Importing libraries

In [1]:
```
import numpy as np
from scipy import linalg as la
import sympy as sp

import sys
```

Question 1

Q:1

$3x + 6y = 1$
$6x + 12y = q$

1) Choose a q value which gives no solution.

   A system of linear equations gives no solution when they are parallel lines. Here, the co-efficient matrix is singular as the columns are dependent. Therefore, this system of equations has infinitely many solutions for any value that is not equal to 2.
   Result : q = -2 or -1 or 0 or 1 or 3 or 4 etc. (basically any value except 2)

2) Choose a q value which gives infinitely many solutions.

   For a system of linear equations to have infinitely many solution, they need to be overlapping lines(essentially the same line). In this case since equation 2 is nothing but 2*(equation 1), the value of q is 2*1.
   Result : q = 2

Question 2

Solve the following system of equations using Gaussian Elimination.

$2x + 3y + z = 12$

$-2x + 3y - 2z = 1$

$x - y + 4z = 16$

Rewrite as an augmented matrix

```
= [   [2    3    1    12]
      [-2   3    -2   1]
      [1    -1   4    16]   ]
```

L1/2 = L1 (Divide row 1 by 2)

```
= [   [1    1.5   0.5   6]
      [-2   3     -2    1]
      [1    -1    4     16]   ]
```

2L1 + L2 = L2 (Multiply row 1 by 2 and add it to row 2)
L3 – L1 = L3 (Subtract row 1 from row 3)

```
= [   [1    1.5    0.5   6]
      [0    6      -1    13]
      [0    -2.5   3.5   10]   ]
```

L2/6 = L2 (Divide row 2 by 6)

```
= [   [1    1.5    0.5    6]
      [0    1      -1/6   13/6]
      [0    -2.5   3.5    10]   ]
```

L3 + 2.5*L2 = L3 (Multiply L2 by 2.5 and add it to L3)

```
= [   [1    1.5    0.5    6]
      [0    1      -1/6   13/6]
      [0    0      37/12  185/12]   ]
```

L3/(37/12) = L3 (Divide L3 by 37/12)

```
= [   [1    1.5    0.5    6]
      [0    1      -1/6   13/6]
      [0    0      1      5]   ]
```

We can now solve this upper triangular matrix by back substitution
z = 5
y – z/6 = 13/6
Substitute z=5 in equation 2 and multiply both sides by 6)
6y – 5 = 13
Solve for y
y = (13+5)/6 = 18/6 = 3
y = 3
x+1.5*y + 0.5*z = 6
Substitute z=5, y=3 in equation 1
x + 4.5 + 2.5 = 6
Solve for x
x = 6 – 7 = -1
x = -1

Result : x = -1,   y = 3,   z = 5

Python validation

In [2]:

```python
#function to solve system of equations using Guassian elimination

n = int(input('Enter number of unknowns: '))
a = np.zeros((n,n+1))
x = np.zeros(n)
print('Enter Augmented Matrix Coefficients:')
for i in range(n):
    for j in range(n+1):
        a[i][j] = float(input( 'a['+str(i)+']['+ str(j)+']='))
for i in range(n):
    if a[i][i] == 0.0:
        sys.exit('Divide by zero detected!')

    for j in range(i+1, n):
        ratio = a[j][i]/a[i][i]

        for k in range(n+1):
            a[j][k] = a[j][k] - ratio * a[i][k]

x[n-1] = a[n-1][n]/a[n-1][n-1]

for i in range(n-2,-1,-1):
    x[i] = a[i][n]

    for j in range(i+1,n):
        x[i] = x[i] - a[i][j]*x[j]
```

```
    x[i] = x[i]/a[i][i]

print('\nThe solution is: ')
for i in range(n):
    print('X%d = %0.2f' %(i,x[i]), end = '\t')
```

```
Enter number of unknowns: 3
Enter Augmented Matrix Coefficients:
a[0][0]=2
a[0][1]=3
a[0][2]=1
a[0][3]=12
a[1][0]=-2
a[1][1]=3
a[1][2]=-2
a[1][3]=1
a[2][0]=1
a[2][1]=-1
a[2][2]=4
a[2][3]=16

The solution is:
X0 = -1.00	X1 = 3.00	X2 = 5.00
```

Question 3

Find the rank of each of the following matrices.

a) [[1 3 1 2 0]
[0 0 2 1 3]
[0 0 0 3 2]
[0 0 0 3 -1]]

L3 – L4 = L4

= [[1 3 1 2 0]
[0 0 2 1 3]
[0 0 0 3 2]
[0 0 0 0 3]]

The number of non-zero rows in the row echelon form in 4
Hence, **the rank of the matrix is 4**

In [3]:
```
#a)
A = np.array([[1,3,1,2,0],
              [0,0,2,1,3],
              [0,0,0,3,2],
              [0,0,0,3,-1]])

# find the reduced row echelon form
print("Reduced row echelon form :", sp.Matrix(A).rref())
print('\n')

# find the rank of matrix
print("a) Rank of matrix A :",sp.Matrix(A).rank())
```

```
Reduced row echelon form : (Matrix([
[1, 3, 0, 0, 0],
[0, 0, 1, 0, 0],
[0, 0, 0, 1, 0],
[0, 0, 0, 0, 1]]), (0, 2, 3, 4))


a) Rank of matrix A : 4
```

b) [[-1 1 0 -1] [-2 2 1 -4] [-1 1 -2 3]]

-2*L1 + L2 = L2
L3 - L1 = L3

= [[-1 1 0 -1] [0 0 1 -2] [0 0 -2 4]]

2*L2 + L3 = L3

= [[-1 1 0 -1] [0 0 1 -2] [0 0 0 0]]

The number of non-zero rows in the row echelon form in 2
Hence, **the rank of the matrix is 2**

In [4]:

```python
#b)
A = np.array([[-1,1,0,-1],
              [-2,2,1,-4],
              [-1,1,-2,3]])

# find the reduced row echelon form
print("Reduced row echelon form :", sp.Matrix(A).rref())
print('\n')

# find the rank of matrix
print("b) Rank of matrix B :",sp.Matrix(A).rank())
```

```
Reduced row echelon form : (Matrix([
[1, -1, 0,  1],
[0,  0, 1, -2],
[0,  0, 0,  0]]), (0, 2))


b) Rank of matrix B : 2
```