Transcendence in Positive Characteristic

Brief Outline

The story of transcendence over function fields started with proof of the non-algebraicity of the Carlitz analogue of π. We will see how the work of J. Yu ported the whole modern formulation of transcendence machinery for commutative algebraic groups over to the function field setting of Drinfeld modules and t-modules. This achieved surprisingly precise parity with the theory in characteristic zero.

Many new impulses in the field arose with the work of Sinha on values of Thakur's function field Gamma function. Analogues of Sinha's transcendence result remain far beyond what is currently known in the classical setting. We will see how those values arise as (coordinates of) periods of certain "soliton" t-motives of Anderson. This approach was extended to linear independence by Brownawell & Papanikolas via a notion of complex multiplication.

Further progress, to algebraic independence, came from widening the algebraic-geometric scope even further culminating in Papanikolas's Tannakian categories, and, at the same time, narrowing the technical transcendence tool. We will retrace some of the basic steps and derive some of the applications of Anderson-Brownawell-Papanikolas, Papanikolas, and/or Chang-Yu.

Finally, we will take a look at Denis' ingenious approach to the algebraic independence of Carlitz logarithms of algebraic numbers. It involves essentially no new machinery beyond the classical "Mahler's Method", but great insight.

Projects

1. Derive a quantitative version of Denis' independence result, using the general set-up of P-G. Becker.

2. Figure out how the Tannakian machinery of Papanikolas applies to give independence results for periods of t-motives, for t-modules to be specified.