[Total No. of CO's :05] Seat No:
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G. H. Raisoni College of Engineering and Management, Pune.
(An Autonomous Institution affiliated to Savitribai Phule ,Pune University)
F.Y. B. Tech (All Branches) (Term-II)
Summer-2021 (2020 Pattern)
Matrices & Differential Calculus(UBSL103)

[Time:2 Hours]
[Max. Marks-50]

COURSE OUTCOME:

1. **Understand and use the theory of Matrices to solve the system of linear equations and engineering problems in respective disciplines.**
2. **Determine the Eigen values and Eigen vectors of a matrix and apply to various engineering problems in respective disciplines.**
3. **Apply concepts of differentiation in solving engineering problems..**
4. **Use applications of partial differentiation to solve various problems in engineering**
5. **Apply the Knowledge of vector differentiation to solve various problems in engineering.**

Instructions to the candidates:

1) *(CO1/CO2/CO….)at the beginning of question/sub question indicates the course outcome related to the question.*
2) *All questions compulsory.*
3) *Neat diagrams must be drawn wherever necessary.*
4) *Figures to the right indicate full marks.*
5) *Assume suitable data, if necessary.*
6) *Other Instructions, if any.*

CO	Sub Question		Marks	BL
CO1	a)	Determine the rank of the following matrix $$A=\begin{bmatrix}1&2&3\\1&4&2\\2&6&5\end{bmatrix}$$	[2]	L2
	b)	Under what condition non homogenous system has infinite number of solutions	[1]	L1
	c)	Examine for linear dependence or independence of vectors $(2,-1,3,2),(1,3,4,2)$ and$(3,-5,2,2)$. Find a relation between them if dependent	[3]	L3
	d)	Examine for consistency the following system of equations and if consistent, solve it. $2x-y+3z=1\,,\ 3x+2y+3z=3\,,\ x-4y+5z=-1$	[4]	L4
CO2	a)	Find the Eigen values of the matrix $$A=\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$	[1]	L1

	b)	A square matrix A is called idempotent if $A^2 = A$What are the possible eigenvalues of an idempotent matrix?	[2]	L2
	c)	Find then Eigen values and Eigen vector corresponding to the highest Eigen value of the matrix $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$	[4]	L4
	d)	Verify Cayley Hamilton Theorem for A = $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$	[3]	L3
CO3	a)	State mean value theorem	[1]	L1
	b)	Find the nth derivative of $(cos2x \cos 6x)$	[2]	L2
	c)	Expand $f(x) = e^{sinx}$ by Maclaurin's series	[3]	L3
	d)	If $y = a\cos(\log x) + b\sin(\log x)$, then show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$	[4]	L4
CO4	a)	If $x = rcos\ \theta, y = rsin\theta$ then find $\frac{\partial(x,y}{\partial(r,\theta)}$	[1]	L1
	b)	State Chain Rule for function of three variables	[1]	L1
	c)	Discuss the maxima and minima of the function $f(x,y) = 3x^2 - y^2 + x^3$	[4]	L3
		OR		
	d)	If $u = xyz\ , v = x^2 + y^2 + z^2\ , w = x + y + z$ find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$	[4]	L3
	e)	If $u = sin^{-1}\left(\frac{x^{\frac{1}{3}}+y^{\frac{1}{3}}}{x^{\frac{1}{2}}-y^{\frac{1}{2}}}\right)^{\frac{1}{2}}$, Prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}(13 + tan^2 u)$	[4]	L4
CO5	a)	Find the maximum value directional derivative of $\phi = x^3y^2z$ at (1,-2,3)	[2]	L2
	b)	Find the directional derivative of $\emptyset = xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t\ , y = t^2\ , z = t^3$ at the point (1,1,1)	[4]	L3
		OR		
	c)	Show that $\bar{F} = (ye^{xy}\cos z)\bar{\imath} + (xe^{xy}\cos z)\bar{\jmath} - (e^{xy}\sin z)\bar{k}$ is irrotational. Find its scalar potential 3	[4]	L3
	d)	Find the Value of n for which the vector $r^n\bar{r}$ is solenoidal , where $\bar{r} = xi + yj + zk, r = \sqrt{(x^2 + y^2 + z^2}$	[4]	L4