UNIT-I
ELECTRON BALLISTICS

UNIT I- ELECTRON BALLISTICS

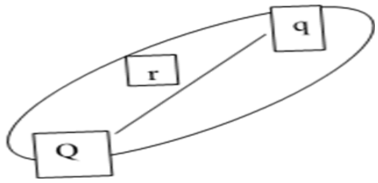
Basic Definitions:

- <u>Electric Charge</u>:-Any particle or object that establishes an electric field in its surrounding space is said to have a charge onit.

- <u>Coulomb Force</u>:-The force of interaction between two point charges q and Q which is directly proportional to the product of two charges and inversely proportional to the square of the distance between them. It is given by

$$F = \frac{1}{4\Pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

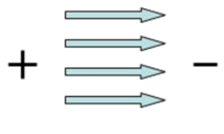
Where ε_0-permittivity of free space



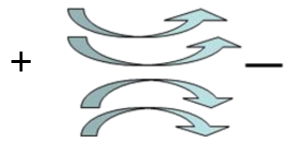
- <u>Electric field/ Electric fieldstrength/Intensity</u>:-
  Electric field/ Electric field strength/Intensity at some point is defined as the electric force F experienced by a unit positive test charge placed at that point.

$$E = \frac{F}{q} = \left(\frac{1}{4\Pi\varepsilon_0}\frac{q_1 q_2}{r^2}\right)\frac{1}{q} = \frac{1}{4\Pi\varepsilon_0}\frac{q_1}{r^2}\,(\therefore q_1 = q_1 = q)$$

<u>Electric field lines in Uniform electric field:-</u>



<u>•Electric field lines in non-uniform electric field:-</u>



- **Electric potential (V):-** Electric potential (V) at a point in space is defined as the work done by an external agent in carrying a unit positive test charge from infinity to that point against the electric force. Thuselectric potential at the point is

$$V = \frac{w}{q} = \frac{Fd}{q} = Ed \quad \because F = qE$$

$$\therefore E = \frac{V}{d}$$

- **Electron-volt :-**The electron-volt is an amount of energy acquired by an electron accelerated through a potential of onevolt

OR

The electron-volt is an amount of energy of an electron equal to the work done on an electron moved through a potential difference of 1V.

- **1 electron-volt =1.6 x 10^{-19} J**

The amount of an energy gained by an electron by accelerating in electric field is very small compared to a joule. Hence in atomic physics and Nuclear Physics, particle energies are expressed in terms of a small unit called electron – volt (eV).

$$1 \text{ eV} = \text{Charge on electron} \times 1 \text{ V}$$

$$= (1.602 \times 10^{-19} C)(1V)$$

$$1 \text{ eV} = 1.602 \times 10^{-19} J$$

Units of MeV is often used in Nuclear Physics; 1 MeV = 10^6 eV

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1. **Motion of electron in uniform electric field**

   Case 1: Motion of electron parallel to uniform electric field

   Case 2: Motion of electron perpendicular to uniform electric field

   Case 3: Motion of electron at an angle to uniform electric field

2. **Motion of electron in uniform magnetic field**

   Case 1: Motion of electron parallel to uniform magnetic field

   Case 2: Motion of electron perpendicular to uniform magnetic field

   Case 3: Motion of electron at an angle to uniform magnetic field

Motion of Electron in Uniform Electric Field

CASE-I: Motion of electron parallel to uniform electric field:

Consider the two plane parallel plates **A & B** of equal area separated by a distance **'d'**. A fixed potential difference of **'V'** volts is applied between plates **A & B.** Due to this, an electric field **E** is produced in the region between the plates, which is directed from the positive plate toward the negative plate and is given by

$$E = \frac{V}{d} \quad \text{--------------(1)}$$

If an electron of mass m and charge e is placed at rest in the uniform electric field and released. The electron experiences a constant force due to electric field given by

$$F = -eE \text{--------------- (2)}$$

The electron will be uniformly accelerated until it reaches the positive plate **A.**

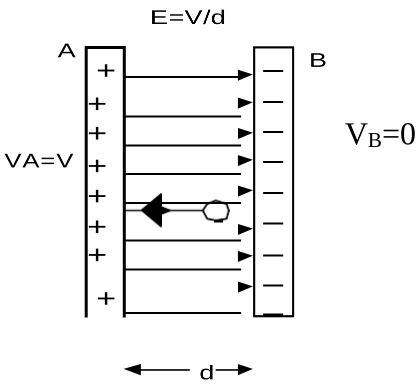


Fig 1: Electron in Uniform Electric Field

According to second law of Newton, the acceleration is given by,

$$a = \frac{F}{m} = \frac{-eE}{m} = \frac{-e}{m}\frac{V}{d} \quad \text{.......................... (3)}$$

The equations of kinematics in one dimension are given as,

$$\begin{bmatrix} v = v_0 + at \\ S = v_0 t + \frac{1}{2}at^2 \\ v^2 = v_0^2 + 2aS \end{bmatrix} \quad \text{.............................. (4)}$$

Using the initial conditions, $x_0 = 0$ and $v_0 = 0$, and substituting for 'a', the eqs. reduces to,

$$\begin{bmatrix} v = \dfrac{eE}{m}t \\ S = \dfrac{eE}{2m}t^2 \\ v^2 = 2\dfrac{eE}{m}S \end{bmatrix} \quad \text{..............................} (5)$$

The Kinetic Energy attained by the electron after moving a distance x is given by,

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{2eEd}{m} = eEd$$

As $Ed = V$, $K.E. = eV$ therefore, $eV = \frac{1}{2}mv^2$

<u>CASE -II</u>: Motion of electron perpendicular (transverse) to uniform electric field:

Let A and B be the two plane parallel metal plates of length 'l' oriented horizontally , separated by a distance 'd'. A potential difference V applied between the plates produces a vertically acting uniform electric field E which is directed, say from plate A to plate B. The strength of electric field acting in the region between the plates is given by $E = v / d$ .... (1)

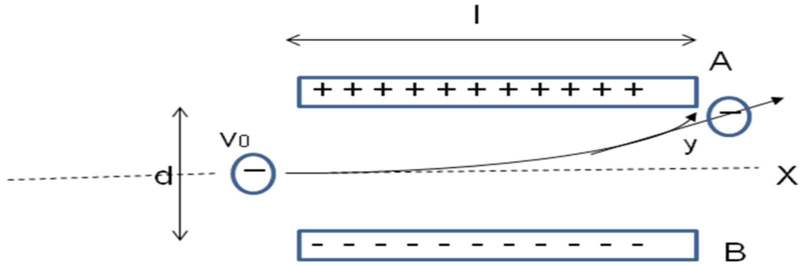


Fig 2

Consider an electron moving along positive direction of x-axis with initial velocity $\mathbf{v_0}$**,** entering in an electric field between plates A and B, which is at right angles to the direction of electron.

While passing through the field, the electron will experience a vertical acceleration, hence will be deflected vertically towards positive plate A. After emerging from the field, it will travel along a straight line and strike the screen at point X. The axial velocity component $\mathbf{v_0}$ remains unchanged during its passage between the plates. However it is continuously attracted towards the plate **A** and attains final velocity $\mathbf{v_y}$ when it just leaves the plate. From this point onward the electron will move in a straight line with a resultant velocity having components $\mathbf{v_x}$ & $\mathbf{v_y}$.

Duetouniformelectricfield**E**aconstantforceF=eE------------ (2) draggs theelectron upwards.

Therefore Vertical acceleration is given by

$$a_y = \frac{F}{m} = \frac{eE}{m} = \frac{eV}{md} \text{ -----(3)}$$

The displacement of electron in Y-direction in the region of electric field at any time 't' is given by

$$y = \frac{1}{2} a_y t^2 = \frac{1}{2} \frac{eE}{m} t^2 \text{ -----(4)}$$

The displacement **x** traveled by the electron in the time interval **'t'** depends on the initial velocity and is given by $t = \frac{x}{v_0}$ ------------(5)

Substituting in **Eqn. (4)** we get $y = \frac{1}{2md} eV \frac{x^2}{v_0^2} = \frac{1}{2} \frac{eEx^2}{m v_0^2}$

$$\therefore y = \frac{eE}{2mv_0^2} x^2$$

$$\therefore y = kx^2 \text{ -----(6)}$$

where, $k = \frac{eE}{2mv_0^2}$

Eqn. (6) shows that the path of electron entering in uniform electric field at right angles to the field lines and traveling through the field is **parabolic.**

Home work:

Find the formula for Kinetic Energy of the electron moving perpendicular to uniform electric field in terms of

(a) Electric field (E) and
(b) Electric Potential (V).

<u>CASE-III</u> : Electron projected at an angle in uniform electric field :

Suppose an electron is projected in to a uniform electric field at an acute angle with the field direction and with an initial velocity $\mathbf{v_0}$. The electric field acts in positive **y-** direction and the electron gets accelerated in the negative y-direction. The acceleration is given by $a = \frac{F}{m} = \frac{eE}{m}$ and is constant. Thus the motion of electron will be very much similar to that of projectile in gravitational field.

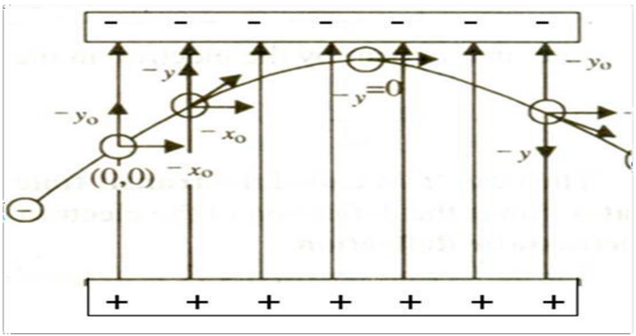


Fig.4: Projectile Motion of an electron in uniform electric field.

Initial velocity v_o is resolved into twocomponents

$$v_{x0} = v_0 Cos\theta_0$$

$$v_{y0} = v_0 Sin\theta_0$$

The velocity component in x-direction $\mathbf{v_x}$ remains constant while $\mathbf{v_y}$ decreases initially and again increases when the electron reverses its path. Therefore the components are given by,

$$\begin{bmatrix} v_x = v_{x0} + at = v_{x0} + 0 = v_0 Cos\theta_0 = Constant \\ v_y = v_{y0} + at = v_0\, Sin\theta_0 + at \end{bmatrix} \text{----------------(1)}$$

Using above equations we can obtain coordinates for the electron at any time **t** as

$$x = v_{xo}\, t + \frac{1}{2} at^2 = v_{x0} t + 0 = v_0 \cos\theta_0 t \text{ ——— (2)}$$

$$y = v_{yo}\, t + \frac{1}{2} at^2 = (v_0\, Sin\theta_0) t + \frac{1}{2} at^2 \text{———(3)}$$

From eq^n (2) we get

$$x = (v_0 Cos\theta_0)\, t \Rightarrow t = \frac{x}{v_0 Cos\theta_0}$$

Putting this value of **t** in **Eqn. (3)** we get

$$y = (v_0 sin\theta_0)\left(\frac{x}{v_0 cos\theta_0}\right) + \frac{1}{2}a\left(\frac{x^2}{v_o^2 cos^2\theta_0}\right)$$

$$y = (tan\theta_0)x + \left(\frac{a}{2v_o^2 cos^2\theta_0}\right)x^2 \text{ ----------------------------(4)}$$

Eq^n (4) is of the form $y = ax + bx^2$, which represents the equation of parabola. Therefore the trajectory of an electron projected into a uniform electric field is a parabola.

The various parameter of projected charge particle in uniform electric field can be obtained as follows.

1. The time taken by the charged particle to reach maximum height in y-Direction can be determined by Eq^n $v = u + at$
   At maximum height v = 0,
   $$\therefore 0 = v_o \sin\theta_o - at$$
   $$t = \frac{v_0 Sin\theta_0}{a} \text{ ------ (5)}$$

2. The time of Flight: The time taken by the charged particle to return to its original level along x- direction is given by
   $$T = 2t = \frac{2v_o \sin\theta_o}{a} \text{ ....(6)}$$
   The maximum height that a charged particle reaches in y-direction is given by putting $\mathbf{v_y} = 0$ in **Eqn.**
   $$v^2 - u^2 = 2aS$$
   $$0 - (v_o^2 \sin^2\theta) = -2aH_{max}$$

$$\therefore H_{max} = \frac{v_o{}^2 Sin^2\theta_o}{2a} \text{ ----------(7)}$$

3. The range, that is the distance traveled along x-direction by the charged particle from the starting point to the point at which it returns to its original level along x-direction is givenby

$$R = v_{xo}\, T = (v_o \cos\theta_o)\left(\frac{2v_o \sin\theta_o}{a}\right)$$

$$R = \frac{v_o^2 \sin 2\theta}{a} \quad \text{.................................} \ (8)$$

Horizontal range will be maximum if $\sin 2\theta_o = 1$i. e. $2\theta_o = \pi/2$

$$\theta_o = \frac{\pi}{4} = 45^o$$

Range will be maximum, when the particle is projected at an angle of 45° to the horizontal.

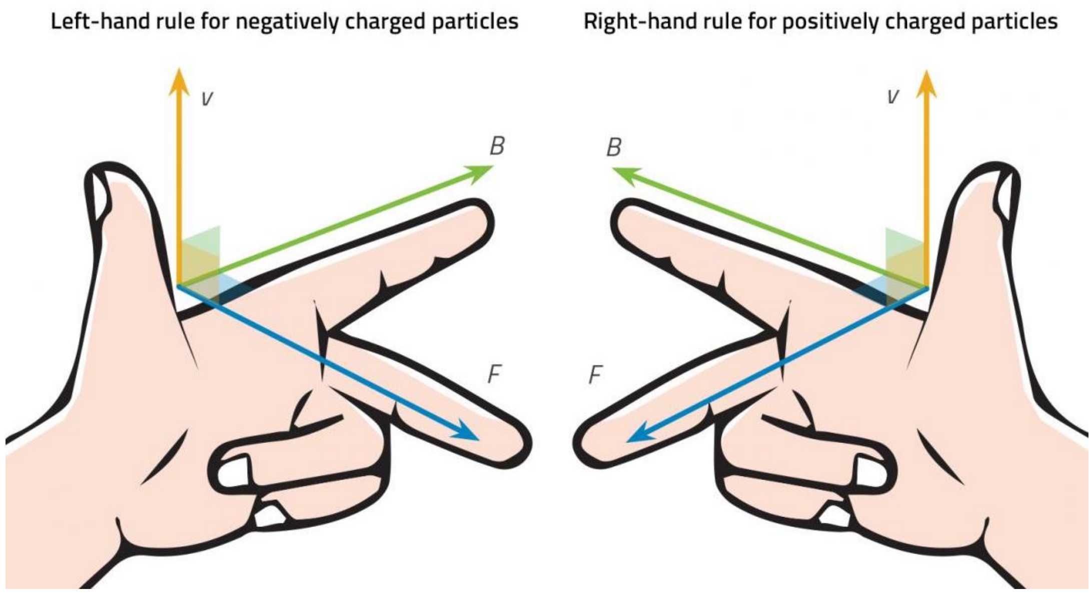
Motion of Electron in Uniform Magnetic Field

Magnetic force

If an electron moving with a velocity v enters in to the region of a magnetic field of strength B then it experiences a force called Lorentz force and is given by

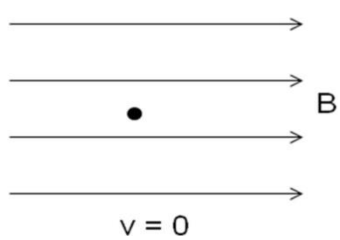
$$F_L = e(\bar{v} \times \bar{B}) = evB \sin\theta$$

Force vector will be at right angles to the plane containing velocity vector and field vector.



When the Charged Particle is at Rest

If the charged particle is at rest in the magnetic field,



The force on the charged particle is

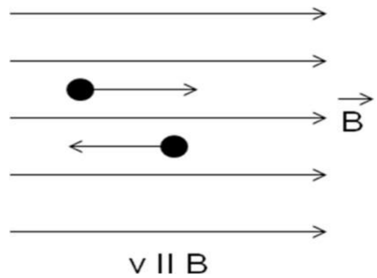
$$F_L = e(\bar{v} \times \bar{B}) = evB \sin\theta$$

If $v = 0$, then,

$$F_L = e(\bar{v} \times \bar{B}) = evB \sin\theta = e.0.B \sin\theta = 0$$

indicates that magnetic force does not act on static electron or electron at rest.

CASE-I: Motion of Electron parallel to uniform Magnetic field



If an electron moves parallel (i.e. $\theta = 0$) to the magnetic field lines, he force on the charged particle is,

$$F_L = e(\bar{v} \times \bar{B}) = evB \sin\theta = evB \sin 0 = 0$$

If an electron moves anti-parallel (i.e. $\theta = 180^o$ or π) to the magnetic field lines, he force on the charged particle is,

$$F_L = e(\bar{v} \times \bar{B}) = evB \sin\theta = evB \sin 180 = 0$$

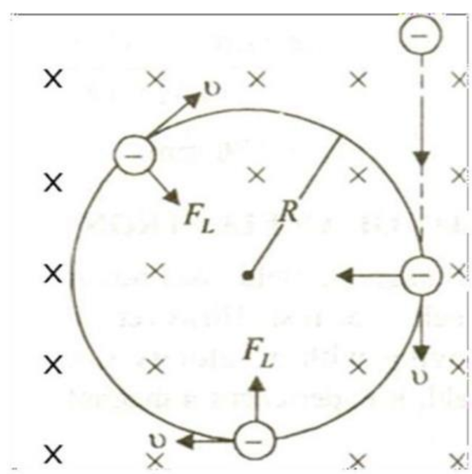
The above condition indicates that magnetic force does not act on electron if it enters the magnetic field either parallel or anti parallel to the lines of induction and it will continue to move along the field lines with initial velocity and direction.

CASE-II: Motion of electron perpendicular (Transverse) to uniform Magnetic field

Let an electron enter with a uniform velocity *v* normally in to a magnetic field of strength *B* as shown in Figure. The direction of *B* is perpendicular to the direction of motion of electron and plane of paper.

Thus the force due to magnetic field is given by $F_L = evB$

This force cannot change the magnitude of electron velocity but deflects the electron continuously along a curvilinear path (Circular path).



Then the centripetal force required for orbital motion is supplied by the magnetic force F_L, which is given by,

$$F_C = \frac{mv^2}{R}$$

Where *R* is the radius of circular orbit.

On equating, $F_L = F_C$

$$evB = \frac{mv^2}{R}$$

$$R = \frac{mv}{eB}$$

Since all the parameters in the above relation are constant R = constant

Time period for orbital motion is

$$T = \frac{\text{Distance travelled by electron in one revolution}}{\text{Speed of Electron}}$$

$$T = \frac{2\pi R}{v} = \frac{2\pi}{v} \times \frac{mv}{eB} = \frac{2\pi m}{eB}$$

and frequency of revolution f and angular frequency ω of an electron are given as

$$f = \frac{1}{T} = \frac{eB}{2\pi m}$$

$$\omega = 2\pi f = 2\pi \left(\frac{eB}{2\pi m}\right) = \frac{eB}{m}$$

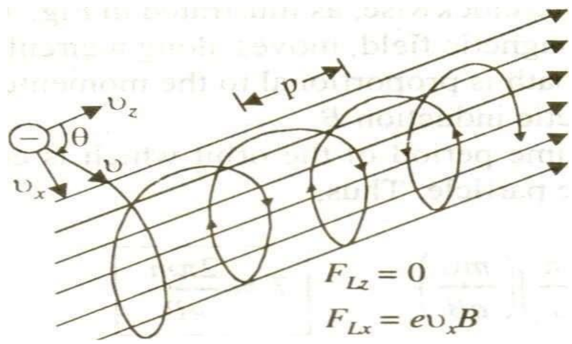
It is seen that the time period, frequency of revolution and angular frequency of electron are independent of velocity and radius of circular orbit.

It implies that slower particles move in smaller circles while faster particles move in larger circles but all of them take same time for completing one revolution.

However these parameters are strongly dependent on its charge to mass ratio (e/m) and the strength of the magnetic field B.

CASE-III: Motion of electron at an angle to uniform Magnetic field

Let an electron with a uniform velocity v enters the magnetic field B neither parallel nor at normal but enters it by making an angle θ with the field direction B. Assuming that the field is acting in the Z- direction, the electron velocity may be resolved into rectangular components v_X and v_Z.



The component of the velocity of the electron parallel (along *z*-axis) to the magnetic induction is

$$v_{\parallel} = v_z = v\cos\theta$$

is not influenced by the field, as

$$F_{\parallel} = F_{Lz} = e(\bar{v}_z \times \bar{B}) = ev_zB\sin\theta = 0 \qquad (as\ \theta\ is\ 0)$$

Hence electron will continue to move with a constant velocity in the z direction.

However the component of velocity in the direction of *x*-axis is

$$v_{\perp} = v_x = v\,sin\,\theta$$

Which give rise to a force,

$$F_{\perp} = F_{Lx} = e(\bar{v}_x \times \bar{B}) = ev_xB\,sin\,\theta = ev_xB \qquad (as\ \theta\ is\ \pi/2)$$

Under the action of this force, the electron tends to describe a circular path in a plane perpendicular to magnetic field.

The radius of the circular path is given by

$$R = \frac{mv_x}{eB} = \frac{mv\,sin\,\theta}{eB}$$

Thus the time taken to complete one revolution is given by,

$$T = \frac{2\pi R}{v_x} = \frac{2\pi}{v_x} \times \frac{mv_x}{eB} = \frac{2\pi m}{eB}$$

The resultant path describe by the electron is obtained by the superposition of the uniform translational motion parallel to B and the uniform circular motion in a plane normal to B. The resultant motion occurs along a helical (spiral) path the axis of the helix being the field direction.

Pitch of the Helix: Pitch of the Helix is the distance covered by the electron along the field direction in one revolution.

Thus the pitch of the helix is given by,

$$P = v_{\parallel}T = v_z T$$

$$P = (v \cos\theta)\left(\frac{2\pi m}{eB}\right) = \frac{2\pi m v \cos\theta}{eB}$$

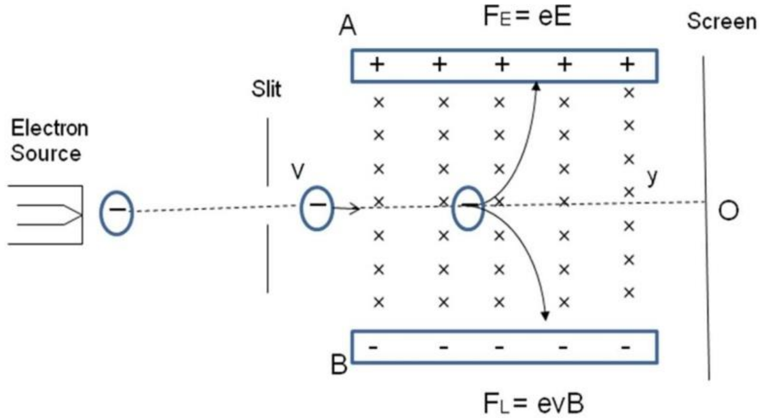
This relation shows that for small values of θ the pitch of the helix is independent of the angle θ. This property can be used for focusing electron beam produced by electron gun in CRT.

Electric and Magnetic Field in Crossed Configuration:

When uniform electric and magnetic fields are perpendicular to each other and act over the same region they are said to be in crossed configuration.

Let two charged plane parallel plates set up a uniform electric field E in the vertical (y-direction) direction and a uniform magnetic induction B is also set up in the same region between the plates in Z-direction.

Let a stream of electrons enter the crossed field configuration with a velocity v. The electric field deflects the electrons upward where as the magnetic field deflects them downward.



The Force due to electric field is

$$F_E = eE$$

And the force due to magnetic Field is

$$F_L = evB$$

If the magnitudes of the fields E and B are adjusted such that the force they exert on electron become equal, the electron will not experience any force. Thus,

$$F_E = F_L$$

$$eE = evB$$

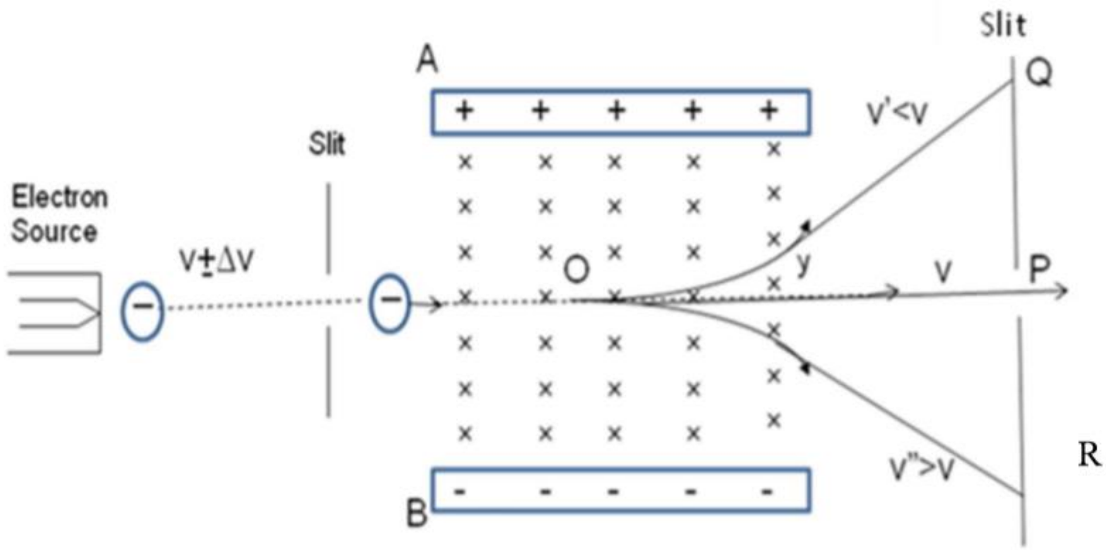
$$v = \frac{E}{B}$$

The electrons experience a zero net force as the two forces balance each other and they will not deviate from their original straight line path and travel without change in the velocity v and strike the screen at point O.

J. J. Thomson used this method in 1897 for the determination of electron beam velocity.

Velocity Filter / Velocity Selector

Defn: The velocity filter is an electro optic device, which uses uniform electric and magnetic field in crossed configuration for selecting a stream of charged particles of single velocity from a beam of charged particles having a wide range of velocities.



Suppose an Electron velocities spread around a central value v enter the crossed field configuration. The electric field E is produced in the vertical direction by a set of charged parallel plate and the uniform magnetic field B is applied perpendicular to it (acting into the page). If the fields are adjusted such that the electric force balances the magnetic force acting on the electrons moving with velocity v, then those electrons are not deflected and continue to travel along a straight path subsequently they pass through the slit at P. Electrons moving with a lesser velocity (v'<v) will get deflected upward along OQ due to electric force and those moving with greater velocity (v">v) will get deflected downward along OR due to magnetic force. The electrons deflected away are absorbed by the slit walls. Thus a strictly homogeneous single velocity electrons beam travelling along OP is obtained with the help of crossed fields. This arrangement is therefore known as a velocity filter or a velocity selector. The velocity filter forms an essential component in Bainbridge Mass Spectrograph.