

# RECALL

$$P > 2r+2$$

r < g

L-T, McP

$$\# X(Q) \leq \# X^m(\mathbb{F}_P) + 2g - 2$$

Stoll: Sps  $X$  hyperelliptic +  $r \leq g-3$ .

Then  $\# X(Q) \leq \# 3(r+4)(g-1)$   
 $+ \max\{1, 4r\} \cdot g$

Katz-Rabinoff-ZB Sps  $r \leq g-3$ .

then  $\# X(Q) \leq 84g^2 - 98g + 28$

# RECALL

$X$  regular  $\Leftrightarrow \forall p \in X,$   
 $\dim_{k(p)} P/P^2 = \dim_p X$

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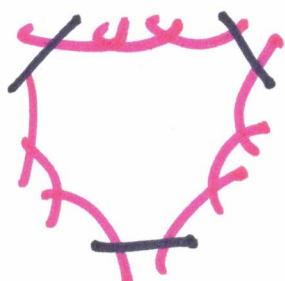
Regular  $\Rightarrow \text{im}(\mathcal{X}(\mathcal{O}_p) \rightarrow \mathcal{X}(F_p))$

$$\leq \mathcal{X}^{sm}(F_p)$$

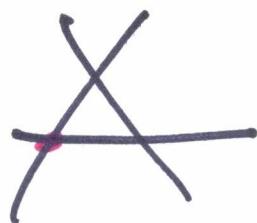
$$\overline{\text{HL}}$$

Problem:  $\mathcal{E}^{\text{sm}}(\mathbb{F}_p)$  is unbounded

$$xyz = p^n(x^3 + y^3 + z^3)$$



3n-gon



$$\text{locally } xy = p^n$$



Sfall:

- work w/ non-regular mesh
- contract chains at  $p^n$ 's

$\Rightarrow \mathcal{E}(\mathbb{F}_p)$  is bounded  
(v.g.  $p+g$ )

Prob's Set up + local analysis

is hard

# Main tool (KZB)

Systematic use of Berkovich f  
tropical geometry

Setup: 2 types of  $S$

$$\begin{matrix} AB \\ \zeta_w \end{matrix} + \begin{matrix} BC \\ \zeta_w \end{matrix}$$

- comes from Lc  $J$
- $\frac{\zeta_{AB}}{P} = 0$
- Compatable
- not equal to  $S$

Difference factors thru  $\text{Trop } T \simeq \mathbb{R}^i$

$$1 \in \widehat{J} \xrightarrow{\text{dN}} J$$

$\downarrow$   
A good red

# Local analysis

13

potential theory on  $X^{\text{an}}$

"Gilahti" step:

"combinatorial optimization" for  
sections of "Tropical canonical bundle"

Tay version:  $|K_P| \geq D,$

bound the min. # of strings for which  
witnesses  $D \sim K_P$

Time travel to AwS 2007

4

$M(R) = \{ R \xrightarrow{\text{I-H}} M \text{ bounded multiplicative} \}$

$\downarrow \text{vert } \overline{I}$

$R \quad |f|$

semi norms

$x \in M(R)$ ,  $f \in R$ , " $f(x)$ " :=  $|f|$

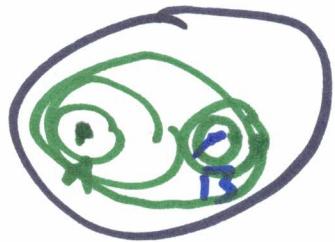
Example:  $\oplus_p \{+3\} \ni \sum_{i=0}^{\infty} a_i \cdot p^i \quad a_i \in \mathbb{Q}_p$   
 $|a_i| \rightarrow 0$

$M(\oplus_p \{+3\}) \ni | - |_x$

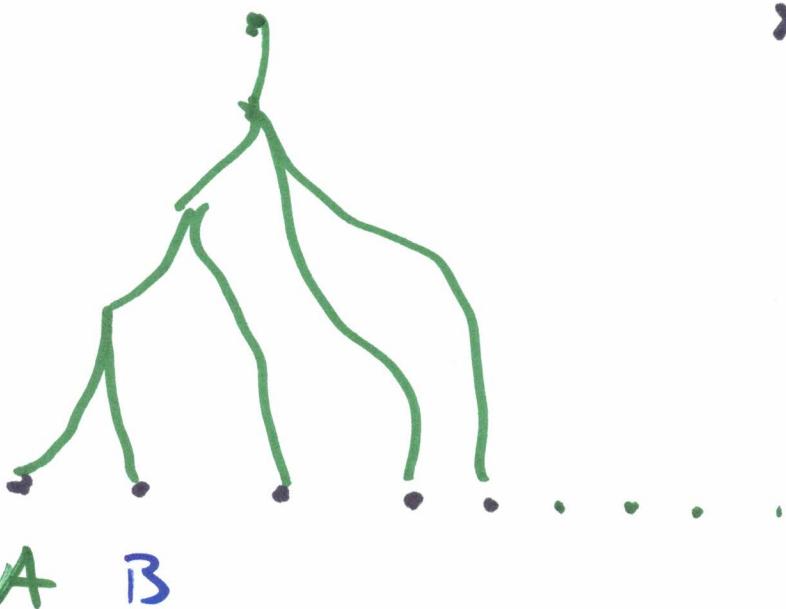
$|f| = |f(x)|$

Max Spec  $\oplus_p \{+3\}$

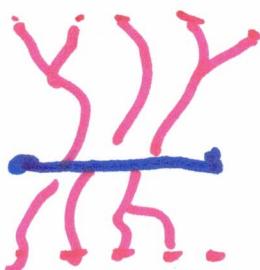
$x \in \overline{\mathbb{Z}_p} \subset \overline{\mathbb{Q}_p}$



More pts:  $|f|_{B(x,r)} = \sup_{x \in B} |f(x)|$



$$xy = p^n$$



Glob. Glue, sheaves, coh, etc

# Fundamental THM of Trop. Geometry

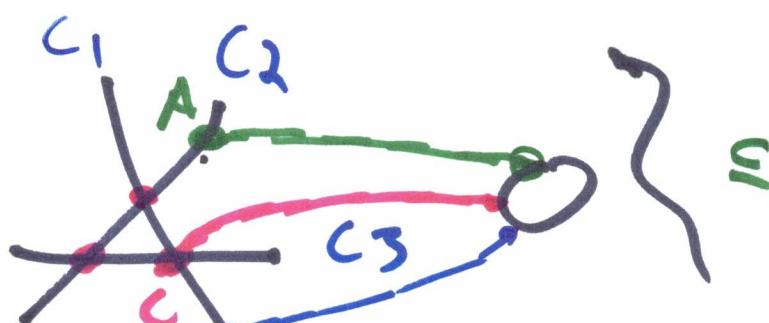
6

Baker - Payne - Rabinoff

Berkovich, Thuillier

$\mathcal{X}/\mathbb{Z}_p$  s-stable

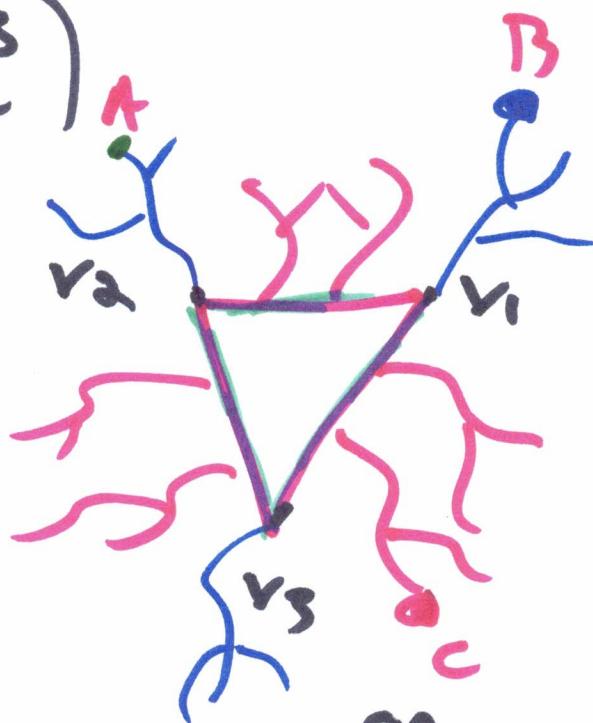
$$xyz = p(x^3 + y^3 + z^3)$$



$$|t| \mapsto v_i(c_i)$$

$\mathcal{X}_{\text{fp}}$

$\mathcal{X}_{\text{gp}}$



$\mathcal{X}^{\text{an}}$

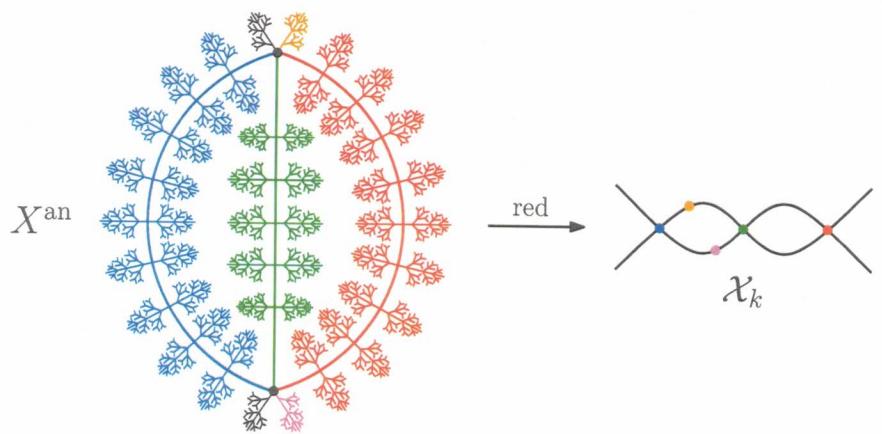
$\mathcal{U}_1$

$\mathcal{X}_T$

Cartado

$\mathcal{Z}_1$

+  
Compartible w/  
Reduction



Potential THM  $\xrightarrow{x^n f \rightarrow p'}$   $\sqrt{S^7}$

$$F(x) := -\log |f(x)|$$

$F/r$  is plw linear

THM:  $\gamma_x \operatorname{div} f = \operatorname{div} F$

$$:= \sum \left( \begin{array}{c} \text{zincally} \\ \text{slopes} \\ e \\ P \end{array} \right)$$

Exampl

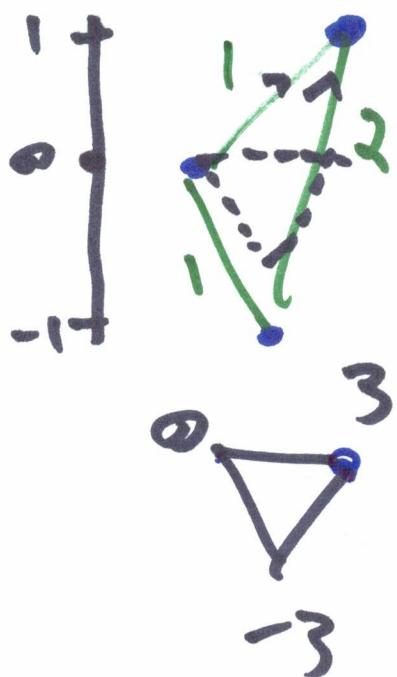
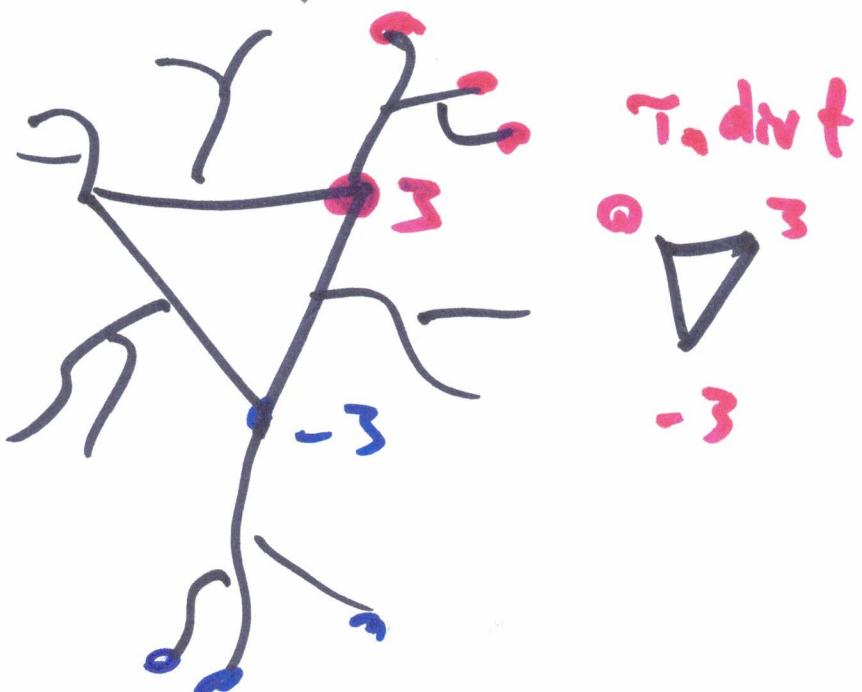
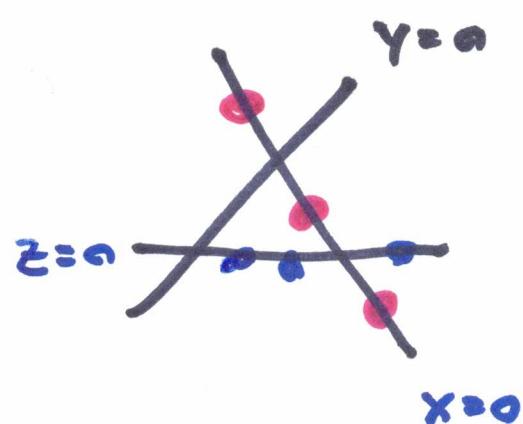
Example:  $f(x,y,z) := \begin{cases} x+z & z \neq 0 \\ 0 & z=0 \end{cases}$

8

$$\operatorname{div} f = [0: \xi_0: 1] + \dots - ([1: \xi_0: 0] + \dots)$$

$$y^3 + z^3 = 0$$

$$x^3 + y^3 = 0$$



KREBS: h, s, II-II

$$\operatorname{div} s = \operatorname{div} F + \|G\|$$
$$F = -\log |S|$$

(KZ2B) If, 1.1 metrized L.B. , S

17

$$\pi_r d\omega_S = d\eta F + \|h\|^2 \quad \text{↑ volume}$$

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$$x = \mathbb{C}^n$$

$$\zeta : P(x) \times \mathbb{C}^n \rightarrow \mathbb{C}^p$$

## Axioms for integration

$$\cdot X \overset{\text{open}}{\ni} \cup \text{ polydisc, } \omega|_U = df$$

$$\int_{\gamma} df = f(\gamma(1)) - f(\gamma(0))$$

$$\cdot \int_{\gamma} \omega \text{ only depends on } [\gamma]$$

. linear.

# Still choices

18

$$X = \text{fin m}, \quad w = \frac{dT}{T}$$

$\approx *$

$H_{\text{eff}} \neq 0$

<sup>BC</sup>

$$S_w = \log T$$

$$\left. \begin{array}{c} \uparrow \\ \mathcal{C}_P^* \supseteq \mathcal{O}_{C_P}^* \supseteq \mathcal{O}_{C_P}^{(1)} \xrightarrow{\log} C_P \\ \downarrow \quad \quad \quad \downarrow \\ P \quad \quad \quad F \end{array} \right\} \text{via} \quad \text{Poin scores}$$

gp basis

Must choose  $\log P$

Date:  $\overline{\mathcal{C}_P^* / \mathcal{O}_{C_P}^*} \simeq \mathbb{R} = T_{\text{reg}} C_m$

(Calc III) Regular Functionality (Math SS Joke) 19

$$x \xrightarrow{f} y, w \in Z'_{dR}(y)$$
$$\gamma \in P(x)$$

$$\sum_{\sigma} f^* \omega = \sum_{\sigma} \omega$$
$$f_* \gamma$$

Apply to  $C \hookrightarrow J$  AJ

$$\sum_{\sigma} i^* \omega = \sum_{\sigma} \omega$$
$$i_* \gamma$$

Apply to  $[n]$  on  $J$

$$\sum_0^n \omega = \frac{1}{n} \sum_a^n \omega$$

$$[n]^* \omega = n \omega$$

Call this  $\sum^{\text{Ab}}$ , use for CHAB

$$\sum_P \omega = 0 \text{ for } P, Q \in C(C)$$

THM: (Cohen, de-Shalit, Berk, Dwork) yo

Fix  $\log P$ , regular Fun für Frch.

Then this uniquely determines an integral  $\int_{\text{BC}}^{\text{AC}}$  s.t.

$$A \cup \subseteq X^n, \forall \sigma, \int_{\text{open}}^{\text{an}} d\sigma = f(x_{n1}) - f(x_0)$$

$\uparrow$  can be anali,

or even "wide open"  $\hookrightarrow$  Joe Pr.

IE, useful for computation.

$$\text{BD}, \int_{\text{BC}}^{\text{AC}} \neq \int_{\text{AB}}$$

$$\text{IE } \int_P^Q w \neq 0 \text{ für } P, Q \in X(Q)$$

# THM (Coleman)

"

If  $X$  has good reduction,  
then  $\zeta_{BC} = \zeta_{AB}$ .

Pf (Kriz)  $X^{\text{an}}$  is contractible (D)

$$\text{Jen B: } \frac{\int_w}{\text{Frob } Q} = \frac{\int_w}{\text{Frob } R}$$

"Analytic continuation along Frob's"

# Final Remarks:

why  $r \leq g-3$ ?

THM (Stoll): The difference

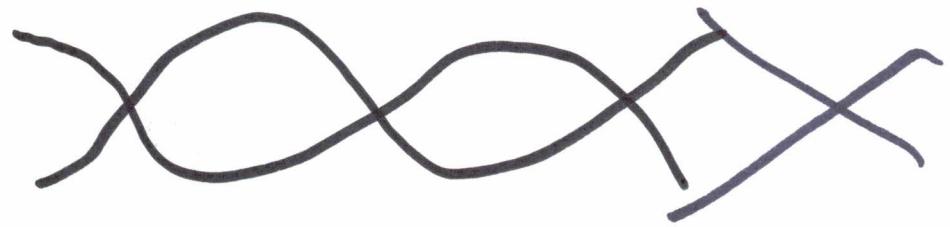
$\zeta_w^{AB} - \zeta_w^{BC}$  is linear in  
 w  
 regular stable

# → ~~Q~~  $\infty$  char in annuli

$$w|_{\mathbb{H}\Omega} = \sum q_i t^i = df + \frac{q_1}{t}$$

not exact!

"Rolle's"? Computing  $d_N F \leftrightarrow$  locally  
 computing w/  
 $N$ -polygons



$$X^{\text{an}} = U_x \cup U_y$$

A diagram illustrating the decomposition of an analytic space  $X^{\text{an}}$  into two open sets  $U_x$  and  $U_y$ . The left side shows a circular domain with a fractal-like boundary, labeled  $\Gamma$ , containing points  $x$  and  $y$ . The right side shows the same domain split into two regions:  $U_x$  (left half) and  $U_y$  (right half), separated by a boundary  $S_x$  and  $S_y$  respectively. The regions are filled with a similar fractal pattern as the original boundary.

$$\text{O}(\text{O}(\text{O})) = \text{O}(\text{O}(\text{O})) \cup \text{O}(\text{O}(\text{O}))$$

A diagram illustrating the decomposition of a curve  $O(O(O))$  into two parts. The left side shows a single closed loop. The right side shows the same loop split into two separate components, each consisting of two nested loops, separated by a central vertical dashed line.