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G. H. RAISONI COLLEGE OF ENGINEERING AND MANAGEMENT, WAGHOLI PUNE

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ROLL NO.: C70 — NAME- SWAYAM TERODE
Registration No.: D020AIFT1101047

CO1 a.) A vector space is a set which is closed under finite vector addition & scalar multiplication.

b.) Let λ_1 and λ_2 be two scalars. Consider linear combination

$\lambda_1 x_1 + \lambda_2 x_2 = 0.$

$\therefore \lambda_1(1,2) + \lambda_2(2,4) = 0.$

$S = \{(1,2),(2,4)\}$ in $V = \mathbb{R}^2$ is linearly dependent because $V_2 = (2,4) = 2(1,2) = 2V_1$.

we have.

$$\left.\begin{aligned} \lambda_1 + 2\lambda_2 &= 0 \\ 2\lambda_1 + 4\lambda_2 &= 0 \end{aligned}\right\} \rightarrow (1).$$

The matrix form of above eq^n is

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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$\therefore A\lambda = B$

The augmented matrix form C, is (Reducing to upper triangular form).

$$C = [A \vdots B] = \begin{bmatrix} 1 & 2 & \vdots & 0 \\ 2 & 4 & \vdots & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$.

$$\begin{bmatrix} 1 & 2 & \vdots & 0 \\ 0 & 0 & \vdots & 0 \end{bmatrix}$$

which is $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\lambda_1 + 2\lambda_2 = 0$

$0\lambda_1 + 0\lambda_2 = 0$.

$\lambda_2 = 0$ and $\lambda_1 = 0$.

Thus $\lambda_1 = \lambda_2 = 0$ which is trivial solution.

So given vectors λ_1 and λ_2 are linearly ~~in~~dependent.

CO1(C) Let $c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + c_3 \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

So now we have.

$1c_1 + 0c_2 + (-2)c_3 = x$.

$2c_1 + 1c_2 + 0c_3 = y$

$3c_1 + 2c_2 + 1c_3 = z$

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So now we have eq. we take into matrix form we have -

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & x \\ 2 & 1 & 0 & y \\ 3 & 2 & 1 & z \end{array}\right]$$

So, after making some transformations.

$R_2 \to 2R_1 \to R_2$, $R_3 \to R_3 - 3R_1$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & x \\ 0 & 1 & 4 & y-2x \\ 0 & 2 & 7 & z-3x \end{array}\right]$$

$R_3 - 2R_2 \to R_3$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -2 & x \\ 0 & 1 & 4 & y-2x \\ 0 & 0 & -1 & z-2y+x \end{array}\right]$$

So from the above matrix we can conclude that this system is consistent for every x, y and z so therefore S spans R^3.

Q1 d.) (a) Every point on the line $x + 2y = 0$ has the form $(-2y, y)$.
Let $u = (u_1, u_2) = (-2u_2, u_2)$
$v = (v_1, v_2) = (-2v_2, v_2)$.
be any two points on the line.

Then $u + v = (-2u_2, u_2) + (-2v_2, v_2)$
$= (-2(u_2 + v_2), u_2 + v_2)$.

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$\Rightarrow u+v(-2y, y)$ lies on the line where $y = u_2 + v_2$.

$\therefore$ let points on the line $x+2y=0$ is $\because$ addition.

Let c be the scalar.

Then $cu = c(-2u_2, u_2)$

$= (-2cu_2, cu_2)$

$= (-2y, y)$ lies on the line

where $y = cu_2$.

$\therefore$ Set of points on the line $x+2y=0$ is closed under multiplication

$\therefore$ Set of points on the line $x+2y=0$ is subspace of R^2

(b) point $(0,0)$ does not lie on $x+2y=1$.

$\therefore$ set of points on the line not a subspace of R^2.

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(Q2 a.) A square matrix is equal to its transpose is known as symmetric matrix.
Only square matrix are symmetric because only equal matrices have equal dimensions.

$A = A^T$

Example:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ -1 & 0 & 5 \end{bmatrix}$$

The transpose of A $(A^T) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ -1 & 0 & 5 \end{bmatrix}$

$\therefore$ $A = A^T$ which is symmetric matrix.

(Q2 b) Start form forming a new matrix by substracting λ from the diagonal entries of the given matrix:

$$\begin{bmatrix} 2-\lambda & 3 & 1 \\ 0 & -\lambda-1 & 2 \\ 0 & 0 & 3-\lambda \end{bmatrix}$$

Now, Derminant:

$$\begin{vmatrix} 2-\lambda & 3 & 1 \\ 0 & -\lambda-1 & 2 \\ 0 & 0 & 3-\lambda \end{vmatrix} = (0)\,(-1)^{3+1} \begin{vmatrix} 3 & 1 \\ -\lambda-1 & 2 \end{vmatrix} + (3-\lambda)(-1) \begin{vmatrix} 2-\lambda & 3 \\ 0 & -\lambda-1 \end{vmatrix}$$

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$= (3-\lambda) \begin{vmatrix} 2-\lambda & 3 \\ 0 & -\lambda-1 \end{vmatrix} \Rightarrow (3-\lambda).(\lambda^2-\lambda-2)$

$= (\lambda-3)(-\lambda^2+\lambda+2)$.

Now Solving above equation gives.

$(\lambda-3)(-\lambda^2+\lambda+2)=0$.

The roots are $\lambda_1=3$, $\lambda_2=2$, $\lambda_3=-1$.

These are the eigen values.
Now, let's find eigen vectors.

• $\lambda=3$

$\begin{bmatrix} 2-\lambda & 3 & 1 \\ 0 & -\lambda-1 & 2 \\ 0 & 0 & 3-\lambda \end{bmatrix} = \begin{bmatrix} -1 & 3 & 1 \\ 0 & -4 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

The nullspace of this matrix is $\begin{bmatrix} \frac{5}{2} \\ \frac{1}{2} \\ 1 \end{bmatrix}$

This is the eigen vector.

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For $\lambda = 2$.

$$\begin{bmatrix} 2-\lambda & 3 & 1 \\ 0 & -\lambda-1 & 2 \\ 0 & 0 & 3-\lambda \end{bmatrix} = \begin{bmatrix} 0 & 3 & 1 \\ 0 & -3 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

The null space of the matrix is $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

This is the eigenvector

For $\lambda = -1$.

$$\begin{bmatrix} 2-\lambda & 3 & 1 \\ 0 & -\lambda-1 & 2 \\ 0 & 0 & 3-\lambda \end{bmatrix} = \begin{bmatrix} 3 & 3 & 1 \\ 0 & 0 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

The null space of this matrix is $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

This the eigenvector.

$\therefore$ Eigenvalue : 3 A,

Multiplicity : 1 A,

eigenvector : $\begin{bmatrix} \frac{5}{2} \\ 1/2 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.5 \\ 0.5 \\ 1 \end{bmatrix}$ A

Eigen value : 2 A

Multiplicity : 1 A

Eigen vector : $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ A

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Eigen value : -1 A

Multiplicity : 1 A.

Eigen vector : $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ A.