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F.Y. B.TECH
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Q.1.) a) Dependent System of Equation means that there are an infinite no. of solutions.

b) A linear equation $Ax=b$ is said to be homogeneous if $b=0$, and non-homogeneous if $b=0$.

Q. A system of linear equations is homogeneous if all of the constant terms are zero.

A system of linear equation is non-homogeneous it has a single (unique) solution or more than one solution. or has no solution at all.

Que 3) Reduce the matrix to Row-Echelon form and find rank of

(b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$

Soln: Given, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix}$.

The above matrix is of the 3×3 order $\therefore \rho(A) \le 3$.
Reducing the matrix A using Row-Echelon form.

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 5 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & -1 & -2 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - R_2$

Hence the above form is in Row-Echelon form.
The no. of non zero rows is 2, so $\rho(A) = 2$

$\therefore$ Rank of matrix $A = \rho(A) = 2$.

Rough Work

$3-4=-1$, $4-6=-2$

$5-6=-1$, $7-9=-2$

0, 0, 0

Que 3) Find Eigen value of $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and Eigen vector of largest Eigen value.

Soln: Given, $A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

The characteristic eqn is. $\det(A - \lambda I) = 0$.

$$\begin{vmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & 1 \\ 0 & 1 & 1-\lambda \end{vmatrix}$$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0.$$

where,

S_1 = sum of principal diagonal element of $A = 4 \; [(1+2+1)]$

S_2 = Sum of minors of principal diagonal elements of A.

$S_2 = M_{11} + M_{22} + M_{33}$

$$M_{11} = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 2 - 1 = 1.$$

$$M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1.$$

$$M_{33} = \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix} = 2 - 1 = 1.$$

$S_2 = M_{11} + M_{22} + M_{33} = 1 + 1 + 1 = 3.$

$$S_3 = |A| = \begin{vmatrix} 1 & -1 & 0 \\ -1 & 2 & 1 \\ 0 & +1 & 1 \end{vmatrix}$$

$$= 1(2-1) + 1(-1-0) + 0$$
$$= (1 \times 1) - 1$$
$$= 1 - 1 = 0$$

Hence, The characteristic eqn is

$$\lambda^3 - 4\lambda^2 + 3\lambda - 0 = 0 \Rightarrow \lambda^3 - 4\lambda^2 + 3\lambda = 0$$

The $|A| = 0$.

Q.4) Verify Cayley-Hamilton theorem and hence find value of A^{-1} & A^4 of

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Soln: $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

The characteristic eqn is $\det(A-\lambda I)=0$.

$$\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0.$$

where S_1 = Sum of the principal diagonal elements of $A = 6$.

S_2 = Sum of minors of principal diagonal elements of A.

$$= \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$$

$$= (4-1)+(4-1)+(4-1) = 9$$

$$S_3 = \det(A) = \begin{vmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix} = 2(4-1)+(1)(-2+1)+1(1-2)$$
$$= 6-1-1 = 4.$$

Hence, the characteristic eqn is

$$\lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0.$$

$$A^2 = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix}$$

$$A^3 - 6A^2 + 9A - 4I.$$

$$= \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{bmatrix} - \begin{bmatrix} 36 & -30 & 30 \\ -30 & 36 & -30 \\ 30 & -30 & 36 \end{bmatrix} + \begin{bmatrix} 18 & -9 & 9 \\ -9 & 18 & -9 \\ 9 & -9 & 18 \end{bmatrix}$$

$$- \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$\longrightarrow$ (1)

Ans

The matrix A satisfies its own characteristic eq. Hence, the Cayley-Hamilton theorem is verified.

Premultiplying with A^{-1}.

$$A^{-1}(A^3 - 6A^2 + 9A - 4I) = 0.$$

$$A^2 - 6A + 9I - 4A^{-1} = 0.$$

$$4A^{-1} = (A^2 - 6A + 9I).$$

$$= \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -6 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

Multiply Eq. (1) by A.

$$A(A^3 - 6A^2 - 9A - 4I) = 0 \Rightarrow A^4 - 6A^3 + 9A^2 - 4A = 0$$

$$A^4 = 6A^3 - 9A^2 + 4A.$$

$$= \begin{bmatrix} 132 & -126 & 126 \\ -126 & 132 & -126 \\ 126 & -126 & 132 \end{bmatrix} - \begin{bmatrix} 54 & -45 & 45 \\ -45 & 54 & -45 \\ 45 & -45 & 54 \end{bmatrix} + \begin{bmatrix} 8 & -4 & 4 \\ -4 & 8 & -4 \\ 4 & -4 & 8 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 86 & -85 & 85 \\ -85 & 86 & -85 \\ 85 & -85 & 86 \end{bmatrix}$$