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①.

$$E_n = p_- unito of F_n, E_n = E_n/W_n (0 \le n \le \infty)$$

Defn. Q'= ring of rational numbers whose denominator is a power of p.

$$\varphi'/z = \varphi_h/z_h$$

We have escact sequence  $(0 \le n < \infty)$   $0 \rightarrow E'_n \rightarrow E'_n \otimes Q' \rightarrow E'_n \otimes Q_1/Z_1 \rightarrow 0$ 

Pass to inductive limit as n-sos

En is a direct summand of Ex.

$$(\mathcal{E}_{\infty})^{\Gamma_m} = \mathcal{E}_m^1$$

$$\Rightarrow (\xi_{\infty} \otimes \varphi')^{\Gamma_{n}} = \xi_{n} \otimes \varphi'.$$

also

$$H'(Gal(F_m/F_m), E_m \otimes \varphi') = 0$$
 becouse  $E_m' \otimes \varphi'$  is  $\varphi$  - divisible.

$$\Rightarrow H'(\Gamma_m, \mathcal{E}_{\infty} \otimes \mathcal{O}') = 0$$

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Take I'm - cohomology of

0 -> E\_0 -> E\_0 0 0 -> E\_0 0 0 / Z, -0

Proposition. For all n > 0, we have the escapt sequence

0 -> E'n @ Q / Z, -> (E'0 @ Q / Z)

→ H'(Γ, ξ, )→0.

How big is H'(r, 800)?



Proposition. An = p-primary subgroups
of In'/Pn'. For all n >0, we have

H'(r, E) = Ker(A) -> A).

In particular,  $H'(\Gamma_n, E_{00})$  -is always a finite group.

Proof later in lacture

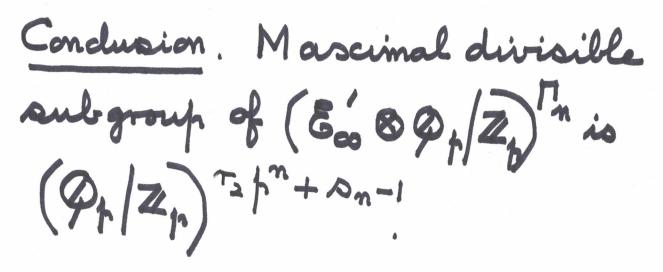
Consequence: Dn = number of primes of Fn above to

Dirichlet - Chev alley

$$\mathcal{E}_{n}'\otimes \varphi_{1}/\mathbb{Z}_{h} = (\varphi_{1}/\mathbb{Z}_{h})^{T_{2}}h^{n} + \delta_{n}-1$$

Combined with escart sequence above guies: —





$$\Longrightarrow (\Upsilon_{\infty})_{\Gamma_{n}} \text{ is dual to } (\Xi_{\infty}' \otimes \mathcal{O}_{1}/Z_{1})^{\Gamma_{n}}$$

by Pontrjagin duality.

Conclusion  $\mathbb{Z}_{h}$ -rank of  $(\Upsilon_{\infty})_{\Gamma_{n}}$  is  $\mathbb{Z}_{h}^{n} + \mathbb{A}_{n} - 1$  for all  $n \gg 0$ .

You is a f.g.  $\Lambda(\Gamma)$ -module since  $(Y'_0)_{\Gamma}$  is a f.g.  $\mathbb{Z}_{h}$ -module.



Fact. There escists no 7,0, such that all primes of Fno above 12 are totally ramifed in Foo.

to totally ramified in Q(µ,00).

Consagnence. Dn = Dno=D for all n>no.

Conclusion. (Too) phas Zy-rank

1. Ta+0-1 for all n 7/10.

Hence structure theory =>

Theorem. Too has  $\Lambda(\Gamma)$ -rank equal to  $T_2$ .

Mco
|
| Kummer theory =>
|
| Noo | Gal (N/Foo) = Hom (E/O) | Zp, upo)
| with natural [-action
|
| Foo

Hence:



Hence we have proven : -

Theorem B. Gal ( $N_{\infty}/F_{\infty}$ ) has  $\Lambda(\Gamma)$ -rank equal to  $T_2 = [F:\varphi]/2$ .

How do we prove

Suffices to prove: \_

Proposition. For all m >> n, we have an isomorphism

Fire a generator & of Gal (Fm/Fm)

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Om = ring of p-integers of Fm

CE Ker (Am -> Am)

Take or E In in class of c

ONO = XO XEO

E = 6x/x

Gbenne:  $\varepsilon \in E_m$ ,  $N_{F_m/F_m}$ 

 $T_{m,m}(c) = cohomology class of$  $<math>E \text{ in } H'(Gal(F_m/F_n), E_m').$ 

Well defined and a homomorphism also Tn, m is injective

Surjectivity. Take any cohomology class in  $H'(Gal(F_m/F_n), E'_m)$  represented by  $\Theta \in E_m$  with  $N_{F_m}/F_m$ 

Hilbert 90: Fac Om with 8 = 25-1

Take OVE I'm defined by OV = & Om.

OV = OV since & = = = = Em.

But all primes of For which do not divide pare unramified in For/For

or is the image of an ideal trin In.

Take c = class of b.

Tn, m (c) = class of 0

Greenberg's thesis gave escamples where Ker (And -> As) +0.

also Iwasawa proves: -

Theorem. Gal (Noo/Foo) is a free Zp-module and, winting t (Gal (Noo/Foo)) for its  $\Lambda$  ( $\Gamma$ ) - torsion submodule, then

Gal  $(N_{\infty}'/F_{\infty})/t$  (Gal  $(N_{\infty}'/F_{\infty})$ )
is a free  $\Lambda(\Gamma)$ -module if and only if  $H'(\Gamma_n, E'_{\infty}) = 0$  for all  $n > n_0$ .

Same for Gal (Moo/Foo)/t (Gal (Moo/Foo)).
Relevant for higher K-theory of OF.

