

MSCA31010: Linear & Non-Linear Models

Winter 2023 Assignment 1

Question 1 (50 points)

The Federal Reserve Act of 1913 created the Federal Reserve System, the central banking system of the United States. This law mandates that the Federal Reserve conduct monetary policy “to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates.”

Even though the act lists three distinct goals of monetary policy, the Fed's mandate for monetary policy is commonly known as the dual mandate. It is because when the people who want to work can get jobs and their wages are livable compared to their expenses, the interest rates will stay at moderate levels. However, policymakers cannot satisfy both mandates at the same time.

Economists have known that there is an inverse relationship between inflation and unemployment, described as the Phillips Curve, in honor of economist Mr. A.W. Phillips, who identified the relationship in 1958. The Phillips Curve exhibits the relationship between inflation and unemployment. When inflation is high, unemployment is low. Conversely, when inflation is low, unemployment levels increase.

There are two common measures of inflation. The first one is the Consumer Price Index (also known as the headline inflation index). The second one is the Personal Consumption Expenditure (this measure is favored by the Federal Open Market Committee in setting the federal fund rate). We provide you both measures and the weekly Initial Jobless Claims data in the `Economy_2020_to_2022.csv`.

1. Year: the reporting year of the measures
2. Month: the reporting month of the measures
3. N_Week: the number of weeks in the reporting month
4. PCEPI: Personal Consumption Expenditures, Monthly, Seasonally Adjusted
5. CPIAUCSL: Consumer Price Index for All Urban Consumers, Monthly, Seasonally Adjusted
6. ICSA_Week1: Initial Jobless Claims Number, First Week, Seasonally Adjusted
7. ICSA_Week2: Initial Jobless Claims Number, Second Week, Seasonally Adjusted
8. ICSA_Week3: Initial Jobless Claims Number, Third Week, Seasonally Adjusted
9. ICSA_Week4: Initial Jobless Claims Number, Fourth Week, Seasonally Adjusted
10. ICSA_Week5: Initial Jobless Claims Number, Fifth Week, Seasonally Adjusted (missing if the month does not have the fifth week)

We will use all the observations to determine whether the Phillips Curve still applies today.

- (a) (10 points). Generate a matrix of scatter plot (SPLOM) of these seven features: PCEPI, CPIAUCSL, ICSA_Week1, ICSA_Week2, ICSA_Week3, ICSA_Week4, and ICSA_Week5. You must properly label the axes and add grid lines to all the scatter plots.
- (b) (10 points). Calculate the Pearson correlations for each pair of the seven features. Display your result up to four decimal places appropriately as a matrix.
- (c) (10 points). Calculate the Spearman rank-order correlations for each pair of the seven features. Display your result up to four decimal places appropriately as a matrix.
- (d) (10 points). Calculate the Kendall's Tau-b correlations for each pair of the seven features. Display your result up to four decimal places appropriately as a matrix.
- (e) (10 points). Calculate the Distance correlations for each pair of the seven features. Display your result up to four decimal places appropriately as a matrix.

Question 2 (50 points)

The Babylonians used base 60 and a famous tablet to calculate the square root of two to about six decimal places around 1000 BC. It was an original application of the Newton-Raphson method to solve this equation $f(x) = x^2 - a$.

- a) (10 points). What is the first derivative of the function $f(x) = x^2 - a$ with respect of x ?
- b) (10 points). You will use the Newton-Raphson method to solve the equation $f(x) = x^2 - a = 0$.
What is the formula for updating the estimate?
- c) (10 points). Suppose $a = 9$ and the initial estimate is $x_0 = 1$. The iteration will converge if $|x_{k+1} - x_k| \leq 10^{-13}$. Please show the iteration history.
- d) (10 points). Suppose $a = 9000$ and the initial estimate is $x_0 = 1$. The iteration will converge if $|x_{k+1} - x_k| \leq 10^{-13}$. Please show the iteration history.
- e) (10 points). Suppose $a = 0.0000009$ and the initial estimate is $x_0 = 1$. The iteration will converge if $|x_{k+1} - x_k| \leq 10^{-13}$. Please show the iteration history.