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GH Raisoni College of Engineering & Management
Waghdi, Pune
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Department	F.Y B.Tech
Term/Section	Term II | Date of Examination: 23/07/21
Subject Name/Code	Integral Calculus and Differential Equations (UBSL
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(CO 3 a.) Green's theorem states that the line integral is equal to the double integral of their quantity over the enclosed region.

The Stokes theorem states that "the surface integral of the curl of a function over a surface bounded by a closed surface is equal to the line integral of the particular vector function around the surface".

(CO3 d.) So, we have to satisfy the condition of Green's Theorem since it is closed and simple and so there really isn't a reason to draw a figure.

Let's first identify P and Q from the line integral.
$P = y^3 \quad Q = -x^3$
As, we know Q has - Sign.
So, Now using Green's Theorem on the line integral given,

$$\oint_C y^3\,dx - x^3\,dy = \iint -3x^2 - 3y^2\,dA$$

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where D is disk of radius 2 centered at the origin. Since D is a disk it seems like the best way to do this integral is to use polar coordinates. Here is the evaluation of the integral

$$\oint y^3\,dx - x^3\,dy = -3\iint_D (x^2+y^2)\,dA$$

$$= -3\int_0^{2\pi}\int_0^{2} r^3\,dr\,d\theta$$

$$= -3\int_0^{2\pi} \frac{1}{4} r^4 \Big|_0^2 \,d\theta$$

$$= -3\int_0^{2\pi} 4\,d\theta = -24\pi.$$

(Q4b) a) If $y_1(x)$ and (x) are any two (linearly independent) solutions of a linear, a homogeneous second order differential equation then the general solution $y_f(x)$ is

$$y_f(x) = Ay_1(x) + By_2(x)$$

where A, B are constants

We see that the second order linear ordinary differential equation has two arbitrary constants in its general solution. The functions $y_1(x)$ and $y_2(x)$ are linearly independent if one is not a multiple of the other. (OR)

A complementary ~~and homogeneous~~ function is the general solution of a homogeneous, linear differential eq. $y_f(x) = Ay_1(x) + By_2(x)$, where A and B are const.

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b.) Given $\frac{d^3y}{dx^3} - 2\frac{dy^2}{dx^2} - \frac{dy}{dx} + 2y = 0$.

$D = \frac{dy}{dx}$.

$\frac{d^3y}{dx} - 2\frac{dy^2}{dx^2} - \frac{dy}{dx} + 2y$ can be written as $\rightarrow$

$(D^3 - 2D^2 - D + 2) = 0$.

$(D-2)\ (D+1)(D-1) = 0.$

$D = 2, -1, 1.$

$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} + C_3 e^{-1x}$

$y = C_1 e^{2x} + C_2 e^{1x} + C_3 e^{-1x}.$

CO4 a.) Let $(e^y + 2xy)dx + (xe^y + x^2)dy = 0 \longrightarrow (1).$

Comparing eq (1) with $M dx + N dy = 0$.

$M = 2xy + e^y$ & $N = x^2 + xe^y$

$dM = \frac{\delta(2xy + e^y)}{dy} = \frac{2d\,xy}{dy} + \frac{de^y}{dy} = 2x + e^y.$

$dN = \frac{\delta(x^2 + xe^y)}{dx} = \frac{dx^2}{dx} + \frac{d\,xe^y}{dx} = 2x + e^y$

$\therefore \frac{dM}{dy} = \frac{dN}{dx}$

So, eq^n (1) is exact diff eq^n.

$$\int M\,dx = \int (2xy + e^y)\,dx \quad (y \text{ is const})$$

$$= \int 2xy\,dx + \int e^y\,dx.$$

$$= 2y\int x\,dx + e^y\int dx.$$

$$= \cancel{2}y\,\frac{x^2}{\cancel{2}} + e^y x + C_1 = x^2y + x\,e^y + C_1 \longrightarrow (2).$$

$$\int N\,dy = \int (x^2 + xe^y)\,dy$$

$$= \int x^2\,dy + \int xe^y\,dy.$$

$$= C_2 \longrightarrow (3)$$

From Eq (2) and (3).

$x^2y + xe^y = 0.$
which is required solution

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$x = 1$
$x = 2$

(Q3 b) Let $y = 1 - x^3$

and $\gamma(t) = x(t), y(t) = (t, 1-t^3)$

$$\int_c F \cdot dx = \int_c 2y\,dx + (1-x)\,dy$$

$$= \int_{-1}^{2} 2 - 2t^3 \frac{dx}{dt}\,dt - \int_{-1}^{2} 1-t \cdot \frac{dy}{dt}\,dt$$

$$= \int_{-1}^{2} (2-2t^3) - (1-t)(0-3t^2)\,dt$$

$$= \int_{-1}^{2} 2 - 2t^3 + 3t^2 - 3t^3\,dt$$

$$= \left[2t - \frac{t^4}{2} - 0t^3 - \frac{3t^4}{4}\right]_{-1}^{2} dt$$

$$\left[4 - 8 - 6 - 12\right] - \left[-2 - \frac{2}{4} + 1 - \frac{3}{4}\right]$$

$$= [-24] - \left[-4 - \frac{5}{4}\right) = [-24] - \left[-\frac{9}{4}\right]$$

$$= -24 + \frac{9}{4} = -\frac{87}{4}.$$