G.H Raisoni College of Engineering and Management, Pune

F.Y B.Tech (Engineering)
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Department - Information Technology (IT)
Term/Section - Term I
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Subject Name - Mathematics
Roll No - C70 Name - Swayam Pramod Terode.

CO3 a) State Leibnitz Theorem.

Answer: The derivative on n^{th} order of the product of two functions can be expressed by formula.

If $y = u.v$ then,

$$y_n = u_n v + {}^nC_1 u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + \dots + {}^nC_n u v_n.$$

b.) Let $y = \dfrac{x^2}{x^2 - 3x + 2}$

Using $\dfrac{\text{dividend}}{\text{Divisor}}$ rule.

$$
\begin{array}{r|l}
 & x+3 \\
\hline
x^2-3x+2 & x^3 \\
 & -x^3 \underset{\oplus}{-} 3x^2 \underset{\ominus}{+} 2x \\
\hline
 & 3x^2-2x \\
 & - \; 3x^2 \underset{\oplus}{-} 9x \underset{\ominus}{+} 6 \\
\hline
 & 7x-6
\end{array}
$$

$$\therefore y = \frac{x^2}{x^2-3x+2} = x+3+\frac{7x-6}{x^2-3x+2}$$

$$= (x+3) + \left[\frac{-1}{x-1} + \frac{8}{x-2}\right]$$

$$\therefore y = x+3+8(x-2)^{-1} - (x-1)^{-1}$$

Taking n^{th} order derivative using

$$y = (ax+b)^{-1}, \quad y_n = \frac{(-1)^n n!\, a^n}{(ax+b)^{n+1}}$$

$$\text{Thus, } y_n = 0+0+8\left[\frac{(-1)^n n!\,(1)^n}{(x-2)^{n+1}}\right] - \left[\frac{(-1)^n n!\,(1)^n}{(x-1)^{n+1}}\right]$$

$$\therefore y = (-1)^n n! \left[\frac{8}{(x-2)^{n+1}} - \frac{1}{(x-1)^{n+1}}\right]$$

c) We have given $y = \sin(m \sin^{-1} x)$ —— 1

Diff. (1) w.r.t x

$\therefore y_1 = m \cos(m \sin^{-1} x) \cdot \frac{1}{\sqrt{1-x^2}}$

$= y_1 \sqrt{1-x^2} = m \cos(m \sin^{-1} x)$

then again diff. w.r.t x, we have,

$y_2 \sqrt{1-x^2} + y_1 = \frac{-2x}{2\sqrt{1-x^2}} = -m^2 \sin(m \sin^{-1} x) \frac{1}{\sqrt{1-x^2}}$

$y_2(1-x^2) - y_1 \cdot x = -m^2 y$, by (1).

Differentiating this eqn w.r.t x we get.

$\therefore \left[y_2(1-x^2)\right]_n - (y_1 x)_n + m^2 y_n = 0.$

By Libnitz's theorem.

$y_{n+2}(1-x^2) + \frac{n}{1} y_{n+1} \cdot (-2x) + \frac{n(n-1)}{1 \times 2} \cdot y_n \cdot (-2)$

$- \left(y_{n+1} \cdot x + \frac{n}{1} \cdot y_n^1\right) + m^2 y_n = 0.$

$(1-x^2) y_{n+2} - (2n+1) x y_{n+1} - (n^2 - n + n - m^2) y_n = 0.$

$\therefore (1-x^2) y_{n+2} - (2n+1) x y_{n+1} \cdot (n^2 - m^2) y_n = 0.$

Q4 a) If $z = f(x,y)$ is continous and possess continous partial derivative then second order mixed partial derivatives follow commutative property that is:

$$\frac{\partial^2 z}{\partial y\, dx} = \frac{\partial^2 x}{\partial x \partial y}$$

b) Given $u = ax^2 + 2hxy + by^2$
So, diff. w.r.t x.

$$\frac{dy}{dx} = \frac{\partial (ax^2 + 2hxy + by^2)}{\partial x} \quad \text{then}$$

$$\frac{\partial v}{\partial x} = 2ax + 2hy.$$

So now diff w.r.t y..

$$\frac{\partial v}{\partial y} = \frac{\partial (ax^2 + 2hxy + by^2)}{\partial y}$$

$$\frac{\partial x}{\partial y} = [2hx + 2by]$$

Hence proved.

CO4

c. Given

$$u = \log(\tan x + \tan y + \tan z)$$

So Now,

$$eu = (\tan x + \tan y + \tan z) \quad \text{——} \quad (1)$$

Taking partial derivatives with respect to x, y, z resp. We get. differ. w.r.t x.

$$\frac{\partial}{\partial x}(eu) = \frac{\partial}{\partial x}(\tan x + \tan y + \tan z).$$

So,

$$(eu)\frac{\partial u}{\partial x} = \sec^2 x \quad \text{——} \quad (2)$$

differ. w.r.to y, then.

$$\frac{\partial}{\partial y}(eu) = \frac{\partial}{\partial y}(\tan x + \tan y + \tan z).$$

So,

$$e^u \frac{\partial u}{\partial y} = \sec^2 y \quad \text{——} \quad (3)$$

$$\frac{\partial(e^u)}{\partial z} = \frac{d}{dy}(\tan x + \tan y + \tan z)$$

So,

$$\frac{e^u\, du}{dz} = \sec^2 z \quad \text{——— } 4$$

Now, if we multiply (2), (3), (4) by $\sin 2x$, $\sin 2y$, $\sin 2z$ resp. and add them. Then we get.

$$\sin 2x\left(e^u \frac{\partial u}{\partial x}\right) + \sin 2y\left(e^u \frac{\partial u}{\partial y}\right) + \sin 2z\left(e^u \frac{\partial u}{\partial z}\right)$$

$$= \sin 2x \sec^2 x + \sin 2y \sec^2 y + \sin 2z \sec^2 z.$$

$$= (2\sin x \cos x \sec^2 x) + (2 \sin y \cos y \sec^2 y) + (2 \sin z \cos z \sec^2 z)$$

$$= (2)(\tan x + \tan y + \tan z).$$

$$\Rightarrow e^u\left(\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z}\right) = 2e^u,$$

So by eq (1).

$$\Rightarrow \sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z} = 2.$$

Hence proved.