G. H. Raisoni Institute of Engineering & Technology, Pune

(An Autonomous institute affiliated to Savitribai Pune University)

Department: First Year Engineering

Subject: Matrices and Differential Calculus

AY.: 2020-21 (Sem-I)

Class: FY B Tech

CAE-II Question Bank

Unit-II **CO-2**

1. Determine the eigen values & corresponding eigen vectors of the following matrices

   a. $\begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$

   b. $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

   c. $\begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$

   d. $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 2 & 0 \end{bmatrix}$

   e. $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

2. Verify Cayley Hamilton theorem for matrix A and use it to find A^{-1} where
$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$$
3. Verify Cayley Hamilton theorem for matrix A and use it to find A^{-1} where
$$A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$
4. Using Sylvester Theorem prove that: $\sin^2 \mathrm{A} + \cos^2 \mathrm{A} = \mathrm{I}$, where $\mathrm{A} = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$
5. Show that the matrix A is diagonalizable, where $\mathrm{A} = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$
6. Show that the following Matrix A is diagonalizable, where $\mathrm{A} = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$
7. Using Sylvester Theorem, find inverse of matrix A where $\mathrm{A} = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$
8. Using Sylvester Theorem prove that: $3 \tan \mathrm{A} = (\tan 3)\mathrm{A}$, where $\mathrm{A} = \begin{bmatrix} -1 & 4 \\ 2 & 1 \end{bmatrix}$

Unit-III **CO-3**

1. Verify Rolle's Theorem for the following function $f(x) = x^2(1 - x^2)$, $x \in [0, 1]$
2. Verify Lagrange's Mean value theorem for the function

$$f(x) = x(x - 1)(x - 2),\ x \in [0, 1/2]$$

3. Verify Cauchy's Mean value theorem for the following function

a) $\mathrm{f(x)} = \mathrm{Sin\,x}$ and $\mathrm{g(x)} = \mathrm{Cos\,x}$ in $\left[0, \frac{\pi}{2}\right]$

b) $\mathrm{f(x)} = \mathrm{x}^2 + 2$ and $\mathrm{g(x)} = \mathrm{x}^3 - 1$ in $[1, 2]$

4. If $y = \sin[\log(x^2 + 2x + 1)]$ then prove that:

$$(x + 1)^2 y_{n+2} + (2n + 1)(x + 1)y_{n+1} + (n^2 + 4)y_n = 0$$

5. Find the n^{th} derivative of each of the following

   a. $x^2 e^x \cos x$

   b. $x^2 \tan^{-1} x$

   c. $\cos^4 x$

   d. $\mathrm{e^{ax} \sin bx \cos cx}$

   e. $\frac{2x+3}{5x+7}$

6. If $y = e^{\tan^{-1} x}$ then prove that:

$$(1 + x^2)y_{n+1} + (2nx - 1)y_n + n(n - 1)y_{n-1} = 0$$

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