topology

arithmetic ⟷ alg. geom.

"discriminant" ⤳ stabilize

pts on a line. ↗

Hypersurfaces

motivation: ①

② probability $n \in \mathbb{Z}$
square-free.

$$\frac{6}{\pi^2} = \frac{1}{\zeta(2)}$$

$\underline{\underline{Q}}$: X proj. variety/$\mathbb{F}_q$.

(smooth)

Does there exist some hypersurface meeting X in a __smooth__ subvariety?

$$\lim_{d \to \infty} \left(\begin{array}{c} \text{odds hypersuface degree } d \\ \text{meets } X \text{ smoothly} \end{array} \right)$$

$$= \frac{1}{3_X (\dim X) \atop +1}$$

④ Topology

Vassiliev. (ICM lecture)
smooth divisors on $\mathbb{C}^n = \mathbb{A}^n$

Space:
$$h^0 = 1$$
$$h^{\square} = 1$$
$$h^i = 0 \text{ otherwise}$$

Thm. X smooth projective $^{(q-)}$

dim n. $\mathcal{L}$ ample

Probability. section $\mathcal{L}^{\otimes} \blacksquare z$

$\boxed{S}$ $\boxed{\underline{\text{smooth}}}$ has $\underline{\text{exactly}}$

m singularities : $k \rightsquigarrow$ exactly m pts

$$\lim_{z \to \infty} = \cfrac{\# \ldots + \delta_{pm} X\, t^{\delta} + \ldots}{Z_X\left(1/\mathbb{L}^{\oplus n + 1}\right) = \ldots + \left(\operatorname{Sym}^k X\right) t^k}$$

$$+ \ldots$$

$$\boxed{t = \left(\mathbb{L}^{-n-1}\right)}$$

Rks. 1) independent of ℓ

2) uses finite fields

3) $m = 0$ $\dfrac{1}{Z_X\left(\frac{1}{\mathbb{L}^{\dim X + 1}}\right)}$

4) enhanced versions of ...
Poonen's result. $\nearrow$
Joseph Gunther.

5). $\mathcal{L}_1$ ample. $/\mathbb{C}$

X (g) proj.

smooth?

$\nearrow$ sections. $\rightsquigarrow$ singular?

(on singr.).

(homotopy type ??)

smooth plane curves: $0 \underset{\nearrow}{3} \underset{\nearrow}{5} 8$

O. Tommasi $\rightsquigarrow$ More!

Kupes-Miller-Tran.
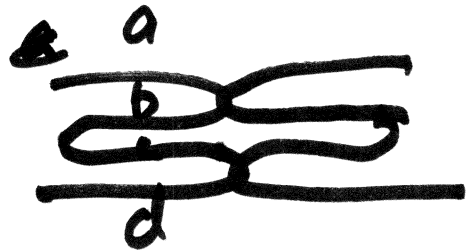
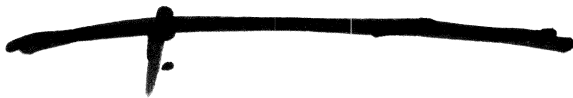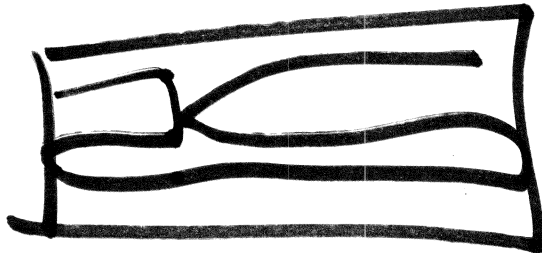# Bhargavology

Fix $d > 1$.

number fields degree d ?

how grow with discrimant $< N$

$(\sim ? N)$.

$d = 2, 3$ Bh: $4, 5$

(Riemann-)zeta values.

$a \, b \, c \, d$

$a \, c \, e \, d$

$a \, d \, e \, b \, c$

a

b

d

Spec $\mathbb{Z}$

$\mathbb{P}^1$

$\mathbb{P}^2$

$g \to \infty$

trigonal.

$\downarrow$

$\mathbb{P}^1$

/Aut $\mathbb{P}^1$

F_g

Zhu

motivic
zeta-value
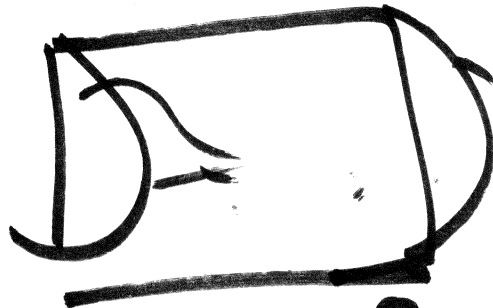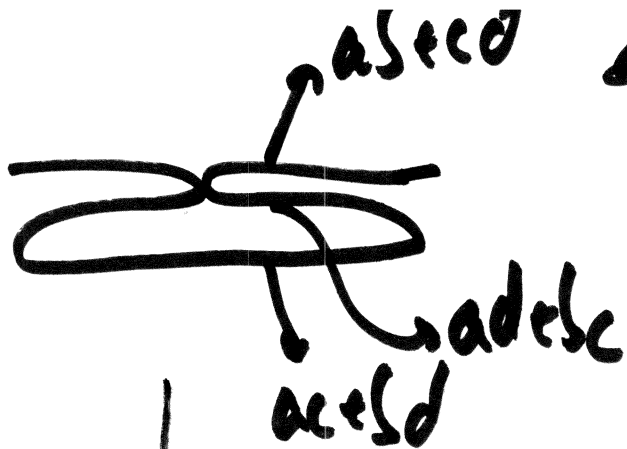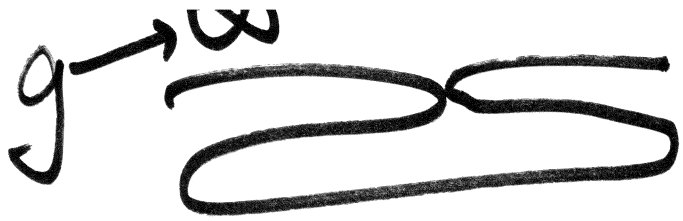
Answ:

Trigonal ✓

deg = 4 Tetragonal ⤳ ▷ ☑

deg = 5 Pentagonal ⤳ ▷ ✓
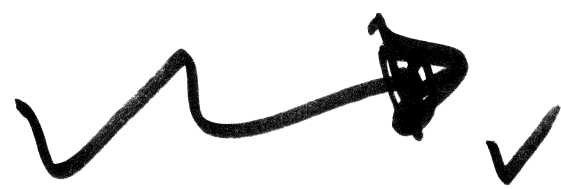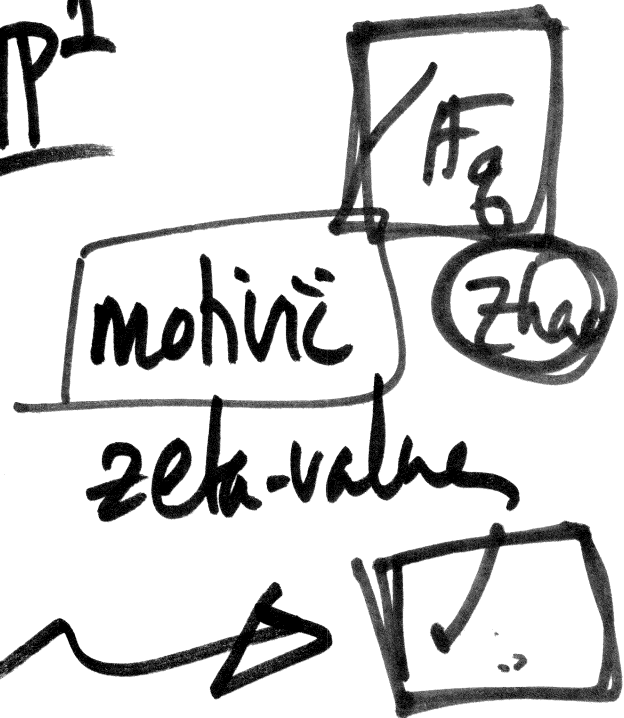
How many genus 2 curves are there /Q ?

2	3	4	5	6	7
deg 2	deg 4 K3	deg 6	deg 8		
$2{:}1 \downarrow \mathbb{P}^2$	Quartic $\mathbb{P}^3$	$Q \cap C$ in $\mathbb{P}^4$	$Q_1 \cap Q_2 \cap Q_3$ in $\mathbb{P}^5$ $\longrightarrow$		

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genus 2	genus 3	genus 4	genus 5		
$2{:}1 \to \mathbb{P}^1$	deg 4 in $\mathbb{P}^2$	$Q \cap C$ in $\mathbb{P}^3$	$Q_1 \cap Q_2 \cap Q_3$ $\longrightarrow$		

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	2 pts	3 pts	4 pts	???	
	$\vdots \mathbb{P}^0$	C in $\mathbb{P}^1$	$Q_1 \cap Q_2$ in $\mathbb{P}^2$		

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	deg 2	genus 1 deg 3	deg 4		
	$2{:}1 \downarrow \mathbb{P}^1$	C in $\mathbb{P}^2$	$Q_1 \cap Q_2$ $\longrightarrow$		

O_K.

~~$\mathbb{PPZ}$~~ $\oplus 2$

rank. 3.

$\downarrow$

$\mathbb{Z}$

$\mathbb{P}$ rank 2
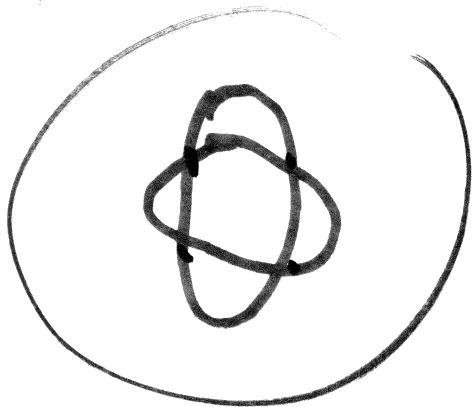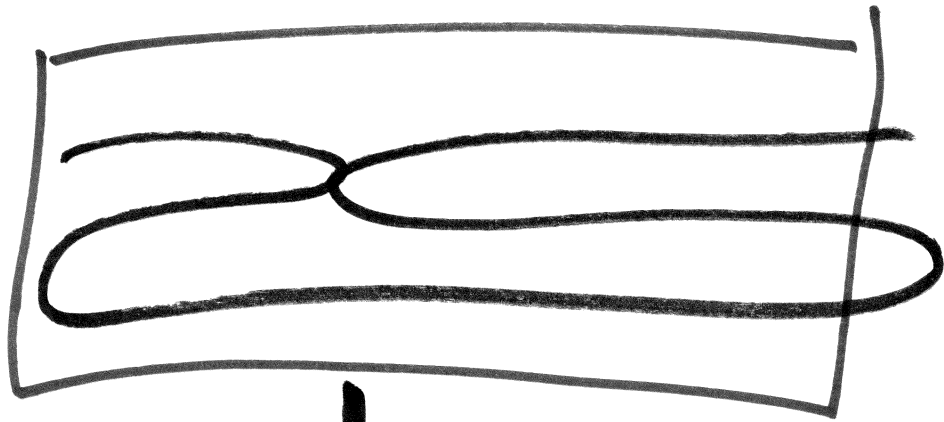
rank

$\mathbb{P}^2$-bundle.

$\mathbb{P}^3$-bundle.

dy 4.

dy 5

$\boxed{\text{deg. 5}}$ $\dot{G}(2,5)^{\dim \mathbf{6}} \subset \mathbb{P}^9.$

Plücker.

~~Slice with 5 hyperplanes.~~

~~deg 5 curve in $\mathbb{P}^4$~~

Slice with quadric

deg 10 $\dim \not{S} I \subset \mathbb{P}^{\not{q}}$ 6.

$\not{A}$ hyperplanes. $\longrightarrow$

~~K3 surface.~~

canonical curve
genus 6

$G(2,5)^{\deg \cdot 5}$ in $\mathbb{P}^9$.   6 pts

$OG(5,10)$ in $\mathbb{P}^{14}$

$\vdots$

$\widehat{R}_{\text{ll}}$