G. H. Raisoni Institute of Engineering & Technology, Pune

(An Autonomous institute affiliated to Savitribai Pune University)

Department: First Year Engineering
Course Name: Matrices and Differential Calculus
AY 2020-21 (Sem-I)
Class: FY B Tech

Question Bank

Unit-1 **CO1**

1. Reduce the Matrix A to its Echelon form and find its rank, where $A=\begin{bmatrix} 4 & 2 & -1 & 2 \\ 1 & -1 & 2 & 1 \\ 2 & 2 & -2 & 0 \end{bmatrix}$
2. Reduce the following Matrix to its normal form and find its rank,

   Where $A=\begin{bmatrix} 3 & 4 & 5 & 6 \\ 1 & 3 & 4 & 5 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$
3. Find inverse of matrix by adjoint Method. $A=\begin{bmatrix} 9 & 7 & 3 \\ 5 & -1 & 14 \\ 3 & 5 & 1 \end{bmatrix}$
4. Examine the consistence of following equations, if consistent solve

   $x+y-z=0, \quad 2x-y+z=3, \quad 4x+2y-2z=2$
5. Examine for linear dependence or independence of vectors $(1,-1,1),(2,1,1),(3,0,2)$. If dependent find the relation between them.
6. Examine for linear dependence or independence of vectors, if dependence find the relation between them $x_1=(2,\ 1,-1,\ 1)$, $x_2=(1,\ 2,\ 1,-1)$ and $x_3=(1,\ 1,\ 2,1)$
7. Reduce the following Matrix to its normal form and find its rank.

   $$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

Unit-2 **CO 2**

1. Determine the eigen values and corresponding eigen vectors of the following matrix

   a. $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$

   b. $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$

   c. $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 2 & 0 \end{bmatrix}$

   d. $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

2. Verify Cayley Hamilton theorem for matrix A and use it to find A^{-1} where
$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$
3. Verify Cayley Hamilton theorem for matrix A and use it to find A^{-1} where $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$
4. Using Sylvester Theorem prove that: $\sin^2 A + \cos^2 A = I$, where $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$
5. Show that the matrix A is diagonalizable where $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$
6. Show that the following Matrix A is diagonalizable where $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$
7. Determine the eigen values and eigen vector of the following matrix Where $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$
8. Using Sylvester Theorem, find inverse of matrix A where $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$
9. Using Sylvester Theorem prove that: $3 \tan A = (\tan 3)A$, where $A = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$

Unit-3: Differential Calculus of single variable **CO3**

Mean Value Theorems

Q.1 Verify Rolle's Theorem for the following function $f(x) = x^2(1 - x^2)$, $x \in [0, 1]$

Q.2 Verify Lagrange's Mean value theorem for $f(x) = x(x-1)(x-2)$, $x \in [0, 1/2]$

Q.3 Verify Cauchy's Mean value theorem for the following function

a) $f(x) = \operatorname{Sin} x$ and $g(x) = \operatorname{Cos} x$ in $\left[0, \frac{\pi}{2}\right]$

b) $f(x) = x^2 + 2$ and $g(x) = x^3 - 1$ in $[1, 2]$

Successive Differentiation and Leibnitz theorem

Q.4 If $y = \sin[\log(x^2 + 2x + 1)]$ then prove that

$$(x+1)^2 y_{n+2} + (2n+1)(x+1)y_{n+1} + (n^2+4)y_n = 0$$

Q.5 Find the n^{th} derivative of 1) $x^2 e^x \cos x$ 2) $x^2 \tan^{-1} x$ 3) $\cos^4 x$

4) $e^{ax} \sin bx \cos cx$ 5) $\frac{2x+3}{5x+7}$

Q. 6 If $y = e^{\tan^{-1} x}$ then prove that: $(1+x^2)y_{n+1} + (2nx - 1)y_n + n(n-1)y_{n-1} = 0$

Taylors and Maclaurin' s Series

Q.5 Expand $\log[1 + e^x]$ by Maclaurin's theorem as far as the term in x^4

Q.7. Expand $\sec x$ by Maclaurin's theorem as far as the term in x^4

Q.6. Using Taylor's theorem express $2\,\mathrm{x}^3 + 7\,\mathrm{x}^2 + \mathrm{x} - 6$ in Powers of $(x-2)$

Q.7. Using Taylor's theorem express $(\mathrm{x}-2)^4 - 3(\mathrm{x}-2)^3 + 4(\mathrm{x}-2)^2 + 5$ in

Powers of x

Unit-IV: Differential calculus of Functions of Several Variables **CO4**

First and higher order Partial derivatives

1. If $u = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$, Find $\frac{\partial^2 u}{\partial x \partial y}$
2. IF $u = x^y$, then verify that $\frac{\partial^2 u}{\partial x\, \partial y} = \frac{\partial^2 u}{\partial y\, \partial x}$
3. IF $u = \log(x^2 + y^2)$ Verify that $u_{xy} = u_{yx}$
4. Find the value of n for which $z = t^n\, e^{-\frac{r^2}{4t}}$ satisfies the partial differential equation
$$\frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2 \frac{\partial z}{\partial r}\right)\right] = \frac{\partial z}{\partial t}$$

Euler's Theorem

5. If $u = cosec^{-1}\sqrt{\frac{x^{1/2}+y^{1/2}}{x^{1/3}+y^{1/3}}}$, show that
$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$$
6. If $u = tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$ show that
$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}(1 - 4\sin^2 u)\sin 2u$$

Jacobians

7. If $x = r\cos\theta\,, y = r\sin\theta$ then evaluate $\frac{\partial(x,y)}{\partial(r,\theta)}$ and $\frac{\partial(r,\theta)}{\partial(x,y)}$
8. If $u = xyz,\ \ v = x^2 + y^2 + z^2,\ \ w = x + y + z$ find $\frac{\partial(u,\ v,\ w)}{\partial(x,\ y,\ z)}$
9. For the transformation $x = e^u \cos v\ \ , y = e^u \sin v$ prove that $\frac{\partial(x,y)}{\partial(u,v)} . \frac{\partial(u,v)}{\partial(x,y)} = 1$

Maxima and Minima of Function of two Variables

10. Determine the points where the function $x^3 + y^3 - 3axy$ has a maximum or minimum
11. Discuss maxima and minima of $x^3y^3(1 - x - y)$
12. Locate the stationary points of $x^4 + y^4 - 2x + 4xy - 2y^2$ and determine their nature.

Lagrange's Method of Undetermined Multipliers

13. Prove that the stationary values of $x^m y^n z^p$ under the condition $x + y + z = a$ is

$$m^m n^n p^p \left(\frac{a}{m+n+p}\right)^{m+n+p}$$

14. As the dimensions of a triangle ABC are varied, show that the maximum value of $\cos A \,.\, \cos B \,.\, \cos C$ is obtained when the triangle is equilateral.

Unit V: Vector Calculus **CO5**

1. Find the angle between the tangents to the curve $\bar{r} = (t^2+1)i + (4t-3)j + (2t^2-6t)k$ at $t = 0$ and $t = 2$.
   a. Find $\nabla\emptyset$ for a) $\emptyset = \log(x^2+y^2+z^2)$ b) $\emptyset = 2xz^4 - x^2y$ at $(2,-2,1)$
2. Find $\nabla\emptyset$ for
   a. $\emptyset = x^2+y^2+z^2 \ at \ (1,1,1)$
   b. $\emptyset = r^m$ where $\bar{r} = xi + yj + zk$
   c. $\emptyset = e^{-r}r^3$
   d. prove that $\nabla f(r) = \frac{f'(r)}{r}\bar{r}$
3. Find the directional derivative of the function $\emptyset = e^{2x}\cos yz$ at a point $(0,0,0)$ in the direction of the tangent to the curve $x = a\sin t$, $y = a\cos t$, $z = at$, at $t = \frac{\pi}{4}$.
4. Find the directional derivative of the function $\emptyset = x\,y^2 + yz^3$ at a point $(1,-1,1)$ in t
   a. Along the vector $2i - 3j + 6k$
   b. Towards the point (2,-1,-1)
   c. Along the direction of the normal to the surface $x^2+y^2+z^2 = 9$ at (1,2,2).
5. Given that $\bar{u} = xyz\,i + (2x^2z - y^2x)j + xz^3k$, $\bar{v} = x^2i + 2yzj + (1+2z)k$ and $\emptyset = xy + yz + z^2$. Find the following at $(1,0,-1)$
   1) $\nabla \circ \bar{u}$ 2) $\nabla \times \bar{v}$ 3) $\nabla\circ(\emptyset\,\bar{u})$ 4) $\nabla \times (\emptyset\,\bar{v})$

6. For the constant vector $\bar{a}$ show that
   a) $\nabla(\bar{a}\cdot\bar{r}) = \bar{a}$
   b) $\nabla \times (\bar{a}\times\bar{r}) = 2\bar{a}$
   c) $\nabla\left(\frac{\bar{a}\cdot\bar{r}}{r^n}\right) = \frac{\bar{a}}{r^n} - \frac{n\,(\bar{a}\cdot\bar{r})}{r^{n+2}}\,\bar{r}$
7. Show that vectors $\bar{F} = (6xy + z^3)i + (3x^2 - z)j + (3xz^2 - y)k$ is irrotational. Also find the Scalar Potential Function ϕ such that $\bar{F} = \nabla\phi$
8. Show that the vector $\left(\frac{\bar{a}\times\bar{r}}{r^n}\right)$ is solenoidal
9. Show that $\nabla f(r) = \frac{f'(r)}{r}\,\bar{r}$ and hence prove that $\nabla^4 e^r = e^r + \frac{4}{r}\,e^r$
10. Show that $\nabla \times \left[\frac{\bar{a}\times\bar{r}}{r^3}\right] = -\frac{\bar{a}}{r^3} + \frac{3(\bar{a}.\bar{r})}{r^5}\,\bar{r}$
11. For the scalar functions $\emptyset$ and Ψ, show that:
    a. $\nabla\cdot(\emptyset\nabla\Psi - \Psi\nabla\emptyset) = \emptyset\nabla^2\Psi - \Psi\nabla^2\emptyset$
    b. $\nabla^2(\emptyset\Psi) = \emptyset\nabla^2\Psi + 2\,(\nabla\emptyset\cdot\nabla\Psi) + \Psi\nabla^2\emptyset$

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