U

## Mod p reps. of p-adic groups

FIOP finite

$$U_F/(\varpi) \cong I_{F}$$
  $(q = p^F)$ 
 $G := GL_n(F)$ 
 $K := GL_n(V_F)$  max.cpt.

 $V_{A} := 1 + \varpi M_n(V_F)$ 
 $V_{A} := 1 + \varpi^2 M_n(V_F)$ 
 $V_{A} := 1 + \varpi^2 M_n(V_F)$ 

Copt. upon

(define top.)

smooth ladm: as in other lectures.

V = UVH dim VH < 00 H < G (cpt.) open Motivation: Local langlands

classical
irred. sm. reps. over I

"Galois reps.

GallFIF) -> GLn(C)"

mod p

Sm. reps. of G over 1Fp

Galois reps.

Gal (FIF) -> GLn ()Fp)

## Challenges

G has no C-valued Haar measure!

$$\left( \int_{K_{r}} = P - \int_{K_{r+1}} = 0 \right)$$

- no analytic tools!

A If H & G open opt.,

H bloos not act
semisimply!

+ V > VH is not

Warmup 
$$(n=1)$$

$$G = GL_{1}(F) = F^{\times}$$

$$= \varpi^{2} \times F_{4}^{\times} \times (1+\varpi U_{F}).$$
abelian

abrecian =) an irr sm. G-rep is 1-dim.

$$\chi: G \longrightarrow C^{\times}$$

Smooth (=) Ker x is upen

$$(*)$$
 = 1, as char(c)=p

$$\chi(\varpi) \in C^{\times}, \quad |F_{\uparrow}^{\times} \to C^{\times}.$$

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Generalise (\*):

V Smooth reproof pro-p gp.

H. (over C)

$$\Rightarrow V^{H} \neq 0.$$

Idea: reduce · H fin. p-9p.

- The function

· C = 1Fp.

· V f.d.

ev: H -> GLa(IFp)

image < Sylow (0-\*)

Cor (notes):

If IT is sm. G-rep., H & G,
open
pro-p. π adm. (=) πH f.d. Applications to irreducibility.

Ex.1:  $T = GL_2(lfg) \cap V = Sym^{2}(C^{2})$  q=p ((X,Y) = f(aX+cY,bX+dY) ((A,Y) = f(aX+cY,bX+dY)

Sketch:

$$\Delta := \begin{pmatrix} 1 \\ | F_{4} \end{pmatrix} \leq T \left( \int y w w \right)$$

(1)  $V^{\Delta}$  is 1-dim.  $\frac{1}{2}$  calculation (2)  $\langle \Gamma, V^{\Delta} \rangle = V$ 

(2) 
$$\langle \Gamma, V^{\Delta} \rangle = V$$
  
(gen. by  
 $\Gamma$ -action)

If 0 + WCV (T-subrep.)

-> Cor: The irreps. of I over C

are Sym'(C2) & dets 0 5 1 5 4-1 S & 2/(9-1). (total = 9(9-1)) "Serre weights". Ex. 2: K = GL2(UF). K, pro-p.

If V irred. sm. rep. of K,

=> 0 + V K, c V ... also a V

Lemma K-subrep.

$$\Rightarrow$$
  $V^{K_1} = V$ , so  $V$  is an irrep. of  $K/K_1 \cong \Gamma$ , a Serre weight.

Ex3: 
$$G = GL_2(F)$$
 $B = (**) \leq G$ 

Therefore  $A = Min. parab$ .

$$B = T \times U, T = (**)$$

$$U = (!*).$$

Recall:  $0 = 0, \otimes 0_2 : T \longrightarrow C^{\times}$ smooth.

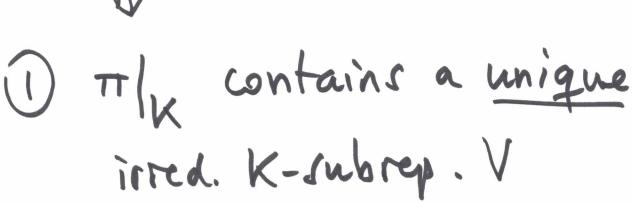
Ind
$$G(\theta) =$$

is an adm. smooth G-rep.

Thm (Barthel-Livné):

If 
$$0, \neq 0_2$$
, then  $|nod_B^G(0)|$  irred.

Idea: Assume  $0, |o_F^{\times} \neq 0_2|_{O_F^{\times}}$ 



$$(2) < G \cdot \lor \rangle = \pi$$

If 
$$0 \neq \pi' \in \pi$$
 (G-subrep.),

then  $\pi'|_{K}$  contains an irred.

 $K-sub$ 
 $V \subset \pi'$ 

 $\frac{1}{2} V C \pi'$   $\pi = \langle G, V \rangle C \pi' \Rightarrow \pi' = \pi$ 

Rk: If  $\beta_1 = \theta_2$ ,  $\theta_3 := \theta_1 = \theta_1$ .

Ind  $G(\theta_1 \otimes \theta_2) = (\theta_0 \circ def) \otimes Ind G(1)$ .

0 - 11 - Inde(1) - st - so
what.

When there

Thm. (B-L)

The irred adm. rep. of GLz (F):

- · O. odet (1-dim)
- · Inda (0,000), 0, 702
- · St & ( Boodet)
- + and no nontriv. isoms. between them.