$$A = \int_2^4 (f(x) - g(x))\,dx = \int_2^4 f(x)\,dx - \int_2^4 g(x)\,dx =$$

$$= \int_2^4 \frac{1}{4}x^2\,dx - \int_2^4 (x-2)\,dx = \left[\frac{1}{12}x^3 - \frac{1}{2}x^2 + 2x\right]_2^4$$

$$= \frac{64}{12} - \frac{16}{2} + 8 - \frac{8}{12} + \frac{4}{2} - 4 = \frac{8}{3}$$

i) Bestimme Inhalt der Flächen A_1 und A_2, die die Graphen der Funktionen

$$f: x \to f(x) = -x^3 + 3x^2 = x^2(-x+3)$$
$$g: x \to g(x) = x^2 - 3x = x(x-3)$$

einschließen.

- Nullstellen $f(x) = -x^3 + 3x^2 = x^2(-x+3)$

  $N_1(0,0)$ doppelt $\to$ kein VZW

  $N_2(3,0)$ einfach $\to$ VZW

- Extrema von $f(x)$: $f'(x) = -3x^2 + 6x = x(-3x+6) \overset{!}{=} 0$

$\to$ $x_1 = 0$ $\quad y_1 = 0$

$x_2 = 2$ $\quad y_2 = -8 + 12 = 4$

$f''(x) = -6x + 6$

$f''(0) = 6 > 0 \Rightarrow$ Minimum $\quad$ Min $(0,0)$

$f''(2) = -6 < 0 \Rightarrow$ Maximum $\quad$ Max $(2,4)$

• Nullstellen von $g(x) = x^2 - 3x = x(x-3)$

$N_1'(0,0)$ einfach → VZW

$N_2'(3,0)$ einfach → VZW

• Extrema von $g(x)$: $g'(x) = 2x - 3 \stackrel{!}{=} 0 \Rightarrow x = \frac{3}{2}$

$g''(x) = 2$ $\quad g''\left(\frac{3}{2}\right) > 0 \Rightarrow$ Minimum $Min'\left(\frac{3}{2}, -2\frac{1}{4}\right)$

$y = \left(\frac{3}{2}\right)^4 - 3 \cdot \frac{3}{2} =$

$= \frac{9}{4} - \frac{9}{2} = \frac{9}{4} - \frac{18}{4} = -\frac{9}{4}$

$= -2\frac{1}{4}$

• Bestimme Schnittpunkte der Graphen:

$f(x) \stackrel{!}{=} g(x)$

$-x^3 + 3x^2 = x^2 - 3x$

$x^3 - 2x^2 - 3x = 0$

$x\,(x^2 - 2x - 3) = 0$

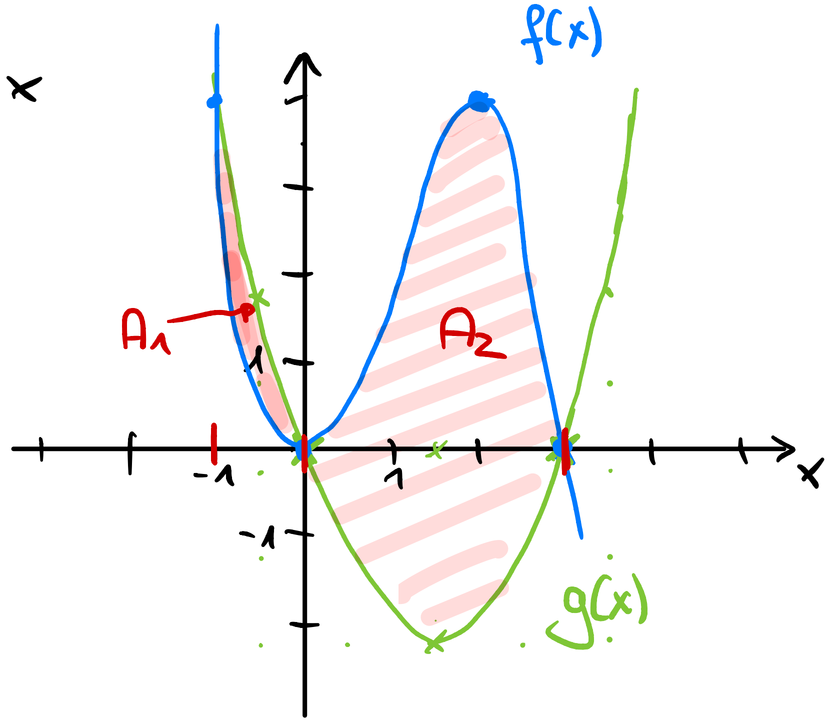
$x_1 = 0$

$x_{2,3} = \frac{+2 \pm \sqrt{4 + 4 \cdot 3}}{2}$

$= \frac{+2 \pm 4}{2}$

$x_2 = 3$

$x_3 = -1$



$\Rightarrow S_1(-1, 4), S_2(0,0), S_3(3,0)$

$$f(x) = -x^3 + 3x^2$$
$$g(x) = x^2 - 3x$$

- Fläche A_1 :

$$\int_{-1}^{0} (g(x) - f(x))\,dx = \int_{-1}^{0} (x^2 - 3x + x^3 - 3x^2)\,dx$$

$$= \int_{-1}^{0} (x^3 - 2x^2 - 3x)\,dx = \left[+\frac{1}{4}x^4 - \frac{2}{3}x^3 - \frac{3}{2}x^2\right]_{-1}^{0}$$

$$= 0 - \left(\frac{1}{4} + \frac{2}{3} - \frac{3}{2}\right) = \frac{-3-8+18}{12} = \frac{7}{12} \,//$$

- Fläche A_2 :

$$\int_{0}^{3} (f(x) - g(x))\,dx = \ldots = \left[-\frac{1}{4}x^4 + \frac{2}{3}x^3 + \frac{3}{2}x^2\right]_{0}^{3} =$$

$$= \left(-\frac{81}{4} + \frac{54}{3} + \frac{27}{2}\right) - 0 = \frac{45}{4} \,//$$