Probability Assessment

Basic Probability

Q1) A coin is tossed once. What is the probability of getting head?

→ P(H) = $\frac{\text{No. of favorable outcomes}}{\text{Total possible outcome}} = \frac{1}{2} = 0.5$ or 50%

2) A dice is rolled once. What is the probability of getting a number > 4?

→ Fav S = {5, 6} out of Total = {1, 2, 3, 4, 5, 6}

P = $\frac{2}{6} = \frac{1}{3} = 33.33\%$.

3) Conditional probability

~~A card is drawn from a standard deck.~~

3) A coin is tossed twice. What is probability of getting heads on the first toss given that second toss is a tail?

→ Possible outcomes :- {HH, HT, TH, TT}

Second toss is tail :- {HT, TT}

First toss is heads {HT}

∴ P(1st toss is head / 2nd toss is tail) = $\frac{1}{2}$.

4) A card is drawn from a standard deck. Given card is red what is probability that it is a heart?

→ Total deck size = 52.

No. of red cards = 26. (Heart + Diamonds)

No. of heart = 13.

P(Heart | Red) = $\frac{13}{26} = \frac{1}{2}$.

5) A die is rolled. Given that the number is even, what is probability it is a 2?

→ Even outcomes : {2, 4, 6}

P(getting a 2 | even numbers) = $\frac{1}{3}$

Bayes Theorem - Class

$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$

6) There are 2 bags:-
- Bag 1 contains fair coin
- Bag 2 contains double-headed coin.

One bag is chosen at random & a coin is tossed. The result is head. What is the probability the coin was from Bag 2

→ $P(\text{Bag 2} | \text{Head}) = \frac{P(\text{Head}|\text{Bag 2}) \cdot P(\text{Bag 2})}{P(\text{Head})}$

$P(\text{Bag 2}) = 1/2$ (there are 2 bags)

$P(\text{Head}|\text{Bag 2}) = 1$ (double-headed coin)

$P(\text{Head}) = P(\text{Head}|\text{Bag 1}) \cdot P(\text{Bag 1}) + P(\text{Head}|\text{Bag 2}) \cdot P(\text{Bag 2})$

$= \frac{1}{2} \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \frac{1}{4} + \frac{1}{2} = \frac{1+2}{4} = \frac{3}{4}$ (concept of Total probability)

$P(\text{Bag 2}|\text{Head}) = \frac{1 \cdot 1/2}{3/4} = \frac{\frac{1}{2}}{\frac{3}{4}} = \frac{2}{3} \simeq 66.67\%$

7) A disease affects 1 in 1000 people. A test for the disease is 99% accurate. If a person tests +ve, what is the probability they actually have a disease?

→ $P(\text{Disease}|\text{Test result +ve}) = \frac{P(\text{Test +ve}|\text{Disease}) \cdot P(\text{Disease})}{P(\text{Test +ve})}$

$P(\text{Disease}) = \frac{1}{1000} = 0.001$

$P(\text{No disease}) = 1 - 0.001 = 0.999$

$P(\text{+ve test}|\text{Disease}) = 0.99$; $P(\text{+ve test}|\text{No disease}) = 1 - 0.99 = 0.01$

$P(\text{+ve test}) = P(\text{+ve test}|\text{Disease}) \cdot P(\text{Disease}) + P(\text{+ve Test}|\text{No disease}) \cdot P(\text{No disease})$

$= 0.99 \times 0.001 + 0.01 \times 0.999 = 0.01098$

$P(\text{Disease}|\text{+ve test}) = \frac{0.99 \times 0.001}{0.01098} = \frac{0.00099}{0.01098} = 0.0901 \simeq 9.01\%$

8. A weather forecast says there is 70% chance of rain. If it rains, there is a 90% chance the forecast was correct. If it doesn't rain, there's a 20% chance the forecast was wrong. Given that it rained, what is the probability forecast was correct.

→ P(Forecast) = 0.7

P(Correct / Rain) = 90% (already given)

9) 70% of the student prepare for an exam. Among prepared students, 90% pass. Among unprepared students, only 30% pass. If a student passes, what is probability they prepared?

→ P - prepared

$P(P) = 0.7$

$P(\bar{P}) = 1 - 0.7 = 0.3$ (not prepared)

$P(Pass / P) = 0.9$

$P(Pass / \bar{P}) = 0.3$

$$P(P / Pass) = \frac{P(Pass / P) \times P(P)}{P(Pass)}$$

$$P(Pass) = P(Pass / P) \times P(P) + P(Pass / \bar{P}) \times P(\bar{P})$$
$$= 0.9 \times 0.7 + 0.3 \times 0.3$$
$$= 0.72$$

$$P(P / Pass) = \frac{0.9 \times 0.7}{0.72} = 0.875 = 87.5\%$$

10) In a class 60% are boys, 40% are girls. 70% of boys & 50% of girls like maths. If a student is randomly selected & is known to like maths, what is the probability student is a boy?

→ $P(B) = 0.6$, $P(G) = 0.4$

$P(Likes\ maths / B) = 0.7$, $P(Likes\ maths / G) = 0.5$

$$P(Boy / Likes\ maths) = \frac{P(Likes\ maths / B) \times P(Boy)}{P(Likes\ maths)} = \frac{0.7 \times 0.5}{(0.7 \times 0.6) + (0.5 \times 0.4)} = 0.677 = 67.7\%$$