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G H Raisoni College of Engineering & Management, Wagholi Pune	
ESE SUMMER 2021	
Department	First Year.
Term/Section	Term II Date of Examination: 17/08/2021
Subject Name	Integral Calculus and Differential Equations (UBSL 104)
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CO1 a) Differentiation under Integral Sign (DUIS).

$\boxed{L1}$ Rule 1: Integrals with constant limits

If $I(\alpha) = \int_a^b f(x,\alpha)\,dx$

then, $\frac{d}{dx}\int_a^b f(x,\alpha)\,dx = \int_a^b \frac{\partial f}{\partial \alpha}(x,\alpha)\,dx$

Rule 2: Integrals with limits as function of parameter

If a and b are functions of parameter α.

i.e. $I(\alpha) = \int_{a(\alpha)}^{b(\alpha)} f(x,\alpha)\,dx.$

then $\frac{d}{dx}\int_a^b f(x,\alpha)\,dx = \int_a^b \frac{\partial}{\partial x} f(x,\alpha)\,dx + f(b,\alpha)\frac{db}{d\alpha} - f(x,\alpha).$

(02 b) $I = \int_0^1 \int_0^x (x^2 + 3y)\, dy \cdot dx.$

$\boxed{L2}$

$$= \int_{x=0}^{1} \left[\int_{y=0}^{x} (x^2+3y)\, dy \right] dx.$$

$$= \int_{x=0}^{1} \left[x^2 y + \frac{3y^2}{2} \right]_{y=0}^{x} dx.$$

$$= \int_{x=0}^{1} x^3 + \frac{3x^2}{2}\, dx \Rightarrow \left[\frac{x^2}{4} + \frac{3x^3}{2\times 3} \right]_{x=0}^{1}.$$

$$\Rightarrow \left[\frac{x^4}{4} + \frac{\cancel{3}x^3}{\cancel{6}_2} \right]_{x=0}^{1} \Rightarrow \frac{1}{4} + \frac{1}{2} = \cancel{0.75} = \frac{6}{8} = \frac{3}{4}$$

(03(b) → Green's Lemma Theorem:

According to Green's Lemma Theorem we say that –

$$\int u\, dx + \mu\, dy = \iint_A \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) dx\, dy.$$

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Gauss divergence theorem:

The gauss divergence theorem states that the vector's outward flux through a closed surface is equal to the volume integral of the divergence over the area within the surface.

(CO1 a) Beta function is defined as $B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$; $m>0,\ n>0$. it is called as Euler's Integral.

L2

Properties:

① $B(m,n) = B(n,m)$.

We know that: $B(m,n) = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$.

But $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

$\therefore B(m,n) = \int_0^1 (1-x)^{m-1}[1-(1-x)]^{n-1}\,dx$.

$= \int_0^1 (1-x)^{m-1}\,\boxed{x}^{n-1}\,dx = \int_0^1 x^{n-1}(1-x)^{m-1}\,dx$.

$B(m,n) = B(n,m)$.

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The same formula can also be written in the form:

1.) $B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx.$

2.) $B(m,n) = 2 \int_0^\infty \sin^{2m-1}\theta . \cos^{2n-1}\theta \, d\theta.$

Beta function Properties

1.) $B(m,n) = B(m,n+1) + B(m+1,n).$
2.) $B(m,n+1) = B(m,n) . [n/m+n].$
3.) $B(m+1,n) = B(m,n) . [m/m+n].$
4.) $B(m,n) . B(m+n,1-n) = \pi / m \sin(\pi n).$

(04c.) Given
$\boxed{L2}$

$M = e^x + 2xy^2 + y^3$	$N = ay^2 + 2x^2y + 3xy^2$
$\frac{\partial M}{\partial y} = 0 + 2x(2y) + 3y^2$	$\frac{\partial N}{\partial x} = 0 + 2y(2x) + 3y^2(1)$
$= 4xy + 3y^2$	$= 4xy + 3y^2$

$\therefore$ It can be written as $\int M\,dx + \int N\,dy = c.$

$= \int (e^x + 2xy^2 + y^3)\,dx + \int a^y\,dy = c.$

$e^x + 2y^2\frac{x^2}{2} + y^3(x) + \frac{a^y}{\log a} = c.$

$$e^x + x^2y^2 + xy^3 + \frac{a^4}{\log a} = c.$$

(Q1 b.)

$\boxed{L4}$

$$I = \int_0^\infty \frac{x^2}{(1+x^2)^{7/2}}\,dx.$$

Beta and Gamma function

The relationship between beta and gamma function.

$$\beta(m,n) = \frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}.$$

Put $x^2 = t$

$$2x\,dx = dt$$

$$dx = \frac{dt}{2x} \Rightarrow \frac{dt}{2\sqrt{t}}$$

$$\therefore I = \int_0^\infty \frac{t}{(1+t)^{7/2}}\,\frac{dt}{2\sqrt{t}}$$

$$= \frac{1}{2}\int_0^\infty \frac{t^{1/2}}{(1+t)^{7/2}}\,dt.$$

$$= \frac{1}{2}\cdot B\left(\frac{3}{2}, \frac{4}{2}\right)$$

$$\left[\because B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}\,dx\right]$$

$$I = \frac{1}{2}\,\frac{\Gamma 3/2 \cdot \Gamma 2}{\Gamma 7/2}.$$

(Q3 c.) As it is given that $z = 0$ $\therefore dz = 0$.

$\boxed{L4}$

$$\therefore \vec{F} = (x^2y)i + (y)j$$

$$\int_c \vec{F}\cdot\vec{dr} = \int_c (xy^2\,dx + y\,dy)\ d \rightarrow (1).$$

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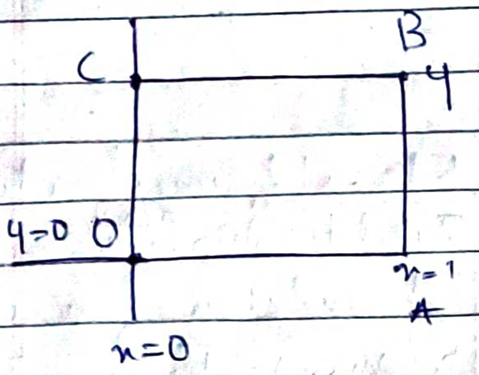
(I) For Rectangle boundary



For path OA: $y=0$, $dy=0$.
For path AB: $x=1$, $dx=0$.
For path BC: $y=2$, $dy=0$
For path CO: $x=0$, $dx=0$.

Thus considering above conditions the surface area can be found by adding all 4 paths, OA+AB+BC+CO.

$$\int_C \vec{F}\,\vec{dr} = \int_C (xy^2 dx + y\,dy)$$

$$= \int_{OA} 0\,dx + \int_{AB} y^2 \times 0 + 4\int_{BC} x\,dx + \int_{CO} y\,dy \rightarrow (2).$$

For path AB – $y=0$ to $y=2$.
For path BC – $x=1$ to $x=0$.
For path CO – $y=2$ to $y=0$.

$$\therefore \int_C F\,\vec{dr} = \int_0^2 y\,dy + 4\int_1^6 x\,dx + \int_2^0 y\,dy$$

$$= \left.\frac{y^2}{2}\right]_0^2 + 4\left.\frac{x^2}{2}\right]_1^6 + \left.\frac{y^2}{2}\right]_2^0$$

$$= (2-0) + 2(0-1) + (0-2)$$

$$\int_C \vec{F}\,\vec{dr} = -2 \longrightarrow (3)$$

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(II) For $\iint_S \nabla\times \vec{F}\cdot \hat{n}\, dn.$

$F = k$

$= \iint \nabla \times \vec{F}\cdot k\, ds.$

$\therefore \vec{\nabla}\times\vec{F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy^2 & y & 0 \end{vmatrix}$

$= 0\hat{i} + 0\hat{j} - 2xy\hat{k} = -2xy\hat{k}$

$\therefore \iint_S \nabla\times\vec{F}\cdot\hat{n}\, ds = \iint_S -2xy\,.\,\hat{k}\,.\,\hat{k}\, ds.$

$= -2\iint_S xy\, ds.$

$= -2\int_{x=0}\int_{y=0}^{2} xy\, dx\, dy.$

$= -2\left[\frac{x^2}{2}\right]_0^1 \cdot \left[\frac{y^2}{2}\right]_0^2 = -2\times\frac{1}{2}\times 2.$

$\therefore \iint_S \nabla\times\vec{F}\cdot\hat{n}\, dn = -2 \longrightarrow (4).$

From Eq (3) & (4).

$\iint_S (\nabla\times F)\,\hat{n}\, ds = \int_C \vec{F}\cdot\vec{dn}$

Thus Stokes Theorem is verified.

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(CO5)
(c)
[L1] • Cauchy Equation: It is linear homogeneous ordinary differential equation with variable coefficients. It is also known as the equidimensional Equation.

• Legendre's equation: The legendre differential equation is a second order ordinary differential equation. It has two linearly independent solution. A solution which is regular at finite points is called a legendre function of the second kind.

(CO3 b.) By Green's theorem, we have.
[L3]

$$\oint_c (\phi\, dx + \psi\, dy) = \iint_R \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dx\, dy$$

$$= \int_{-1}^{1}\int_{-1}^{1} \left[\frac{\partial}{\partial x}(x^2+y^2) - \frac{\partial}{\partial y}(x^2+xy)\right] dx\,dy$$

$$= \int_{-1}^{1}\int_{-1}^{1} (2x-x)\, dx\, dy \Rightarrow \int_{-1}^{1}\int_{-1}^{1} x\, dx\, dy.$$

$$= \int_{-1}^{1} x\, dx \int_{-1}^{1} dy \Rightarrow \int_{-1}^{1} x\, dx\, (y)_{-1}^{1}$$

$$= \int_{-1}^{1} x\, dx\,(1+1) = \int_{-1}^{1} 2x\, dx$$

$$\Rightarrow (x^2)_{-1}^{1} = 1-1$$

$$= \boxed{0}$$

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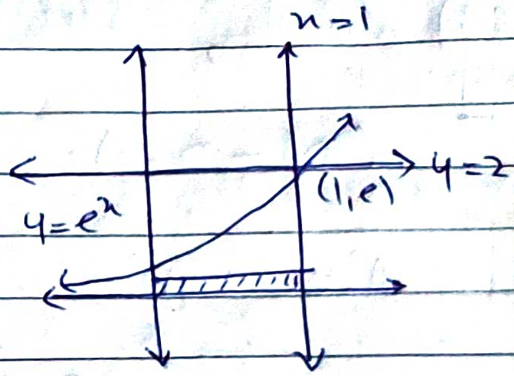
(Q2 a.) $\int_0^1 \int_{e^x}^{e} \frac{1}{\log y}\, dx\, dy$.

$\boxed{L3}$



$x=0$ to $x=1$

$y=e^x$ to $y=e$

Consider the strip ∥ to x-axis.

$x=0$ to $x=\log y$

$y=0$ to $y=e$.

$$I = \int_1^e \left[\int_0^{\log y} \frac{1}{\log y}\, dx \right] dy$$

$$= \int_1^e \frac{[x]_0^{\log y}}{\log y}\, dy$$

$$\cancel{= \int_0^e \frac{\log y}{\log y}\, dy}$$

$$\cancel{= \int_0^e dy}$$

$$\cancel{= [y]_0^e}$$

$$= \int_0^{\log y} \frac{1}{\log(y)}\, dx = 1.$$

$$= \int_1^e 1\, dy \Rightarrow \int_1^e 1\, dy = e-1$$

$\boxed{= e-1}$ ∴ $\boxed{I = e}$

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(04) Given,
$\boxed{13}$

$$(D^2 - D - 2)y = 2\log x + \frac{1}{x} + \frac{1}{x^2}$$

A.E. :- $D^2 - D - 2 = 0$ $\quad \therefore (D-2)(D+1) = 0.$

$$\therefore y_c = C_1 e^{2x} + C_2 e^{-x}.$$

$$y_p = \frac{1}{(D-2)(D+1)}\left(2\log x + \frac{1}{x} + \frac{1}{x^2}\right)$$

$$= \frac{1}{D-2}\left[e^{-x}\int e^{x}\left(2\log x + \frac{1}{x} + \frac{1}{x^2}\right)dx\right]$$

$$= \frac{1}{D-2}\left[e^{-x}\int e^{x}\left\{2\log x + \frac{2}{x} - \frac{1}{x} + \frac{1}{x^2}\right\}dx\right]$$

$$= \frac{1}{D-2}\left\{e^{-x}\int e^{x}\left[\left(2\log - \frac{1}{x}\right) + \left(\frac{2}{x} + \frac{1}{x^2}\right)\right]dx\right\}$$

$$= \frac{1}{D-2}\, e^{-x} \cdot e^{x}\left(2\log x - \frac{1}{x}\right) = \frac{1}{D-2}\left(2\log x - \frac{1}{x}\right)$$

$$= e^{2x}\int e^{-2x}\left(2\log x - \frac{1}{x}\right)dx$$

$$= e^{2x}\left\{\int 2\log x \cdot e^{-2x}\,dx - \int e^{-2x}\frac{1}{x}\,dx\right\}$$

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$$= e^{2x}\left\{ 2\log x \left(\frac{e^{-2x}}{2}\right) - \int \frac{2}{x}\left(\frac{e^{-2x}}{-2}\right)dx - \int e^{-2x}\cdot\frac{1}{x}\,dx \right\}$$

$$= e^{2x}\left\{ -\log x \cdot e^{-2x} + \int e^{-2x}\frac{1}{x}\,dx - \int e^{-2x}\frac{1}{x}\,dx \right\}$$

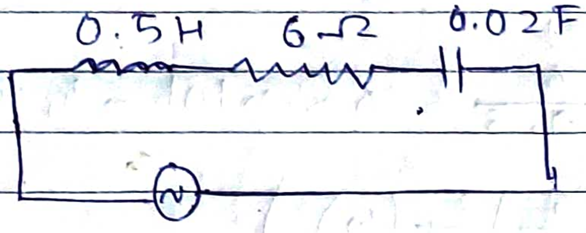
$$= e^{2x}\left\{ -\log x\, e^{-2x} \right\} = -\log x.$$

$$y = C.F. + P.I = y_c + y_p$$

$$y = c_1 e^{2x} + c_2 e^{-x} - \log x.$$

(Q5 d.)
$\boxed{L4}$

Circuit Diagram:



$$V_s = IR + L\frac{di}{dt} + \frac{Q}{C}$$

$$V_s = iR + L\frac{di}{dt} + \frac{1}{C}\frac{di}{dt}$$

$$24\sin(10t) = i\times 6 + 0.5\frac{di}{dt} + \frac{1}{0.02}\frac{di}{dt}$$

$$24\sin(10t) = 6i + 0.5\frac{di}{dt} + 50\frac{di}{dt}$$

$$24\sin 10t = 6i + (50.5)\frac{di}{dt}$$

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$$\frac{di}{dt} + 0.118\frac{di}{dt} = 24\sin 10t$$

b.) If the charge on capacitor is zero.

$$\frac{di}{dt} + \frac{6}{0.5}i = \frac{24}{0.5}\sin 10t.$$

Solution of D.E is.

$$i \times e^{\int \frac{6}{0.5}dt} = \int e^{\int \frac{6}{0.5}dt} \sin 10t\, dt + k$$

$$\left[\int e^{\int ax} \sin bx\, dx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)\right]$$

$$\therefore i e^{\frac{6t}{0.5}} = \frac{e^{\frac{6}{0.5}t}\; 6\sin 10t - 10\cos 10t + k}{\left(\left(\frac{6}{0.5}\right)^2 + (10)^2\right)}$$

$$i = \frac{1}{488}\left[12\sin 10t - \cos\right].$$

$$i = \frac{1}{488}\left[12\sin 10t - \cos 10t\right] + ke^{-12t}$$

for finding current put $i = 0$ & $t = 0$.

$$0 = \frac{1}{488}\left[-\cos 0\right] + ke^{0} \Rightarrow k = \frac{1}{488}$$

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