightarrow y(n-k)$

overall.

$x(n) \longrightarrow y(n)$.

$x(n-k) \rightarrow y(n-k)$.

$\therefore T = T_1 T_2$ is time invariant

(c). If T_1 and T_2 are causal, then T is causal.

True. T_1 is causal $\Rightarrow v(n)$ dependsonly $x(k)$ for $k \le n$. T_2 is causal n. Therefore, $y(n)$ depends only on $x(k)$ for $k \le n$. Hence, T is causal.

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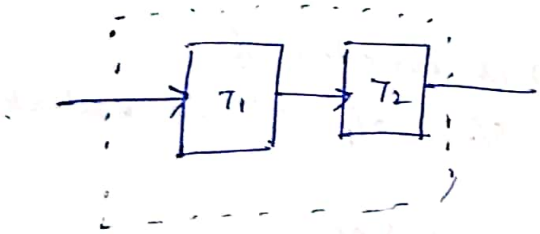
(d) If T_1 and T_2 are linear and time invariant, the same holds for T.



T.

proof from above b and c.

(e) If T_1 and T_2 are linear and time invariant, then interchanging their order does not change the S/m.

True. $h_1(n) * h_2(n) = h_2(n) * h_1(n)$.

(f) As in part (e) except that T_1, T_2 are now time varying.

(Hint: use an example).

It will not be possible.

Here in this case it

$T_1 : y(n) = n x(n)$ and

$T_2 : y(n) = n x(n+1)$.

here the two s/m s are not in time varying

Then

if $\delta(n)$ is the i/p.

$$T_2[T_1[\delta(n)]] = T_2[0] = 0.$$

$$T_1[T_2[\delta(n)]] = T_1[\delta(n+1)]$$

$$= -\delta(n+1)$$

$$\neq 0.$$

h. If T_1 and T_2 are stable, then T is stable.

$v(n)$ is bounded i/p $a(n)$ is bounded o/p.

$a(n)$ is input to the next S/m. o/p will be boubende
(bounded)

$x(n)$ bounded i/p. $y(n)$ is bounded o/p.

So overall S/m a stable s/m.

i. Show by an example that the inverses of parts (c) and (b) do not hold in general.

inverse of part (c) is

if T is causal then only T_1 and T_2 are causal.

if T is stable then only T_1 and T_2 are stable.

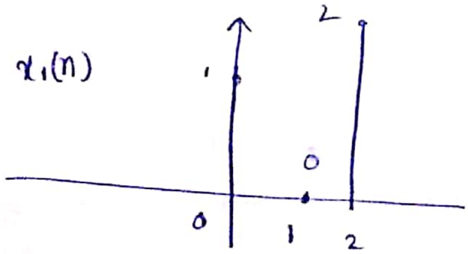
2.10. The following input-output pairs have been observed during operation of a time invariant s/m:

$x_1(n) = \{\underset{\uparrow}{1}, 0, 2\} \xleftrightarrow{T} y_1(n) = \{0, \underset{\uparrow}{1}, 2\}$

$x_2(n) = \{\underset{\uparrow}{0}, 0, 3\} \xleftrightarrow{T} y_2(n) = \{0, \underset{\uparrow}{1}, 0, 2\}$

$x_3(n) = \{\underset{\uparrow}{0}, 0, 0, 1\} \xleftrightarrow{T} y_3(n) = \{1, \underset{\uparrow}{2}, 1\}$

Can you draw any conclusions regarding the linearity the s/m. what is the impulse response of the s/m.



2.16

(b) Compute the convolution $y(n) = x(n) * h(n)$ of the following signals and check the correctness of the results by using the test in (a).

(1) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$

$h(n) \backslash x(n)$	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4.

$y[n] = \{1, 3, 7, 7, 7, 6, 4\}$.

the

(2). $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$.

$h(n) \backslash x(n)$	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1.

$y(n) = \{1, 4, 2, -4, 1\}$

(3) $x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{1/2, 1/2, 1, 1/2\}$

$h(n) \backslash x(n)$	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2
1/2	0	1/2	-1	3/2	-2
1	0	1	-2	3	-4
1/2	0	1/2	-1	3/2	-2

$y[n] = \{0, 1/2, -1/2, 3/2, 0, -5/2, -$

(4). $x(n) = \{1, 2, 3, 4, 5\}$, $h(n) = \{1\}$

$h(n)$ \ $x(n)$	1	2	3	4	5
1	1	2	3	4	5

5.) $x(n) = \{\underset{\uparrow}{1}, -2, 3\}$, $h(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$

$h(n)$ \ $x(n)$	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

$y[n] = \{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$

(6). $x(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$ $n=0$, $h(n) = \{1, \underset{\uparrow}{-2}, 3\}$ $n=0$

$h(n)$ \ $x(n)$	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

$y[n] = \{0, \underset{\uparrow}{0}, 1, -1, 2, 2, 1, 3\}$

(7). $x(n) = \{\underset{\uparrow}{0}, 1, 4, -3\}$, $h(n) = \{\underset{\uparrow}{1}, 0, -1, -1\}$.

$h(n)$ \ $x(n)$	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
1	0	1	4	-3.

$y[n] = \{\underset{\uparrow}{0}, 1, 4, -4, -3, 7, -3\}$.

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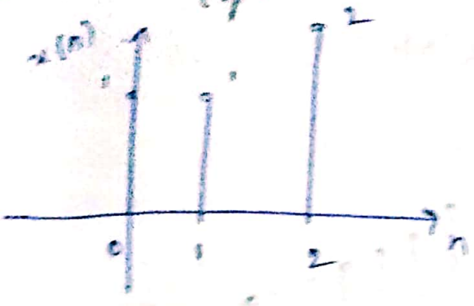
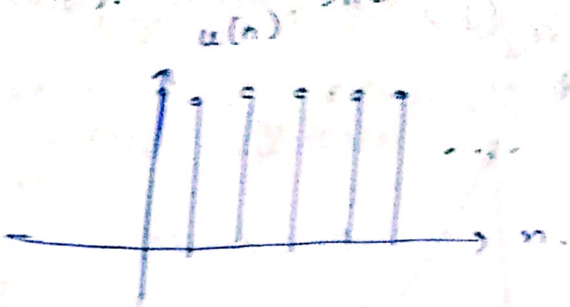
⑧ $x(n) = \{\underset{\uparrow}{1}, 1, 2\}$, $h(n) = u(n)$.





$n < 0 \quad y[n] = 0$

$n = 0 \quad y[n] = 1$

$n > 0 \quad \sum_{n=0}^{2} x[k] \cdot h[n-k]$

$1\times1 + 1\times1 + 2\times1$

$1+1+2 = 4$

$y[n] = 4$ for $n > 0$.

⑨ $x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}$, $h(n) = \{1, -2, -1, \underset{\uparrow}{4}\}$.

h(n) \ x(n)	1	1	0	1	1
1	1	1	0	1	1
−2	−2	−2	0	−2	−2
−1	−1	−1	0	−1	−1
4	4	4	0	4	4

$y[n] = \{1, -1, -5, 1, 2, \underset{\uparrow}{-5}, 1, 4\}$

⑩ $x(n) = \{1, 2, \underset{\uparrow}{0}, 2, 1\}$ $h(n) = x(n)$.

h(n) \ x(n)	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	1
0	0	0	0	0	0
2	2	4	0	4	1
1	1	2	0	2	1

$y[n] = \{1, 4, 4, 4, \underset{\uparrow}{10}, 3, 4, 3, 1\}$

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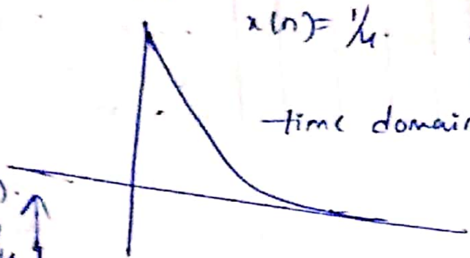
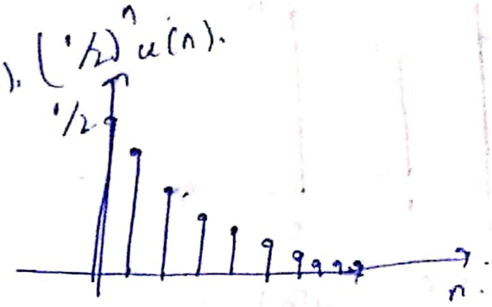
(11). $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$.

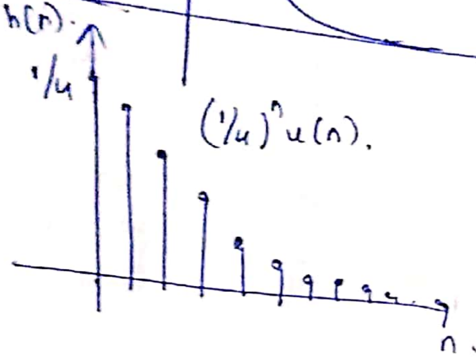
$n = 1$

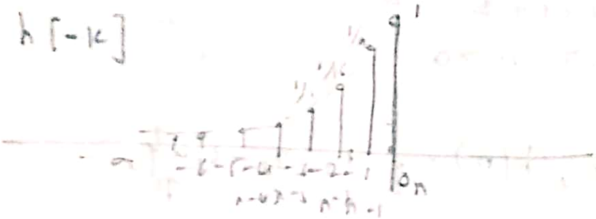
$x(n) = 1/2$ $n = 2$

$x(n) = 1/4$.









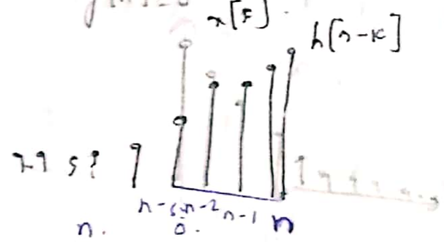
$n = 0$

$1 \times 1 = 1 = y[n]$

$n < 0$

$y[n] = 0$

$n > 0$



$$\sum_{n=0}^{n} (1/2)^n \cdot \left(\frac{1}{4}\right)^n \quad (1/2)^n \cdot \left(\frac{1}{2}\right)^{2n} \quad (1/2)^{3n}$$

for $n \geq 0$.

$y[n] = (1/2)^{3n}$.

for $n = 0$.

$y[n] = 1$.

for $n < 0$ $y[n] = 0$.

(a). If $y(n) = x(n) * h(n)$, show that $\mathcal{E}_y = \mathcal{E}_x \mathcal{E}_h$, where $\mathcal{E}_x = \sum_{n=-\infty}^{\infty} x(n)$.

$$y[n] = x(n) * h(n) = \sum_{n=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$$= \sum_{n=-\infty}^{\infty} h[k]\, x[n-k]$$

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$\sum_n y(n) = \sum_n \sum_k h(k) \cdot x(n-k) = \sum_k h(k) \cdot \boxed{\sum_{n=-\infty}^{\infty} x(n-k)}$

applying summation on both sides.

$= \sum_k h(k) \cdot \sum_n x(n)$.

$x(n-k) = x(n)$. [amplitudes are equal].

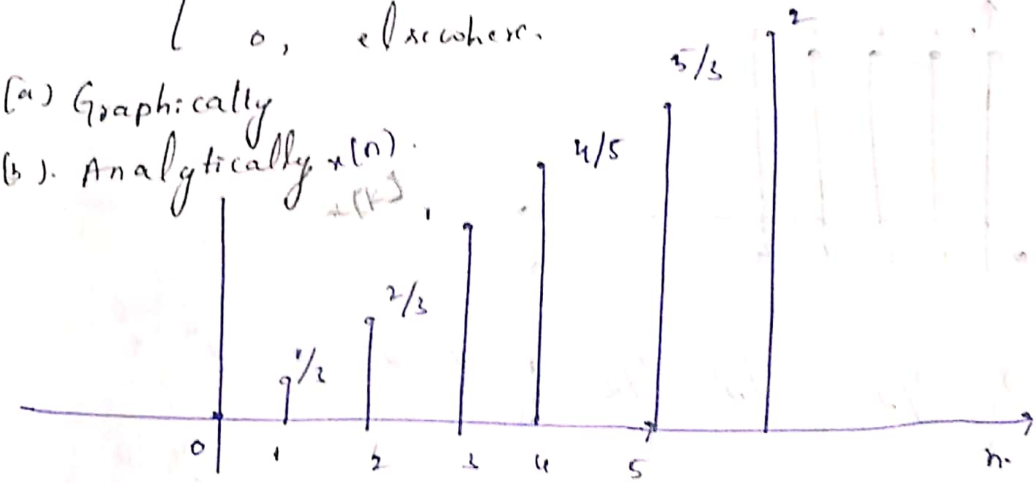
2.18. Determine and sketch the convolution $y(n)$ of the signals

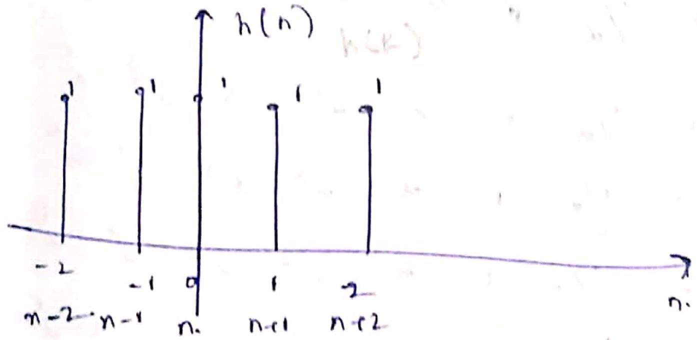
$$x(n) = \begin{cases} \frac{1}{3}n, & 0 \le n \le 6 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, & -2 \le n \le 2 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Graphically

(b). Analytically





$$y[n] = \left\{ \frac{1}{3}, 1, 2, \frac{10}{3}, \frac{15}{3}, \frac{20}{3}, 6, 5, \frac{11}{3}, 2 \right\}$$

if $n < 0$.

h(n) \ x(n)	1/3	2/3	1	4/3	5/3	2
1	1/3	2/3	1	4/3	5/3	2
1	1/3	2/3	1	4/3	5/3	2
1	1/3	2/3	1	4/3	5/3	2
1	1/3	2/3	1	4/3	5/3	2
1	1/3	2/3	1	4/3	5/3	2

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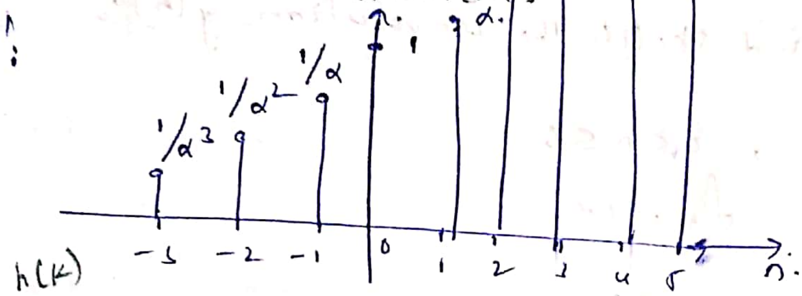
2.19. Compute the convolution $y(n)$ of the signals.

$$x(n) = \begin{cases} \alpha^n, & -3 \le n \le 5 \\ 0, & \text{elsewhere} \end{cases}$$

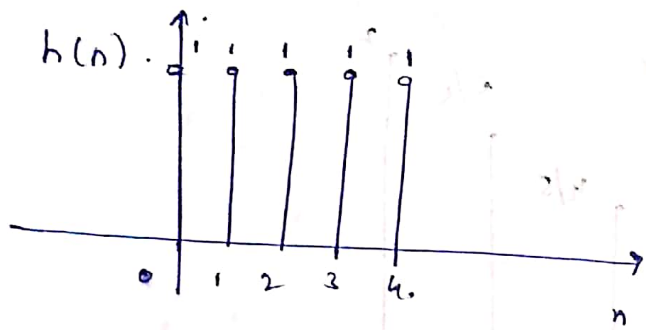
$$h(n) = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

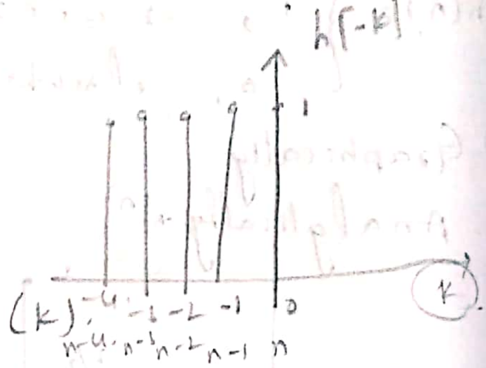
$\alpha^{-1} = 1/\alpha^3$ $\qquad \alpha^{-2} = 1/\alpha^2$

sol:



h(K)





h(n) \ x(n)	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
1	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5
	$1/\alpha^3$	$1/\alpha^2$	$1/\alpha$	1	α	α^2	α^3	α^4	α^5

$$y[n] = \left\{ \frac{1}{\alpha^3},\ \frac{1}{\alpha^2} + \frac{1}{\alpha^3},\ \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha},\ \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1,\ \underset{\uparrow}{\alpha + 1 + \frac{1}{\alpha} + \frac{1}{\alpha^2} + \frac{1}{\alpha^3}},\ \frac{1}{\alpha^3} + \frac{1}{\alpha^2} + \frac{1}{\alpha} + 1 + \alpha,\ \alpha^2 + \alpha + 1 + 1/\alpha + 1,\right.$$

$$1 + \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5,\ \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5,\ \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5,$$

$$\left. \alpha^3 + \alpha^4 + \alpha^5,\ \alpha^4 + \alpha^5,\ \alpha^5 \right\}.$$

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2-20 Consider the following three operations.

(a) Multiply the integer numbers : 131 and 122.

(b). Compute the convolution of signals : $\{1, 3, 1\} * \{1, 2, 2\}$.

(c) Multiply the polynomials : $1 + 3z + z^2$ and $1 + 2z + 2z^2$.

(d) Repeat part (a) for the numbers 1.31 and 12.2

(e). comment on your results.

(a). 15982

(b).

$h(n)$ \ $x(n)$	1	3	1
1	1	3	1
2	2	6	2
2	2	6	2

$y[n] = \{1, 5, 9, 8, 2\}$.

(c). $(1 + 3z + z^2) . (1 + 2z + 2z^2)$

$1 + 3z + z^2 + 2z + 6z^2 + 2z^3 + 2z^2 + 6z^3 + 2z^4$

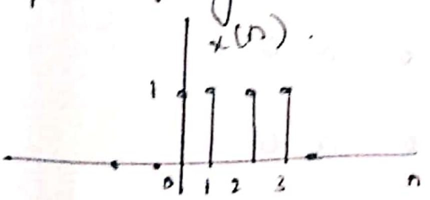
$= 9z^2 + 8z^3 + 2z^4 + 5z + 1$

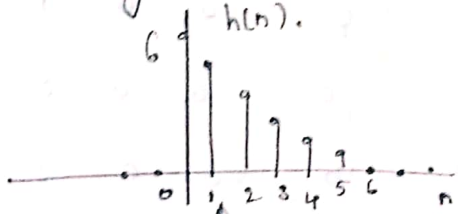
$\Rightarrow 2z^4 + 8z^3 + 9z^2 + 5z + 1$

(d). $1.31 \times 12.2 = 15.982$

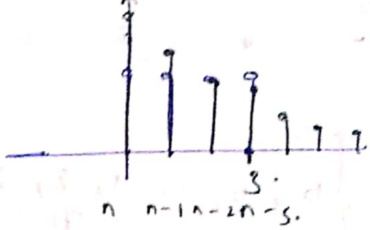
(e). There are different ways to perform convolution.

Compute and plot the convolution $x(n) * h(n)$ and $h(n) * x(n)$ the pairs of signals shown in Fig P2.17.





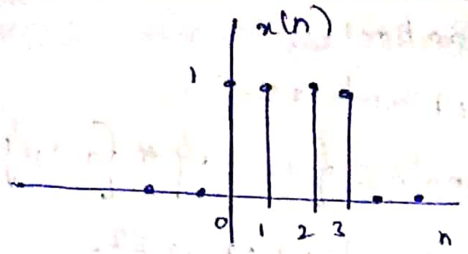
$n = 0$. $y[n] = 6$.

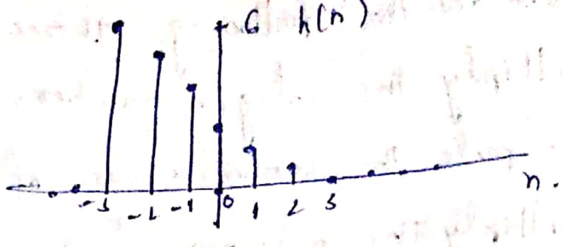


$n < 0$. $y[n] = 0$.

$n > 0$.

$$\sum_{n=0}^{n-3} x(k) . h(n-k).$$





for n < 0.

y[n] = 0.

for n = 0.

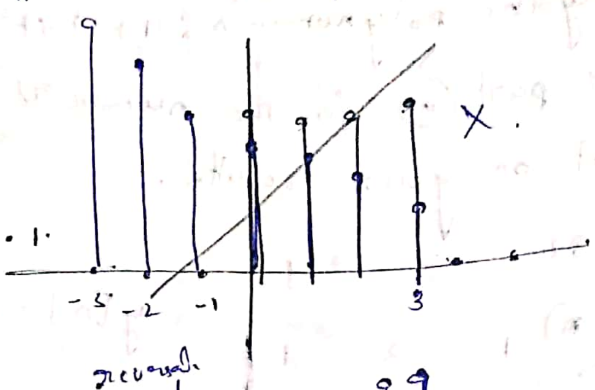
y[n] = h[0] · 1.

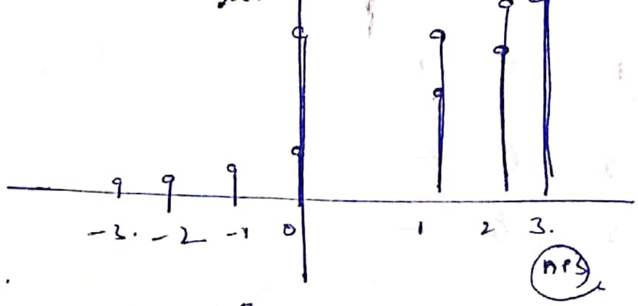
for n > 0.

(n+3) 3.

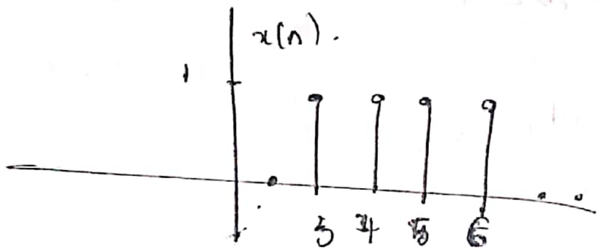
Σ

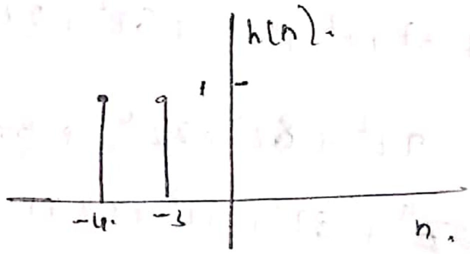
n = 0





$$y[n] = \sum_{n=0}^{n+3} h[n-k]\, x[k].$$





h(n) \ x(n)	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
1	0	0	0	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0.	0	0	0	0	0	0	0
–	0	0	0	0	0	0	0

$$y[n] = \{0, 0, 0, 1, \underset{\uparrow}{2}, 2, 2, 1, 0, 0, 0\}$$

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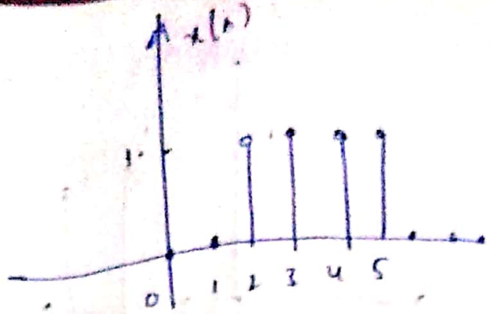
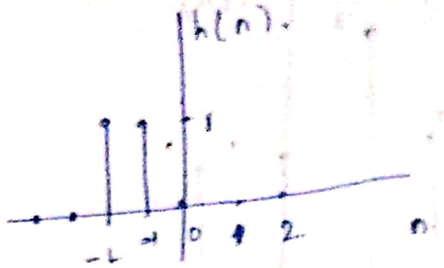




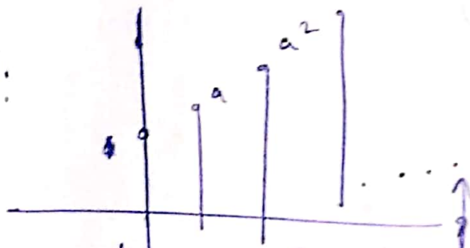
h(n) \ x(n)	0	0	1	1	1	1
1	0	0	1	1	1	1
1	0	0	1	1	1	1
0	0	0	0	0	0	0

$$y(n) = \{0, 0, \underset{\uparrow}{1}, 2, 2, 2, 1, 0\}.$$

2.21 Compute the convolution $y(n) = x(n) * h(n)$ of the following pairs of signals.

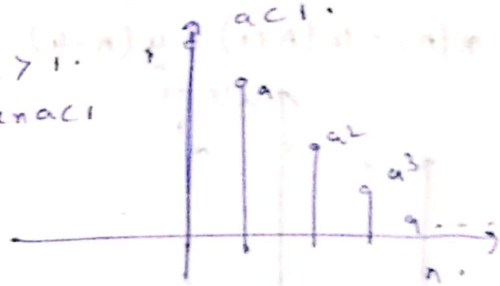
(a) $x(n) = a^n u(n)$, $h(n) = b^n u(n)$ where $a \neq b$ and when $a = b$.

Sol.

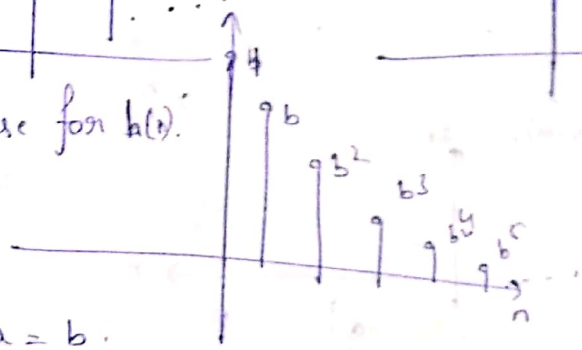


when $a > 1$.

when $a < 1$



Similar case for $h(n)$.



when $a = b$.

$n < 0$. $y[n] = 0$ $n \geq 0$.

here 0. $y[n] = 1$. $\quad y[n] = \sum_{n=0}^{n} a^k u(k) \cdot b^{n-k} u(n-k)$

$$= b^n \sum_{k=0}^{n} (ab^{-1})^k.$$

$a^k (b^{-1})^k$ or $a^k (b^k)^{-1}$ $(ab^{-1})^k$

$a = b$. $\quad \sum_{k=0}^{n} b^n \cdot a(n)$. $\boxed{(n)}$ when $a = b$.

(b). $x(n) = \begin{cases} 1, & n = -2, 0, 1 \\ 2, & n = -1 \\ 0, & \text{elsewhere} \end{cases}$ $\quad h[n] = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-3)$.

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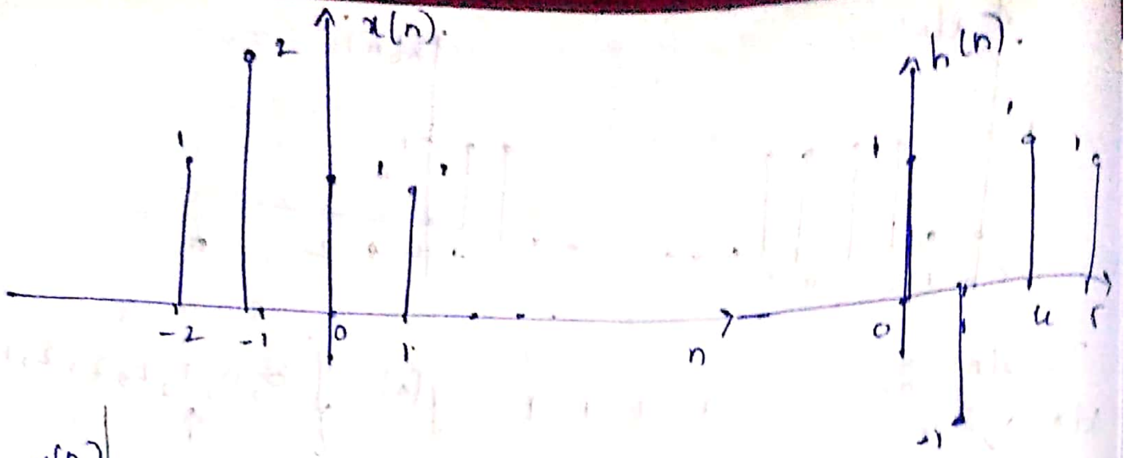
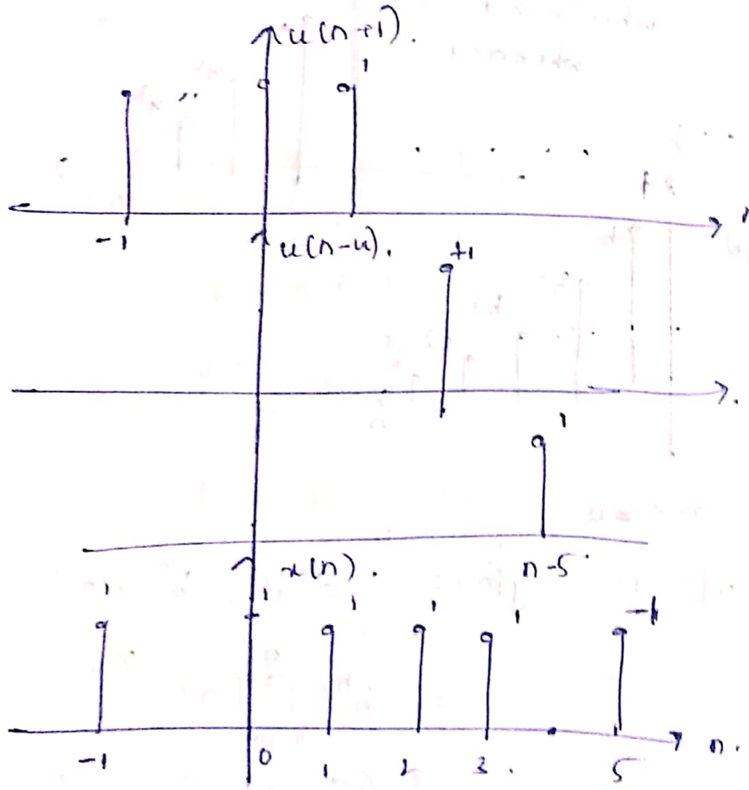


x(n) / h(n)	1	2	1	1
1	1	2	1	1
−1	−1	−2	−1	−1
1	1	2	1	1
1	1	2	1	1

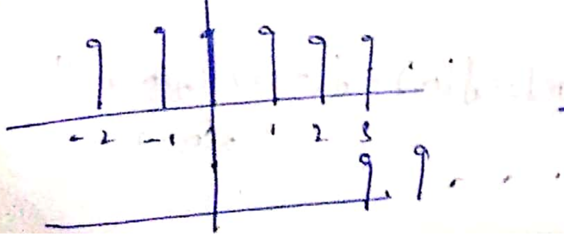
$y[n] = \{1, 1, \underset{\uparrow}{0}, 3, 2, 2, 1\}$

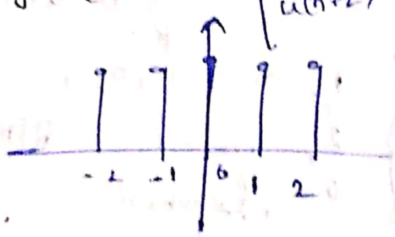
$\delta(n) =$

(c) $x(n) = u(n+1) - u(n-4) - \delta(n-5)$



$h(n) = [u(n+2) - u(n-3)] \cdot (3 - |n|)$ $\quad [u(n+2) - u(n-3)]$.





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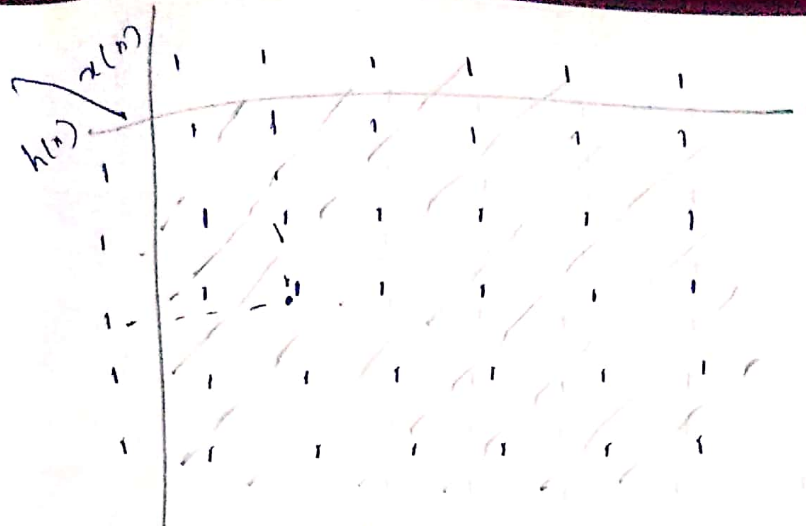
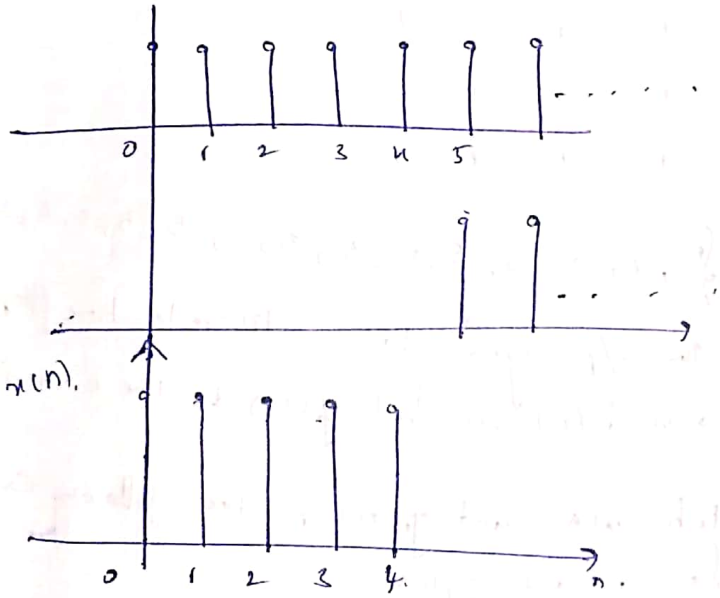
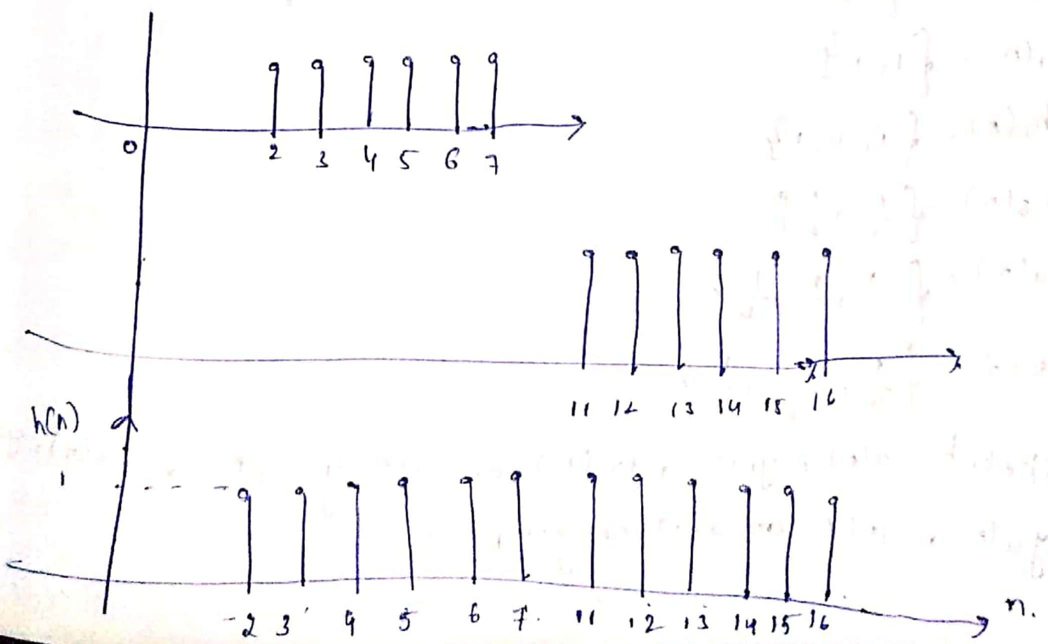


$$y[n] = \{1, 2, 3, \underset{\uparrow}{4}, 5, 6, 4, 3, 2, 1\}.$$

(d) $x(n) = u(n) - u(n-5)$.



$h(n) = u(n-2) - u(n-8) + u(n-11) - u(n-17)$.



$h(n) \backslash x(n)$	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1
1	1	1	1	1	1

$y[n] = \{1, 2, 3, 4, 5, 5, 5, 5, 5, 5, 5, 5, 4, 3, 2,$

2.22 Let $x(n)$ be the i/p signal to a discrete-time filter with impulse response $h_i(n)$ and let $y_i(n)$ be the corresponding output

(a) Compute and sketch $x(n)$ and $y_i(n)$ in the following case using the same scale in all figures.

$x(n) = \{1, 4, 2, 3, 5, 3, 3, 4, 5, 7, 6, 9\}$.

$h_1(n) = \{1, 1\}$

$h_2(n) = \{1, 2, 1\}$

$h_3(n) = \{\frac{1}{2}, \frac{1}{2}\}$

$h_4(n) = \{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\}$

$h_5(n) = \{\frac{1}{4}, -\frac{1}{2}, \frac{1}{4}\}$.

sketch $x(n)$, $y_1(n)$, $y_2(n)$ on one graph and $x(n)$, $y_3(n)$ $y_4(n)$, $y_5(n)$ on another graph.

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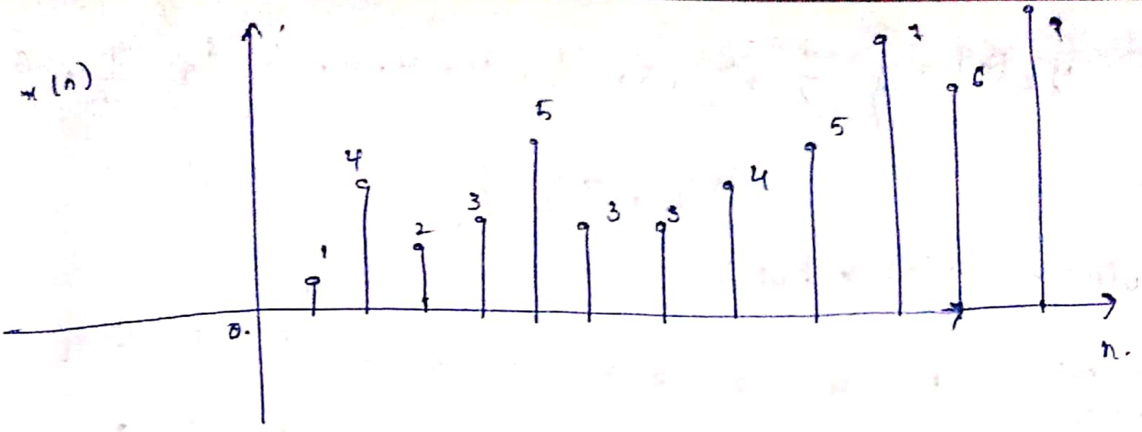

$y_1(n) = x(n) * h_1(n)$.

h(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9

$y_1(n) = \{1, 5, 6, 5, 8, 8, 6, 7, 9, 12, 13, 15, 9\}$.

$y_2(n) = x(n) * h_2(n)$.

x(n)	1	4	2	3	5	3	3	4	5	7	6	9
1	1	4	2	3	5	3	3	4	5	7	6	9
2	2	8	4	6	10	6	6	8	10	14	12	18
1	1	4	2	3	5	3	3	4	5	7	6	9.

$y_2(n) = \{1, 6, 11, 11, 13, 16, 14, 13, 16, 21, 25, 28, 24, 9\}$.

$y_3(n) = x(n) * y_3(n)$.

h(n) \ x(n)	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{2}$	$\frac{1}{2}$	2	1	3/2	5/2	3/2	3/2	2	5/2	7/2	3	9/2
$\frac{1}{2}$	1/2	2	1	3/2	5/2	3/2	3/2	2	5/2	7/2	3	9/2

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$$y_2[n] = \left\{ \frac{1}{2}, \frac{5}{2}, 3, 5/2, 4, 4, 4, 3, 7/2, \frac{9}{2}, 6, 15/2, 7/2 \right\}.$$

$$y_4(n) = x(n) * h_4(n)$$

$h(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{1}{2}$	$\frac{3}{4}$	$5/4$	$3/4$	$3/4$	1	$5/4$	$\frac{7}{4}$	$3/2$	$\frac{9}{4}$
$\frac{1}{2}$	$1/2$	2	1	$3/2$	$5/2$	$3/2$	$3/2$	2	$5/2$	$7/2$	3	$9/2$
$\frac{1}{4}$	$1/4$	1	$1/2$	$3/4$	$5/4$	$3/4$	$3/4$	1	$5/4$	$7/4$	$3/2$	$9/4$

$$y_4(n) = \left\{ \frac{1}{4}, \frac{3}{2}, \frac{11}{4}, \frac{11}{4}, \frac{13}{4}, 4, \frac{7}{2}, \frac{13}{4}, 4, \frac{21}{4}, \frac{25}{4}, 6, \frac{9}{4} \right\}.$$

$$y_5(n) = x(n) * h_5(n)$$

$h(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{3}{2}$	$\frac{9}{4}$
$\frac{-1}{2}$	$-\frac{1}{2}$	-2	-1	3 $\frac{-3}{2}$	$\frac{-5}{2}$	$\frac{-3}{2}$	$\frac{-3}{2}$	-2	$\frac{-5}{2}$	$\frac{-7}{2}$	-3	$\frac{-9}{2}$
$\frac{1}{4}$	$\frac{1}{4}$	1	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{5}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	1	$\frac{5}{4}$	$\frac{7}{4}$	$\frac{3}{2}$	$\frac{9}{4}$

$$y_5(n) = \left\{ \frac{1}{4}, 1/2, \frac{-5}{4}, \frac{3}{4}, \frac{1}{4}, -1, \frac{1}{2}, \frac{1}{4}, 0, \frac{1}{4}, \frac{-3}{4}, 1, -3, \frac{9}{4} \right\}.$$

(b) what is The difference between $y_1(n)$ and $y_2(n)$, and between $y_3(n)$ and $y_4(n)$?

$y_3(n) = \frac{1}{2} y_1(n)$ $\qquad$ $y_4(n) = \frac{1}{4} y_2(n)$.

$h_3(n) = \frac{1}{2} h_1(n)$. $\qquad$ $h_4(n) = \frac{1}{4} h_2(n)$.

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(c) Comment on the smoothness of $y_2(n)$ and $y_4(n)$. which factors aeffect the smoothness?

$y_2(n)$ and $y_4(n)$ are smoother than $y_1(n)$, but $y_4(n)$ will appear even smoother because of the smaller scale fator.

(d). Compare $y_4(n)$ with $y_5(n)$. what is the difference? can you explain?

$y_4(n)$ is smoother than $y_5(n)$. $h_5(0)$ is -ve. This results a non smoothe waveform.

(e). Let $h_6(n) = \left\{ \frac{1}{2}, -\frac{1}{2} \right\}$. Compute $y_6(n)$. Sketch $x(n), y_2(n)$ and $y_6(n)$ on the same figure and comment on the results.

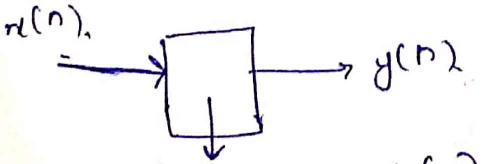
$h_6(n) = \left\{ \frac{1}{2}, -\frac{1}{2} \right\}$.

$h(n)$ \ $x(n)$	1	4	2	3	5	3	3	4	5	7	6	9
$\frac{1}{2}$	$\frac{1}{2}$	2	1	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$	$\frac{7}{2}$	3	$\frac{9}{2}$
$-\frac{1}{2}$	$-\frac{1}{2}$	-2	-1	$-\frac{3}{2}$	$-\frac{5}{2}$	$-\frac{3}{2}$	$-\frac{3}{2}$	-2	$-\frac{5}{2}$	$-\frac{7}{2}$	-3	$-\frac{9}{2}$

$y_6[n] = \left\{ \frac{1}{2}, \frac{3}{2}, -1, \frac{1}{2}, 1, -1, 0, \frac{5}{2}, \frac{1}{2}, 1, -\frac{1}{2}, \frac{3}{2}, -\frac{9}{2} \right\}$

$y_2(n)$ is smoother than $y_6(n)$.

2.23 Express the output $y(n)$ of a linear time-invariant system with impulse response $h(n)$ in terms of its step response $s(n) = h(n) * u(n)$ and the input $x(n)$.



here impulse response is interms of step response.

$s(n) = h(n) * u(n)$

$y(n) = x(n) * s(n)$.

$= x(n) * h(n) * u(n)$.

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2.24 The discrete-time system

$$y(n) = n y(n-1) + x(n), \quad n \geq 0.$$

is at rest [i.e., $y(-1) = 0$]. check if the system is linear invariant and BIBO stable

Sol:- linearity:-

$$y_1(n) = n y_1(n-1) + x_1(n).$$

$$y_2(n) = n y_2(n-1) + x_2(n).$$

$$a y_1(n) + b y_2(n) = a n y_1(n-1) + a x_1(n) + b n y_2(n-1) + b x_2(n)$$

$$x(n) = a x_1(n) + b x_2(n).$$

$$y(n) = n y(n-1) + a x_1(n) + b x_2(n).$$

$$y'(n) = \underbrace{a n y_1(n-1) + b n y_2(n-1)} + a x_1(n) + b x_2(n).$$

$$n \cdot (a y_1(n-1) + b y_2(n-1)) + a x_1(n) + b x_2(n)$$

The given s/m is linear.

time invariant:-

delaying i/p.

$$y(n) = n y(n-1) + x(n - n_0).$$

$n \geq 0.$

delay o/p.

$$y(n-n_0) = y'(n) = (n-n_0) y(n-1-n_0) + x(n-n_0).$$

$$y'(n) \neq y(n).$$

the given s/m is time variant s/m.

Stability:-

let $x(n) = u(n)$ is the bounded i/p. Then $|x(n)| \leq 1$.

for the bounded i/p o/p is

$$y(0) = 0 \cdot y(0-1) + u(0).$$

$y(0) = 1.$

$y(1) = 1 \cdot y(0) + u(1)$

$= 1 + 1. \Rightarrow y(1) = 2$

$y(2) = 2 \times y(1) + u(2)$

$= 2 \times 2 + 1$

$= 5 \ldots$

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2.26 Determine the zero-input response of the s/m described by the second-order difference equation.

$x(n) - 3y(n-1) - 4y(n-2) = 0.$

Sol:- given that we need to find the zero-i/p response

$x(n) = 0.$

$-3y(n-1) - 4y(n-2) = 0.$

$3y(n-1) + 4y(n-2) = 0$

$y(n-1) + \frac{4}{3}y(n-2) = 0.$

$y(n-1) = -\frac{4}{3}y(n-2).$

$n=0.$

$y(-1) = -\frac{4}{3}y(-2)$ — ①.

$y(0) = -\frac{4}{3}\ y(1-2).$

$y(0) = -\frac{4}{3} \cdot y(-1).$

$y(0) = -\frac{4}{3} \times -\frac{4}{3}\ y(-2).$ — ②.

$y(1) = -\frac{4}{3}y(0).$

$y(1) = \left(-\frac{4}{3}\right)^3 y(-2).$

$y(2-1).$

$\left(\frac{-4}{3}\right)$

$\vdots$

$y(k) = \left(-\frac{4}{3}\right)^{k+2} \cdot y(-2) \leftarrow$ zero-i/p response.

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2.27 Determine the particular solution of the difference equation

$$y(n) = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$$

when the forcing function is $x(n) = 2^n u(n)$.

Sol: continuous differentiation in time domain corresponds to difference in discrete domain.

for a differential non-homogeneous equation two solutions exist.

$$y(t) = y_c(t) + y_p(t)$$

$\downarrow$ Complementary solution. $\downarrow$ particular solution

Similarly for difference equation.

when $x(n)=0$.

$$y(n) - \frac{5}{6} y(n-1) + \frac{1}{6} y(n-2) = 0.$$

let $y[n] = c\lambda^n$ mathematical equation suitable to solve the difference equation.

$$c\left[\lambda^n + \frac{5}{6}\lambda^{n-1} + \frac{1}{6}\lambda^{n-2}\right]$$

$$c\lambda^{n-2}\left[\lambda^2 + \frac{5}{6}\lambda + \frac{1}{6}\right] = 0.$$

$$\lambda^2 + \frac{5}{6}\lambda + \frac{1}{6} = 0. \qquad \lambda_1 = \frac{1}{2} \quad \lambda_2 = +\frac{1}{3}.$$

$$y_h[n] = c_1\left(\frac{1}{2}\right)^n + c_2\left(+\frac{1}{3}\right)^n.$$

$$y_p(n) = k\,2^n u(n). \qquad x(n) = 2^n u(n).$$

$$k\,2^n u(n) - \frac{5}{6} 2^{n-1} u(n-1) + k\frac{1}{6}\cdot 2^{n-2} u(n-2) = 2^n u(n).$$

for $n=2$

$$4k - \frac{5}{\cancel{6}\,3} \times \cancel{2}k + k\cdot\frac{1}{6} = 2^2$$

$$4k - \frac{5}{3}k + \frac{k}{6} - 4 = 0. \qquad \frac{5}{2}k = 4 \qquad \underline{k = 8/5}$$

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$y(n) = y_p(n) + y_h(n) = \frac{8}{5} 2^n u(n) + C_1 \left(\frac{1}{2}\right)^n u(n) + C_2 \left(\frac{1}{3}\right)^n u(n)$

for -ve value $y(n) = 0$.

$y(0)$. These are the initial conditions.

$1 = \frac{8}{5} + C_1 + C_2$

$y(1) = \frac{8}{5} \times 2 + C_1 \frac{1}{2} + C_2 \frac{1}{3}$.

$y(1) = \frac{16}{5} + \frac{C_1}{2} + \frac{C_2}{3}$

$C_1 + C_2 = -\frac{3}{5}$ —(1).

$\frac{C_1}{2} + \frac{C_2}{3} + \frac{8}{5} = \frac{17}{6}$.

$3C_1 + 2C_2 = \left(\frac{17 \times 2}{6} - \frac{8}{5}\right) \cdot 6$.

$3C_1 + 2C_2 = \frac{27}{5}$ —(2).

solving eq ① and ②.

$3\cancel{c_1} + 3c_2 = \frac{-9}{5}$

$\cancel{3c_1} + 2c_2 = \frac{37}{5}$

$(-) \quad (-) \quad (-)$

$c_2 = \frac{-46}{5}$

$\cancel{2c_1 + 2c_2} = -\frac{6}{5}$

$3c_1 + \cancel{2}c_2 = \frac{37}{5}$

$(-) \quad (-) \quad (-)$

$\cancel{7}c_1 = \frac{\cancel{7}43}{5}$

$c_1 = \frac{43}{5}$

$y[n] = \frac{8}{5} 2^n u(n) + \frac{43}{5}\left(\frac{1}{2}\right)^n u(n) - \frac{46}{5}\left(\frac{1}{3}\right)^n u(n)$

2.30. Determine the response $y(n), n \geq 0$ of the system described by the second-order difference equation.

$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1).$

to the i/p $x(n) = 4^n u(n)$.

Sol: $y(n) - 3y(n-1) - 4.y(n-2) = 0.$

$y(n) = c\lambda^n.$

$c\lambda^n - 3c\lambda^{n-1} - c4\lambda^{n-2} = 0.$

$c(\lambda^{n-2})(\lambda^2 - 3\lambda - 4) = 0.$

$\lambda^2 - 3\lambda - 4 = 0. \qquad \lambda_1 = 4 \qquad \lambda_2 = -1.$

$y_h[n] = c_1.(4)^n + c_2(-1)^n.$

$y_h[n] = c_1(4)^n u(n) + c_2(-1)^n u(n)$

$\cancel{D - 3D - 4D = x(n) + 2x(n-1).}$

$= 4^n u(n) + 2.4^{n-1} u(n-1).$

$D - 3D - 4D = 4^n u(n) + 2 4^{n-1} u(n-1).$

for $n = 2$

$D - 3D - 4D = 16 + 2 \times 4^{2-1}$

$D - 3D - 4D = 16 + 2 \times 4$

$= 16 + 8.$

$D - 7D = 24$

$-6D = 24$

$D = \frac{24}{-6} = -4$

$nK4^n u(n) - 3K4^{n-1}(n-1) u(n-1) - 4K4^{n-2}(n-2) u(n-2)$

$= 4^n u(n) + 2.4^{n-1} u(n-1)$

for $n=2$

$2 \times 16K - \cancel{3}12K - \cancel{4K}$

$= 16 + 8.$

$(32K - 12K) = 16 + 8$

$K(20) = 24$

$K = \frac{24}{20} = \frac{6}{5}$

$K = 6/5$

$$y[n] = y_c[n] + y_p[n] = \left[\frac{6}{5} n 4^n + c_1 4^n + c_2 (-1)^n \right] u(n)$$

$y(0) = x(0)$ } initial conditions.

$x(0) = 4^0 u(0) = 1.$ $\quad x(1) = 4^1 u(1)$

$y(1) =$

$3y(0) + x(1) + 2x(0).$

$= 3 \cdot 1 + 4 + 2 \cdot 1$

$= 3 + 4 + 2 = 7 + 2 = 9$

$y(1) = 9.$

$1 = c_1 + c_2 \quad -①.$

$9 = \frac{6}{5} \times 1 \times 4 + c_1 4 - c_2$

$9 - \frac{24}{5} = 4c_1 - c_2$

$\frac{21}{5} = 4c_1 - c_2 \quad - ②.$

$1 = c_1 + c_2$

$\frac{21}{5} = 4c_1 - c_2$

$5c_1 = \frac{26}{5}$

$c_1 = \frac{26}{25}$.

$c_1 + c_2 = 1$

$c_2 = 1 - \frac{26}{25} = \frac{-1}{25}$

$4 \times \frac{26}{25} - c_2 = \frac{21}{5}$

$c_2 = \frac{-1}{25}$

$$y[n] = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25} (-1)^n \right] u(n).$$

Determine the impulse response of the following causal s/m:

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

Sol:- Impulse response means. i/p is impulse o/p is impulse response.

$x(n) = \delta(n)$.

$y_c[n] = c_1\, 4^n + c_2\, (-1)^n$.

$y_p[n] =$

$k \cdot n\,\delta[n] + (n-1)\,3 \cdot \delta[n-1] - k(n-2) \cdot 4 \cdot \delta[n-2] = \delta[n] + 2 \cdot \delta[n-2]$

for $n = 2$

$0 \cdot \quad \dfrac{-k(2-2)}{0} \quad 0 = \delta[n] + 2 \cdot \delta[n-2]$.

$\delta[n] \quad \underline{k = 0}$.

$y[n] = c_1 4^n + c_2 (-1)^n$.

$1 = c_1 + c_2 \quad -①$.

$y(0) = x(0)$

$y(0) = y(0) = 1$.

$y(1) = 3y(0) - 0 + \delta(1) + 2\delta(1-1)$

$= 3(1) + 0 + 2(0) = 5$.

$5 = c_1 4 - c_2 \quad ②$.

$c_1 + \cancel{c_2} = 1$

$4c_1 - \cancel{c_2} = 5$

$(+)$

$5c_1 = 6$

$c_1 = 6/5$

$c_2 = 1 - 6/5 = \frac{5-6}{5} = \frac{-1}{5}$

$$\boxed{y[n] = \left[\frac{6}{5} 4^n - \frac{1}{5}(-1)^n\right] u(n)}.$$

2.29 Determine the impulse response for the cased of two linear time-invariant systems having impulse responses,

$h_1[n] = a^n\left[u(n) - u(n-N)\right]$ and $h_2(n) = \left[u(n) - u(n-N)\right]$.

Sol:

$h_1[n] * h_2[n]$.

$x_1(n) \cdot \rightarrow \boxed{h_1[n]} \xrightarrow{x_2(n)} \boxed{h_2[n]} \xrightarrow{y_2(n)}$

$x_2(n) = y_1(n)$.

$y_2(n) = y_1(n) * h_2(n)$.

$y_1(n) = x_1(n) * h_1(n)$.

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$y_2(n) = x_1(n) * h_1(n) * h_2(n)$.

$x_1(n) = \delta(n)$.

$y_2[n] = \delta(n) * a^n[u(n) - u(n-N)] * [u(n) - u(n-M)]$

2.32 let $x(n)$, $N_1 \le n \le N_2$ and $h(n)$, $M_1 \le n \le M_2$ be two finite duration signals.

(a) Determine the range $L_1 \le n \le L_2$ of their convolution in terms of N_1, N_2, M_1 and M_2.

(b) Determine the limits of the case of partial overlaps from the left, full overlap, and partial overlap from the right. For convenience, assume that $h(n)$ has shorter duration than $x(n)$.

(c) Illustrate the validity of your results by computing the convolution of the signals.

$$x(n) = \begin{cases} 1 & -2 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 2, & -1 \le n \le 2 \\ 0, & \text{elsewhere.} \end{cases}$$

(a) $y(n) = x(n) * h(n)$ (o/p of convolution).

range will be

$N_1 + M_1 \le n \le N_2 + M_2$

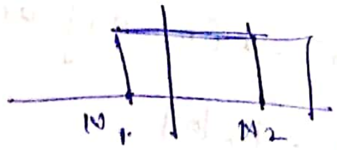
$L_1 = N_1 + M_1 \quad L_2 = N_2 + M_2$

(b).

$h(n)$ has shorter duration than $x(n)$.

partial overlap from the the left

low $N_1 + M_1$ high $N_1 + M_2 - 1$



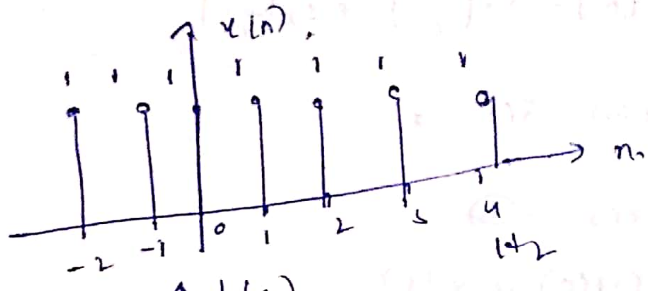
foull overalap

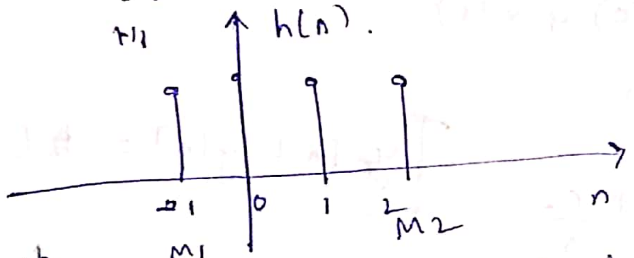
$N_1 + M_2$ $N_2 + M_1$ M_1 $N_2 + M_2 - 1$

Partial overlap from right

low $N_2 + M_1 + 1$ high $N_2 + M_2$

(c)
sol.





h(n) \ x(n)	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

$$y[n] = \{1, 2, 3, 4, 4, 4, 4, 3, 2, 1\}$$

$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$N_2 + M_2$

6

$l_1 = -3. \qquad l_2 = 6$

2023

Determine the impulse response and the unit step response of the s/ms described by the difference equation.

(a) $y(n) = 0.6\,y(n-1) - 0.08\,y(n-2) + x(n)$.

sol. $x(n) = \delta(n)$.

$y(n) - 0.6\,y(n-1) + 0.08\,y(n-2) = x(n)$.

$\lambda^2 - 0.6\lambda + 0.08 = 0. \qquad \lambda_1 = 2/5 \qquad \lambda_2 = \frac{1}{5}$

$y_c[n] = c_1\left(\frac{2}{5}\right)^n + c_2\left(\frac{1}{5}\right)^n.$

$y_p(n) = ? \qquad nk\,\delta[n] - (n-1)k\,0.6\,\delta[n-1] + 0.08\,k\,(n-2)\,\delta[n-2] = \delta[n]$

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for $n=2$.

$y_p[n]=0$.

$$y[n] = y_c[n] = c_1\left(\frac{2}{5}\right)^n + c_2\left(\frac{1}{5}\right)^n.$$

$y(0) = x(0) = \delta(0) = 1$.

$1 = c_1 + c_2$ — ①.

$y(1) = 0.6\,y(0) + x(1)$.

$= 0.6(1)$.

$$0.6 = \frac{2c_1}{5} + \frac{c_2}{5}$$

$$\boxed{y_k[n] = y[n] = 2\left(\frac{2}{5}\right)^n - \left(\frac{1}{5}\right)^n}$$

$2c_1 + c_2 = 3$ — ②.

$c_1 + \cancel{c_2} = 1$

$2c_1 + \cancel{c_2} = 3$

$(-)\ (-)\quad (-)$

$-c_1 = -2$

$c_1 = +2$

$c_2 = 1 - 2$

$c_2 = -1$.

$x(n) = u(n)$.

$nk\,u(n) - k(n-1)\,0.6\,u(n-1) + 0.8k(n-2)u(n-2) = u(n)$.

$y(0) = 1$.

$y(1) = 0.6 + 1 = 1.6$.

$n=2$

$2k - k\,0.6 + 0 = \frac{u(2)}{1}$.

$2k - 0.6k = 1$.

$1.4k = 1$

$k = \frac{1}{1.4} = \frac{5}{7}$

$$y[n] = \frac{5}{7}\,n\,u(n) + \left[c_1\left(\frac{2}{5}\right)^n + c_2\left(\frac{1}{5}\right)^n\right]u(n).$$

$1 = \frac{5}{7}\,0 + c_1 + c_2$ — ①.

$1.6 = \frac{5}{7} + c_1 \times \frac{2}{5} + c_2 \cdot \frac{1}{5}$

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$\frac{31}{35} =$

$\frac{31}{7} = 2C_1 + C_2$ — ①.

$2C_1 + \cancel{C_2} = \frac{31}{7}$

$C_1 + \cancel{C_2} = 1$

(−):

$C_1 = \frac{24}{7}$ $\quad C_2 = 1 - \frac{24}{7}$

$C_2 = -\frac{17}{7}$

$$y[n] = \left[\frac{5}{7} n + \frac{24}{7}\left(\frac{2}{8}\right)^n - \frac{17}{7}\left(\frac{1}{8}\right)^n\right] u(n).$$

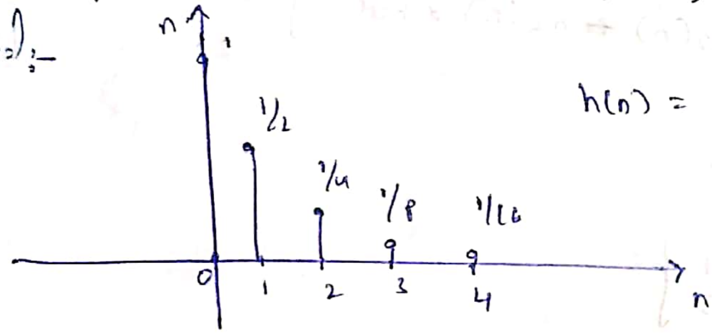
2.3.4 Consider a S/m with impulse response

$$h(n) = \begin{cases} \left(\frac{1}{2}\right)^n, & 0 \le n \le 4 \\ 0, & \text{elsewhere} \end{cases}$$

determine the i/p x(n) for $0 \le n \le 8$ that will generate the o/p sequence.

$y(n) = \{1, 2, 2.5, 3, 3, 3, 2, 1, 0, \ldots\}$.

Sol:-



$h(n) = \{1, 1/2, 1/4, 1/8, 1/16\}$.

$y(n) = x(n) * h(n)$.

$x(n) * h(n) = y(n)$.

$$y(n) = \sum_{n=-\infty}^{\infty} x(n) . h(n-k)$$

$x(n) \ast 1$

$x(0) \cdot h(0) = y(0) \Rightarrow x(0) = 1.$

$x(n)$

$\frac{1}{2} x(0) + x(1) = y(1) \Rightarrow x(1) = 3/2$

$$x(n) = \left\{1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}, \frac{3}{2} \ldots\right\}$$

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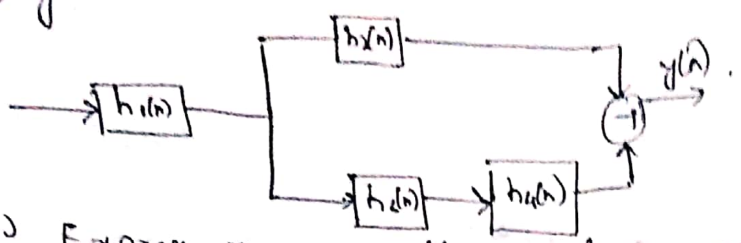
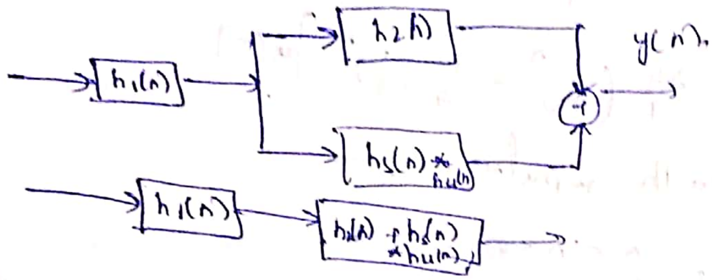
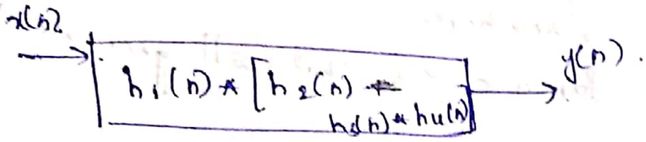
2.35

Consider a s/m with impulse respo LTI s/m as shown in fig.



(a) Express the overall impulse response in terms of $h_1(n)$, $h_2(n)$, $h_3(n)$, and $h_4(n)$.





impulse response

$$y(n) = h_1(n) * [h_2(n) + h_3(n) * h_4(n)] * x(n)$$

(b). determine $h(n)$ when

$$h_1(n) = \left\{ \frac{1}{2}, \frac{1}{4}, \frac{1}{2} \right\}$$

$$h_2(n) = h_3(n) = (n+1)\,u(n).$$

$$h_4(n) = \delta(n-2).$$

$$h(n) = h_3(n) * h_4(n) = (n+1)u(n) * \delta(n-2).$$

$$h(n).\ \left[\frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-1)\right] * \left[(n+1)u(n) - (n+1)u(n) * \delta(n-2)\right]$$

(C). determine the response of the s/m in part (b) if

$$x(n) = \delta(n+2) + 3\delta(n-1) - 4\delta(n-3).$$

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$x(n) = \{1, 3, -4\}$

$\{ \underset{-2}{1}, \underset{-1}{0}, \underset{\uparrow 0}{0}, \underset{1}{3}, \underset{2}{0}, \underset{3}{-4} \}$

$x(n) = \{1, 0, 0, 3, 0, -4\}$.

$y(n) = x(n) * h(n)$.

2.34 :-

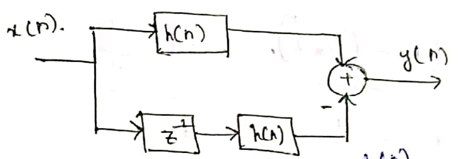
Sol:

$x(n)$ / $h(n)$	$x(0)$	$x(1)$	$x(2)$	$x(3)$	$x(4)$	$x(5)$	$x(6)$	$x(7)$	$x(8)$
$h(0)$	$x(0)h(0)$	$h(0)x(1)$	$h(0)x(2)$	$h(0)x(3)$	$h(0)x(4)$	$h(0)x(5)$	$h(0)x(6)$	$h(0)x(7)$	$h(0)x(8)$
$h(1)$	$h(1)x(0)$	$h(1)x(1)$	$h(1)x(2)$	$h(1)x(3)$	$h(1)x(4)$	$h(1)x(5)$	$h(1)x(6)$	$h(1)x(7)$	[illegible]
$h(2)$	$h(2)x(0)$	$h(2)x(1)$	$h(2)x(2)$	$h(2)x(3)$	$h(2)x(4)$	$h(2)x(5)$	$h(2)x(6)$	$h(2)x(7)$	[illegible]
$h(3)$	$h(3)x(0)$	$h(3)x(1)$	$h(3)x(2)$	$h(3)x(3)$	$h(3)x(4)$	$h(3)x(5)$	$h(3)x(6)$	$h(3)x(7)$	[illegible]
$h(4)$	$h(4)x(0)$	$h(4)x(1)$	$h(4)x(2)$	$h(4)x(3)$	$h(4)x(4)$	$h(4)x(5)$	$h(4)x(6)$	$h(4)x(7)$	[illegible]

$x(0)h(0) = y(0)$. $h(1)x(0) + h(0)x(1) = y(1)$.

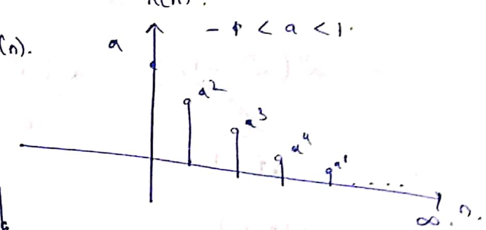
$x(0) = 1$.

2.36 Consider the s/m in Fig. P2.36 with $h(n) = a^n u(n)$, $-1 < a < 1$. Determine the response $y(n)$ of the s/m to the excitation $x(n) = u(n+5) - u(n-10)$.

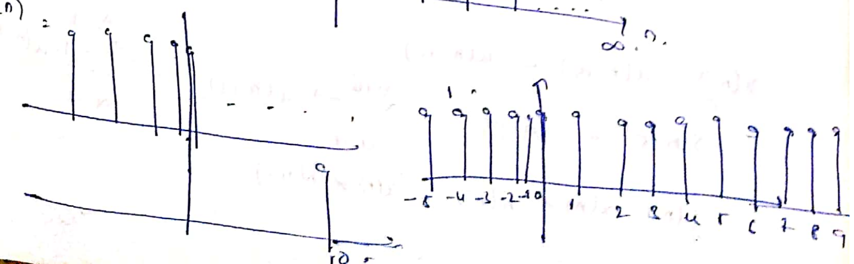


Sol:-

$h(n) = a^n u(n)$.



$x(n) =$



$\left[h(n) - \frac{1}{z^2} * h(n)\right]$

$y(n) = x(n) * \left[h(n) - \frac{1}{z^2} * h(n)\right]$

$= \left[u(n+5) - u(n-10)\right] * \left[a^n u(n) - \frac{1}{z^2} * a^n u(n)\right]$

$s(n) = u(n) * h(n).$

$s(n) = \sum_{k=0}^{\infty} u(k) h(n-k)$

$= \sum_{k=0}^{\infty} h(n-k) \Rightarrow \sum_{k=0}^{\infty} a^{n-k}$

$= a^n + a^{n-1} + a^{n-2} + a^{n-3} + \dots$

$\frac{a^{n-1}}{a^n} = \frac{a^n}{\frac{a}{a^n}} = \frac{\cancel{a^n}}{a \cdot \cancel{a^n}} = 1/a$

$r = 1/a.$

It seems to be a geometric series

$\frac{a(1-r^n)}{1-r}$

$= \frac{a^n\left(1 - (1/a)^n\right)}{1 - \frac{1}{a}} = \frac{a^n\left(1 - \frac{1}{a^n}\right)}{\frac{a-1}{a}}$

$= \frac{\frac{\cancel{a^n}(a^n - 1)}{\cancel{a^n}}}{\frac{a-1}{a}} = \frac{a(a^n - 1)}{a-1} = \frac{a^{n+1} - a}{a-1}$

$x(n) = u(n+5) - u(n-10).$

$s(n+5) - s(n-10) = \frac{a^{n+6} - a}{a-1} u(n+5) - \frac{a^{n-9} - 1}{a-1} u(n-10)$

$y(n) = x(n) * h(n) - x(n) * h(n-2)$

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$$n) = \frac{a^{n+6} - a}{a-1} u(n+4) - \frac{a^{n-9} - 1}{a-1} u(n-10)$$

$$- \frac{a^{n+4} - a}{a-1} u(n+2) + \frac{a^{n-11} - a}{a-1} u(n-12).$$

2.37 Compute and sketch the step response of the s/m

$$y(n) = \frac{1}{M} \sum_{k=0}^{M-1} x(n-k).$$

Sol:

$$x(n) = u(n).$$

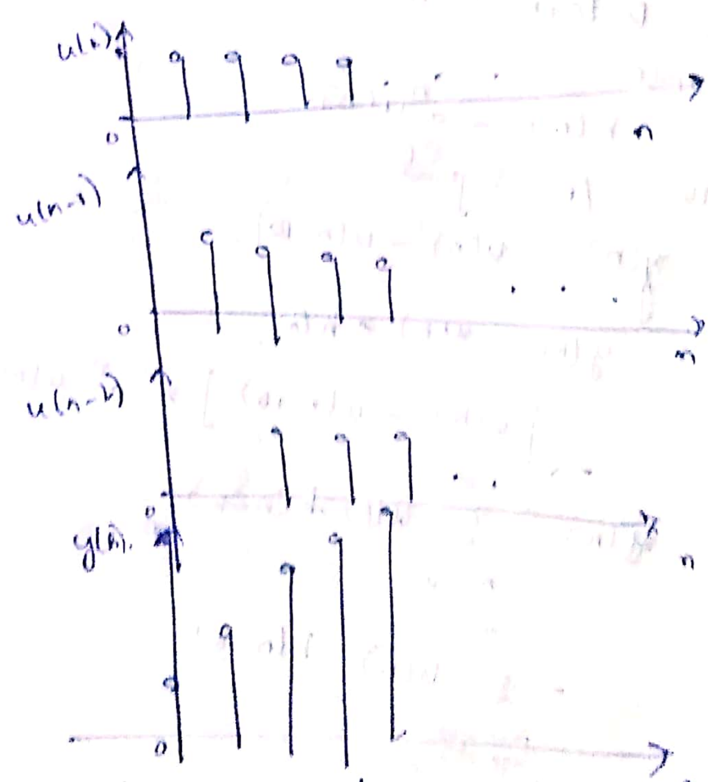
$$y(n) = h(n) * x(n).$$

$$y(n) = \frac{1}{m} \sum_{k=0}^{m-1} x(n-k)$$

$$= \frac{1}{m} \sum_{k=0}^{m-1} u(n-k)$$

$$= \frac{1}{m} \sum_{k=0}^{m-1} u(n-k) = \frac{1}{m} \Big[u(n) + u(n-1) + u(n-2) + u(n-3) + \dots + u(n-m+1). \Big]$$

$$= \frac{1}{m} \Big[$$



$$n) = \boxed{\frac{n+1}{m}}$$

2.38 Determine the range of values of the parameter a for which the linear time-invariant s/m with impulse response

$$h(n) = \begin{cases} a^n, & n \ge 0, n\text{ even}. \\ 0, & \text{otherwise} \end{cases}$$

is stable.

Sol:

given LTI s/m is stable.

$|x(n)| < \infty$

$|h(n)| < \infty$.

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0,\text{even}}^{\infty} |a|^n.$$

$$= \sum_{n=0}^{\infty} |a|^{2n}.$$ it denotes the even numbers,

$$= 1 + |a|^2 + |a|^4 + |a|^6 + \cdots$$

$$= \frac{1}{1 - |a|^2}$$

stable if $|a| < 1$.

2.39. Determine the response of the s/m with impulse response

$h(n) = a^n u(n)$.

to the i/p signal

$x(n) = u(n) - u(n-10)$.

Sol:

$y(n) = x(n) * h(n)$.

$= [u(n) - u(n-10).] * a^n u(n)$.

$$y(n) = \sum_{n=-\infty}^{\infty} x(k) \cdot h(n-k).$$

$$= \sum_{n=-\infty}^{\infty} u(k) \cdot h(n-k).$$

$$= \sum_{k=0}^{\infty} h(n-k).$$

$$= \sum_{k=0}^{\infty} a^{n-k} = a^n + a^{n-1} + a^{n-2} + \cdots$$

$$\frac{a^n}{a^{n-1}}$$

$$\frac{a^n\frac{1}{2} - (\frac{1}{a})^n a}{1 - \frac{1}{a}a} = \frac{a^n(\ldots)}{1-a}$$

$$\frac{a^{n-1}}{a^n} = \frac{a^n}{\frac{a^n}{a^n}} = \frac{1}{n} = 1$$

$$S_n = \frac{a^n\left(1-\left(\frac{1}{a}\right)^n\right)}{1-\frac{1}{a}} = \frac{\cancel{a^n}\left(a^n-1\right)/\cancel{a^n}}{\frac{a-1}{a}} = \frac{a(a^n-1)}{a-1}.$$

$$S_n = \frac{a^{n+1}-a}{a-1}$$ this is for $u(n)$.

$$u(n-10) = \frac{a^{n-10+1}-a}{a-1} = \frac{a^{n-9}-a}{a-1}.$$

$$y(n) = \frac{a^{n+1}-a}{a-1}u(n) - \frac{a^{n-9}-a}{a-1}u(n-10). = \cancel{\frac{a^{n+1}-a-a^{n-9}+a}{a-1}}$$

$$y(n) = \cancel{\frac{a^{n+1}-a^{n-9}}{a-1}}$$

2.40.

Determine the response of the (relaxed) s/m characterized by the impulse response

$h(n) = \left(\frac{1}{2}\right)^n u(n)$ to the i/p signal

$$x(n) = \begin{cases} 1, & 0 \le n < 10 \\ 0, & \text{otherwise} \end{cases}$$

Sol:

$$x(n) = u(n) - u(n-10).$$

$$y(n) = \frac{\left(\frac{1}{2}\right)^{n+1} - \frac{1}{2}}{\frac{1}{2}-1} u(n) - \frac{\left(\frac{1}{2}\right)^{n-9} - \frac{1}{2}}{\frac{1}{2}-1} u(n-10).$$

$$= \cancel{\frac{1}{2}}\, 2\left[\frac{2\left(\frac{1}{2}\right)^{n-9} - 1}{2} u(n-10) - \frac{2\left(\frac{1}{2}\right)^{n+1} - 1}{2} u(n)\right]$$

$$y(n) = \left(2\left(\frac{1}{2}\right)^{n-9} - 1\right)u(n-10) - \left(2\left(\frac{1}{2}\right)^{n+1} - 1\right)u(n)$$

$-a$

a

$\hat{u}$)

$\frac{1}{n}$

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2.41 Determine the response of the (relaxed) s/m characterized by the impulse response

$$h(n) = \left(\tfrac{1}{2}\right)^n u(n)$$ to the i/p signals

(a). $x(n) = 2^n u(n)$

(b) $x(n) = u(-n)$.

(a)

Sol:

$$h(n) = \left(\tfrac{1}{2}\right)^n u(n)$$

$$x(n) = 2^n u(n).$$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=0}^{\infty} 2^k \left(\tfrac{1}{2}\right)^{n-k}$$

$$= \sum_{k=0}^{\infty} 2^k\, 2^{-n+k} = \sum_{k=0}^{\infty} 2^{k-n+k} = \sum_{k=0}^{\infty} 2^{2k-n}$$

$$= 2^{0-n} + 2^{2-n} + 2^{4-n} + 2^{6-n} + \cdots$$

$$S_n = \frac{2^{-n}(1-4^n)}{1-4} \quad \frac{2^{2-n}}{2^{-n}} = \frac{2^2}{\frac{2^n}{\frac{1}{2^n}}} = \frac{2^2}{2^n} \times 2^n = 4.$$

(b).

$$y(n) = \sum_{k=-\infty}^{0} u(-k) \cdot \left(\tfrac{1}{2}\right)^{n-k}$$

$$= \quad u(-k) \quad 0. \quad \left(\tfrac{1}{2}\right)^{n-k} u(k) \cdot n$$

$$-\infty \qquad u(n-k).$$

$$k \geq 0. \quad = 0, \quad k \text{ to } \infty.$$

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2.42 Three S/ms with impulse responses $h_1(n) = \delta(n) - \delta(n-1)$, $h_2(n) = h(n)$, and $h_3(n) = u(n)$, are connected in cascade.

(a). what is the impulse response, $h_c(n)$ of the overall s/m.

$$h_c(n) = h_1(n) * h_2(n) * h_3(n).$$

$$= [\delta(n) - \delta(n-1)] * \cancel{h_2}(n) * u(n).$$

$$= [\cancel{\delta}u(n) - u(n-1)] * h(n).$$

$$= \delta(n) * h(n)$$

$$= h(n-0) = h(n).$$

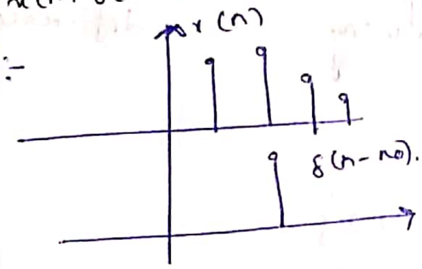
(b). Does the order of the inter connection affect the overall s/m.

NO.

2.43 (a). prove and explain graphically the difference between the relations

$x(n)\,\delta(n-n_0) = x(n_0)\cdot\delta(n-n_0)$ and $x(n) * \delta(n-n_0) = x(n-n_0)$.

sol:-



$x(n)\cdot\delta(n-n_0)$.

this gives the value of $x(n)$ at $n = n_0$

$x(n) * \delta(n-n_0) = x(n-n_0)$. This give the shifted version of sequence.

(b) show that a discrete-time system, which is described by a convolution summation, is LTI and relaxed.

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\cdot h(n-k) = \sum_{k=-\infty}^{\infty} h(k)\cdot x(n-k).$$

$$= h(n) * x(n).$$

linearity: $x_1(n) \rightarrow y_1(n) = h(n) * x_1(n)$

$x_2(n) \rightarrow y_2(n) = h(n) * x_2(n)$.

Then $a x_1(n) + \beta x_2(n) \rightarrow y(n) = h(n) * x(n)$.

$y(n) = h(n) * [a x_1(n) + \beta x_2(n)]$.

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$= a h(n) * x_1(n) + \beta h(n) * x_2(n)$

$y(n) = a . y_1(n) + \beta y_2(n)$

Time Invariant -

$x(n) \longleftrightarrow y(n) = h(n) * x(n)$

$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0).$

$= \sum_k h(k) . x(n-n_0-k).$

$= y(n-n_0).$

(c). what is the impulse response of the s/m described by y(n) $= x(n-n_0)$?

(c) $h(n) = \delta(n-n_0).$

2.45 Compute the zero-state response of the s/m described by the difference equation

$y(n) + \frac{1}{2} y(n-1) = x(n) + 2x(n-2)$ to the i/p.

$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 2, 1\}$ by solving the difference equation recursively

Sol:-

$x(n) = \{ \underset{-2}{1}, \underset{-1}{2}, \underset{\underset{0}{\uparrow}}{3}, \underset{1}{4}, \underset{2}{2}, \underset{3}{1} \}$

$y(-2) = x(-2) + 2x(-4) - \frac{1}{2} y(-3). = 1.$

$y(-1) = x(-1) + 2x(-3) - \frac{1}{2} y(-2). = 2 + 0 + \frac{1}{2} = 3/2$

$y(0) = x(0) + 2x(-2) - \frac{1}{2} y(-1). = 3 + 2(1) - \frac{1}{2}(3/2)$

$= 5 - \frac{3}{2} = \frac{10-3}{2} = \frac{7}{2}$

$y(1)$ $\qquad 5 - \frac{3}{4} \quad \frac{20-3}{4} = \frac{17}{4}$

$y(2)$

$y(3).$

2.46 determine the direct form II realization for each of the following LTI S/ms.

(a) $2y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-5)$

2.54 Compute and sketch the convolution $y_i(n)$ and correlation $r_i(n)$ sequence for the following pair of signals and comment on the results obtained.

(a) $x_1(n) = \{1, 2, 4\}$ $h_1(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

2.48 Consider the s/m described by the difference equation

$y(n) = ay(n-1) + bx(n)$

(a). Determine b in terms of a so that

$$\sum_{n=-\infty}^{\infty} h(n) = 1$$

$h(n) = ba^n u(n)$

$$\sum_{n=-\infty}^{\infty} b \cdot a^n u(n) = \sum_{n=0}^{\infty} b \cdot a^n = \frac{b}{1-a} = 1.$$

$$\frac{b}{1-a} = 1$$

$b = 1 - a.$

2.54.

(a). $x_1(n) = \{\underset{\uparrow 0}{1}, \underset{1}{2}, \underset{2}{4}\}$ $h_1(n) = \{\underset{\uparrow 0}{1}, \underset{1}{1}, \underset{2}{1}, \underset{3}{1}, \underset{4}{1}\}$. max = 4. min.

$h_1(n)$ \ $x_1(n)$	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

Convolution.

$y_1(n) = \{\underset{\uparrow}{1}, 3, 7, 7, 7, 6, 4\}$

Correlation.

$r_1(n) = \{1, 3, 7, 7, 7, 6, 4\}.$
0 0 1 2 3 4

(b) $x_2(n) = \{\underset{\uparrow}{0}, 1, -2, 3, -4\}$

$h_2(n) = \{\frac{1}{2}, 1, \underset{\uparrow}{2}, 1, \frac{1}{2}\}$

$h_2(n) \backslash x_2(n)$	1/2	1	2	1	1/2
0	0	0	0	0	0
1	1/2	1	2	1	1/2
-2	-1	-2	-4	-2	-1
3	3/2	3	6	3	3/2
-4	-2	-4	-8	-4	-2

reverse x.

Convolution:

$y_2(n) = \{0, 1/2, 0, 3/2, -2, 1/2, -6, -\frac{5}{2}, -2\}$.

Correlation

$r_2(n) = \{0, 1/2, 0, 3/2, -2, 1/2, -6, -5/2, -2\}$.

(c) $x_3(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $h_3(n) = \{\underset{\uparrow}{4}, 3, 2, 1\}$.

$h_3(n) \backslash x_3(n)$	1	2	3	4
4	4	8	12	16
3	3	6	9	12
2	2	4	6	8
1	1	2	3	4

Convolution

$y_3(n) = \{\underset{\uparrow}{4}, 11, 20, 30, 20, 11, 4\}$.

Correlation.

$h_3(n) \backslash x_3(n)$	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16.

$r_3(n) = \{1, 4, 10, \underset{\uparrow}{20}, 25, 24, 16\}$

d) $x_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$ $h_4(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$

$h_4(n) \backslash x_4(n)$	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

convolution.

$y_4(n) = \{\underset{\uparrow}{1}, 4, 10, 20, 25, 24, 16\}$.

Correlation:-

$h_4(n) \backslash x_4(n)$	1	2	3	4
4	4	8	12	16
3	3	6	9	12
2	2	4	6	8
1	1	2	3	4.

$r_4(n) = \{4, 11, 20, \underset{\uparrow}{30}, 20, 11, 4\}$

2.55. The zero-state response of a causal LTI S/m to the i/p $x(n) = \{1, 3, 3, 1\}$ is $y(n) = \{1, 4, 6, 4, 1\}$ Determine its impulse response.

Sol:

$h(n) \backslash x(n)$	1	3	3	1.
h_0	$h(0)x(0)$.	$h(0)x(1)$	$h(0)x(2)$	$h(0)x(3)$.
h_1.	$h(1)x(0)$	$h(1)x(1)$	$h(1)x(2)$	$h(1)x(3)$.

$h(1)x(0) + h(0)x(1) = 4$

$h(0)x(0) = 1$.

$h(0) = 1$.

$h(1)(1) + 1.(3) = 4$.

$h(1) = 4 - 3$

$h(1) = 1$.

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2.57. Determine the response $y(n)$, $n \geq 0$ of the s/m described by the second-order difference equation

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$ when the i/p is

$x(n) = (-1)^n u(n)$

and the initial conditions are $y(-1) = y(-2) = 0$.

sol: solving the difference equation solving for complementary solution

$c \cdot r^n - 4c \cdot r^{n-1} + 4c\, r^{n-2} = 0.$

$r^2 - 4r + 4 = 0.$

$r_1 = 2 \quad r_2 = 2$

$y_h(n) = c_1 2^n + c_2 n \cdot 2^n.$

$(c_1 + c_2 n) 2^n.$ as the roots are same.

$y_p(n) = k(-1)^n u(n).$ (particular solution).

$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-2) = x(n) - x(n-1).$

let $n=2$

$k + 4k + 4k = x(2) - x(1).$

$k(1+8) = x(2) - x(1).$

$9k = x(2) - x(1),$

$x(2) = (-1)^2 u(n) = 1.$ $\quad x(1) = -1.$

$9k = 2$

$k = 2/9$

$y[n] = y_h[n] + y_p[n] = \frac{2}{9}(-1)^n u(n) + c_1 2^n + c_2 n 2^n$

$y(0) = \frac{4y(0)}{0.} + 0 + x(0) - 0.$

$y(0) = x(0).$

$y(0) = 1.$ $\quad c_1 + c_2 = 1 - \frac{2}{9} = 7/9$

$1 = \frac{2}{9} + c_1 + c_2 \quad c_1 + c_2 = 7/9 \; - (1)$

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$y(1) = 4y(0) + x(1) - x(0).$

$= 4(1) + 1 - 1$

$= 4 - 2 = 2$

$\boxed{y(1) = 2}$

$2 = -\frac{2}{9} + 2c_1 + 2c_2$

$2(c_1 + c_2) = 2 + \frac{2}{9} = \frac{20}{9}$

$c_1 + c_2 = \frac{\cancel{20}^{10}}{9 \times \cancel{2}} \Rightarrow c_1 + c_2 = \frac{10}{9}$ — (2).

solving (1) and (2).

$c_1 = 7/9.$

$\cancel{c_1 + c_2} = \frac{10}{9}$

$c_2 = \frac{10}{9} - \frac{7}{9} = 3/9 = 1/3.$

~~$c_1 + c_2$~~

$c_2 = 1/3.$

2.58. Determine the impulse response $h(n)$ for the S/m described by the second-order difference equation.

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1).$

Sol:

$h(n) = [c_1 2^n + c_2 n 2^n] u(n).$

$y(0) = 1 \quad y(1) = 3.$

$c_1 = 1$

$2c_1 + 2c_2 = 3$

$c_2 = 1/2$

$h(n) = [2^n + \frac{1}{2} n 2^n] u(n).$

2.59. Show that any discrete-time signal $x(n)$ can be expressed as

$$x(n) = \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] u(n-k).$$

where $u(n-k)$ is a unit step delayed by k units in time, that is

$$u(n-k) = \begin{cases} 1, & n \geq k \\ 0, & \text{otherwise}. \end{cases}$$

Sol:

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$x(n) = x(n) * \delta(n)$

$= x(n) * [u(n) - u(n-1)]$

$= [x(n) - x(n-1)] * u(n)$

$= \sum_{k=-\infty}^{\infty} (x(k) - x(k-1)) u(n-k)$

2.60. show That the o/p of an LTI S/m can be expresse in terms of its unit step response s(n) as follows.

$$y(n) = \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [x(k) - x(k-1)] s(n-k).$$

Sol: Let h(n) be The impulse response of The S/m

$$s(k) = \sum_{m=-\infty}^{k} h(m)$$

$\Rightarrow$ $h(k) = s(k) - s(k-1).$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) x(n-k)$$

$$= \sum_{k=-\infty}^{\infty} [s(k) - s(k-1)] x(n-k).$$

2.61. compute The correlation sequences $r_{xx}(l)$ and $r_{xy}(l)$ for The following signal sequences.

$$x(n) = \begin{cases} 1, & n_0 - N \le n \le n_0 + N \\ 0, & \text{otherwise} \end{cases}$$

$$y(n) = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise.} \end{cases}$$

Sol:

$$x(n) = \begin{cases} 1, & n_0 - N \le n \le n_0 + N \\ 0, & \text{otherwise} \end{cases}$$

$$y(n) = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise.} \end{cases}$$

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$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n).x(n-l).$$

$(n_0 - N \leq n \leq n_0 + N) \quad (x(n)).$

$n_0 - N \leq n - l \leq n_0 + N$

$[n_0 - N + l \leq n \leq n_0 + N + l] \quad x(n-l)$

$x(n).x(n-l)$ summation is non zero value $2N+1-|l|$.

$-2N \leq l \leq 2N.$

ref

$$r_{xx}(l) = \begin{cases} 2N+1-|l| & -2N \leq l \leq 2N \\ 0, & \text{otherwise.} \end{cases}$$

$$r_{xy}(l) = \begin{cases} 2N+1-|l-n_0|, & n_0-2N \leq l \leq n_0+2N \\ 0, & \text{otherwise} \end{cases}$$

2.62 Determine the autocorrelation sequences of the following signals.

(a) $x(n) = \{ \underset{\uparrow}{1}, 2, 1, 1 \}.$

(b) $y(n) = \{ \underset{\uparrow}{1}, 1, 2, 1 \}$

what is your conclution

(a)

$x(n-l)$ \ $x(n)$	1	2	1	1
1	1	2	1	1
1	1	2	1	1
2	2	4	2	2
1	1	2	1	1

$r_{xx}(l) = \{ 1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1 \}.$

$r_{yy}(l) = \{ 1, 3, 5, \underset{\uparrow}{7}, 5, 3, 1 \}.$

(b)

$y(n)$ \ $y(n)$	1	1	2	1
1	1	1	2	1
2	2	2	4	2
1	1	1	2	1
1	1	1	2	1

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2.63 What is the normalised autocorrelation sequence of the signal $x(n)$ given by

$$x(n) = \begin{cases} 1, & -N \le n \le N \\ 0, & \text{otherwise.} \end{cases}$$

Sol:-

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$$

$$= \begin{cases} 2N+1-|l|, & -2N \le l \le 2N \\ 0, & \text{otherwise.} \end{cases}$$

$$r_{xx}(0) = 2N+1.$$

normalized autocorrelation is

$$\rho_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1-|l|), & -2N \le l \le 2N \\ 0, & \text{otherwise.} \end{cases}$$

2.64. An audio signal $s(t)$ generated by a loudspeaker is reflected at two different walls with reflection coefficients r_1 and r_2. The signal $x(t)$ recorded by a microphone close to the loudspeaker, after sampling, is

$$x(n) = s(n) + r_1 s(n-k_1) + r_2 s(n-k_2).$$

where k_1 and k_2 are the delays of the two echoes.

(a) Determine the autocorrelation $r_{xx}(l)$ of the signal $x(n)$.

Sol:

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)x(n-l)$$

$$= \sum_{n=-\infty}^{\infty} [s(n) + r_1(n-k_1) + r_2 s(n-k_2)] \cdot [s(n-l) + r_1(n-l-k_1) + r_2 s(n-l-k_2)]$$

(b) Can we obtain r_1, r_2, k_1 and k_2 by observing $r_{xx}(l)$?

Sol: $r_{xx}(l)$ has peaks at $l = 0, \pm k_1, \pm k_2$ and $\pm(k_1+k_2)$. r_2 and k_2 can be

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e of determined from other peaks.

(c) what happens if $r_2 = 0$?

If $r_2 = 0$, the peaks occur at $l = 0$ and $l = I k_1$ Th

it is easy to obtain q_1 and k_1

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