**Question 1:**

*1) Solve $A\hat{x} = b$ by least squares (i.e., calculate $\hat{x}$).*

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\boxed{\hat{x} = \left(A^T A\right)^{-1} A^T b}$$

$$A^T = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{2x3} . \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}_{3x2}$$

$$= \begin{bmatrix} 1+0+1 & 0+0+1 \\ 0+0+1 & 0+1+1 \end{bmatrix}_{2x2}$$

$$A^T A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}_{2x2}$$

$$A^T B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}_{2x3} . \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}_{3x1}$$

$$= \begin{bmatrix} -1+0+0 \\ 0+1+0 \end{bmatrix}_{2x1}$$

$$A^T B = \begin{bmatrix} -1 \\ 1 \end{bmatrix}_{2x1}$$

we know that $A^{-1} = 1/\det(A) * \text{Adj. } A$

$$(A^T A)^{-1} = 1/3 . \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Therefore, $\hat{x} = (A^T A)^{-1} A^T B$

$$= 1/3 \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} . \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$=1/3 \begin{bmatrix} (2*-1) + (-1*1) \\ (-1*-1) + (2*1) \end{bmatrix}$$

$$=1/3 \begin{bmatrix} -3 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

**Solution :** $\hat{x} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

### Assignment 3

*Q1. (2) Verify your answer using Python.*

```python
import numpy as np

# Defining A and b
A = np.array([[1,0],
              [0,1],
              [1,1]])
b = np.array([[-1],[1],[0]])
```

```python
# Solving A.xHat = b by using xHat = Inv(ATrans.A). ATrans. b

xHat = np.dot((np.dot(np.linalg.inv(np.dot(A.T,A)),A.T)),b)
print(xHat)
```

```
[[-1.]
 [ 1.]]
```

**Question 2:**
*1) Is this a linear transformation? Explain your answer.*

$$T(x,y,z) = (z \cdot x+1, z, y)$$

A transformation is linear if

i) $T(u + v) = T(u) + T(v)$
ii) $T(cv) = cT(v)$

$u = [x_1, y_1, z_1]$ and $v = [x_2, y_2, z_2]$

**Property 1:**

LHS : $T(u+v) = T([x_1, y_1, z_1] + [x_2, y_2, z_2])$
$= T([x_1 + x_2, y_1 + y_2, , z_1 + z_2])$
$= [(z_1 + z_2)*( x_1 + x_2) + 1, z_1 + z_2, y_1 + y_2]$
LHS $=[ z_1 x_1 + z_1 x_2 + z_2 x_1 + z_2 x_2 + 1, z_1 + z_2, y_1 + y_2]$

RHS : $T(u) + T(v) = T([x_1, y_1, z_1]) + T([x_2, y_2, z_2])$

$= [z_1 x_1 + 1, z_1, y_1] + [z_2 x_2 + 1, z_2, y_2]$
$= [z_1 x_1 + 1 + z_2 x_2 + 1, z_1 + z_2, y_1 + y_2]$
RHS $= [z_1 x_1 + z_2 x_2 + 2, z_1 + z_2, y_1 + y_2]$

Since LHS is not equal to RHS $T(x,y,z) = (z*x+1,z,y)$ is does not satisfy the additive property

**Property 2:**

LHS : $T(cu) = T(c[x_1, y_1, z_1])$
$= T([cx_1, cy_1, cz_1])$
$= [cz_1 cx_1 + 1, cz_1, cy_1]$
LHS $= [c^2 z_1 x_1 + 1, cz_1, cy_1]$

RHS $= cT(u) = cT([x_1, y_1, z_1])$

$= c[z_1 x_1 + 1, z_1, y_1]$
RHS $= [cz_1 x_1 + c, cz_1, cy_1]$

Since LHS is not equal to RHS $T(x,y,z) = (z*x+1,z,y)$ does not satisfy the homogeneity property.

**Solution : Therefore, $T(x,y,z) = (z*x+1,z,y)$ is not a linear transformation**

**Question 3:**
*Suppose $T$ is the linear transformation from $R^2$ to $R^2$*

$$T(x,y) = (2x,3y)$$

*1) Find the matrix representation of the given linear transformation based on the standard basis.*

In the case of the Euclidean plane $R^2$ formed by the pairs (x, y) of real numbers, the standard basis is formed by the vectors

$$e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$T(x,y) = (2x, 3y)$

$$T(e_1) = \begin{bmatrix} 2*1 \\ 3*0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$T(e_2) = [2*0 \qquad = [0$

3*1]                    3]

Matrix representation T = [T($e_1$)  |  T($e_2$)]

**Solution :**      **A = [   2          0**
**0          3  ]**

*2)  Is T invertible? Explain your answer.*

Transformation T is said to be invertible if it has a unique inverse $T^{-1}$

We know that only if the determinant of a square matric is non zero, the inverse exists. That is |T| should be equal to 0.

|T| = (2*3) – (0*0) = 6 . Since the determinant value is not equal to 0, T is invertible and the inverse is = det(T) . Adj(T)

= 1/6 . [3        0                        = [1/2            0
0        2]                              0                  1/3]

**Solution :  Yes, T is invertible.**

*3)  Find the Kernel of T*

Kernel of T is the set of all inputs for which T(v) = 0 i.e., the null space

[ 2x      = [ 0
3y ]        0 ]
Since 2x = 0 and 3y = 0

**Solution :  The kernel of T is**    {(x, y)| (2x, 3y)  = (0, 0) } = **{(0, 0 )}**

*4)  Find the Range of T*

Range(T ) = {(z, w) | there exists an (x,y) such that (2       0      (x     = (z,w)}
0        3).    y)

z, w belong to $R^2$

z = 2x +0y,      w = 0x + 3y

x = z/2,           y = w/3

**Solution :**  For all z, w belongs to $R^2$, **the Range of T is (z/2,  w/3)**