Question 1: Lens System

(a)

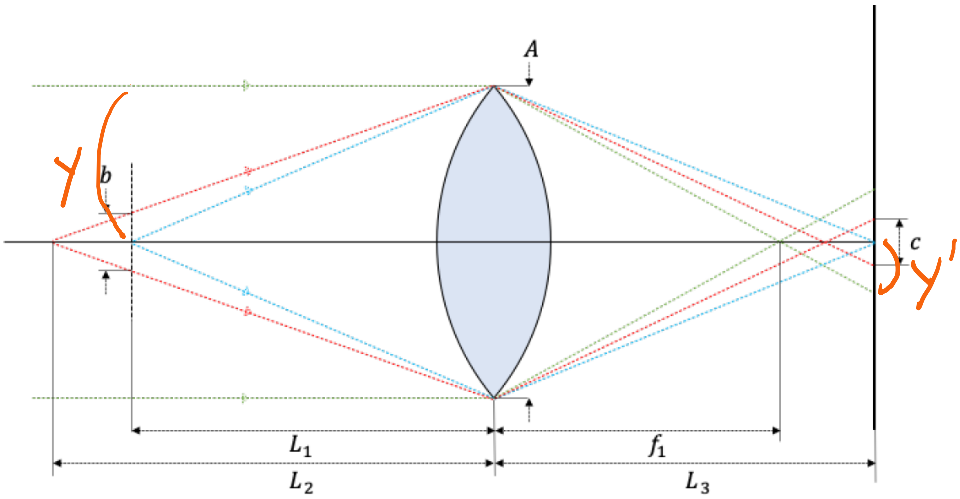

Figure 1: Diagram of circle of confusion diameter c for an out-of-focus subject at distance L_2 when the camera is focused at L_1 ($L_2 > L_1$). f_1 is the focal length of the lens, A is the aperture diameter of the lens, b is a blur circle diameter, and L_3 ($L_3 > f_1$) is a distance between the lens and the camera (*i.e.*, image).

$$\frac{y'}{y} = \frac{L_3}{L_1} = \frac{L_3 - f_1}{f_1} \quad => \quad \frac{1}{L_1} = \frac{L_3 - f_1}{L_3 * f_1} \quad => \quad \frac{1}{L_1} = \frac{1}{f_1} - \frac{1}{L_3} \quad => \quad \frac{1}{L_1} + \frac{1}{L_3} = \frac{1}{f_1}$$

(b)

$$\frac{L_2 - L_1}{b} = \frac{L_2}{A} \quad => \quad b = \frac{A}{L_2}(L_2 - L_1)$$

(c)

Magnification $m = \frac{L_3}{L_1}$

By (a), $L_3 = \frac{L_1 * f_1}{L_1 - f_1}$

$$c = bm = \frac{L_3}{L_1} * \frac{A}{L_2}(L_2 - L_1) = \frac{L_1 * f_1}{L_1 - f_1} * \frac{1}{L_1} * \frac{A}{L_2}(L_2 - L_1) = \frac{f_1}{L_1 - f_1} * \frac{A}{L_2}(L_2 - L_1)$$

(d)

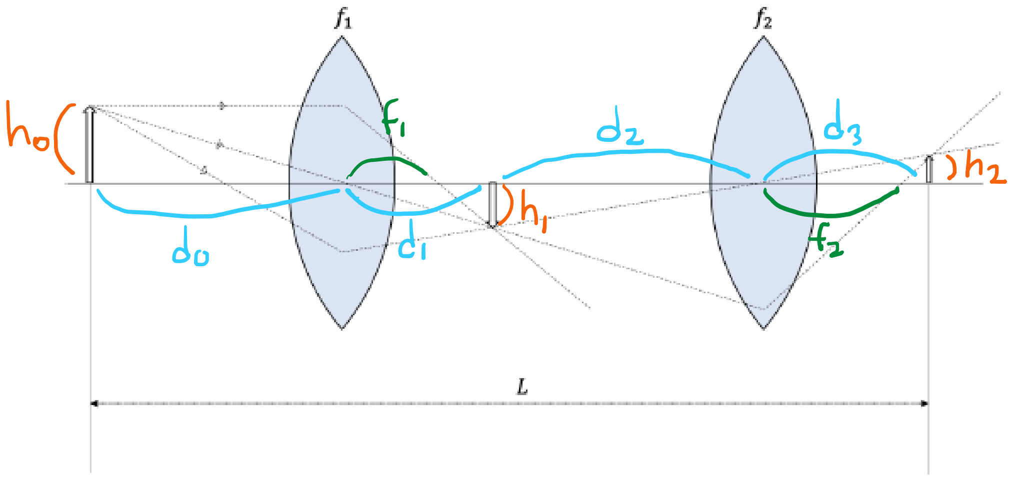

Figure 2: Diagram of two lenses system. f_1 and f_2 are the focal lengths of each lens and L is the distance between an image plane and an object for which to be properly focused.

$$\frac{1}{d_0}+\frac{1}{d_1}=\frac{1}{f_1},\qquad \frac{1}{d_2}+\frac{1}{d_3}=\frac{1}{f_2}$$

$$m_1=\frac{h_1}{h_0}=\frac{d_1}{d_0},\qquad m_2=\frac{h_2}{h_1}=\frac{d_3}{d_2},$$

$$L = d_0+d_1+d_2+d_3$$

$$d_1 = m_1 * d_0,\qquad d_3 = m_2 * d_2$$

$$f_1=\frac{d_0 * d_1}{d_0+d_1}=\frac{d_0 * m_1 * d_0}{(1+m_1) * d_0}=\frac{d_0 * m_1}{(1+m_1)},\qquad f_2=\frac{d_2 * d_3}{d_2+d_3}=\frac{d_2 * m_2 * d_2}{(1+m_2) * d_2}=\frac{d_2 * m_2}{(1+m_2)}$$

$$L = d_0+d_1+d_2+d_3 = (1+m_1) * d_0 + (1+m_2) * d_2$$

$$L = (1+m_1) * \frac{f_1 * (1+m_1)}{m_1} + (1+m_2) * \frac{f_2 * (1+m_2)}{m_2} = \frac{f_1 * (1+m_1)^2}{m_1}+\frac{f_2 * (1+m_2)^2}{m_2}$$

Question 2: Homography

(a)

$$S = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix}$$

Matrix S has size 3 x 3

(b)

Homography matrix S maps point q on plane π_1 of to point p on plane π_2 and since there isn't any constraints on homography and either views, 3 x 3 matrix S has full rank of 3. For example, homography matrix S should cover scaling of size 1, [$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$] which has rank 3.

(c)

At least 4 point correspondences are required to estimate S. This is because homography matrix S has 9 variables with 8 degree of freedom, and each correspondence resolves 2 variables.

(d)

From (2), $p_1^i = Hp_2^i$ can be noted as $\begin{matrix} x_i' \\ y_i' \\ 1 \end{matrix} = \begin{matrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{matrix} * \begin{matrix} x_i \\ y_i \\ 1 \end{matrix}$ where $p_1^i = \begin{matrix} x_i' \\ y_i' \\ 1 \end{matrix}$ and $p_2^i = \begin{matrix} x_i \\ y_i \\ 1 \end{matrix}$

Then putting h = $\begin{matrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{matrix}$, Ah=b can be written with

A =

$$\begin{matrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1'x_1 & -x_1'y_1 & -x_1' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y_1'x_1 & -y_1'y_1 & -y_1' \\ & & & & \vdots & & & & \\ x_N & y_N & 1 & 0 & 0 & 0 & -x_N'x_N & -x_N'y_N & -x_N' \\ 0 & 0 & 0 & x_N & y_N & 1 & -y_N'x_N & -y_N'y_N & -y_N' \end{matrix}$$

b =

$$\begin{matrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{matrix}$$

where size of A is 2N x 9 and size of b is 2N x 1.

Since there are 9 variables, with the constraint that h is a unit vector, only 4 correspondences are needed to solve for h.