SOME ASPECTS OF THE ALGEBRAIC THEORY OF QUADRATIC FORMS

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In this series of lectures, we shall discuss the following topics:

(1) Witt group of quadratic forms, filtration by the fundamental ideal, invariants for quadratic forms with values in Galois cohomology, statement of Milnor conjecture.
(2) Pfister's theory of multiplicative forms, quadratic forms under function field extensions.
(3) Some numerical invariants associated to fields: level, u-invariant, pythagoras number; we shall discuss application of Pfister's theory to the solution of fundamental questions concerning these invariants as well as more recent progress. We shall also discuss some open questions concerning these invariants.

Project Title: Hasse principle for function fields.

Hasse-Minkowski's theorem asserts that a quadratic form over a number field k admits a nontrivial zero if it does over completions at all places of k. One could look for analogues of Hasse principle for function fields. Let F be a field and $F(t)$ the rational function field in one variable over F. Let V_1 denote the set of all discrete valuations of $F(t)$ trivial on F and V the set of all discrete valuations of $F(t)$. We shall discuss analogues of Hasse principle for isotropy of quadratic forms over $F(t)$ with respect to V_1 and V. An affirmative answer to the Hasse principle for $F = \mathbb{Q}_p$ would lead to the fact that every quadratic form in at least 9 variables over $\mathbb{Q}_p(t)$ has a nontrivial zero.